WP 2.3: Extension to 3d geometry

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Motivation:

Why do we need a simplified and fast description of wave particle interaction in 3D?

- predictions of unstable modes in W7-X
- predict/find favorable operation regimes with respect to fast particles in W7-X (e.g. relevant for configuration changes of NBI)
- provide guidance and background physics for expensive fully gyro-kinetic global simulations
- affordably predict fast particle transport due to wave particle interaction

- In analogy to the local version two-dimensional gyro-kinetic code LIGKA, a three-dimensional extension will be developed aiming at solving the related eigenvalue problem. The development will rely on realistic stellarator equilibria calculated with VMEC.
- The kinetic part will make use of knowledge from the drift kinetic code CAS3D-K. Due to the many more additional couplings, considerable numerical problems are expected, therefore the passing particle contribution will be focus of the development. The model will be benchmarked against the analytical model of Kolesnichenko et al. and its validity can be tested by comparison with the EUTERPE and the STAE-K code.

Contents:

1. Kolesnichenko model (LGRO)
2. perturbative global MHD (CAS3D-K)
3. reduced MHD + passing species + FLR + $E_{||}$ (STAE-K)
4. challenges in 3D

valid in the limit of very localized modes and for an isotropic distribution of the hot particles (Kolesnichenko et al. PoP 2001)

hot particle growth rate:

$$\gamma = \frac{3\pi\beta_\alpha}{64k^2r^2} \sum_{\nu,\mu,j} \left|\epsilon^{(\mu\nu)}\right|^2 \frac{w \int_w^{w/\sqrt{\epsilon_{eff}}} duu(u^2+w^2)^2(\omega\partial/\partial u^2+\omega_d)f_0}{\int_0^\infty duu^4 f_0}$$

with

$$w = \left| v_{A*}\left(1+2j\frac{\iota_*-\nu N}{\mu_0\iota_*-\nu_0 N}\right)\right| / v_0 \qquad u = v/v_0$$

$$\iota_* = (2n+\nu N)/(2m+\mu_0) \qquad k = [(m+p)\iota - n + s]R_0^{-1}$$

- proportionality to equilibrium quantities

$$\frac{\gamma}{\omega_0} \propto A^2 \sum_{\mu\nu} |\epsilon^{\kappa}_{\mu\nu}|^2 \approx A^2 \sum_{\mu\nu} |\epsilon^{B}_{\mu\nu}|^2$$

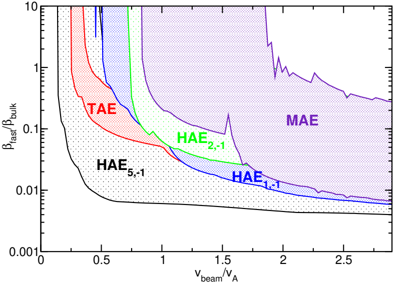
- coupling is approximately given by the structure of B
  $\Rightarrow$ investigate spectrum of B
- note, that for a TAE in a large aspect ratio tokamak: $\frac{\gamma}{\omega_0}$ is independent of the equilibrium
- the resonance condition $\omega - k_{||} v_{th} = 0$ determines

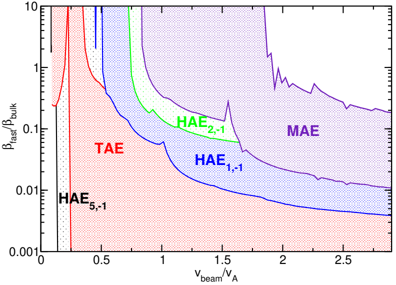
$$v^{\mathrm{res}}_{m'n'} = v_A \left| 1 \pm \frac{m'\iota^* + n' N_p}{m\iota^* + n} \right|^{-1}$$

i.e. well known resonances at $v_0 = v_A$ and $v_0 = v_A/3$ for a Tokamak

TAE mode frequencies and growth/ damping rates from a local computation

$T_i = 3.8keV$: $T_i = 7.6keV$





there is an energy integral considering kinetic effects

$$\delta W_{\mathrm{kin}} = \omega^2 \frac{1}{2} \int d^3\mathbf{x}\, |\xi_\perp|^2 \rho_M = \delta W_{\mathrm{mag}} + \sum_{s=\mathrm{i,e,fast}} \delta W_s(\omega)$$

(Kruskal/Oberman 1958 ... Antonsen/Lee 1984)

$$\delta W_{\mathrm{mag}} = \frac{1}{2} \int d^3\mathbf{x} \left\{ \left| B_\perp^{(1)} \right|^2 + \left| B_\parallel^{(1)} \right|^2 + \mathbf{j}_\parallel \cdot \left(\xi_\perp \times \mathbf{B}_\perp^{(1)} \right) - \frac{B_\parallel^{(1)}}{B} \xi_\perp^* \cdot \nabla p + (\nabla \cdot \xi_\perp^*)\,(\xi_\perp \cdot \nabla p) \right\}$$

the non-adiabatic contributions from the hot and thermal component replace the MHD fluid compression term
the contributions from the thermal plasma ($\delta W_{\mathrm{i,e}}$) and the fast particles δW_{fast}) depend on the perturbed particle Lagrangian $L^{(1)}$

(A. Könies, PoP 2000)

particle- wave- energy- exchange by resonant interaction

$$\delta W_s = \frac{\pi}{M_s^2}\left\{\sum_\sigma\right\}\int ds\int d\varphi\int d\mu\, d\epsilon\left(-\int\frac{d\vartheta}{|v_{||}|}\sqrt{g}B\right)\sum_{\substack{n,m\\ n',m'}}\sum_{p=-\infty}^{\infty} e^{-i\frac{2\pi}{N_p}(n'-n)\varphi}\times$$

$$\times\left(\frac{\partial F_s}{\partial\epsilon}\right)_\mu \frac{\omega-2\pi(\frac{n}{N_p}J-mI)\omega^*}{m\left\langle\omega_d^\vartheta\right\rangle+\frac{n}{N_p}\left\langle\omega_d^\varphi\right\rangle+\left\{\begin{matrix}\sigma(p+nq)\\ p\end{matrix}\right\}\omega_{\left\{\begin{matrix}t\\ b\end{matrix}\right\}}-\omega} L^{(1)*}_{m'n'}\mathcal{M}_{pn}^{m'n'*}L^{(1)}_{mn}\mathcal{M}_{pn}^{mn}$$

definition of $\mathcal{M}_{pn}^{m'n'}$:
for passing particles:

$$\mathcal{M}_{pn}^{m'n'} = \left\langle e^{i[2\pi(m'+n'q)\vartheta''-(p+nq)\omega_t t'']}\right\rangle_{\vartheta''}$$

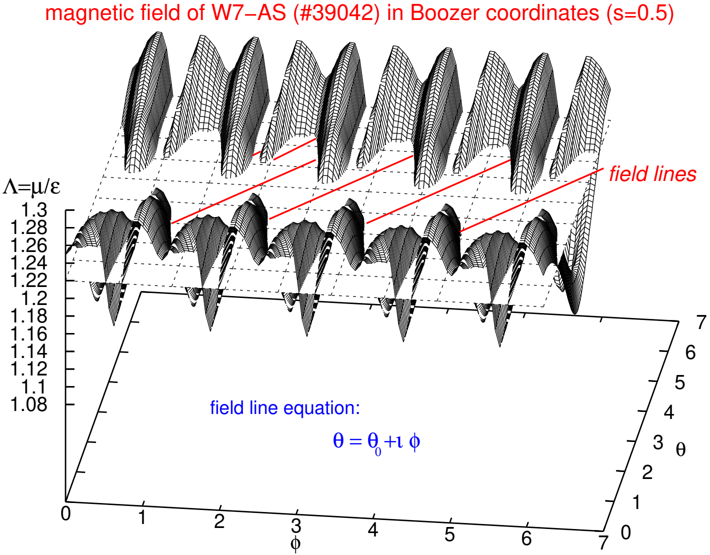
for reflected particles:

$$\mathcal{M}_{pn}^{m'n'} = \left\langle e^{2\pi i(m'+n'q)\vartheta''}[\cos^2(\frac{\pi}{2}p)\cos(p\omega_b t'')-i\sin^2(\frac{\pi}{2}p)\sin(p\omega_b t'')]\right\rangle_{\vartheta''}$$

perturbed particle Lagrangian:

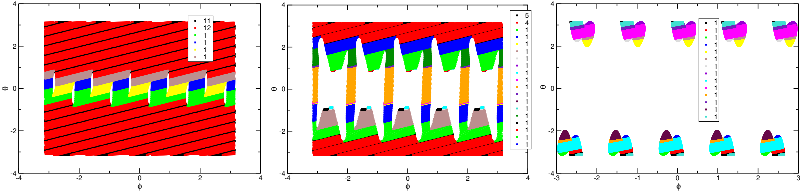
$$L^{(1)} = -(Mv_\parallel^2-\mu B)\xi_\perp\cdot\kappa+\mu B\nabla\cdot\xi_\perp$$

$\langle\dots\rangle$ denotes the transit or bounce average
(A. Könies et al. Varenna Fusion Theory Conf. 2008)


magnetic field of W7−AS (#39042) in Boozer coordinates (s=0.5)
field lines
Λ=μ/ε
1.3
1.28
1.26
1.24
1.22
1.2
1.18
1.16
1.14
1.12
1.1
1.08
field line equation:
θ = θ0 +ι φ
0
1
2
3
4
5
6
7
φ
θ

if bounce expansion is kept
all averages as numerical functions including full geometry
even including reflected particles

field line orbits of reflected particles:



(Shooting code for Toroidicity-induced Alfvén Eigenmodes with Kinetic extensions)

The equation that has to be solved is given as

$$0 = -\left(\mathbf{B}\cdot\nabla\right)\left\{\frac{1}{B^2}\nabla\cdot\left[B^2\nabla_\perp\left(\frac{1}{B^2}\left(\mathbf{B}\cdot\nabla\right)\phi^{(1)}\right)\right]\right\}$$
$$-\left[\mathbf{B}\times\nabla\left(\frac{1}{B^2}\left(\mathbf{B}\cdot\nabla\right)\phi^{(1)}\right)\right]\cdot\nabla\left[\frac{R^2}{I}\nabla\cdot\left(\frac{1}{R^2}\nabla\psi\right)\right]$$
$$-\nabla\cdot\left(\frac{\omega^2}{v_\mathrm{A}^2}\nabla_\perp\phi^{(1)}\right)\underbrace{+\frac{4\pi\mathrm{i}\omega}{c^2}\sum_s e_s\int \mathrm{d}^3v\;\nabla\cdot\left(f_s^{(1)}\mathbf{v}_\mathrm{toroidal}\right)}_{\text{kinetic extension}}.$$

H. L. Berk, J. W. Van Dam, Z. Guo and D. M. Lindberg,
Phys. Fluids **B**, Vol. 4, No. 7, July 1992, page 1806

- calculations can be performed in tokamak- or stellarator geometry
- physical shortcommings:
  - no FOW- and FLR-effects
  - only passing particles
  - at this point: isotropic Maxwellian equilibrium distribution function (It is planned to also include a more r ealistic slowing-down distribution function in the future.)
- advantages:
  - calculations are very fast ⟶ especially suited for parameter scans in the EPM regime
  - easy availability of additional information such as kinetic continuum
  - stability diagrams

C. Slaby et al. Phys. Plasmas, 23: 092501 (2016)

- singular eigenfunctions lead to numerical difficulties

$\Longrightarrow$ sinking of the mode frequency below the continuum boundary must be prevented if higher fast-particle temperatures and densities (EPM regime) are to be reached

$\Longrightarrow$ add

$$\nabla \cdot \left[\nabla_\perp \left\{ g_{Km} \frac{1}{\rho} \nabla \cdot \left(\rho \nabla_\perp \phi^{(1)} \right) \right\} \right] \cong g_{Km} \nabla_\perp^4 \phi^{(1)}$$

with

$$g_{Km} = k_{\|,m} \left[\frac{3}{8} \rho_\mathrm{i}^2 + \frac{1}{2} \rho_\mathrm{s}^2 \frac{1 + \mathrm{i}\hat{\nu} Z(x)}{1 + xZ(x)} \right]$$

ρ_i ... ion gyroradius, ρ_s ... sound gyroradius, $x = \frac{\omega + \mathrm{i}\nu}{k_\| v_\mathrm{th,e}}$, $\hat{\nu} = \frac{\nu}{k_\| v_\mathrm{th,e}}$

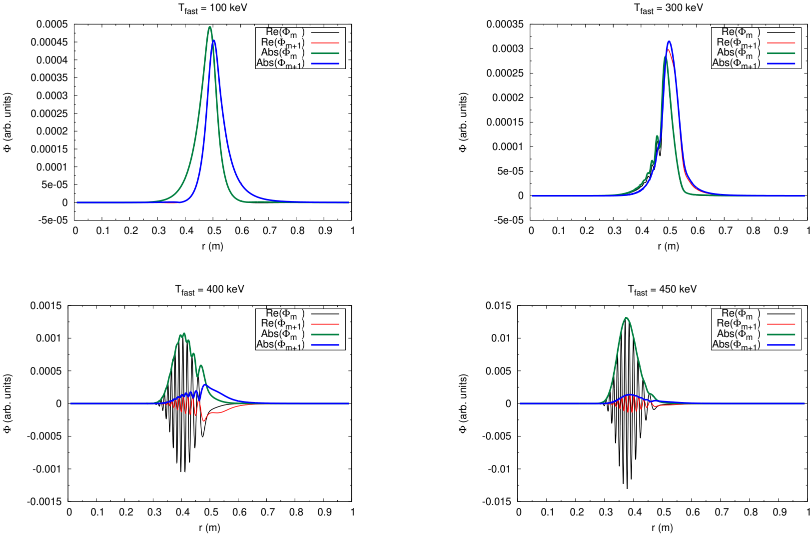
G. Y. Fu *et al.*, Physics of Plasmas, Vol. 12, 082505, 2005

to the equation $\longrightarrow$ gives fourth-order system

$$0 = \underline{M}_4 \boldsymbol{\phi}^{(iv)} + \underline{M}_3 \boldsymbol{\phi}''' + \underline{M}_2 \boldsymbol{\phi}'' + \underline{M}_1 \boldsymbol{\phi}' + \underline{M}_0 \boldsymbol{\phi}$$

- continuum of matrix $\underline{M}_2$ resolved, now that of $\underline{M}_4$ matters instead
- singularities in the eigenfunctions resolved

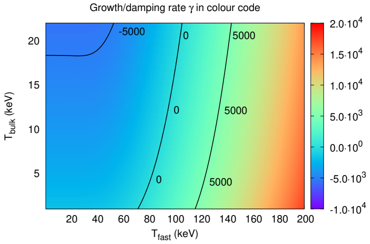
Results including the fourth-order damping term



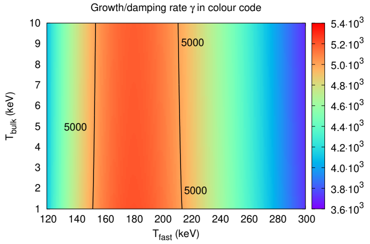
- $T_{\text{bulk}} = 1\,\text{keV}$ for all cases

Stability diagrams

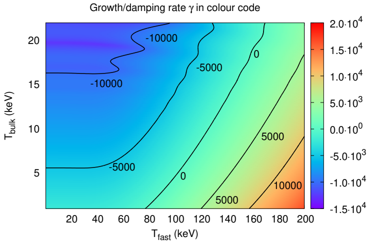
ITPA-benchmark

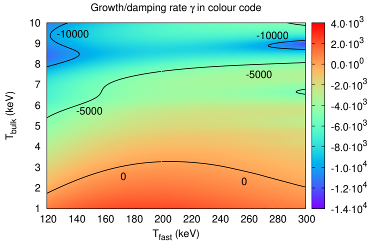


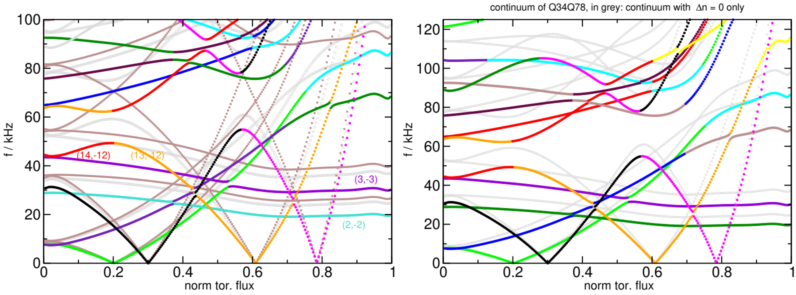
W7-X scenario



including also radiative damping:

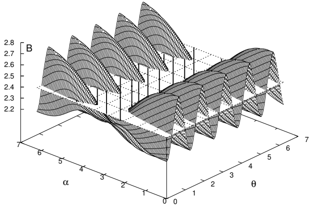


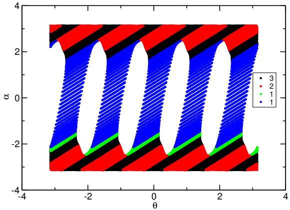




$\Delta n \neq 0$ couplings need to be considered for quantitative agreement of continuum (even for branches relevant to TAE and GAE)
otherwise frequency deviation can be considerable ($5 \ldots 10$kHz)

$\Rightarrow$ look for smallest possible set of harmonics



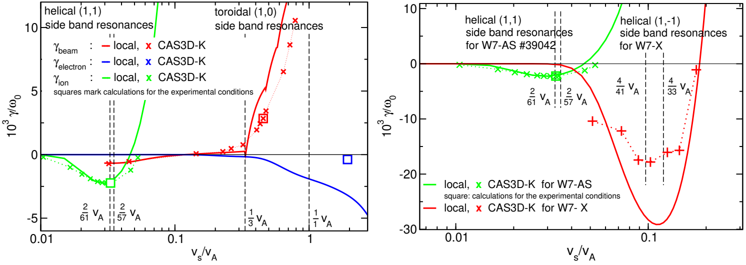


complicated structure of magnetic field
complicated particle orbits
no inherent symmetry

⇒ expand using quasi-symmetry or $J = const$ properties?
⇒ use bounce/transit harmonic from CAS3D-K

mode coupling and complicated orbits pose much more complicated system of equations

$\Rightarrow$ adopt LIGKA scheme ?
$\Rightarrow$ again try shooting method if possible (STAE-K)?



positive perspective:
local and global results did agree quite well
due to small