# Gyrokinetic simulations in stellarators using different computational domains

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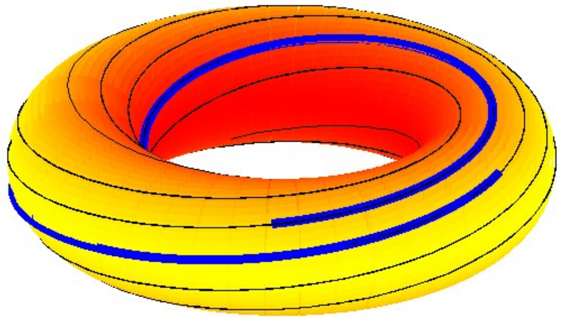
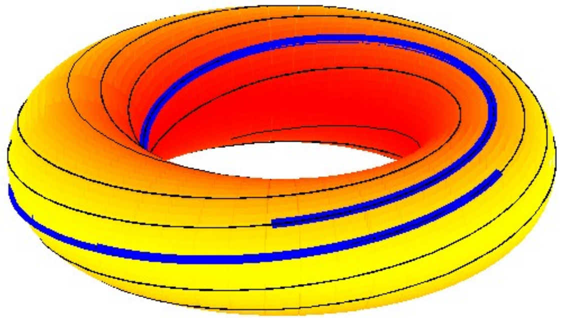
This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement number 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

# Motivation and goal

- **In tokamaks, a flux tube (a finite volume around a field line, with a finite length, usually one poloidal turn in length) is the minimum domain covering the full flux surface.**
  - All the flux tubes in a flux surface are equivalent.



# Motivation and goal



- **In tokamaks, a flux tube (a finite volume around a field line, with a finite length, usually one poloidal turn in length) is the minimum domain covering the full flux surface.**
  - All the flux tubes in a flux surface are equivalent.

- **In stellarators,**

  The magnetic field quantities affecting instabilities and turbulence vary in a three-dimensional basis

  => different flux tubes explore different magnetic geometry.

  The zonal flows in stellarators show properties depending on the full flux surface.

  The radial electric field influence cannot be accounted for in a flux tube domain.

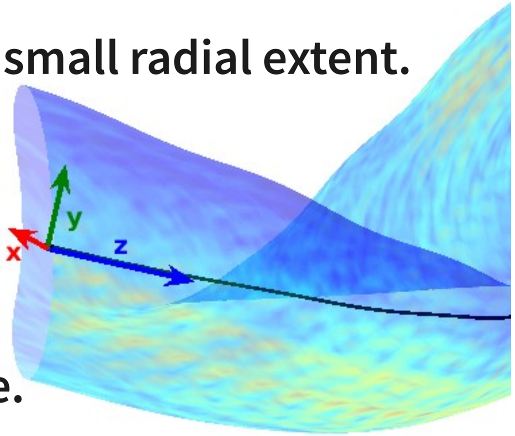
**Which is the minimum computational domain suited for simulating instabilities and turbulence in stellarators?**

# Background

- **In previous works, the properties of the linear collisionless relaxation of ZFs was studied in stellarators.**
  - Monreal et al, PPCF 2016. Study of the long-term ZF residual level.
    - Good agreement between semi-analytical calculations, global (EUTERPE) and full surface (GENE) simulations in W7-X.
  - Monreal et al. PPCF 2017. ZF oscillation frequency in stellarators LHD and W7X.
    - Good agreement between semi-analytical calculations, global (EUTERPE) and flux tube (GENE) simulations.
  - Smoniewski et al. PoP 2021. ZF relaxation in global and flux tube simulations in HSX and NCSX.
    - Differences between different short-length flux tubes.
    - Agreement between global (EUTERPE) and flux tube (GENE) simulations when the flux tube length is enough.
    - The length required for convergence/agreement with global is configuration-dependent.
- **In this work,**
  - The ZF relaxation and the ITG instability are studied in different simulation domains in LHD and W7-X.

# TASK: Compare GK simulations in several computational domains

- **Flux tube (FT) domain: A small volume around a field line covering a finite length (npol).**
  - Local approximation in both radial and binormal direction.
  - Periodic boundary conditions in the radial, binormal and parallel direction (twist and shift).
  - Linear simulations for many $k_z$ modes, one single $k_x$ and $k_y$.
- **Full-flux-surface (FFS) domain: A volume around a flux surface with small radial extent.**
  - Radially local domain, covering the full flux surface.
  - Periodic boundary conditions in the radial direction.
  - Multiple $k_z$, $k_y$ and $k_x$ (depending on the code) modes.
- **Radially global (RG) domain: A finite volume with a finite radial scale.**
  - No periodicity apart from that of the device.
  - Multiple $k_z$, $k_y$, $k_x$ modes even in linear simulations.



# Gyrokinetic codes

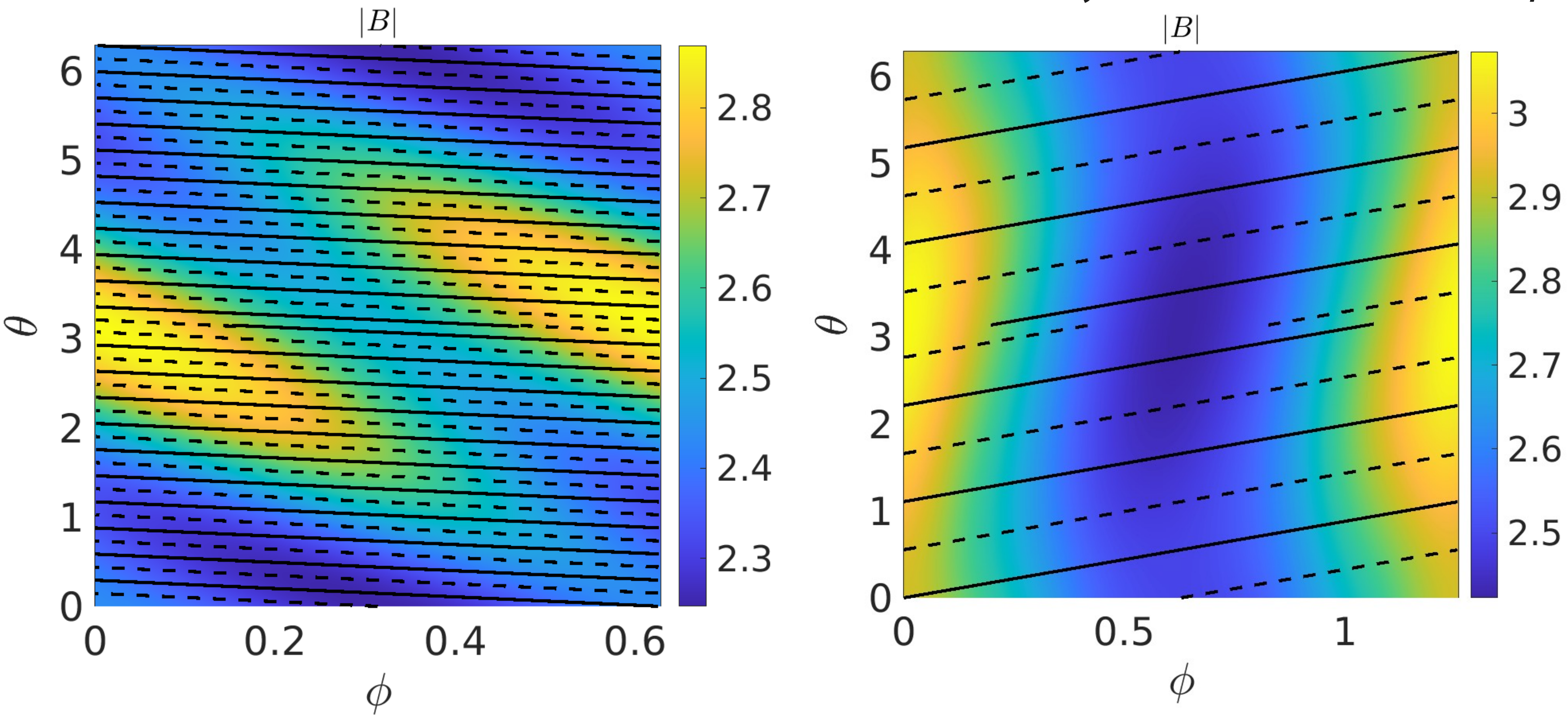
- **EUTERPE (RG)**
  - Particle-in-cell, lagrangian, delta-f.
  - Real space representation in x,y, and z. Full volume or radial annulus simulations.
- **GENE (FT, FFS)**
  - Continuum delta-f code.
  - FT: spectral in x and y.
  - FFS: spectral x and real space in y.
- **GENE-3D (FT, FFS, RG)**
  - Continuum delta-f code.
  - Real space representation in x,y,z
- **Stella (FT)**
  - Continuum delta-f code.
  - Spectral in y and x.

# Magnetic configurations and flux tubes

**Two stellarator symmetric FTs considered:** (—) $\alpha = 0$ (- - - -) $\alpha = \iota\pi/N$

$$\alpha = \theta - \iota\phi$$

**LHD stantard** **W7-X KJM, beta 3%** **r/a=0.5**




LHD is more densely covered by a FT with $n_{pol}$=1 than W7-X.

In LHD the FTs overlap with each other while in W7-X it is not the case.

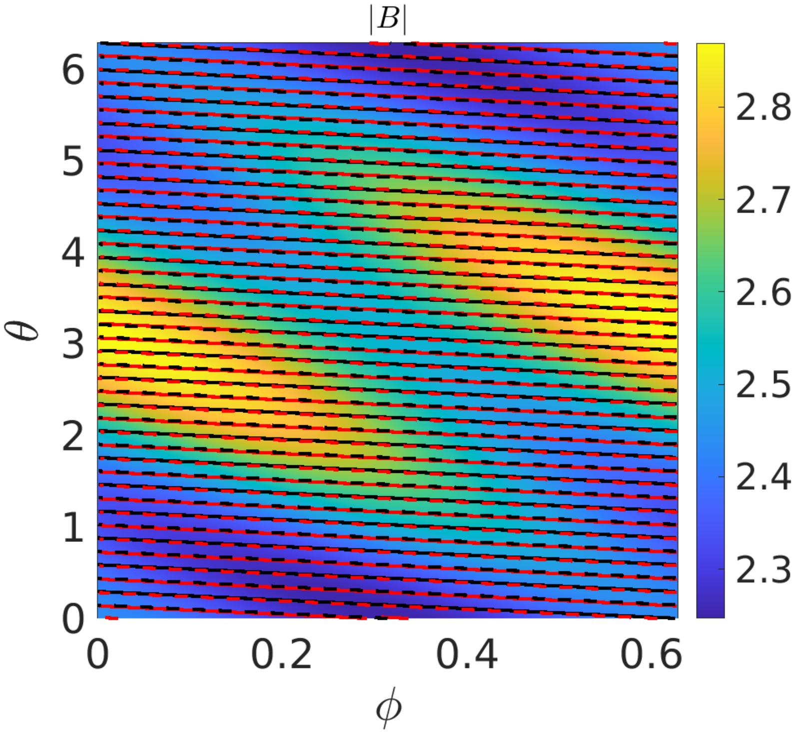
# Magnetic configurations and flux tubes

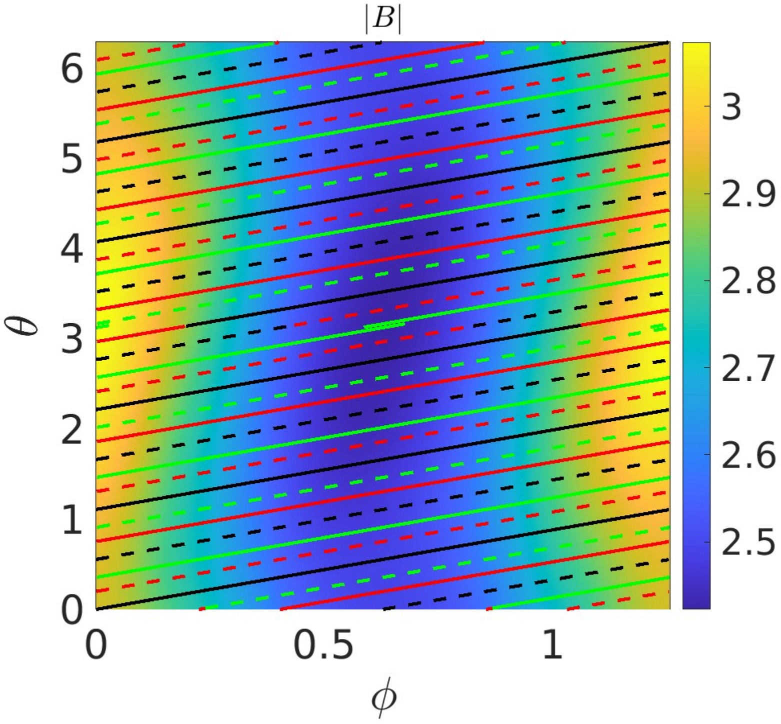
**Two stellarator symmetric FTs considered:** (—) $\alpha = 0$ (- - - -) $\alpha = \iota\pi/N$

$$\alpha = \theta - \iota\phi$$

1 poloidal turn 2 polodal turns 3 poloidal turns

**LHD stantard** **W7-X KJM, beta 3%** **r/a=0.5**



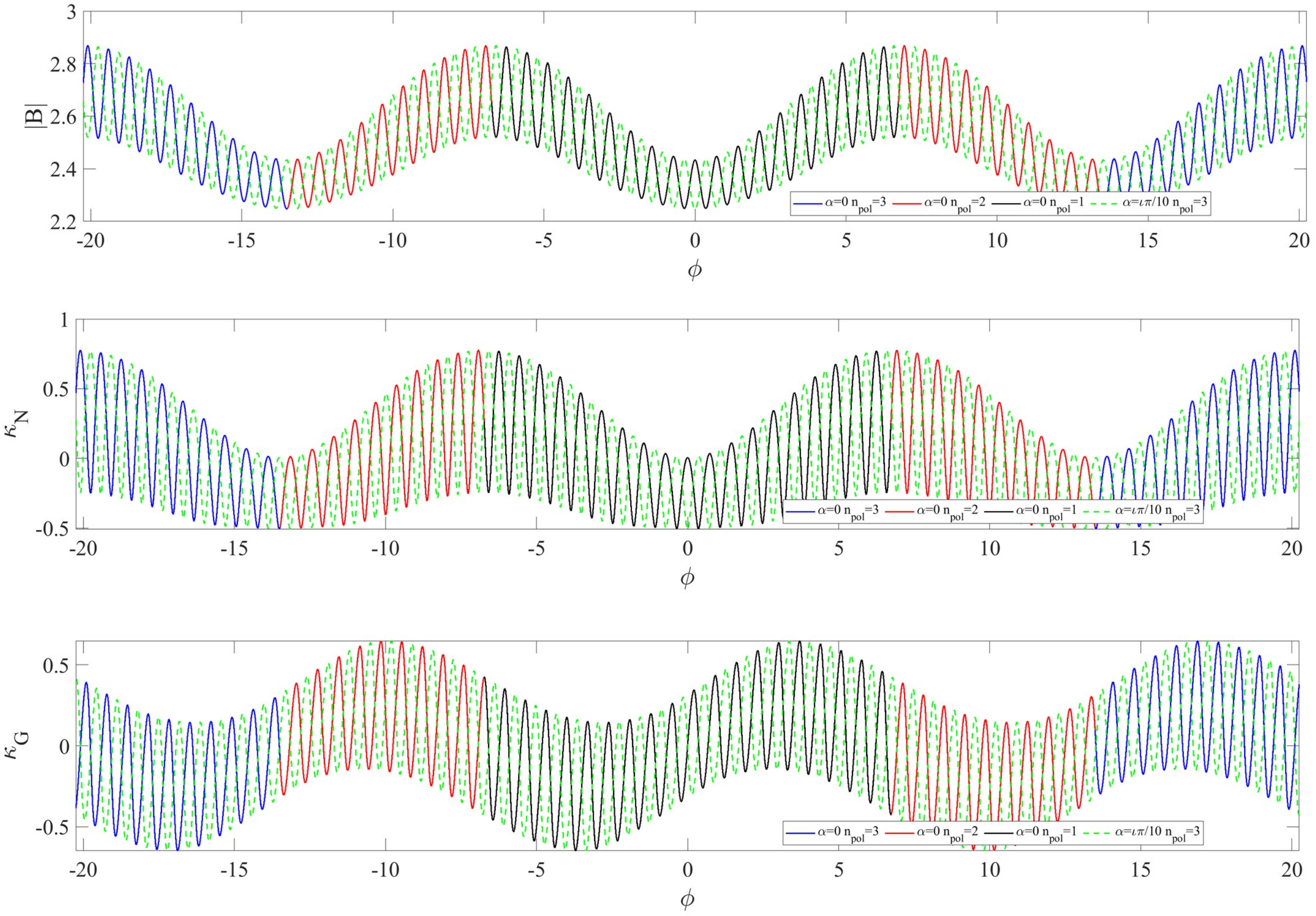




LHD is more densely covered by a FT with $n_{pol}$=1 than W7-X.

In LHD the FTs overlap with each other.

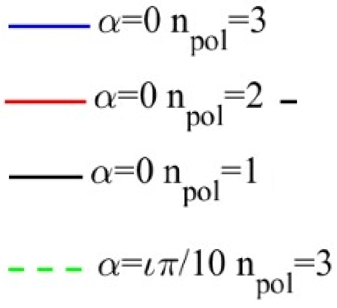
In W7-X, the triangular FT at $n_{pol}$=3 approaches the bean-FT at $n_{pol}$=1.

# FTs in LHD

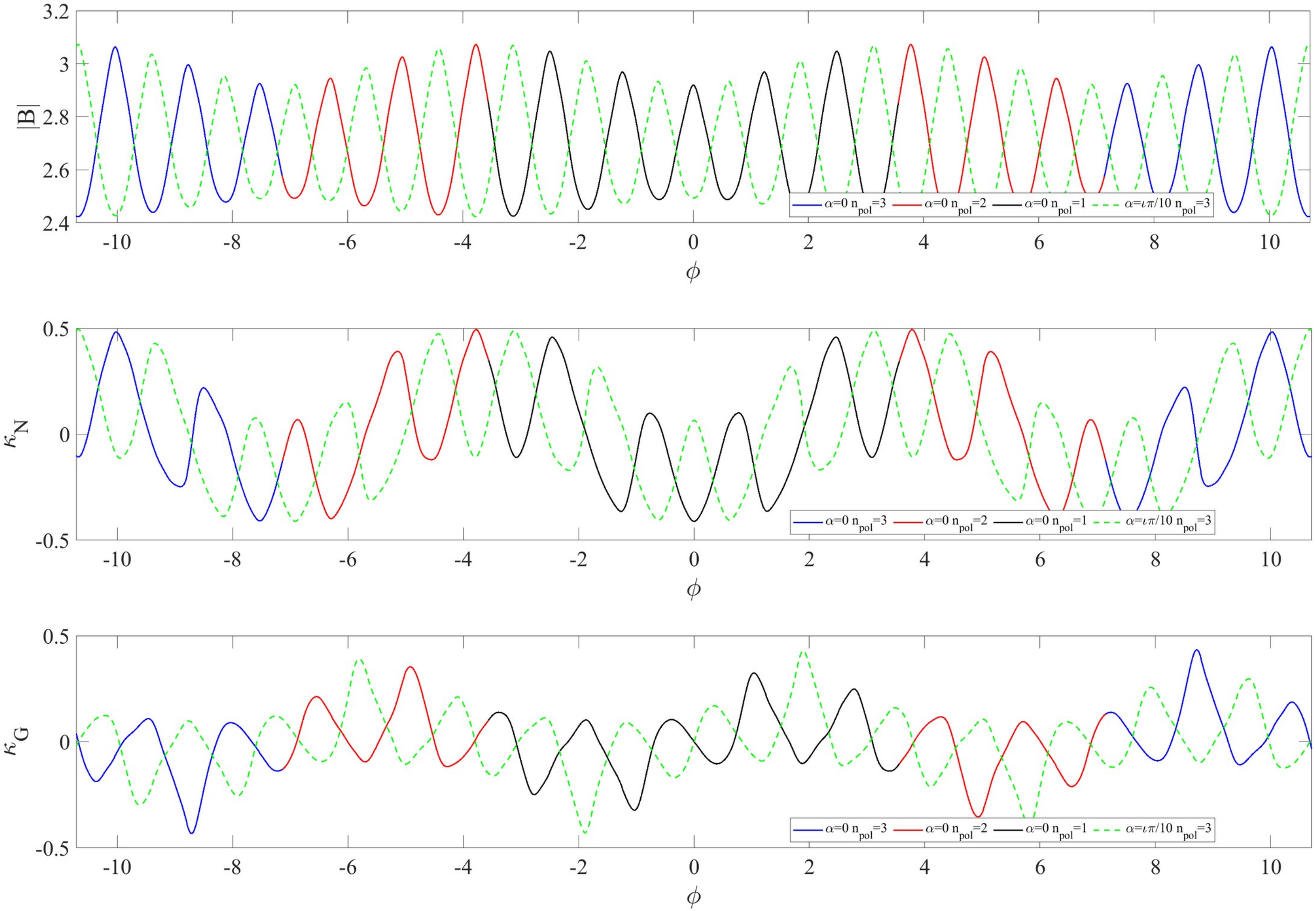



Approximate periodicity of magnetic quantities along the FT

The FTs are approximately shifted in toroidal angle

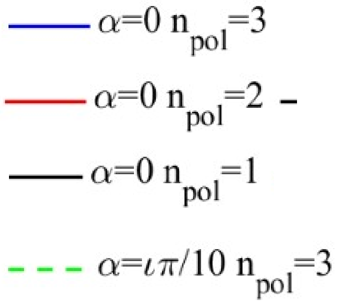


# FTs in W7-X




The magnetic quantities deviate from periodicity along the FT more than in LHD

The differences between bean and triangular FTs are larger in W7-X than in LHD

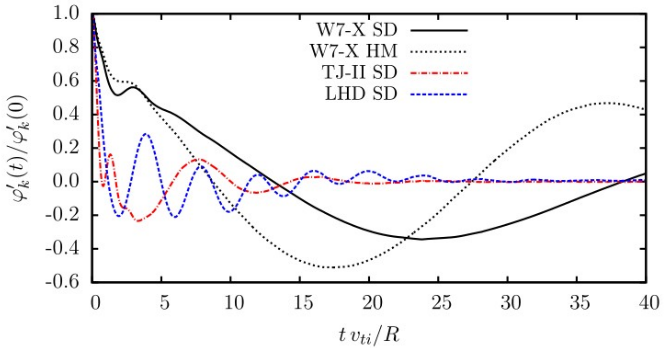


# Two physical problems

- **Linear collisionless relaxation of ZFs**
  - Paradigmatic problem because the linear response of ZFs depends on the full flux surface (Moreal PPCF 2016, Monreal PPCF 2017).
  - Relaxation studied for several radial scales $k_x$ of the perturbation.
  - Residual level and oscillation frequency are the quantities to compare. Extracted from model fit.

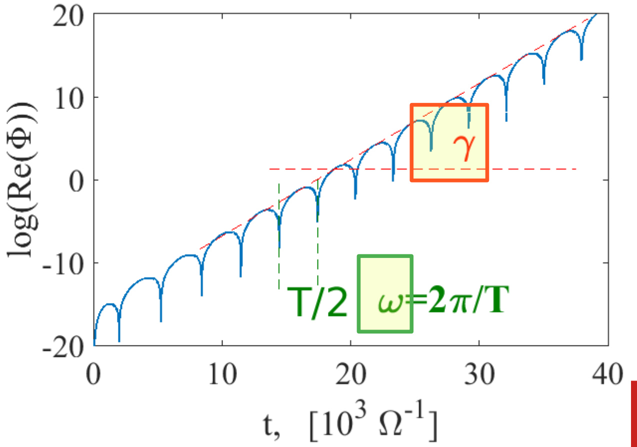
$$\frac{\varphi'(t)}{\varphi'(0)} = A\cos(\Omega t)e^{-\gamma_{ZF}t} + \frac{c}{1+dt^e} + R$$

Monreal et al. PPCF 2017




- **Linear instability of ITGs**
  - Growth rate and frequency of the unstable modes are the quantities to compare.

$$\phi_k(t) = A_k e^{(\gamma t + i\omega)t)}$$



# Linear relaxation of zonal flows

# Simulations settings for ZF relaxation

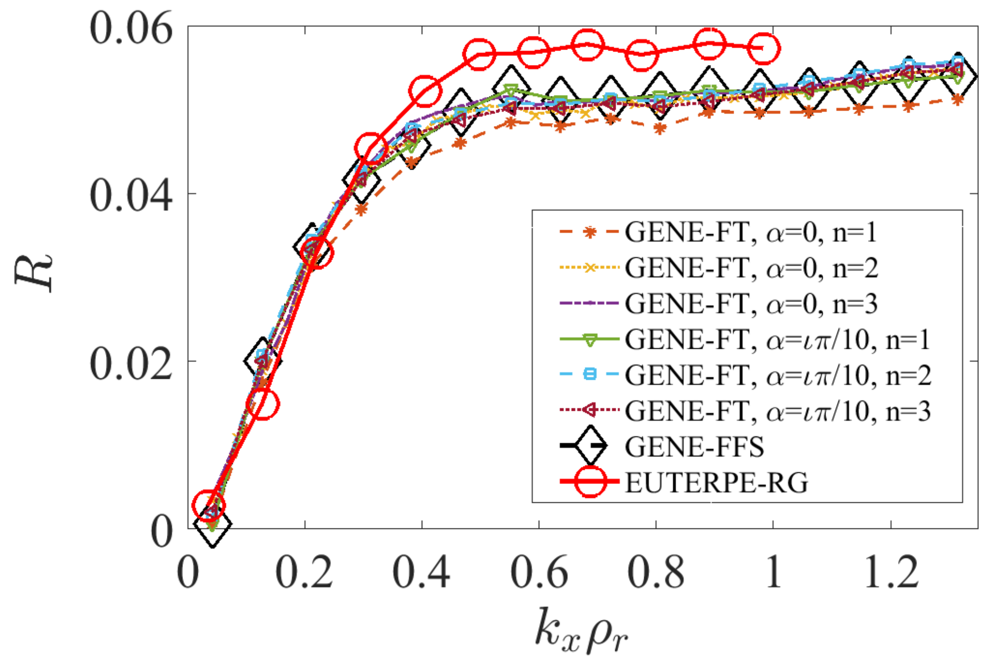
- **RG simulations run with flat n,T profiles**
  - Initialized with density perturbation such as $\varphi \propto \cos(k_s \pi s)$
  - Linear simulation, $n_\phi = 32, n_\theta = 32$ , FS average solver
  - Only modes with |m|<6, |n|<6 are retained
  - Extract time traces at r/a=0.5

- **FT and FFS simulations at r/a=0.5**
  - Same n and T as in the RG simulations at r/a=0.5
  - Only m=0, n=0 with finite $k_x$ mode initialized with finite amplitude

- **Relationship betwen FT/FFS and RG radial mode numbers**

$$k_x = 2k_s \pi r / a^2 \qquad B_r = \Psi_0 / \pi a^2$$

# Linear relaxation of ZFs in LHD: residual level

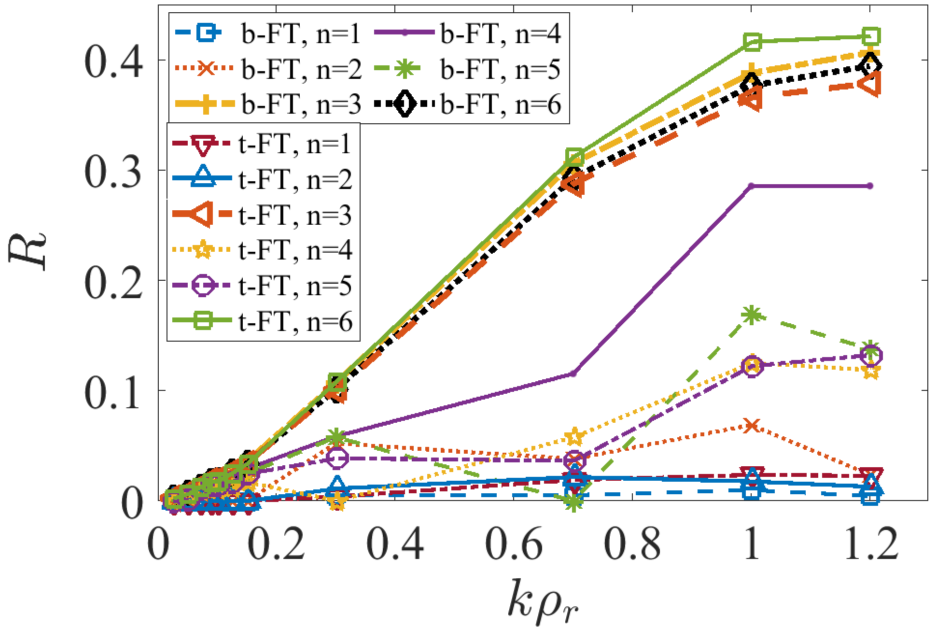



**Comparison of FT, FFS and RG (GENE and EUTERPE)**

- Different FTs, with $n_{pol}$>=2 provide very similar results.
  Only for $n_{pol}$=1 FT results slightly differ.
- FT and FFS results are very similar.
- RG results are similar but slightly larger residual level (~7%).

**=> Consistent with expectations from FT figures** (both FTs overlap for $n_{pol}$=2)

# Relaxation of ZFs in W7-X: residual level




**Comparison of bean and triangular FTs (GENE)**

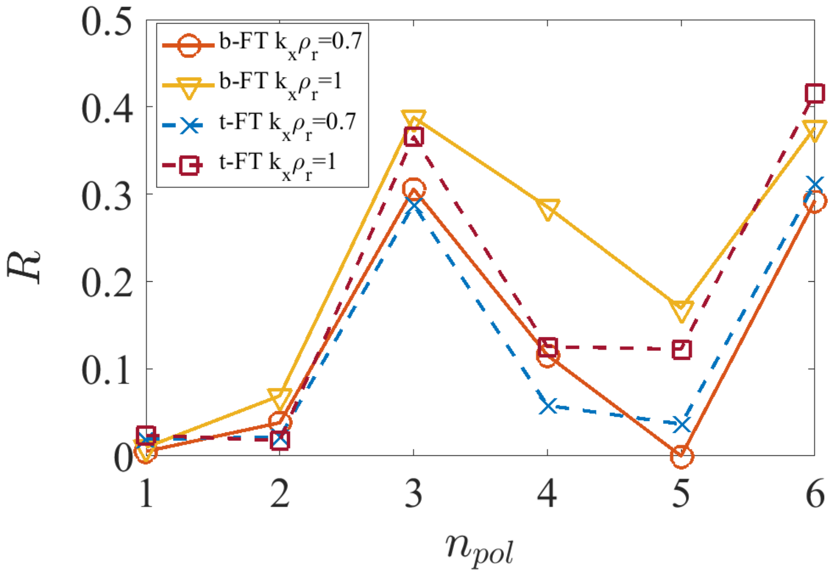
- At $n_{pol}$=1, the residual level is very small.
- At $n_{pol}$<3, different FTs provide different results.
- The residual for the bean and triangular FTs increases for $n_{pol}$ 1→3.
- At $n_{pol}$=3 the results are almost similar.
- At $n_{pol}$=4,5, FT residual separate and decrease again.
- At $n_{pol}$=6 FT results match the npol=3 result.

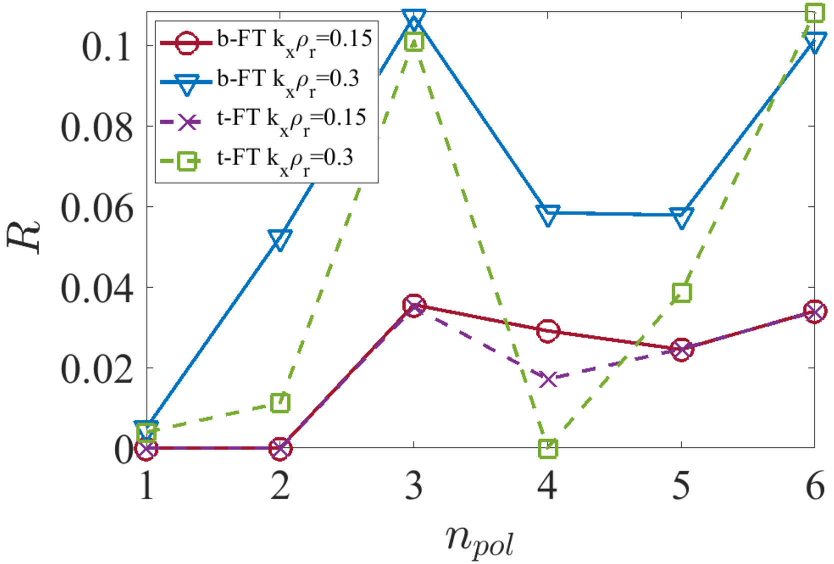
**=> Consistent with expectations from FT figures**

(both FTs map different regions of the FS and get close to each other for $n_{pol}$=3)

# Relaxation of ZFs: residual level vs. FT length (GENE FT)

## Comparison of FTs with different $n_{pol}$ (GENE)

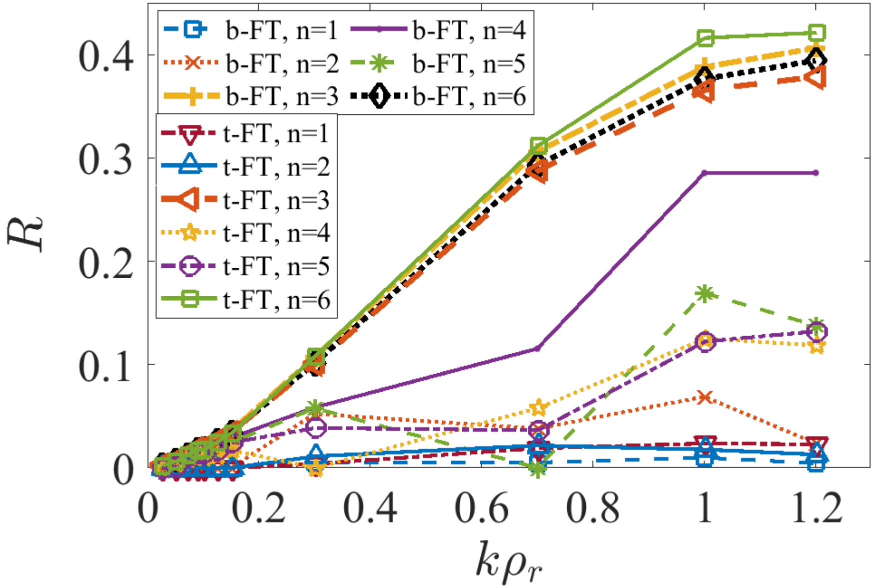


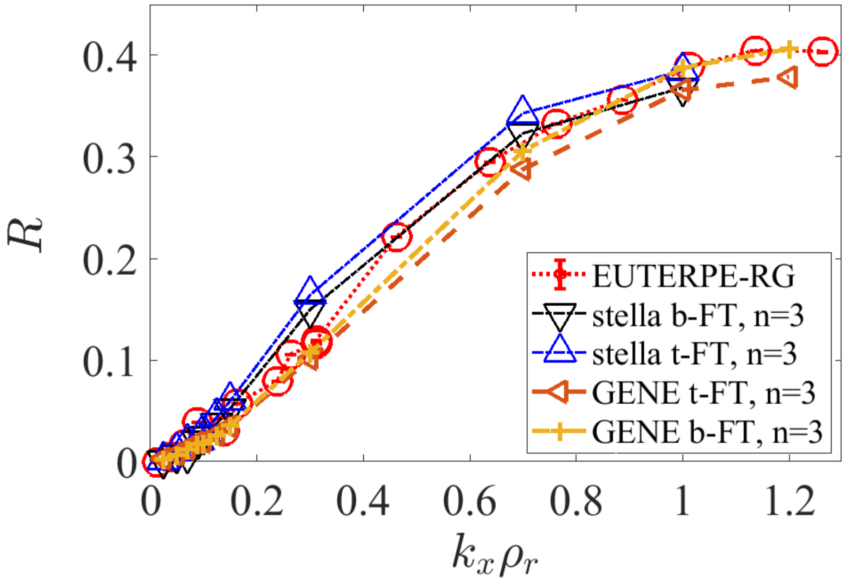

- As the FT length is increased from $n_{pol}$=1 to $n_{pol}$=3, the residual level increases.
- For $n_{pol}$>3, the residual level decreases again.
- For $n_{pol}$=6 the residual level gets close to the $n_{pol}$=3 result.

# Relaxation of ZFs in W7-X: residual level

**GENE (FT, npol=1-6)**



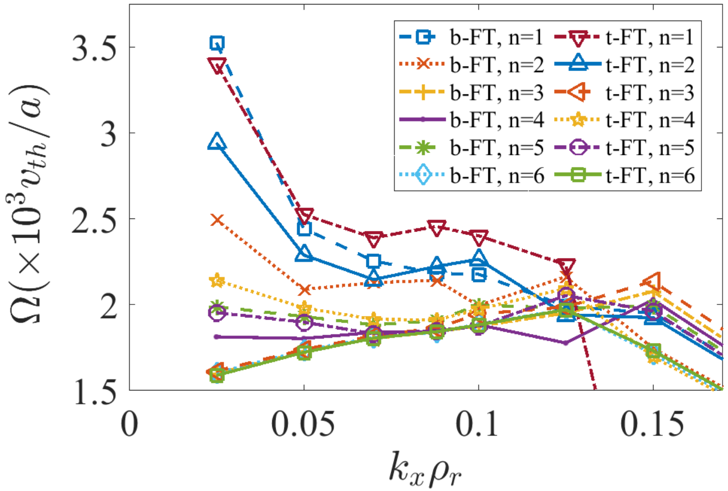

**GENE (FT, npol=3), stella (FT, npol=3)and EUTERPE**



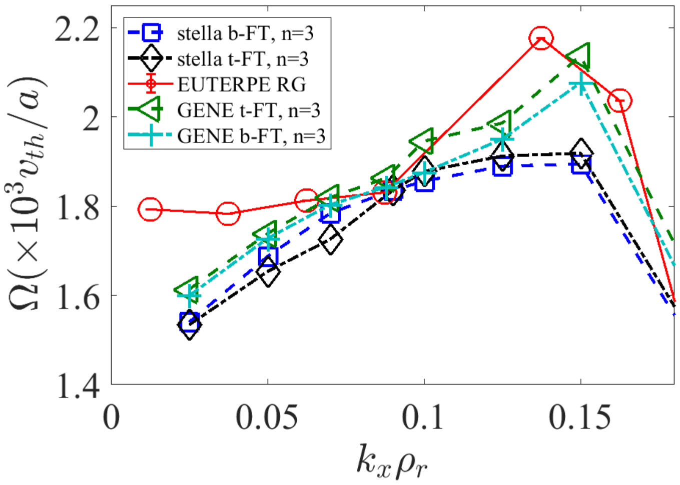

- At $n_{pol}$=3, FT results (GENE and stella) are very close to the RG (EUTERPE) results.

# Relaxation of ZFs in W7-X: oscillation frequency

**GENE (FT, $n_{pol}$=1-6)**

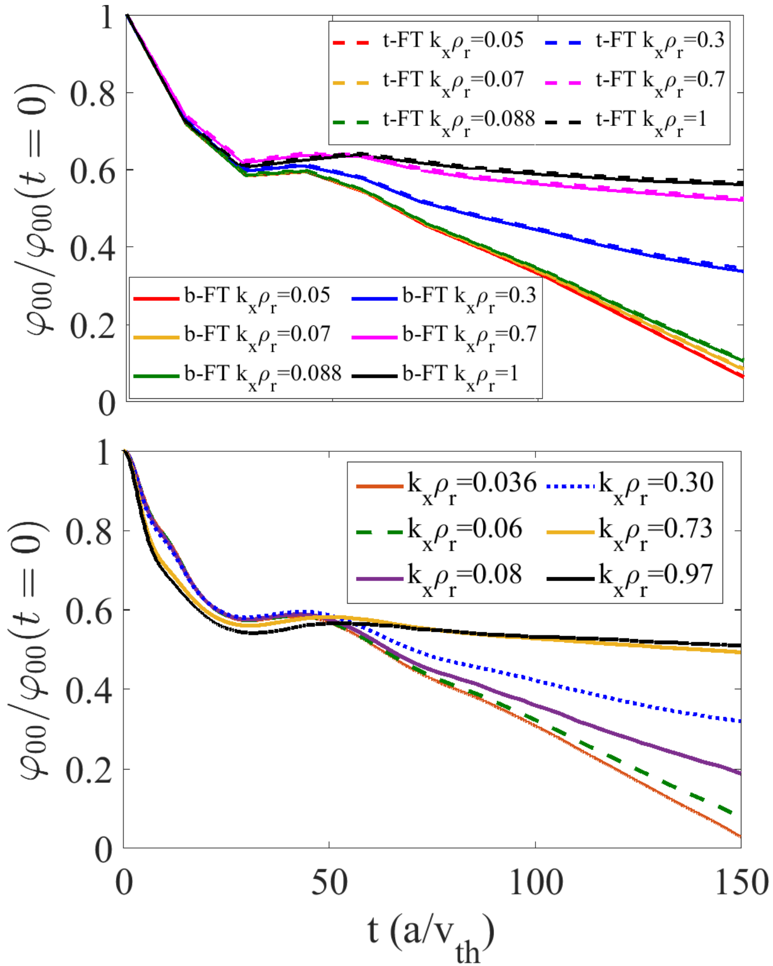



**GENE (FT, $n_{pol}$=3), stella (FT, $n_{pol}$=3)and EUTERPE (RG)**




- At short length ($n_{pol}$<3), different FTs give different results for the oscillation frequency.
- At $n_{pol}$=3, bean and triangular FTs (with stella and GENE) give similar results, close to the RG result (EUTERPE).

# Relaxation of ZFs: short time behaviour




**Comparison of FTs (GENE) and RG (EUTERPE) results.**

- At very short times, there is no difference between FTs.
- At very short times, t<50μs, there is no dependence with the radial scale.
- For longer times, t>50μs, there is a dependence with the radial scale which is similar to that of the long-term residual level.
- Very good agreement between FT and RG results.

# Linear stability of ITGs

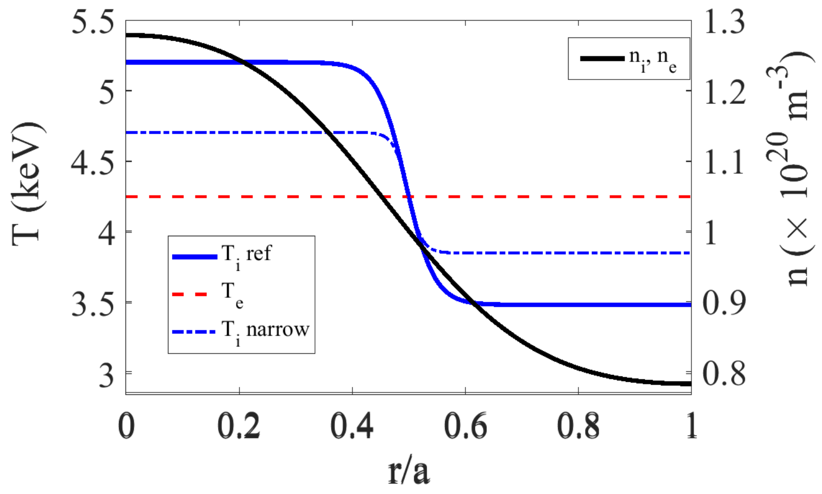
# Simulation settings

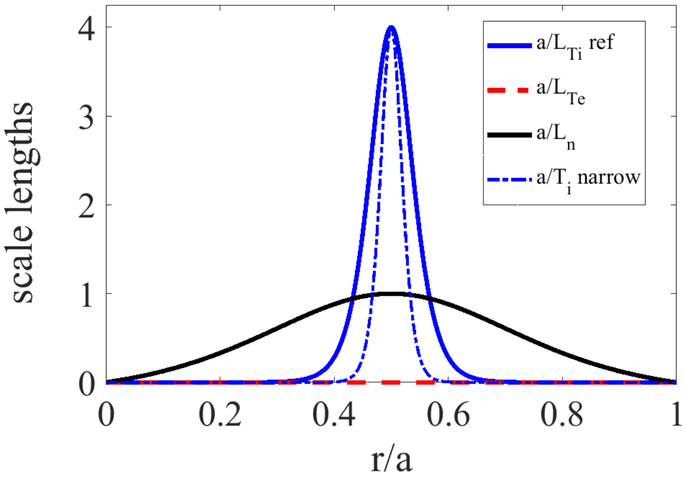
- **RG simulations run with profiles having sharp gradient of Ti**
  - Two kinds of simulations:
    - Scan moving the center of a small Fourier filter (phase factor extraction, EUTERPE)
    - Simulations with large resolution [+wide Fourier filter] (EUTERPE and GENE-3D).
  - Only information on the growth rate and frequency of the most unstable mode in a simulation.
- **FT simulations using the local parameters ($L_n$=1, $L_{Ti}$=4) of the profiles at r/a=0.5**
  - Scan in $k_y$ wavenumbers to extract spectrum of unstable modes.
- **FFS simulations using the local parameters ($L_n$=1, $L_{Ti}$=4) of the profiles at r/a=0.5**
  - Multiple $k_z$, $k_y$ resolved in a simulation.
- **Relationship betwen FT/FFS and RG mode numbers**

$$k_y = m/r$$

# Kinetic profiles for ITG simulations

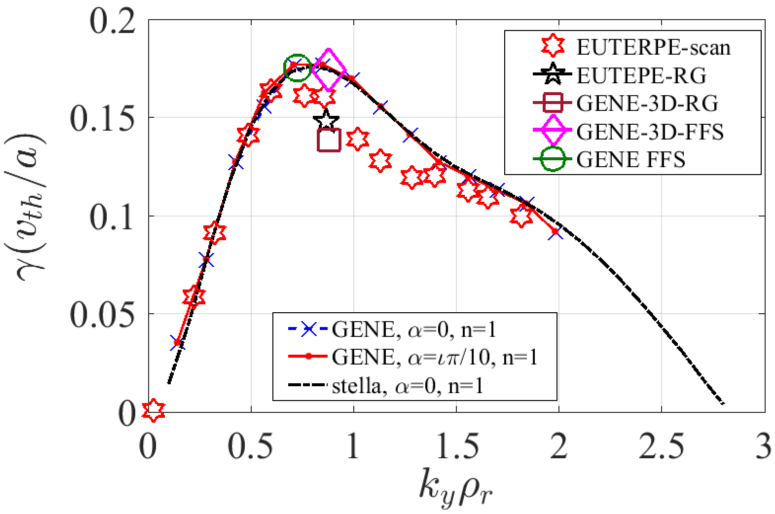
- Model profiles $X = X_* \exp\left[\frac{-\kappa_x}{1-\mathrm{sech}_x^2}\left(\tanh(\frac{r-r_0}{a\Delta_x}) - \mathrm{sech}_X^2\right)\right]$ for Ti and n, w. sharp ion temperature gradient at r/a=0.5.
- Flat electron temperature profile
- Two different ion temperature profiles with different $\eta_i$ width.

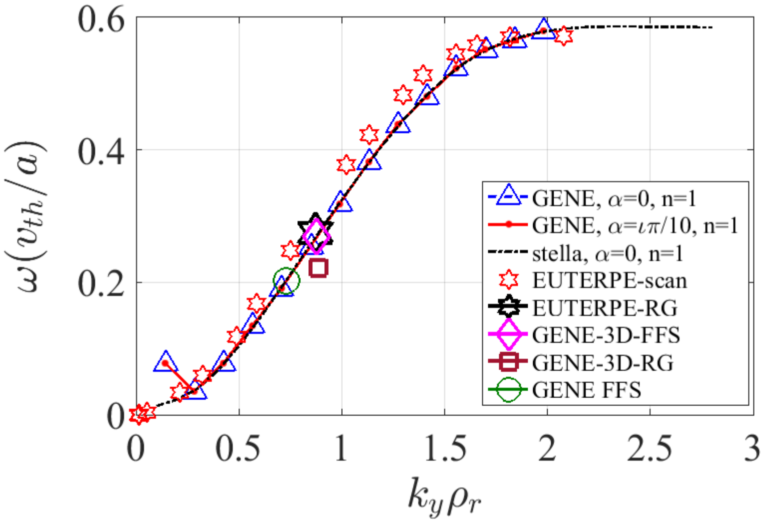


# Simulation of ITGs in LHD

**FT (GENE, stella), FFS (GENE, GENE-3D) and RG (EUTERPE, GENE-3D)**



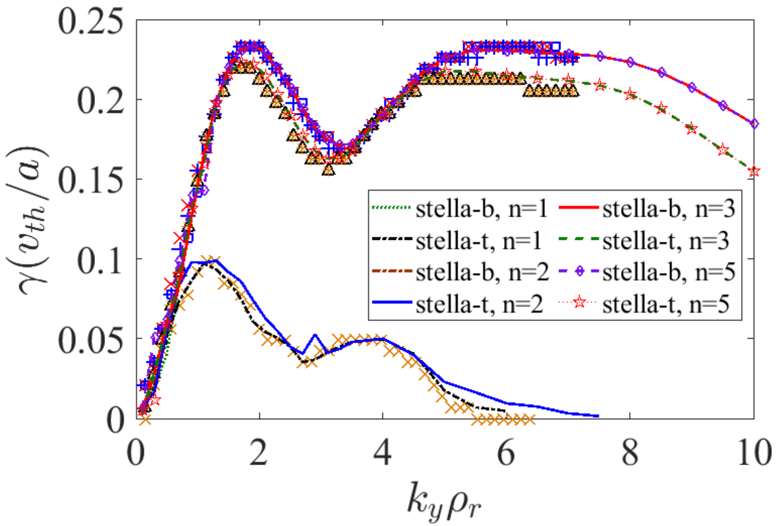



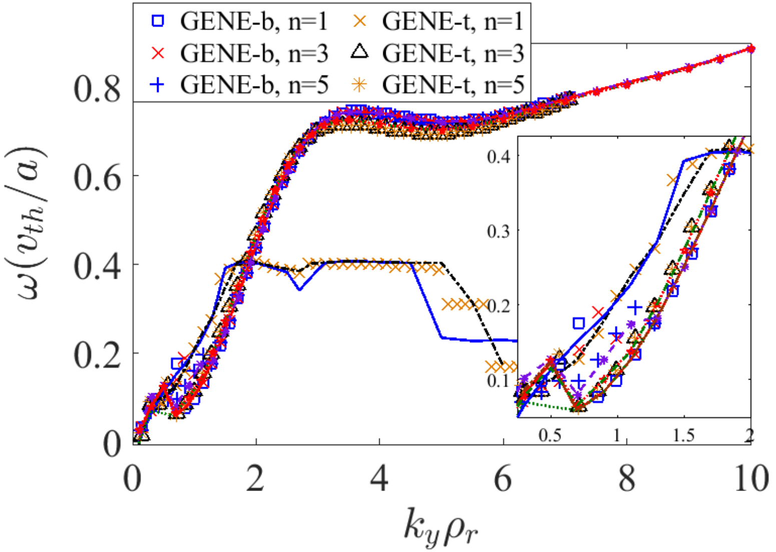

- Different FTs (GENE and stella) provide very similar results.
- FT and FFS results agree very well.
- RG codes (EUTERPE and GENE-3D) give slightly smaller $\gamma$, and frequency very similar to that of FT and FFS.

=> **Consistent with expectations from FT figures** (both FTs overlap for $n_{pol}$=2)

# Simulations of ITGs in W7-X in FT domain

**FT simulations with $k_x$=0 in W7-X (GENE, stella)**



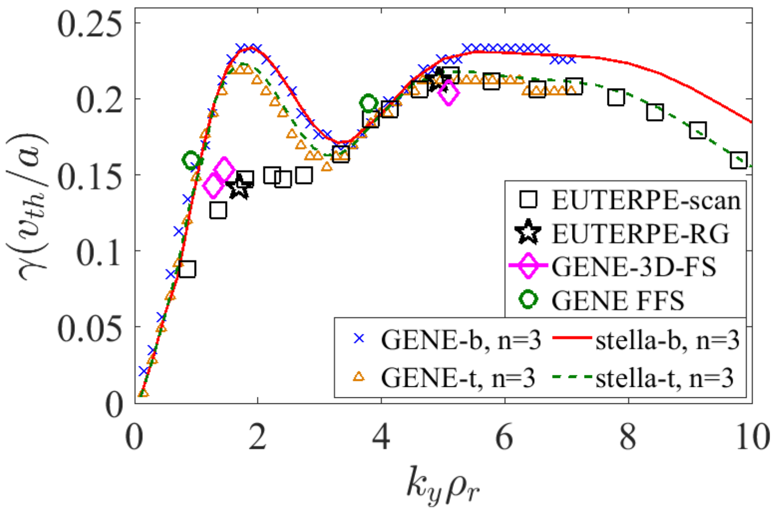

- At short FT length, different FTs (GENE and stella) provide different results.
- At $n_{pol}$>=3, FT results agree very well for the frequency but not completely for $\gamma$.

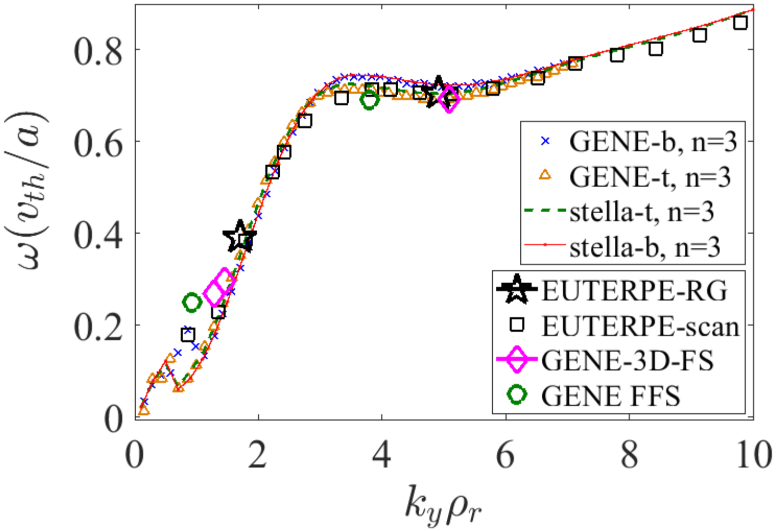
**=> Consistent with expectations from FT figures**

(both FTs map different regions of the FS and get close to each other for $n_{pol}$=3)

# Simulations of ITGs in W7-X

**FT (GENE, stella with npol=3), FFS (GENE, GENE-3D) and RG (EUTERPE, GENE-3D)**

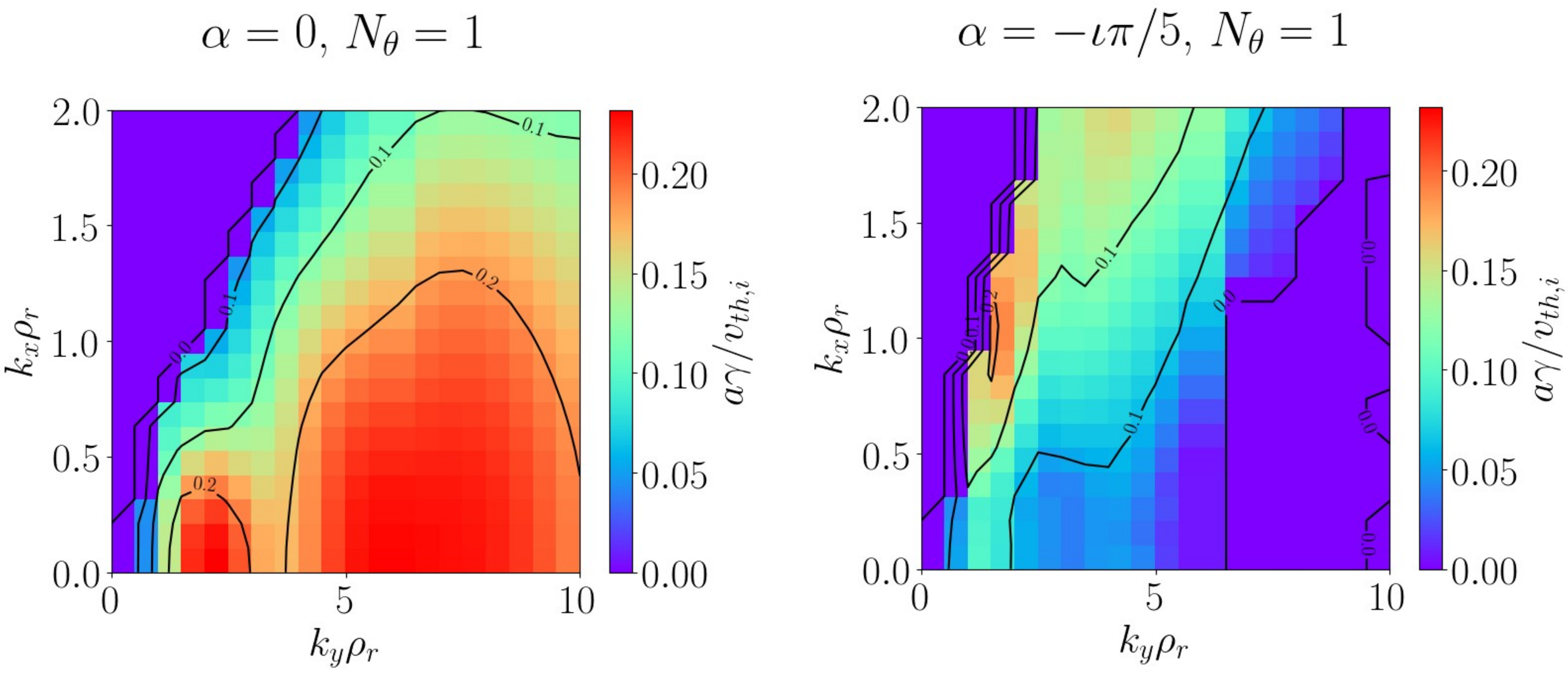


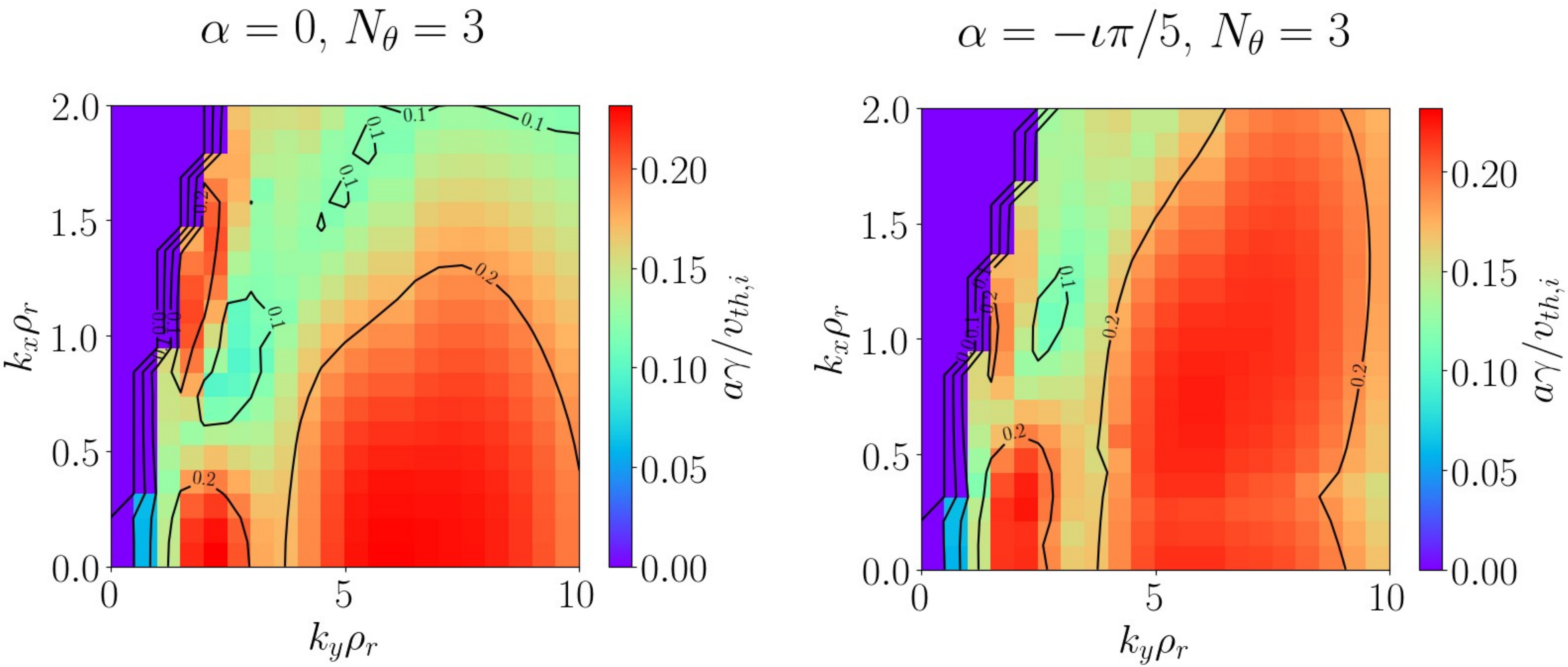

- FFS and RG results reasonably agree on both γ and frequency.
- FFS, RG and FT results reasonable agree on the frequency
- RG codes (EUTERPE and GENE-3D) give **slightly smaller γ for $k_y\rho$<3.**

# Radial scale of the unstable modes

- Set of simulations in FTs with different lengths  with stella and scaning in $k_y$ and $k_x$.

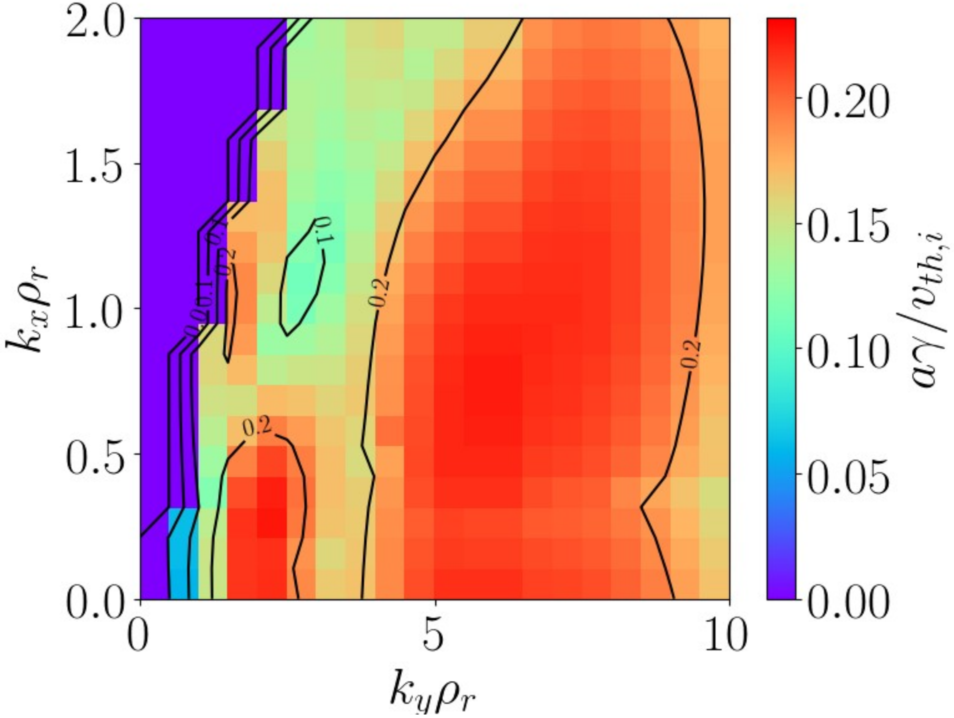


# Radial scale of the unstable modes



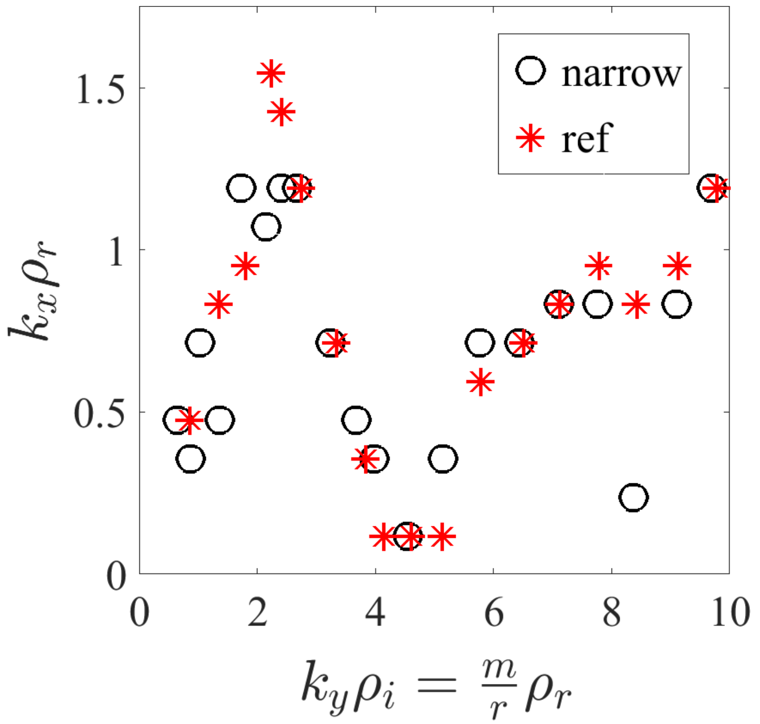
# Radial scale of the unstable modes

$\alpha = -\iota\pi/5,\ N_\theta = 3$




Radially global simulation with EUTERPE.

$k_x$ obtained by Fourier transforming the potential in radial direction




- Basic structures, large $k_x$ in $k_y\rho<4$ and $k_y\rho>5$, $k_x$ valley around $k_y\rho=4$, are captured by FT and RG although with differences.

# Summary: Linear relaxation of ZFs

- **In LHD,**
  - Different flux tubes provide very similar results for the ZF residual level.
  - FT results are close to FFS and RG ones (RG ~7% larger residual).
- **In W7-X,**
  - Short FTs ($n_{pol}$<3) give a small residual level and large oscillation frequency
  - Different flux tubes give different results.
  - The residual increases and the frequency decreases as the FT length $n_{pol}$ is increased 1→3.
  - For $n_{pol}$>3, the resdiual decreases again and the frequency increases again.
  - For $n_{pol}$=6, the results from $n_{pol}$=3 are recovered.
  - For $n_{pol}$=3,6 the results are close to the RG ones.
  - The very short-time evolution (t<50μs) does not show dependence with the FT or the radial scale.
  - At t>50μs time evolution shows a dependency with $k_x$ similar to the residual.

# Summary: Linear stability of ITGs

- **In LHD,**
  - Different flux tubes provide very similar results for $\gamma$ and $\omega$ (perfect agreement for $n_{pol}>2$).
  - FT results are close to FFS and RG ones (RG gives sligthly smaller $\gamma$).
- **In W7-X,**
  - Short FTs ($n_{pol}<3$) give different results for $\gamma$ and $\omega$.
  - At $n_{pol}>3$, convergence with $n_{pol}$ is found but different FT do not exactly match each other.
  - At $n_{pol}>=3$, FT results more or less match the RG and FFS for $k_y\rho>4$.
  - For $k_y\rho<3$, the FFS and RG codes give significantly smaller $\gamma$ than FTs.
  - For $k_y\rho<3$, the agreement of FT ($n_{pol}>3$), FFS and RG on $\omega$ is good.
  - FFS and RG results more or less agree, with slight differeces

    $\gamma_{RG} > \gamma_{FFS}$ for $k_y\rho>4$, while $\gamma_{RG} < \gamma_{FFS}$ for $k_y\rho<3$.
- The radial scale of unstable modes in RG simulations shows $k_y$ dependence in W7-X (not in LHD) in qualitative consistency with FT results.

# Conclusions

- Different FTs provide different results, in general.

  The results of different FTs get closer as the FT length is increased, but the length required for convergence is configuration-dependent

  $n_{pol}$=1-2 in LHD, $n_{pol}$>3 in W7-X, [$n_{pol}$=4 for HSX,and 8 for NCSX, Smoniewski et al. Pop 2020]

- As the length of the FTs is increased, the results approach those of FFS and RG.

  Qualitative agreement is reached at some scales but full quantitative agreement in all wavenumbers is now always possible.

- FFS and RG results are always very similar, with small quantitative differences.

- The FFS domain appears as the minimum computational domain suited for stellarators.

# What next?

- Nonlinear simulations in different domains.
- Compare turbulent transport in different domains.
- Study the role of the ZF in different domains in the NL regime.

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