The Dyson Schrödinger model and Phase Space Zonal Structure theory

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Summary

- This work provides a general description of the self-consistent energetic particle phase space transport in burning plasmas, based on nonlinear gyrokinetic theory [cf. F. Zonca P-15; M. V. Falessi I-14 Varenna20]

- The self consistency is ensured by [cf. F. Zonca et al. JPCS 2021]

  - the evolution equations of the Alfvénic fluctuations are given by nonlinear radial envelope evolution equations ⇒ NLSE-like!

  - the energetic particle fluxes in the phase space are explicitly constructed from long-lived phase space zonal structures, which are undamped by collisionless processes

- As a result, this work provides a viable route to computing fluctuation induced energetic particle transport on long time scales in realistic tokamak plasmas.

⇒ Derived for the first time!

Motivation and Background

- High temperature fusion plasmas are weakly collisional:
  this naturally introduces
  - ⇒ importance of phase space structures for transport processes
  - ⇒ deviation of the system from local thermodynamic equilibrium
  - ⊙ these issues are addressed here

- This work:
  - ⊙ Develops a first principle based reduced model for fluctuation induced transport in fusion plasmas
    - ⇒ Describes evolution equation for spectral density for low-frequency fluctuations (NL Schrödinger-like equation)
    - ⇒ Closes the system with evolution (transport) equations for phase space zonal structures (renormalization of particle response)
    - ⇒ Reduced Dyson Schrödinger transport Model (DSM)

Evolution of the fluctuation spectrum

- Perpendicular pressure balance allows to explicitly solve for $\delta B_\parallel$ [C&Z RMP16] $\Rightarrow$ standard NLGK ordering

$$\nabla_\perp \left(B_0 \delta B_\parallel + 4\pi \delta P_\perp \right) \simeq 0 \ ,$$

- Introducing PSZS, $\bar{F}_0$ (nonlinear equilibrium) [C&Z RMP16], and nonadiabatic particle response, δg, (to be discussed later), other two closing field equations are:

$\Rightarrow$ quasineutrality condition

$$\sum \left\langle \frac{e^2}{m} \frac{\partial \bar{F}_0}{\partial \mathcal{E}} \right\rangle_v \delta\phi + \nabla \cdot \sum \left\langle \frac{e^2}{m} \frac{2\mu}{\Omega^2} \frac{\partial \bar{F}_0}{\partial \mu} \left(\frac{J_0^2 - 1}{\lambda^2} \right) \right\rangle_v \nabla_\perp \delta\phi + \sum \langle e J_0(\lambda) \delta g \rangle_v = 0 \ .$$

$\Rightarrow$ gyrokinetic vorticity equation [C&Z RMP16]

$$B_0\left(\nabla_\parallel+\frac{\delta\boldsymbol{B}_\perp}{B_0}\cdot\boldsymbol{\nabla}\right)\left(\frac{\delta J_\parallel}{B_0}\right)-\boldsymbol{\nabla}\cdot\sum\left\langle\frac{e^2}{m}\frac{2\mu}{\Omega^2}\left(B_0\frac{\partial\bar{F}_0}{\partial\mathcal{E}}+\frac{\partial\bar{F}_0}{\partial\mu}\right)\left(\frac{J_0^2-1}{\lambda^2}\right)\right\rangle_v\boldsymbol{\nabla}_\perp\frac{\partial}{\partial t}\delta\phi$$

$$+\sum ec\boldsymbol{b}_0\times\boldsymbol{\nabla}\left\langle\frac{2\mu}{\Omega^2}\bar{F}_0\left(\frac{J_0^2-1}{\lambda^2}\right)\right\rangle_v\cdot\boldsymbol{\nabla}\nabla_\perp^2\delta\phi+\frac{c}{B_0}\boldsymbol{b}_0\times\boldsymbol{\kappa}\cdot\boldsymbol{\nabla}\sum\left\langle m\left(\mu B_0+v_\parallel^2\right)J_0\delta g\right\rangle_v$$

$$+\delta\boldsymbol{B}_\perp\cdot\boldsymbol{\nabla}\left(\frac{J_{\parallel 0}}{B_0}\right)+\sum e\left\langle J_0\left[\frac{c}{B_0}\boldsymbol{b}_0\times\boldsymbol{\nabla}\left(J_0\delta\phi\right)\cdot\boldsymbol{\nabla}\delta g\right]-\frac{c}{B_0}\boldsymbol{b}_0\times\boldsymbol{\nabla}\delta\phi\cdot\boldsymbol{\nabla}\left(J_0\delta g\right)\right\rangle_v$$

$$+\frac{c}{B_0}\boldsymbol{b}_0\times\boldsymbol{\nabla}\delta\phi\cdot\boldsymbol{\nabla}\left[\boldsymbol{\nabla}\cdot\sum\left\langle\frac{e^2}{m}\frac{2\mu}{\Omega^2}\frac{\partial\bar{F}_0}{\partial\mu}\left(\frac{1-J_0^2}{\lambda^2}\right)\right\rangle_v\boldsymbol{\nabla}_\perp\delta\phi\right]=0\ .$$

- Consistent with the ordering $|\gamma_{Ln}| \sim \tau_{NLn}^{-1} \ll |\omega_n|$ [C&Z RMP16], we can average quasineutrality and nonlinear vorticity equations over linear parallel mode structures, yielding for $\boldsymbol{A}_n(r,t) \equiv \hat{\boldsymbol{e}}_n A_n(r,t)$

$$\underbrace{\hat{\boldsymbol{e}}_n^+ \cdot \boldsymbol{D}\left(r,t,k_{nr},\omega_n\right) \cdot \boldsymbol{A}_n(r,t) e^{iS_n(r,t)}}_{\text{linear response}} = \underbrace{\hat{\boldsymbol{e}}_n^+ \cdot \boldsymbol{F}(r,t)}_{\text{nonlinear response}} \; ,$$

- Introduce $\delta\hat{\phi}_{\|n} \equiv \delta\hat{\phi}_n - \delta\hat{\psi}_n$, radial envelope, A_n and polarization vector $\hat{\boldsymbol{e}}_n$

$$\begin{pmatrix} e\delta\hat{\psi}_n(r,\vartheta;t)/T_{0i} \\ e\delta\hat{\phi}_{\|n}(r,\vartheta;t)/T_{0i} \end{pmatrix} \equiv A_n(r,t)e^{iS_n(r,t)} \begin{pmatrix} e_1(r,t)y_1(r,\vartheta) \\ e_2(r,t)y_2(r,\vartheta) \end{pmatrix} \; .$$

with eikonal representation and normalizations

$$\int_{-\infty}^{\infty} |y_{1,2}(r,\vartheta)|^2 \, d\vartheta = 1 \; . \qquad \hat{\boldsymbol{e}}_n^+ \cdot \hat{\boldsymbol{e}}_n = 1 \; .$$

- The nonlinear term (including ext. forcing) can be formally written as

$$e^{-iS_n} \hat{\boldsymbol{e}}_n^+ \cdot (\boldsymbol{F} - \boldsymbol{F}_{\text{ext}}) = (C_{n,0} + C_{0,n}) \circ A_n(r,t) A_z(r,t) + \sum_{n'+n''=n}^{n',n'' \neq n} C_{n',n''} \circ A_{n'}(r,t) A_{n''}(r,t)$$

where $C_{n',n''}$ imply nonlocal interactions in the n toroidal mode number space and whose composition with $A_n(r,t)$ and/or $A_z(r,t)$ is denoted by $\circ$.

- Consistent with the GFLDR theory [C&Z RMP16],

$$e^{-iS_n} \hat{\boldsymbol{e}}_n^+ \cdot \boldsymbol{F}/A_n = i\Lambda^{NL} - \delta \bar{W}_f^{NL} - \delta \bar{W}_k^{NL} + e^{-iS_n} \hat{\boldsymbol{e}}_n^+ \cdot \boldsymbol{F}_{\text{ext}}/A_n \; .$$

Λ describes the generalized inertia due to the e.m. field behaviors on short length scales, while $\delta \bar{W}_f$ and $\delta \bar{W}_k$ account for "fluid" and "kinetic" potential energy fluctuations due to meso- and macro-scale responses.

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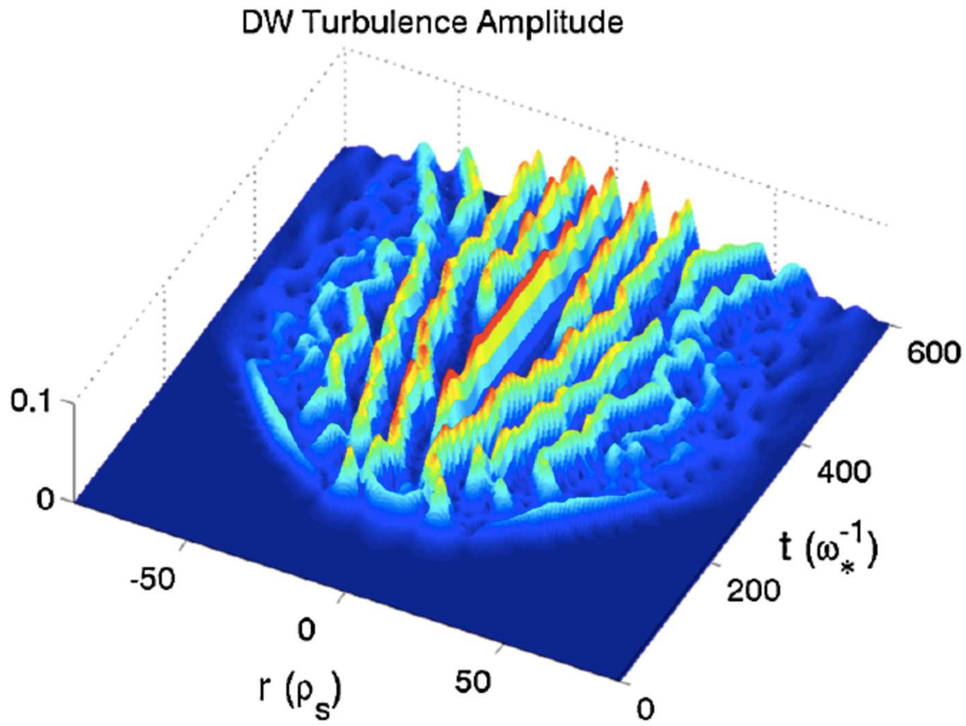
- Nonlinear envelope equation can be written in NLSE-like form

$$\frac{\partial}{\partial t}\left(\frac{\partial D^0_{Rn}}{\partial \omega_n}A_n^2\right)-\frac{\partial}{\partial r}\left(\frac{\partial D^0_{Rn}}{\partial k_{nr}}A_n^2\right)+2D^1_{An}A_n^2-2iD^1_{Rn}A_n^2$$
$$+iA_n\left(\frac{\partial^2 D^0_{Rn}}{\partial k_{nr}^2}+2\frac{\partial \hat{\boldsymbol{e}}_n^+}{\partial k_{nr}}\cdot\boldsymbol{D}^0_{Rn}\cdot\frac{\partial \hat{\boldsymbol{e}}_n}{\partial k_{nr}}\right)\frac{\partial^2 A_n}{\partial r^2}=-2ie^{-iS_n}A_n\hat{\boldsymbol{e}}_n^+\cdot\boldsymbol{F}$$
$$-\left(\hat{\boldsymbol{e}}_n^+\cdot\frac{d}{dt}\hat{\boldsymbol{e}}_n-\frac{d}{dt}\hat{\boldsymbol{e}}_n^+\cdot\hat{\boldsymbol{e}}_n\right)\frac{\partial D^0_{Rn}}{\partial \omega_n}A_n^2+\left(\frac{\partial \hat{\boldsymbol{e}}_n^+}{\partial \omega_n}\cdot\boldsymbol{D}^0_{Rn}\cdot\frac{\partial \hat{\boldsymbol{e}}_n}{\partial t}-\frac{\partial \hat{\boldsymbol{e}}_n^+}{\partial t}\cdot\boldsymbol{D}^0_{Rn}\cdot\frac{\partial \hat{\boldsymbol{e}}_n}{\partial \omega_n}\right)A_n^2$$
$$-\left(\frac{\partial \hat{\boldsymbol{e}}_n^+}{\partial k_{nr}}\cdot\boldsymbol{D}^0_{Rn}\cdot\frac{\partial \hat{\boldsymbol{e}}_n}{\partial r}-\frac{\partial \hat{\boldsymbol{e}}_n^+}{\partial r}\cdot\boldsymbol{D}^0_{Rn}\cdot\frac{\partial \hat{\boldsymbol{e}}_n}{\partial k_{nr}}\right)A_n^2\ .$$

- The NLSE-like structure is of crucial importance for proper analysis of structure formation in strongly magnetized toroidal plasmas, where wave packets can be focused/defocused and back scattered by both nonlinearities as well as by radial nonuniformities [C&Z RMP16].

- This goes beyond the standard wave kinetic equation that is typically adopted in literature, is of fundamental importance not only in the description of EP induced avalanches, such as in the case of energetic particle modes (EPM) [Zonca et al 05] but also for the interaction of zonal fields and drift wave turbulence [Guo et al 09].

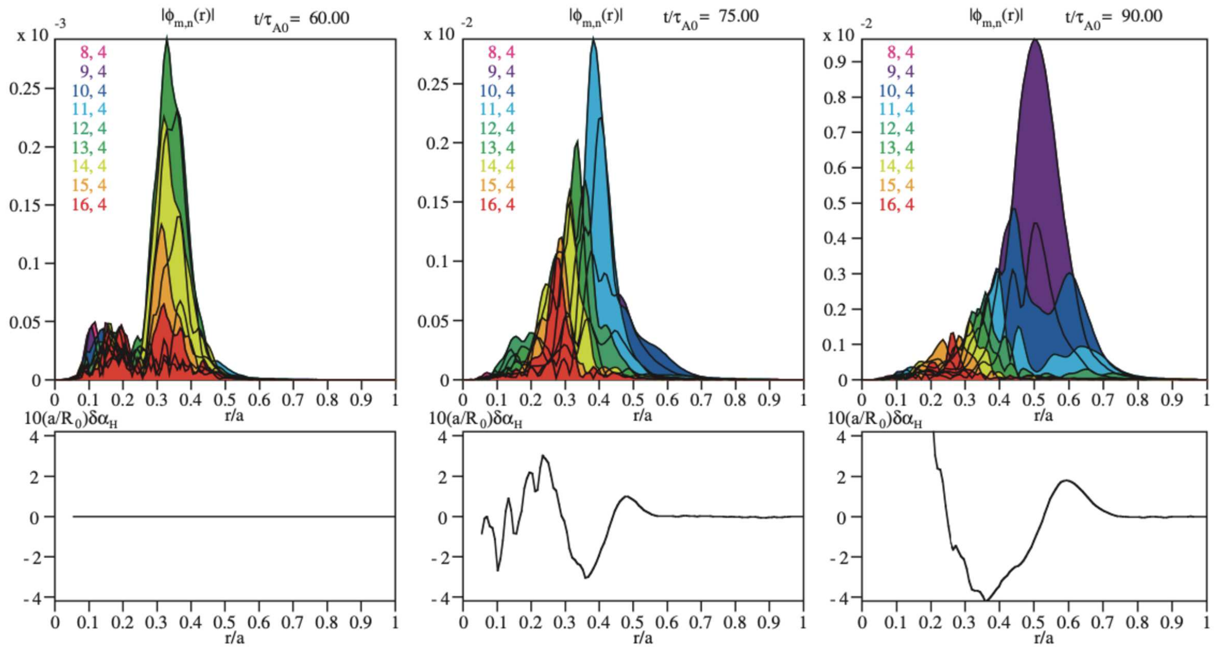


DW turbulence spreading in the presence of growth or damping, dissipation and finite system size effects [courtesy of Z. Guo PRL 103, 055002 (2009)]

- The NLSE-like equation for symmetry breaking fluctuations is closed by the nonlinear evolution equation for $\delta\phi_z$ (quasineutrality; cf. above) and $\delta A_{\parallel z}$ [C&Z RMP16] [M.V. Falessi et al. I-14 Varenna 2020; NJP to be sub.]

$$\frac{\partial}{\partial t}\delta A_{\parallel z} = \left(\frac{c}{B_0} \boldsymbol{b}_0 \times \nabla \delta A_{\parallel} \cdot \nabla \delta \psi \right)_z \; .$$

Evidence of EP avalanche [Z. et al Nucl. Fusion 45 (2005) 477 – 484]



Phase space zonal structures and transport

- There is more than radial corrugations of equilibrium profiles
  - ⇒ Fluctuations force the system away from reference state
  - ⇒ Collisions tend to restore local thermodynamic equilibrium
  - ⇒ Need to consider these processes in phase space on the same footing
- We can describe reference state evolution by means of phase space zonal structures (PSZS), defined as those unaffected by fast collisionless damping ⇒ important on transport time scale [M.V. Falessi I-14 Varenna 2020].
- Since PSZS are undamped by (fast) collisionless dissipation mechanisms, they are naturally expressed as functions of invariants of motion (nearly integrable Hamiltonian system). Separating fast ($[...]_F$) from slow variations,

$$F_z \equiv \bar{F}_0 + e^{-iQ_z}\left(\overline{e^{iQ_z}\delta F_z}\Big|_F + \delta\tilde{F}_{Bz}\right) ,$$

macro- ⊕ meso-scale (CGL) micro-scale collisionless damped

where $\overline{[...]} = \oint d\ell/v_\parallel [...]/\oint d\ell/v_\parallel$, $\overline{\widetilde{[...]}} = 0$, F_z is the $n = 0$ gyrocenter particle distribution function. e^{-iQ_z} ⇒ nonlocal (integral) particle response

- Evolution equation for the PSZS (CGL nonlinear equilibrium) is given by

$$\partial_t \overline{e^{iQ_z}\bar{F}_0} = -\left.\overline{e^{iQ_z}\frac{F(\psi)}{B_0}\partial_t\left\langle\delta A_{\parallel g}\right\rangle_z\frac{\partial}{\partial\bar{\psi}}\bar{F}_0}\right|_S - \frac{1}{\tau_b}\frac{\partial}{\partial\psi}\left[\tau_b\overline{e^{iQ_z}\delta\dot{\psi}_z\delta F_z}\right]_S$$

$$-\frac{1}{\tau_b}\frac{\partial}{\partial\mathcal{E}}\left[\tau_b\overline{e^{iQ_z}\delta\dot{\mathcal{E}}_z\delta F_z}\right]_S - \frac{1}{\tau_b}\frac{\partial}{\partial\psi}\left[\tau_b\overline{e^{iQ_z}\delta\dot{\psi}\delta F}\right]_{zS} - \frac{1}{\tau_b}\frac{\partial}{\partial\mathcal{E}}\left[\tau_b\overline{e^{iQ_z}\delta\dot{\mathcal{E}}\delta F}\right]_{zS}$$

$$+\left.\overline{e^{iQ_z}\left[C_g+\mathcal{S}\right]}\right|_{zS} .$$

[M.V. Falessi I-14 Varenna 2020]

- Introducing

$$\left.\overline{\delta g_{Bz}}\right|_S + \left.\overline{\delta g_{Bz}}\right|_F$$

$$\equiv \left.\overline{e^{iQ_z}\delta F_z}\right|_F - \left.\overline{\frac{e}{m}e^{iQ_z}\left\langle\delta L_g\right\rangle_z\frac{\partial}{\partial\mathcal{E}}}\right|_{\bar{\psi}}\bar{F}_0 + \overline{e^{iQ_z}\frac{F(\psi)}{B_0}\left\langle\delta A_{\parallel g}\right\rangle_z\frac{\partial}{\partial\bar{\psi}}\bar{F}_0},$$

one can derive the evolution equation for $\left.\overline{\delta g_{Bz}}\right|_F$ as well [JPCS 2021]

- Evolution equation for the micro spatiotemporal scale equilibrium variation

$$
\begin{aligned}
\partial_t \overline{\delta g_{Bz}}\Big|_F = & - \overline{e^{iQ_z} \frac{e}{m} \partial_t \left[\langle \delta L_g \rangle_z \frac{\partial}{\partial \mathcal{E}} \Big|_{\bar{\psi}} \bar{F}_0 \right]}\Bigg|_F + \overline{e^{iQ_z} \frac{F(\psi)}{B_0} \langle \delta A_{\| g} \rangle_z \frac{\partial}{\partial \bar{\psi}} \partial_t \bar{F}_0}\Bigg|_F \\
& + \overline{e^{iQ_z} [C_g + \mathcal{S}]}\Big|_{zF} - \frac{1}{\tau_b} \frac{\partial}{\partial \psi} \left[\tau_b \overline{e^{iQ_z} \delta \dot{\psi}_z \delta F_z} \right]_F - \frac{1}{\tau_b} \frac{\partial}{\partial \mathcal{E}} \left[\tau_b \overline{e^{iQ_z} \delta \dot{\mathcal{E}}_z \delta F_z} \right]_F \\
& - \frac{1}{\tau_b} \frac{\partial}{\partial \psi} \left[\tau_b \overline{e^{iQ_z} \delta \dot{\psi} \delta F} \right]_{zF} - \frac{1}{\tau_b} \frac{\partial}{\partial \mathcal{E}} \left[\tau_b \overline{e^{iQ_z} \delta \dot{\mathcal{E}} \delta F} \right]_{zF} .
\end{aligned}
$$

- Analogously, one can derive the evolution equations for $\delta \tilde{g}_{Bz}$ (and, thus, $\delta \tilde{F}_{Bz}$), omitted here for brevity [M.V. Falessi I-14 Varenna 2020].
- In order to self-consistently close the system of equations with the ZFS equations and the NLSE-like equation for the nonlinear envelope evolution of symmetry breaking fluctuations we need, finally, the equation for the non-adiabatic particle response, δg.

$$
\begin{aligned}
\left(\partial_t + \dot{\boldsymbol{X}}_0 \cdot \boldsymbol{\nabla}\right) \delta g \; = \; & -\frac{e}{m}\partial_t \left[\langle \delta L_g \rangle \left.\frac{\partial}{\partial \mathcal{E}}\right|_{\bar{\psi}} \bar{F}_0 \right] + \frac{F(\psi)}{B_0} \left\langle \delta A_{\|g} \right\rangle \frac{\partial}{\partial \bar{\psi}} \partial_t \bar{F}_0 \\
& - c\partial_\zeta \left\langle \delta L_g \right\rangle \frac{\partial}{\partial \bar{\psi}} \bar{F}_0 + [C_g + \mathcal{S}] - \frac{1}{\mathcal{J}D}\frac{\partial}{\partial \theta}\left[\mathcal{J} D \delta \dot{\theta} \delta F_z\right] \\
& - \frac{1}{\mathcal{J}D}\frac{\partial}{\partial \psi}\left[\mathcal{J} D \delta \dot{\psi} \delta F_z\right] - \frac{1}{\mathcal{J}D}\frac{\partial}{\partial \mathcal{E}}\left[\mathcal{J} D \delta \dot{\mathcal{E}} \delta F_z\right] - \frac{1}{\mathcal{J}D}\frac{\partial}{\partial \theta}\left[\mathcal{J} D \delta \dot{\theta}_z \delta F\right] \\
& - \frac{1}{\mathcal{J}D}\frac{\partial}{\partial \psi}\left[\mathcal{J} D \delta \dot{\psi}_z \delta F\right] - \frac{1}{\mathcal{J}D}\frac{\partial}{\partial \mathcal{E}}\left[\mathcal{J} D \delta \dot{\mathcal{E}}_z \delta F\right] - \frac{1}{\mathcal{J}D}\frac{\partial}{\partial \theta}\left[\mathcal{J} D \delta \dot{\theta} \delta F\right] \\
& - \frac{1}{\mathcal{J}D}\frac{\partial}{\partial \psi}\left[\mathcal{J} D \delta \dot{\psi} \delta F\right] - \frac{1}{\mathcal{J}D}\frac{\partial}{\partial \mathcal{E}}\left[\mathcal{J} D \delta \dot{\mathcal{E}} \delta F\right] ,
\end{aligned}
$$

Nonlinear evolution of resonance structures

- Rich non-linear behaviors due to non-perturbative W-P interactions: [C&Z RMP16]
  - resonances may evolve nonlinearly (chirping)
  - phase-space structures may form on spatiotemporal meso-scales (phase-locking, bunching ...) and yield **secular transport** and/or **avalanches** [Zonca et al NF05]; [C&Z RMP16]
  - favorable conditions for this phenomenology
    - continuous spectrum of modes that can be resonantly excited
    - **Non-perturbative W-P interactions** modify lowest order dispersion properties

- Within the present theoretical framework, these physics are **accounted for self-consistently**

  ⇒ renormalized expression of reference state distribution by emission and re-absorption of symmetry breaking perturbations

  ⇒ Dyson-like equation for PSZS [Zonca et al NJP15], [C&Z RMP16]

Dyson-like equation for PSZS

- Dropping for simplicity radial modulations by ZFS, and writing formal solution for the particle response keeping relevant NL terms and considering precession resonance only

$$\partial_t \delta \bar{G}_z \sim -i \sum_{\boldsymbol{k}} \frac{nc}{d\psi/dr} \frac{\partial}{\partial r} \left[\overline{e^{iQ_z} \langle \delta L_g \rangle_{-k} e^{-iQ_k}} \frac{(nc)/(d\psi/dr)}{(\bar{\omega}_{dk} - \omega_k - i\partial_t + i\Delta \ldots)} \overline{e^{iQ_k} \langle \delta L_g \rangle_k e^{-iQ_z}} \frac{\partial}{\partial r} \delta \bar{G}_z \right]$$

- Importance of fluctuation spectrum in determining NL PSZS evolution:
  - broad spectrum $\Rightarrow$ QL diffusion [Al'tshul' & Karpman 65]
  - narrow (quasi-coherent) spectrum: ballistic transport of phase-locked particles and convective amplification of radially propagating wave-packets (NLSE) $\Rightarrow$ Non-adiabatic chirping [C&Z RMP16]

- Simplest example of the Dyson Schrödinger transport Model (DSM) for EP transport in burning plasma that can be constructed in general from the NLSE-like nonlinear envelope evolution equation and the corresponding evolution equation for PSZS.

- Existence of integral operators combined with resonant behaviors [Dupree 66] $\Delta \Rightarrow \Delta + F\partial_r + \partial_r D \partial_r$
  - Physical interpretation [Dupree 66]:
    - Δ: nonlinear complex frequency shift
    - F: nonlinear anti-symmetric resonance distortion (in radius)
    - D: nonlinear resonance broadening (in radius)

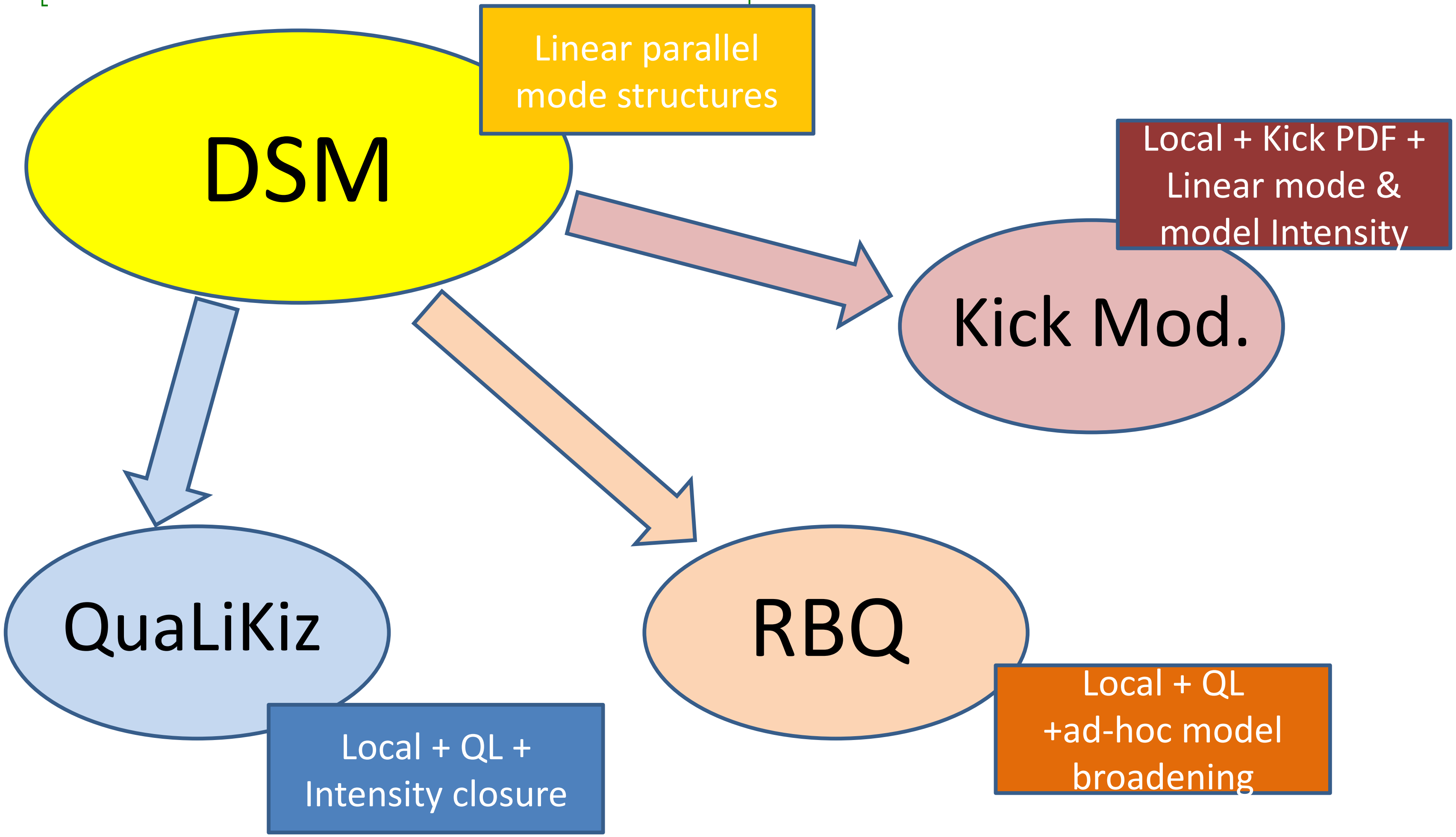
$$\Delta = -\sum_{k\neq -k_0}\left(1+\frac{n}{n_0}\right)\left(\frac{n_0 c}{d\psi/dr}\right)^2 \overline{e^{iQ_{k_0}}\partial_r \left\langle \delta L_g \right\rangle_{-k} e^{-iQ_{k+k_0}}} \frac{i}{(\bar{\omega}_d-\omega)_{k+k_0}} \overline{e^{iQ_{k+k_0}}\partial_r \left\langle \delta L_g \right\rangle_{k} e^{-iQ_{k_0}}}$$

$$F = \sum_{k\neq -k_0} n n_0 \left(\frac{c}{d\psi/dr}\right)^2 \overline{e^{iQ_{k_0}}\partial_r \left\langle \delta L_g \right\rangle_{-k} e^{-iQ_{k+k_0}}} \frac{i}{(\bar{\omega}_d-\omega)_{k+k_0}} \overline{e^{iQ_{k+k_0}} \left\langle \delta L_g \right\rangle_{k} e^{-iQ_{k_0}}}$$

$$- \sum_{k\neq -k_0} n n_0 \left(\frac{c}{d\psi/dr}\right)^2 \overline{e^{iQ_{k_0}} \left\langle \delta L_g \right\rangle_{-k} e^{-iQ_{k+k_0}}} \frac{i}{(\bar{\omega}_d-\omega)_{k+k_0}} \overline{e^{iQ_{k+k_0}}\partial_r \left\langle \delta L_g \right\rangle_{k} e^{-iQ_{k_0}}}$$

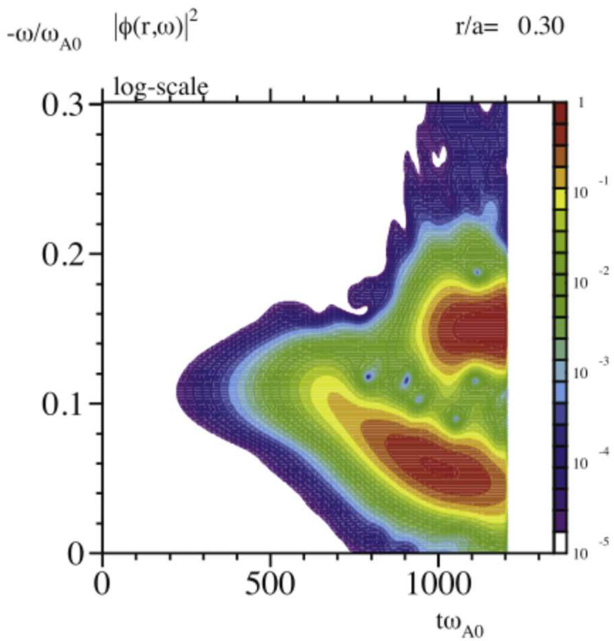
$$D = \sum_{k\neq -k_0} \left(\frac{nc}{d\psi/dr}\right)^2 \overline{e^{iQ_{k_0}} \left\langle \delta L_g \right\rangle_{-k} e^{-iQ_{k+k_0}}} \frac{i}{(\bar{\omega}_d-\omega)_{k+k_0}} \overline{e^{iQ_{k+k_0}} \left\langle \delta L_g \right\rangle_{k} e^{-iQ_{k_0}}}$$

- Recovering the QL limit: ... for a broad spectrum [JPCS 2021] [M.V. Falessi et al. NJP to be sub.]



Applications

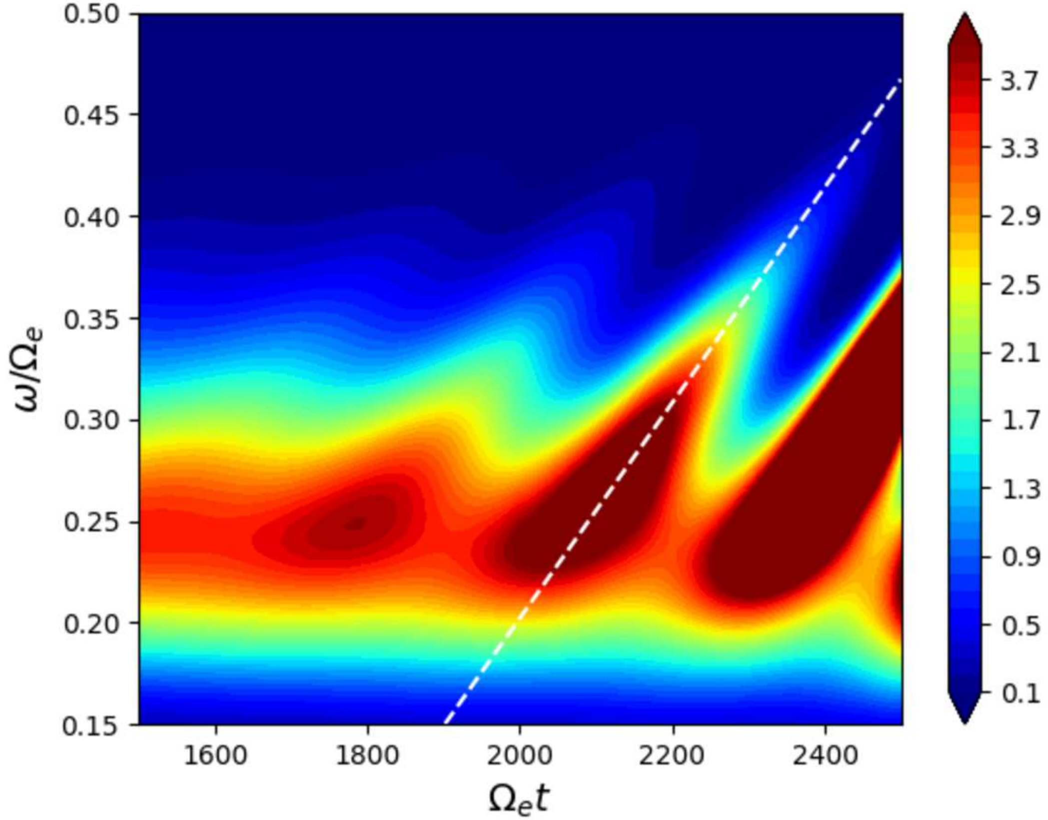
- Various applications made during former ENR projects (NLED/NAT/MET):
  - Energetic Particle Modes: demonstrated convective amplification of the unstable sech-soliton-like front being reduced to the solution of the Z&C NLSE: $\partial_\xi^2 U = \left(\lambda_0 - \epsilon_q^2\right) U - 2iU|U|^2$ [C&Z NJP15] [C&Z RMP16]
  - Fishbones: nonlinear evolution and frequency chirping dictated by maximization of wave particle power transfer [C&Z RMP16] [Vlad et al NJP16] ⇒ ⇒ ⇒



- Next step: Applications to realistic cases with physical sources & sinks

Example from Space Physics: chorus emission

- Adopt the Dyson approach to construct the nonlinear growth rate
  ⊕ Solve WKE [F. Zonca et al JGR to be sub.]



$$\frac{\partial \omega}{\partial t} = R\left(1 - \frac{v_r}{v_g}\right)^{-2} \omega_{tr}^2$$

See also X. Tao, F. Zonca, L. Chen and Y. Wu, Theoretical and numerical studies of chorus waves: A review, Science China Earth Sciences 63, 78-92, (2020).

Summary

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- The self consistency is ensured by [cf. F. Zonca et al. JPCS 2021]

  - the evolution equations of the Alfvénic fluctuations are given by nonlinear radial envelope evolution equations ⇒ NLSE-like!

  - the energetic particle fluxes in the phase space are explicitly constructed from long-lived phase space zonal structures, which are undamped by collisionless processes

- As a result, this work provides a viable route to computing fluctuation induced energetic particle transport on long time scales in realistic tokamak plasmas.

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