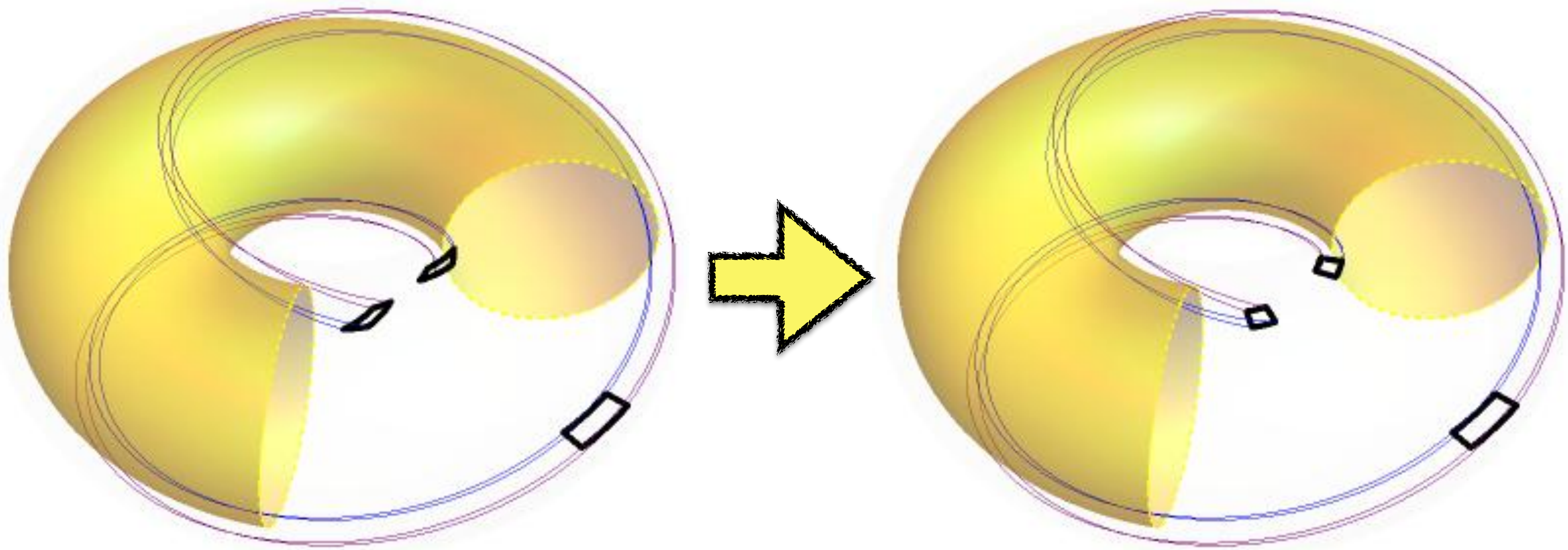
Reimagining the flux tube for the tokamak edge



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TSVV1 Progress Workshop
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Outline

J. Ball and S. Brunner. *PPCF* (2021).

1. The non-twisting flux tube

2. Further generalizations beyond the non-twisting flux tube

3. Including non-uniform magnetic shear in a flux tube

The non-twisting flux tube

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Summarizing the non-twisting flux tube

J. Ball and S. Brunner. *PPCF* (2021).

- The non-twisting flux tube is created by laying down a rectangular grid $K_x = k_x + k_y \frac{\nabla x \cdot \nabla y}{|\nabla x|^2}$, instead k_x

- Prioritizes including Fourier modes with minimal FLR damping, instead of prioritizing following linear modes

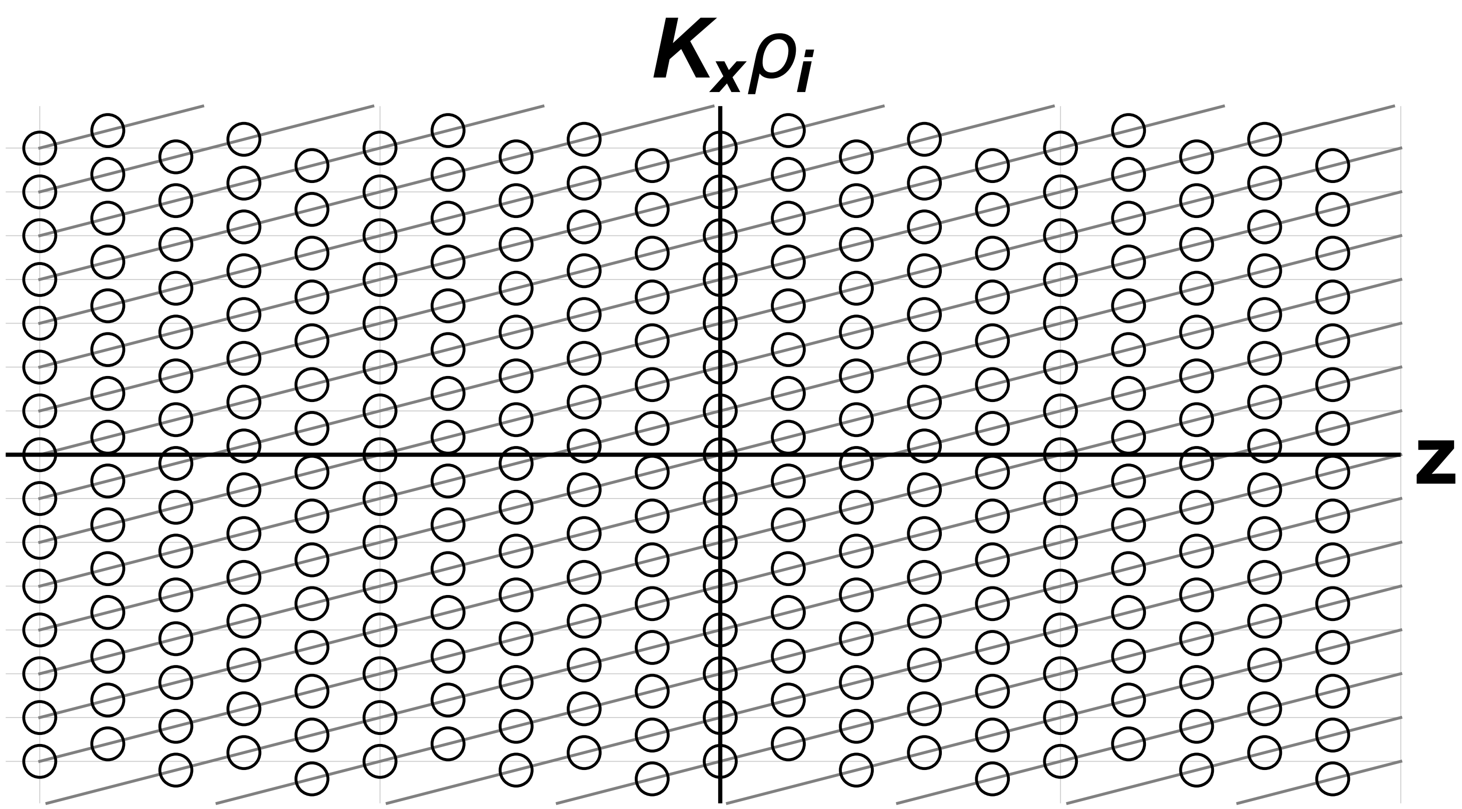
- Nothing physical has changed

Non-twisting flux tube is a different set of gridpoints

J. Ball and S. Brunner. *PPCF* (2021).

- Boundary conditions determine an infinite lattice of allowed Fourier modes according to either $k_x = \frac{2\pi}{L_x} m$ or $K_x = \frac{2\pi}{L_x} m + k_y \frac{\nabla x \cdot \nabla y}{|\nabla x|^2}$ where $m \in \mathbb{Z}$

- These are the same physical perturbations, just labeled differently



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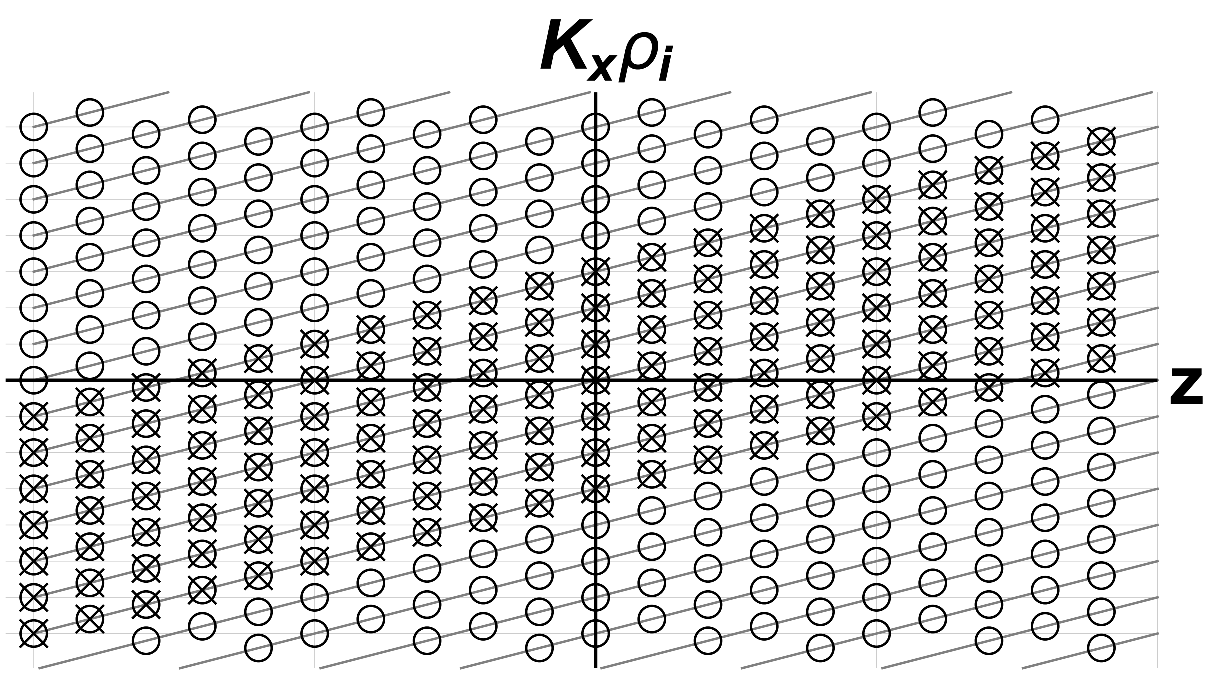
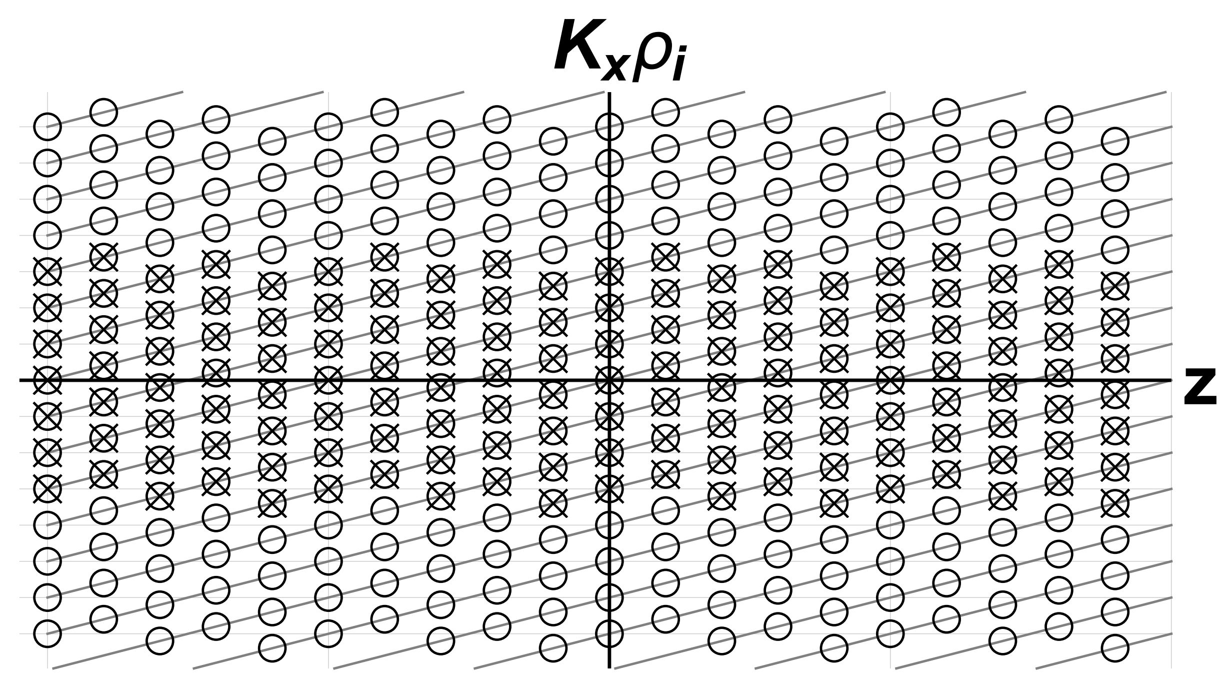
Conventional



Non-twisting



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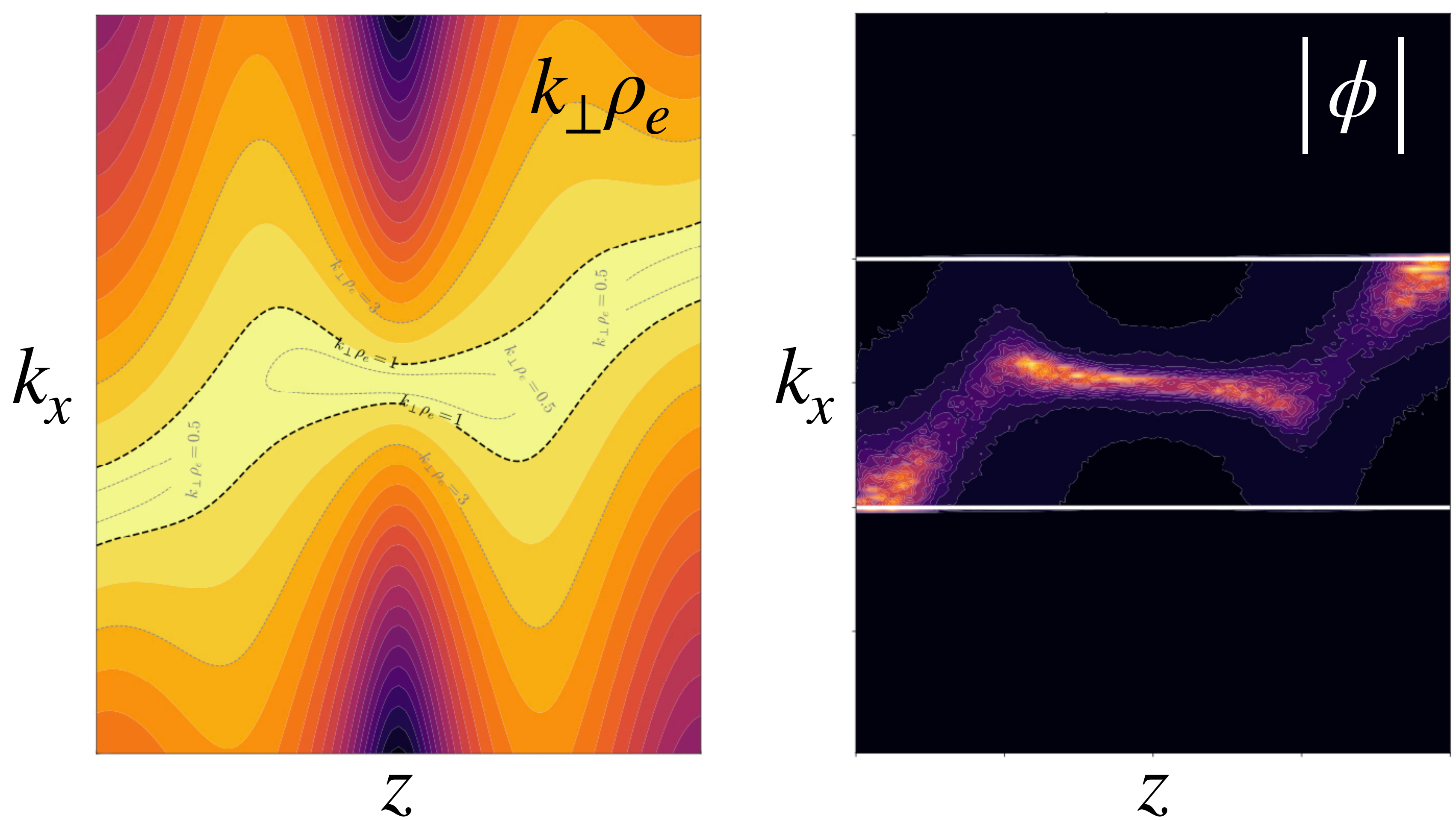
Takeaways from runtime tests

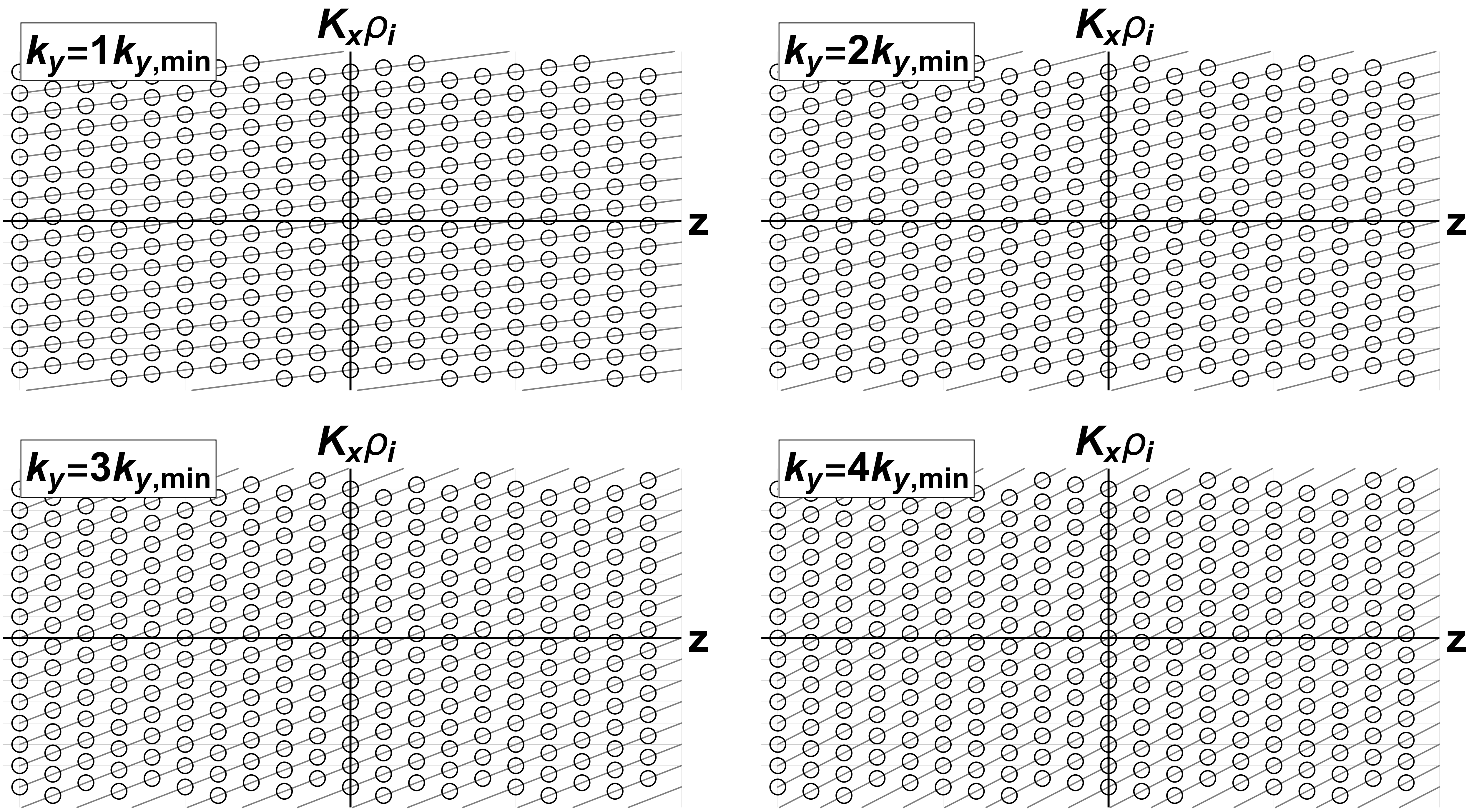
Parisi et al. *In prep.* (2021).

- Most helpful for multiple regions of turbulent drive, e.g. **tokamaks with $N_{pol} > 1$**, **H-mode pedestals**, stellarators
- Prioritizes including Fourier modes with minimal FLR damping
- Not harmful



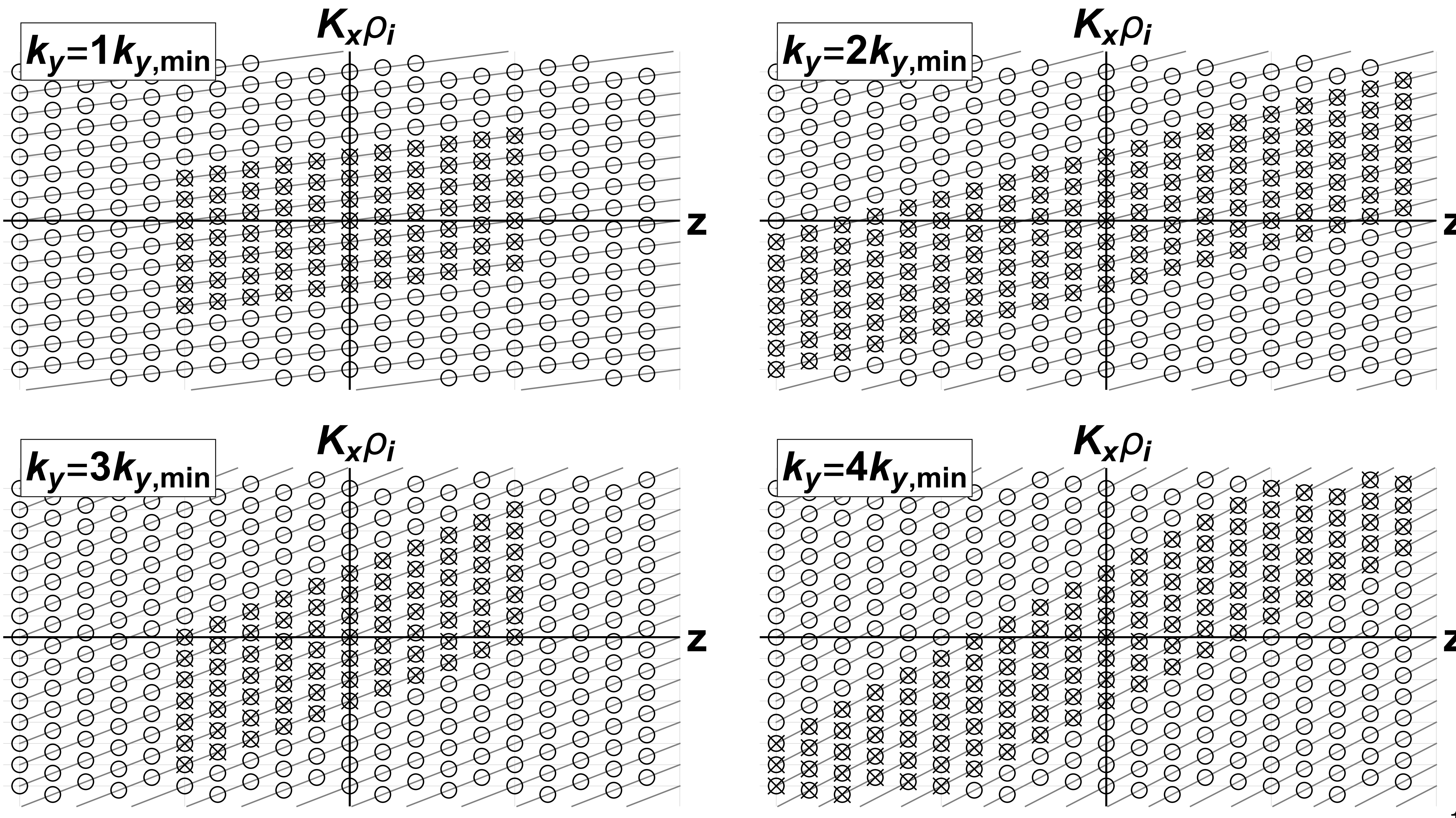
Generalizations of the non-twisting flux tube

Boundary conditions define allowed Fourier modes



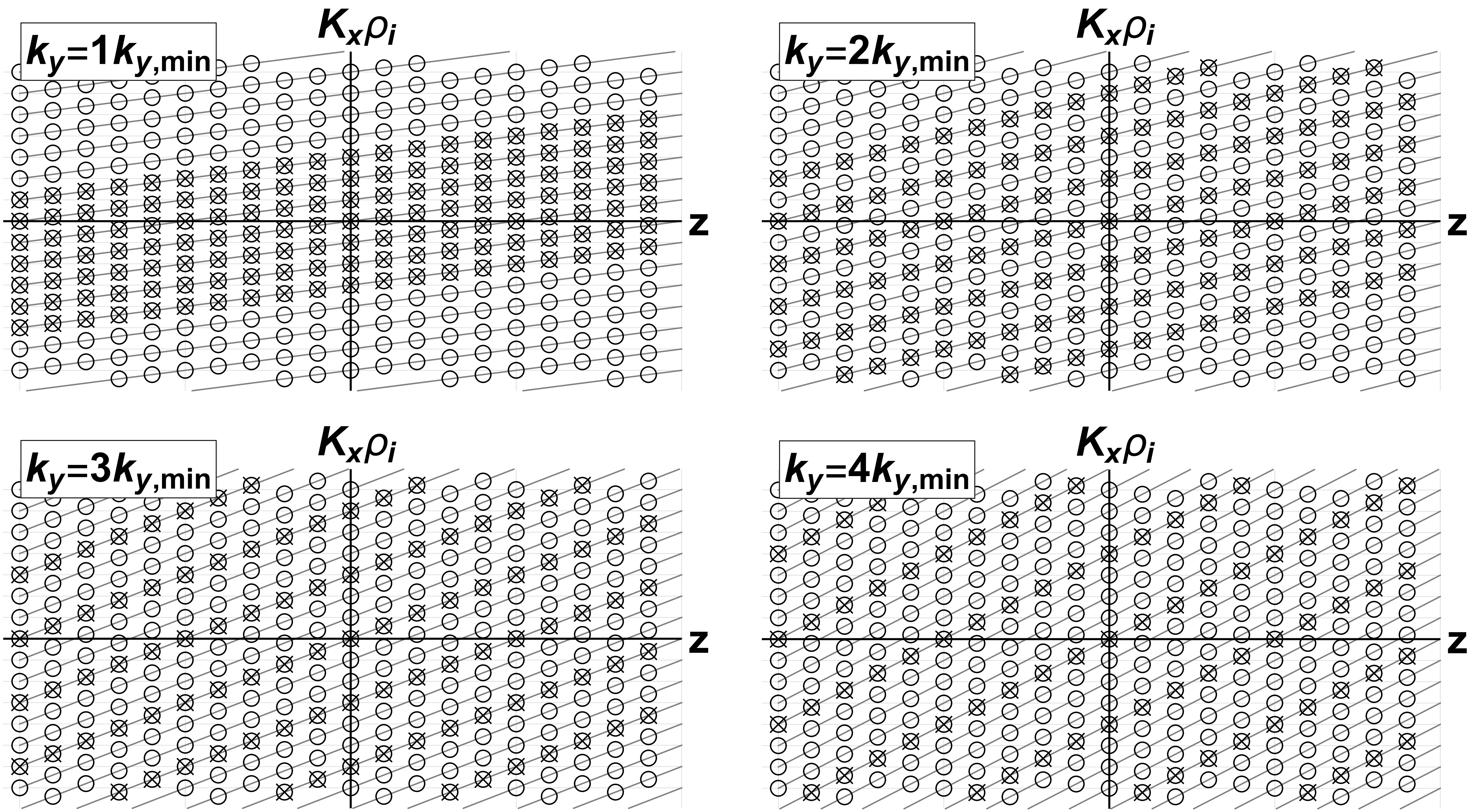
Can choose points freely with different transformations

L_y that varies with z



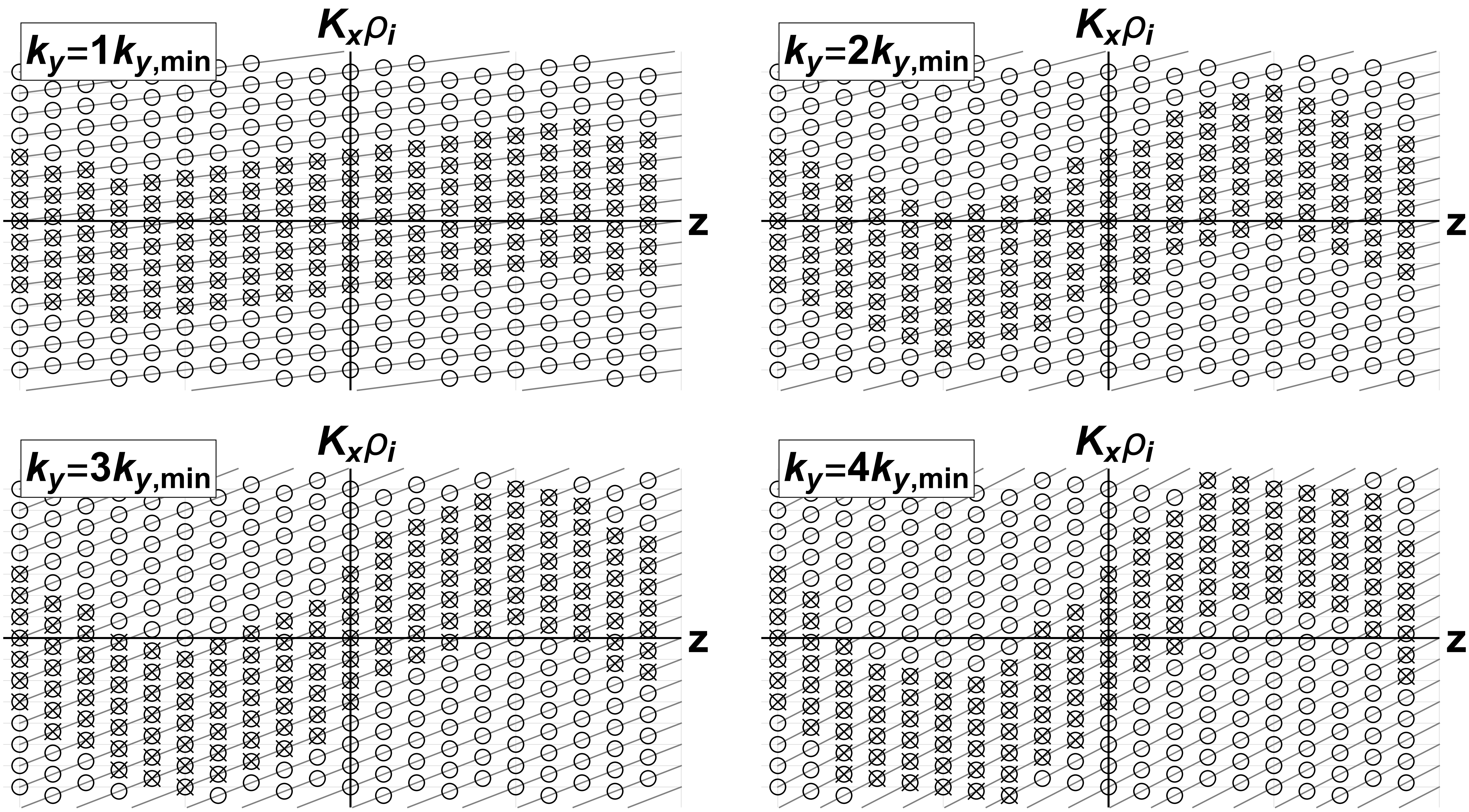
Can choose points freely with different transformations

Uniform radial grid in $\theta_0 \equiv k_x/(k_y\hat{s})$



Can choose points freely with different transformations

Completely control twist with z



Limitations on generalization of the flux tube

Ball et al. *PPCF* (2021).

- Practical limitations due to terms that cause mode coupling:
  - Nonlinear term — need evenly-spaced grid for efficient calculation in real space
- Potential to greatly benefit multi-scale simulations or pedestal simulations with weird turbulence?

Flux tubes with non-uniform magnetic shear

Work in progress!

Consistency with gyrokinetic orderings

- Turbulence-scale variation (~10ρ_i) in the $\hat{s}$ profile can be created by:

    1. **ECCD can provide a localized current source and the q profile evolves on the very slow resistive diffusion timescale**

    2. The bootstrap current in the pedestal

- Order this source $\tilde{S}_{Ip} \sim \nu \rho_* \omega F_s$ such that it appears in the GK equation, but then perform a subsidiary expansion in $\nu \ll 1$ to make the resistive diffusion timescale of the source asymptotically slow

- Thus, $\hat{s}$ within the flux tube can be considered fixed in the GK calc.

How to include non-uniform magnetic shear

- Instead of just the standard linear dependence $\hat{s}x$, we want to add

$$\tilde{s}(x) = \sum_{j=1}^{j_m} \tilde{s}_{Cj} \cos\left(2\pi jx/L_x\right) + \tilde{s}_{Sj} \sin\left(2\pi jx/L_x\right)$$

1. **In the non-twisting derivation, magnetic shear only appears in the parallel streaming term, so simply replace $\hat{s}x \rightarrow \hat{s}x + \tilde{s}(x)$**

2. Include a steady, external $A_{||}^{ext}(x, z)$ perturbation arising from the resistive diffusion timescale equations

3. Use standard coordinate $y|_{\tilde{s}(x)=0}$ without new shear variation, causing the $\hat{b} \cdot \nabla y|_{\tilde{s}(x)=0}$ term to persist in parallel streaming

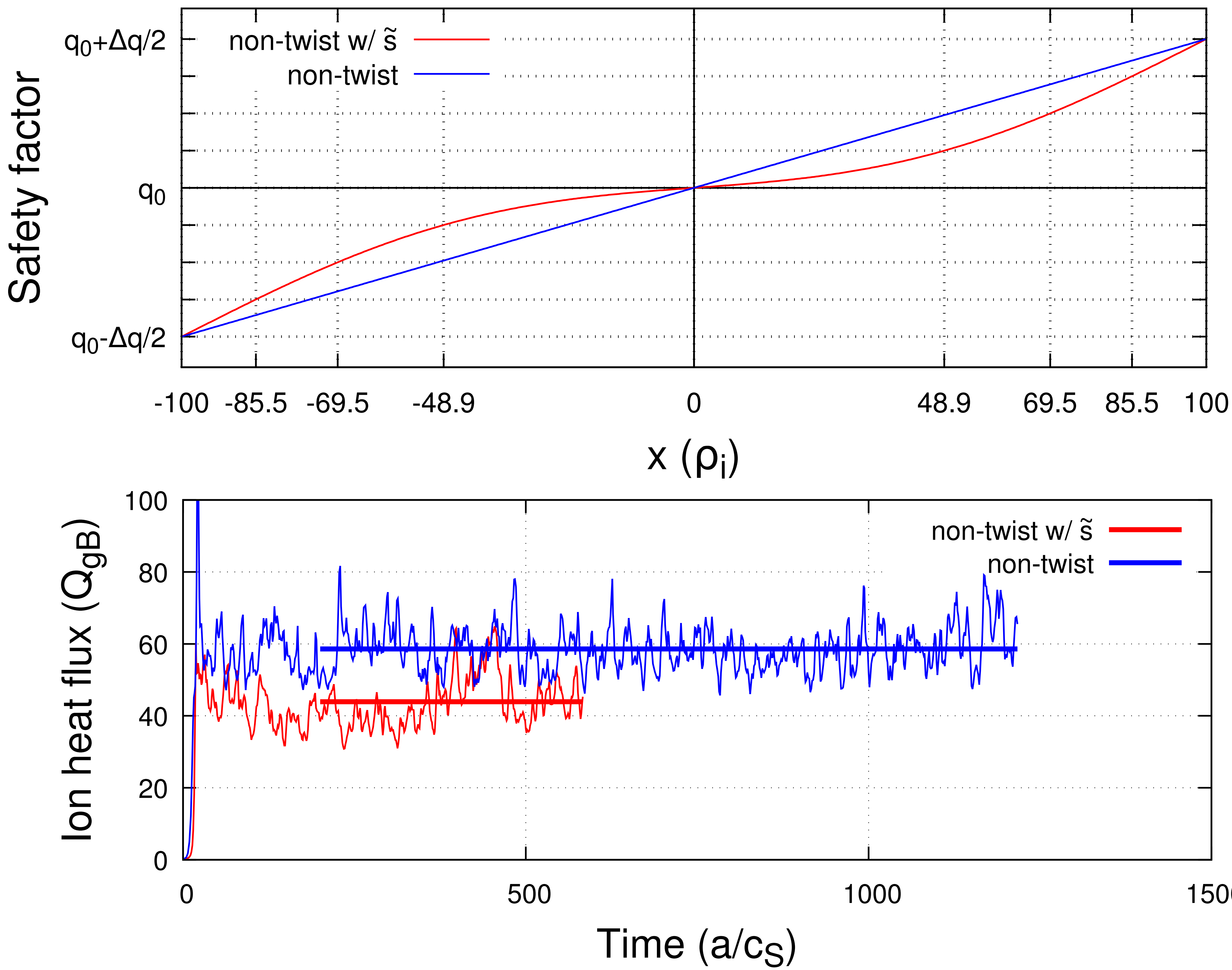
Implementation in GENE

- Surprisingly simple form in Fourier-space

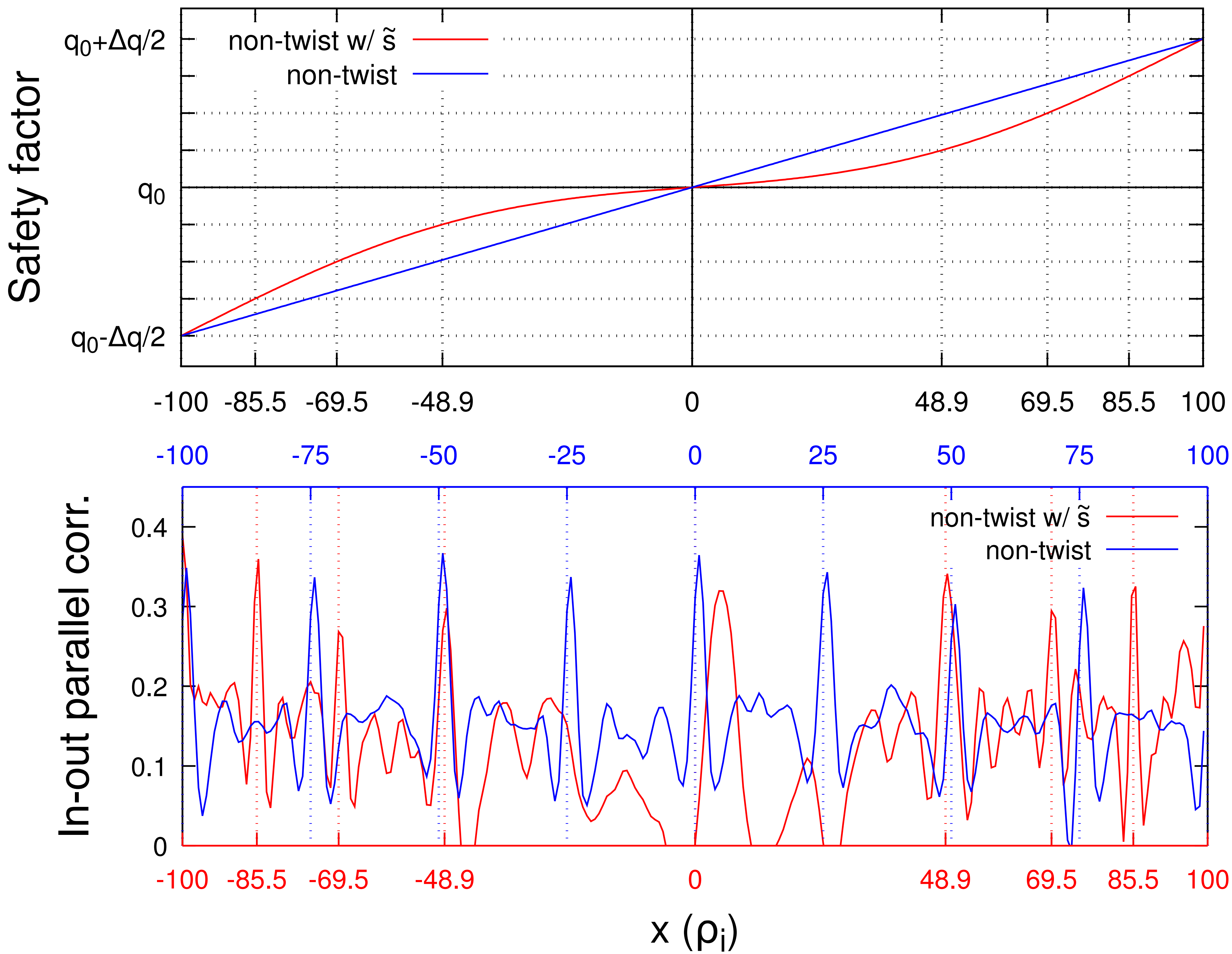
$$\left.\frac{\partial h_s}{\partial z}\right|_{k_x,k_y} \rightarrow \left.\frac{\partial h_s}{\partial z}\right|_{k_x,k_y} + \frac{k_y}{2}\sum_{j=1}^{j_m}\frac{L_x}{2\pi j}\left[\left(\tilde{s}_{Cj} + i\tilde{s}_{Sj}\right) h_s\left(K_x + \frac{2\pi}{L_x}j, k_y, z\right) - \left(\tilde{s}_{Cj} - i\tilde{s}_{Sj}\right) h_s\left(K_x - \frac{2\pi}{L_x}j, k_y, z\right)\right] + \ldots$$

- Straightforward to implement, likely at little computational cost

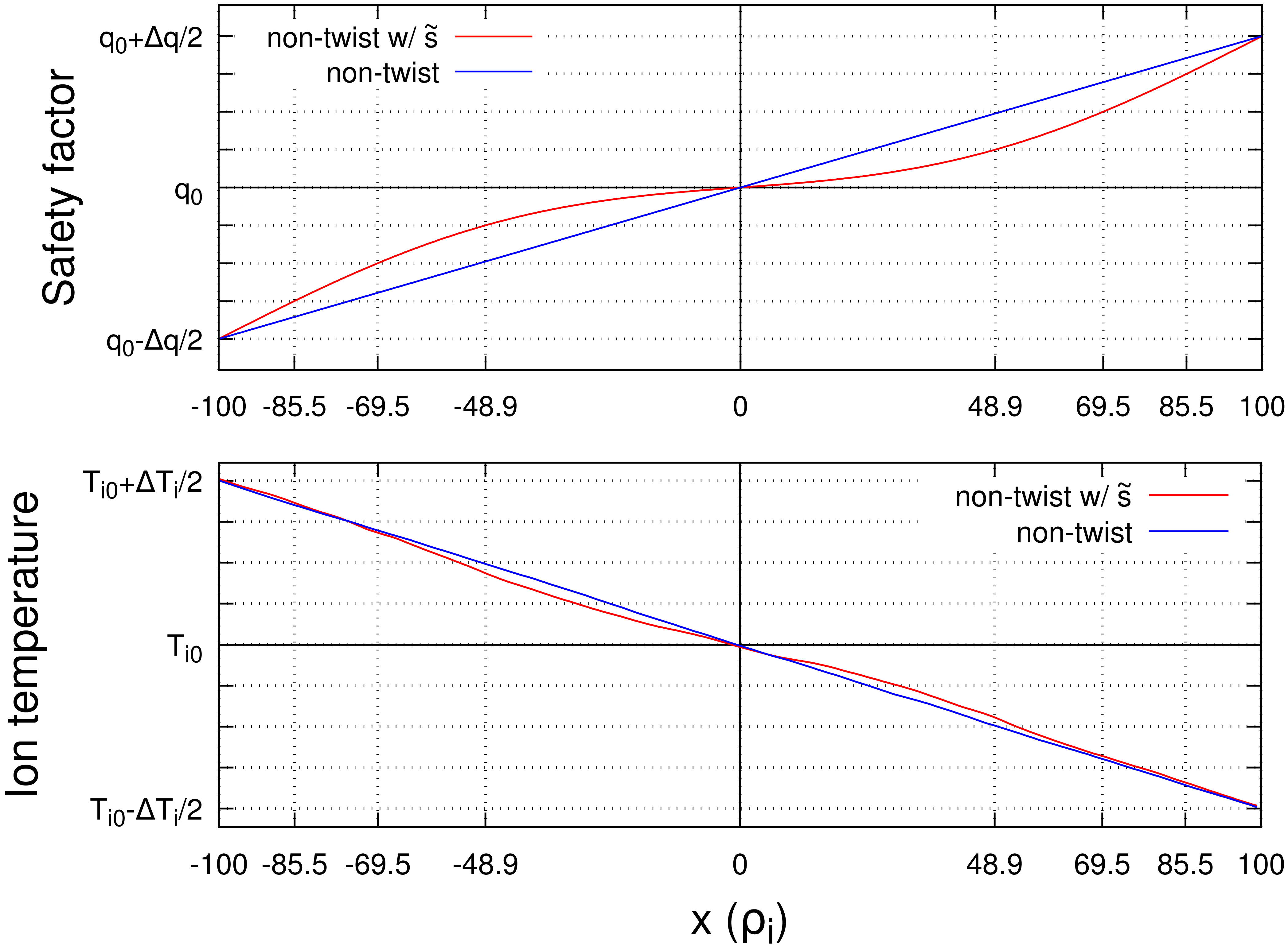
First proof-of-principle simulations



Moves integer surfaces of flux tube as expected



Temperature profile modified, but not as expected



All done.

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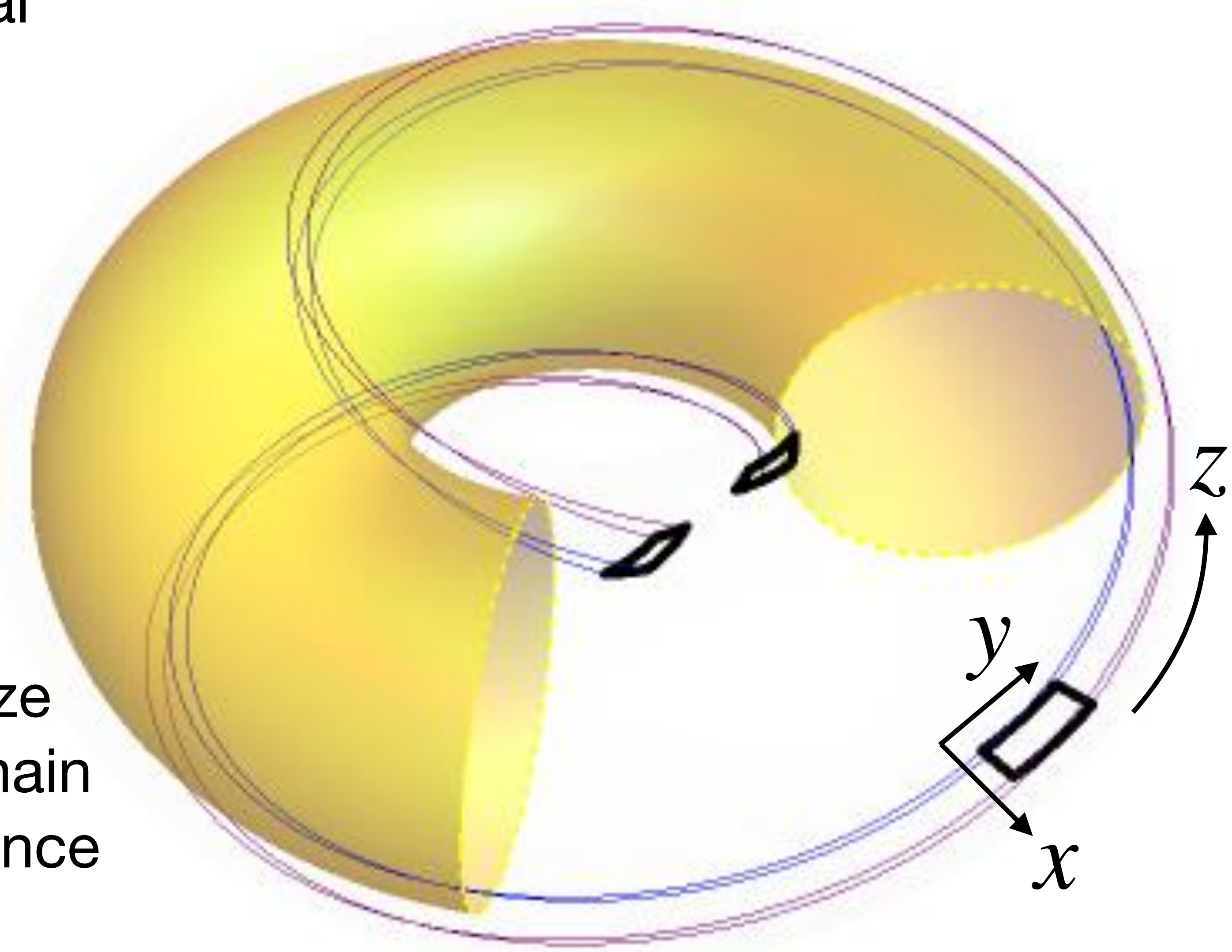
Motivation for the flux tube

Beer et al. *Phys. Plasmas* (1995).

- Crucial to minimize computational cost as much as possible
- Flux tube simulation domain exploits the scale separation assumed by gyrokinetics, $\rho_i / a \ll 1$
- Field-aligned coordinates minimize the volume of the simulation domain by reflecting the shape of turbulence
- Boundary conditions are elegant (i.e. periodicity)



$$q(x) \approx q_0 \left(1 + \hat{s} \frac{x - x_0}{x_0} \right)$$

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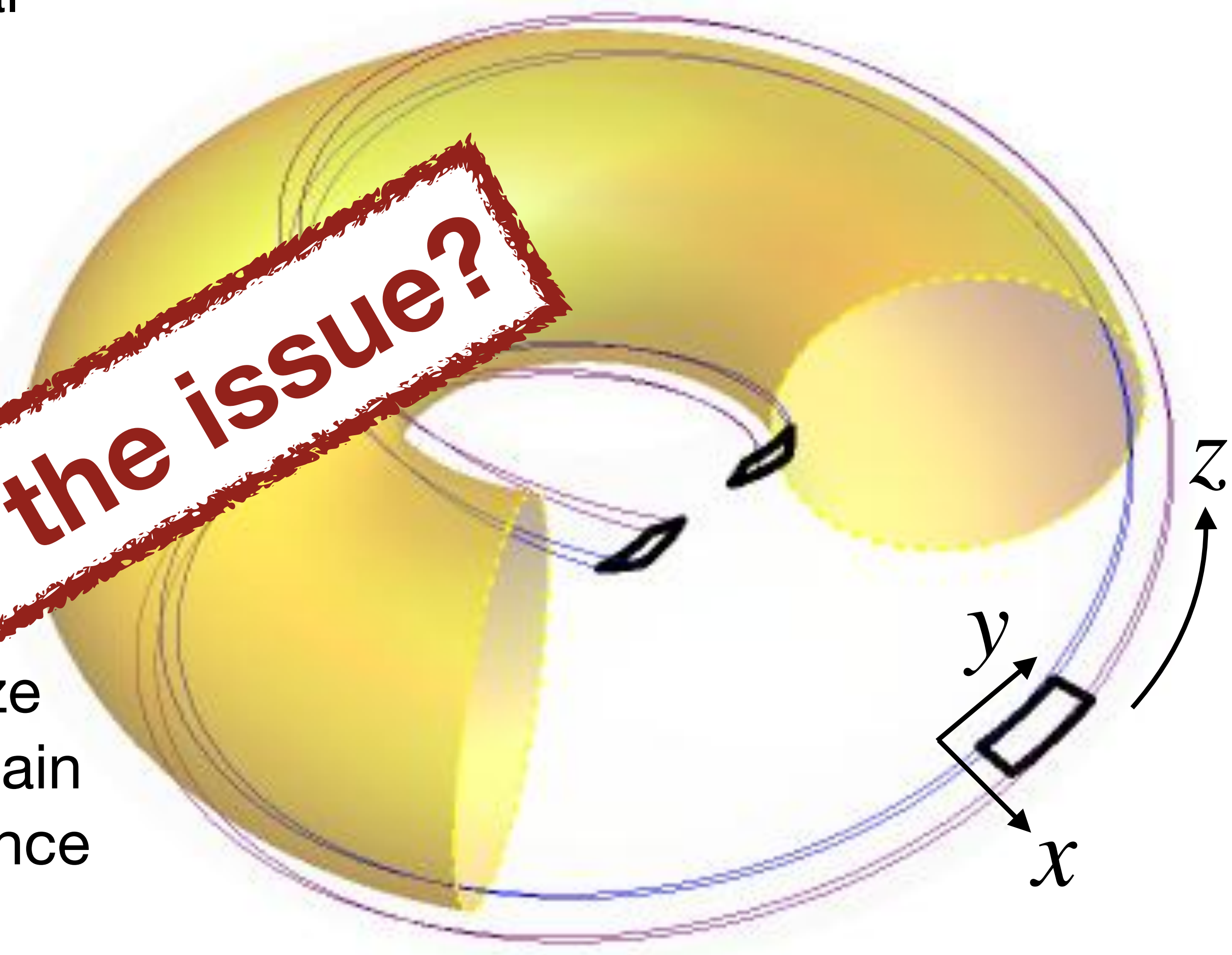
What's the issue?



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Motivation for the flux tube

Beer et al. *Phys. Plasmas* (1995).

$\hat{s} \rightarrow 2\hat{s}$

- Crucial to minimize computational cost as much as possible
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- Field-aligned coordinates minimize the volume of the simulation domain by reflecting the shape of turbulence
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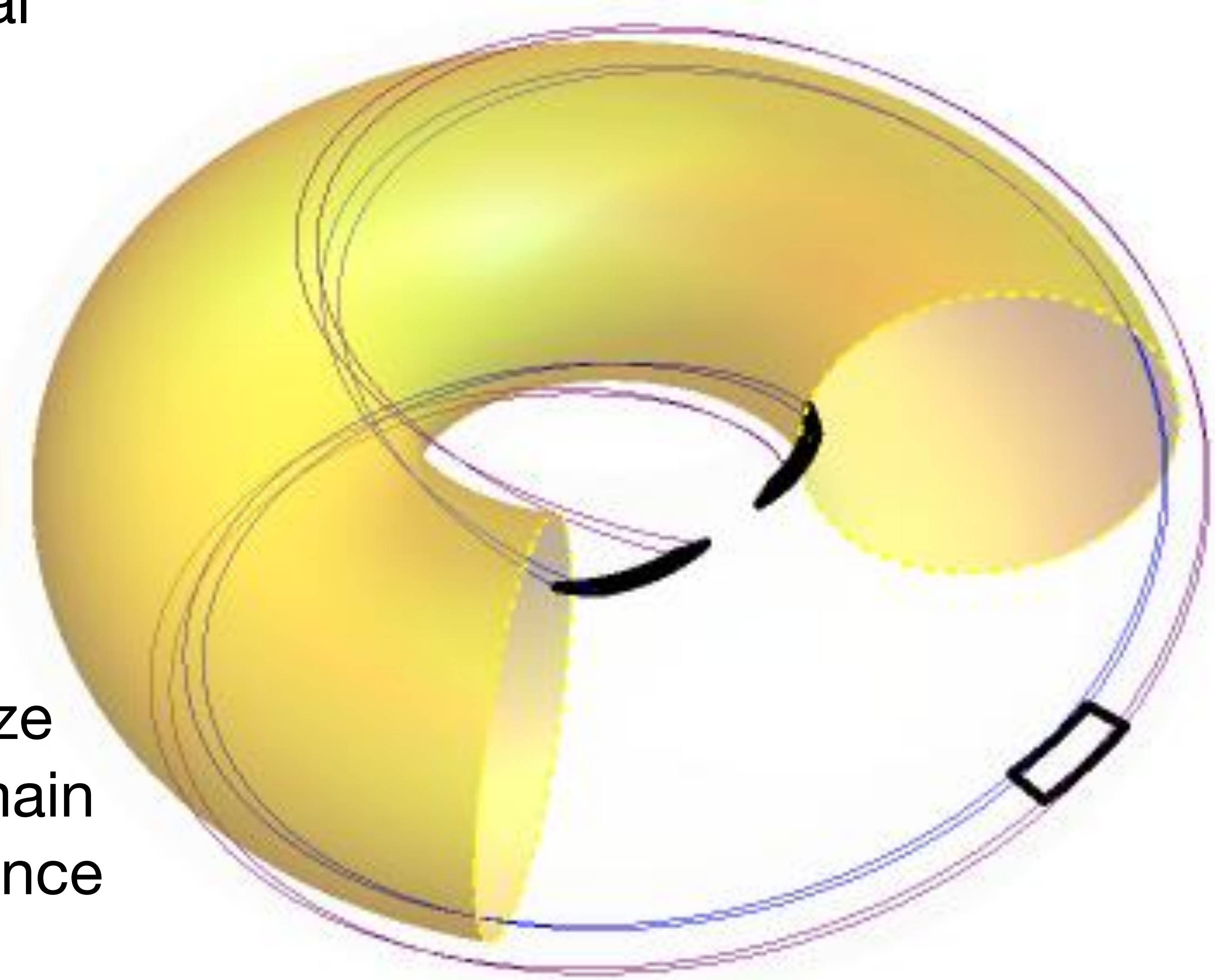

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Motivation for the flux tube

Beer et al. *Phys. Plasmas* (1995).
Ball et al. *JPP* (2020).

- Crucial to minimize computational cost as much as possible
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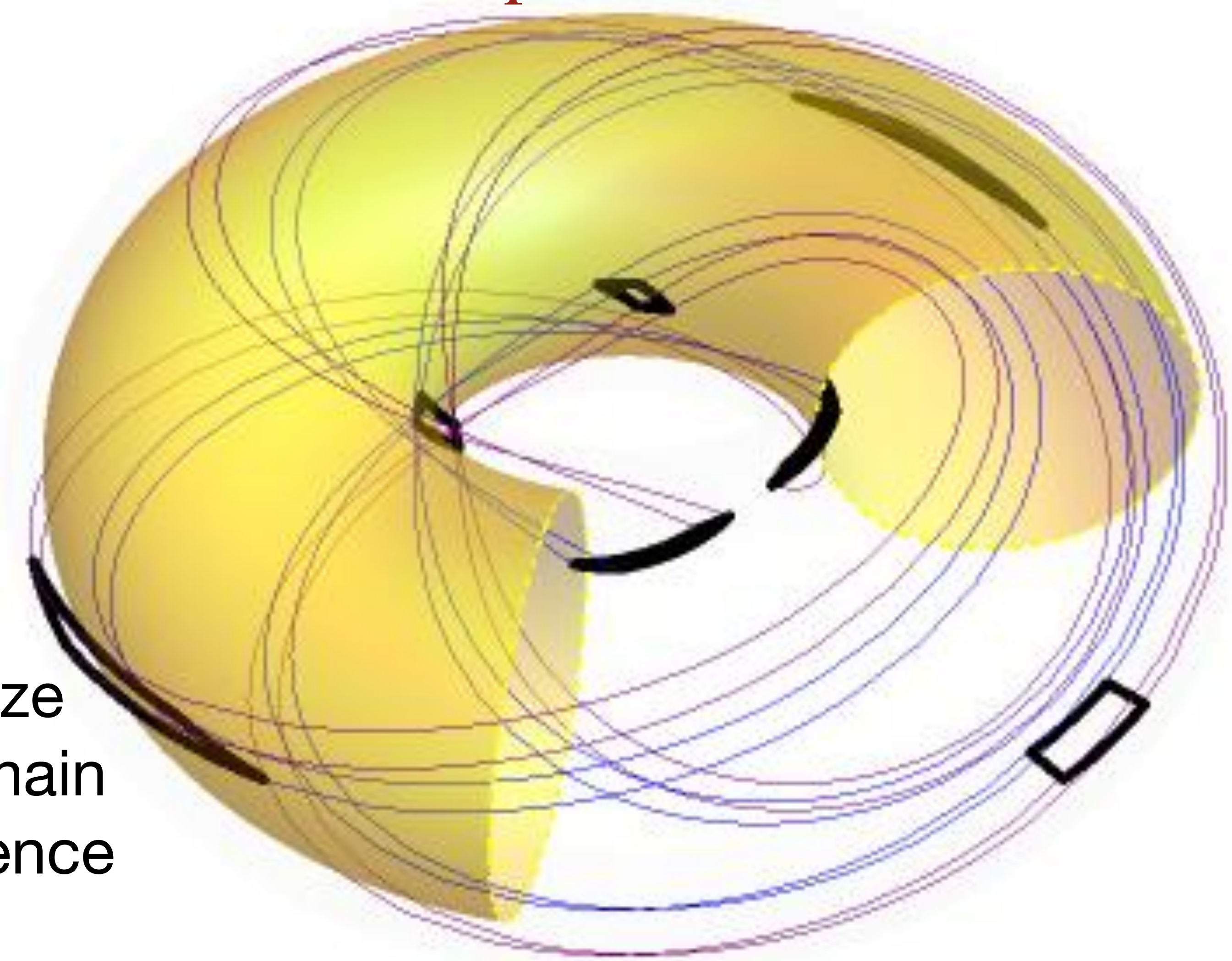
- Field-aligned coordinates minimize the volume of the simulation domain by reflecting the shape of turbulence
- Boundary conditions are elegant (i.e. periodicity)

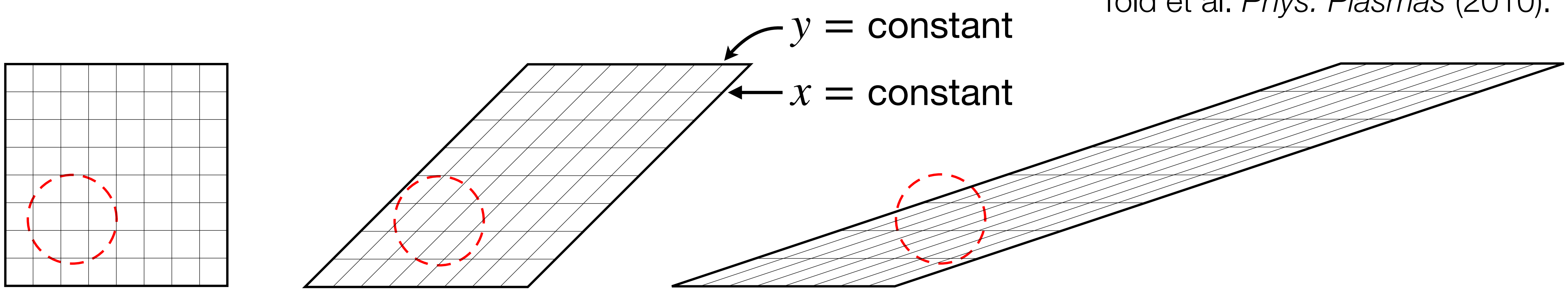
$N_{pol} = 3$



$$q(x) \approx q_0 \left(1 + \hat{s}\frac{x - x_0}{x_0}\right)$$

Motivation for a non-twisting flux tube

Scott et al. *Phys. Plasmas* (2001).
Told et al. *Phys. Plasmas* (2010).



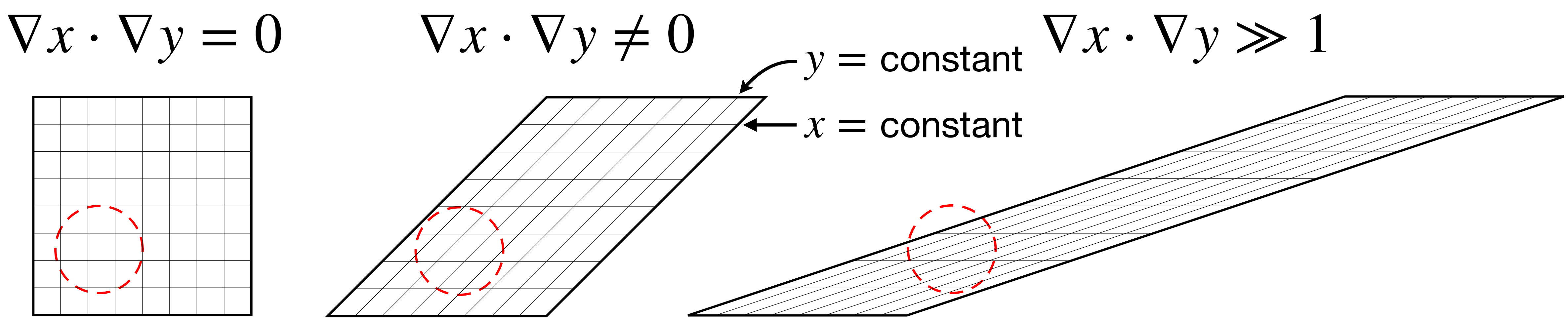
- When $2\pi N_{pol}\hat{s} \gg 1$, the conventional flux tube becomes so twisted that it only supports modes that are strongly damped by FLR effects

- "Shifted metric" approach has been developed, but could not be applied to the standard flux tube domain (i.e. **periodic radial boundary condition and with a Fourier representation**)

- In this talk, we will show how it can be done

Coordinate system transformation

Scott et al. *Phys. Plasmas* (2001).



- Define a new coordinate Y such that $\nabla x \cdot \nabla Y = 0$, so use

$$Y(x, y, z) \equiv y - \frac{\nabla x \cdot \nabla y}{|\nabla x|^2} x$$

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Coordinate system transformation

Ball et al. *PPCF* (2021).

- Key new insight is to transform the boundary conditions consistently and then use them to find the allowed modes in Fourier-space

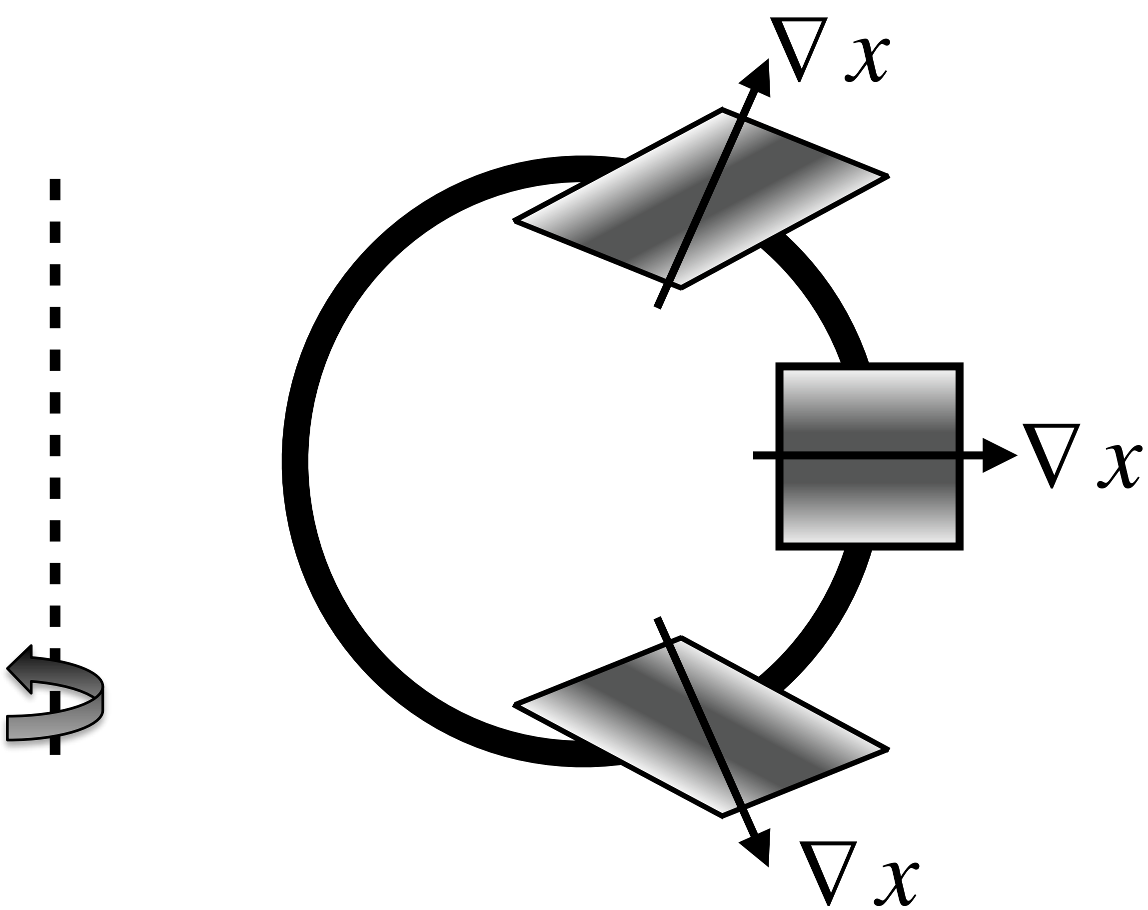
- Instead of the typical $k_x = \frac{2\pi}{L_x} m$, we find

$$K_x = \frac{2\pi}{L_x} m + k_y \frac{\nabla x \cdot \nabla y}{|\nabla x|^2} \quad \text{where} \quad m \in \mathbb{Z}$$

- To make non-twisting flux tube in Fourier-space, **construct a rectangular grid in K_x instead of k_x**

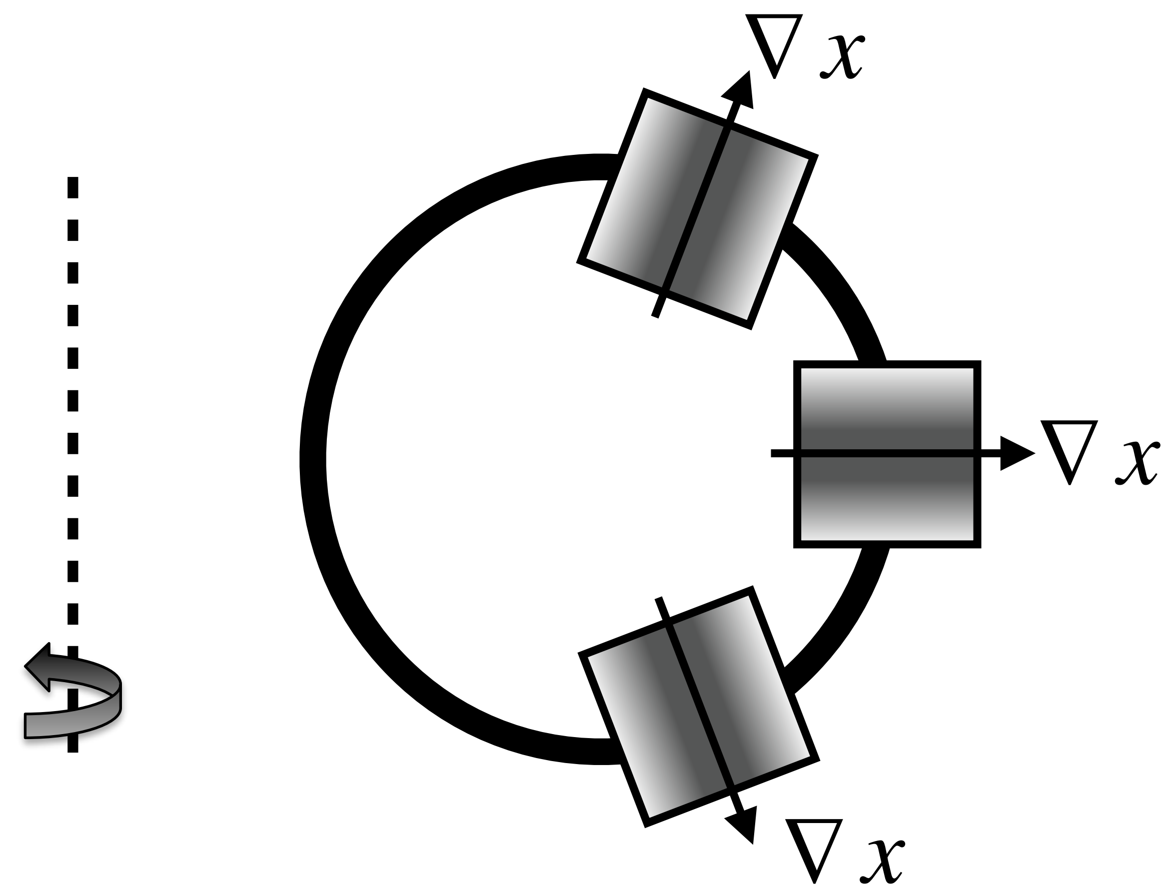
Physical meanings of k_x and K_x

$k_x = 0$



If you trace the perturbation back along field lines to $z = 0$, has no variation in ∇x

$K_x = 0$



Has no variation in ∇x at all z locations

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Transforming the Fourier-analyzed gyrokinetic eq.

$$\frac{\partial h_s}{\partial t} + w_{||}\hat{b}\cdot\nabla z\,\frac{\partial h_s}{\partial z}\bigg|_{K_x - k_y\frac{\nabla x\cdot\nabla y}{|\nabla x|^2}} + i\vec{v}_{ds}\cdot\left(K_x\nabla x + k_y\nabla Y\right)h_s + a_{s||}\frac{\partial h_s}{\partial w_{||}}$$

$$+\{h_s', \phi'' J_0\left(K_\perp''\rho_s\right)\} = \frac{Z_s e F_{Ms}}{T_s}\frac{\partial \phi}{\partial t}J_0\left(K_\perp \rho_s\right) - i\frac{k_y}{JB}\phi J_0\left(K_\perp\rho_s\right)\frac{dF_{Ms}}{dx}$$

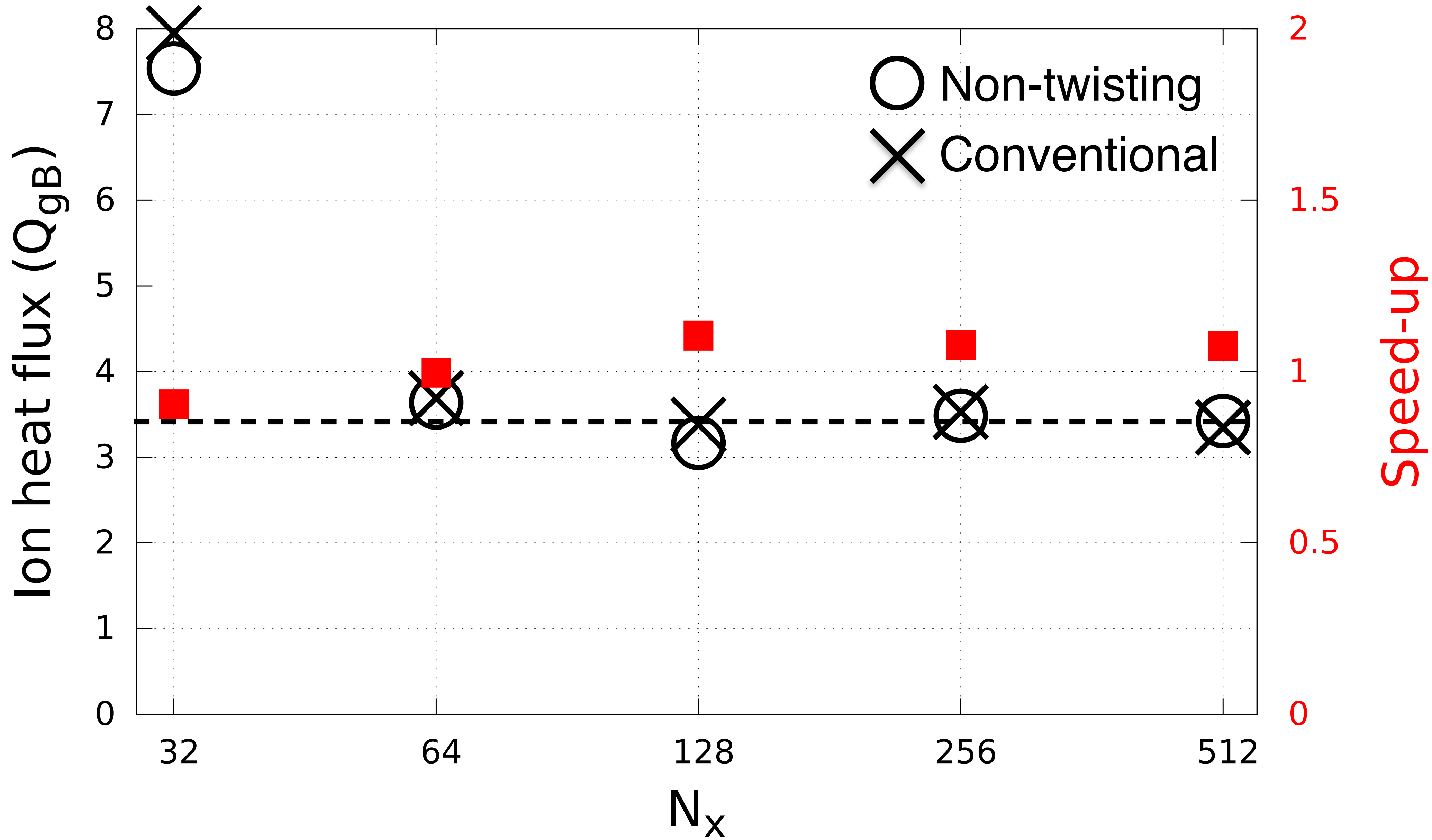
1. The parallel derivative must still be taken at constant $k_x = K_x - k_y\dfrac{\nabla x\cdot\nabla y}{|\nabla x|^2}$
2. The geometric coefficients lose their secular dependence along the field line:

$$k_\perp = \sqrt{k_x^2|\nabla x|^2 + 2k_x k_y \underbrace{\nabla x\cdot\nabla y}_{\propto\,\hat{s}z} + k_y^2\underbrace{|\nabla y|^2}_{\propto\,(\hat{s}z)^2}} \rightarrow K_\perp = \sqrt{K_x^2|\nabla x|^2 + k_y^2|\nabla Y|^2}$$

$\Rightarrow$ Conventional grid prioritizes following linear modes, while non-twisting grid minimizes FLR damping

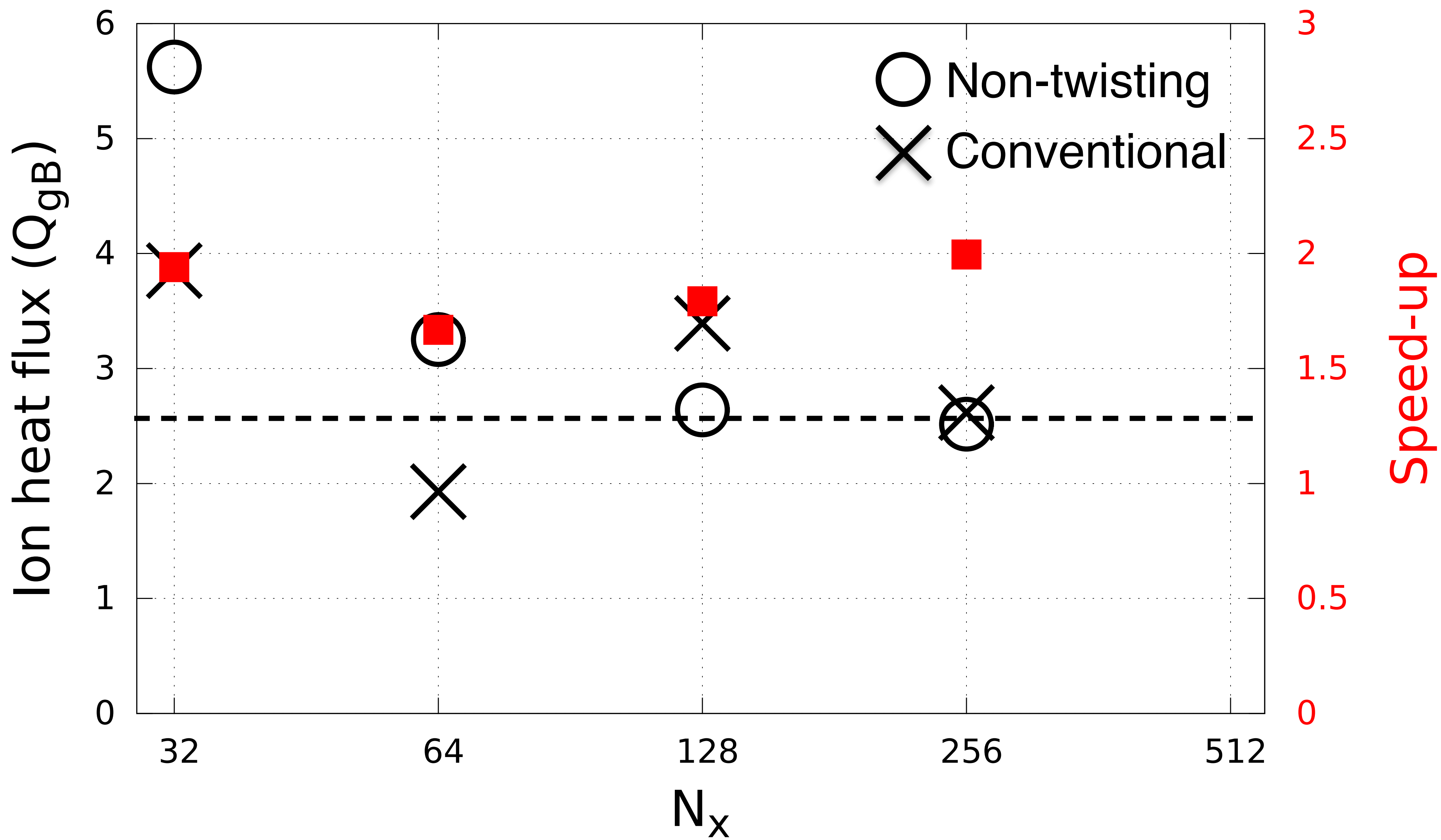
CBC with adiabatic electrons and $\hat{s} = 0.8$

Dimits et al. *Phys. Plasmas* (2000).



- Similar convergence and run time

CBC with adiabatic electrons, $\hat{s} = 0.8$, $N_{\mathrm{pol}} = 3$



- Time step is much larger due to elimination of small radial scales

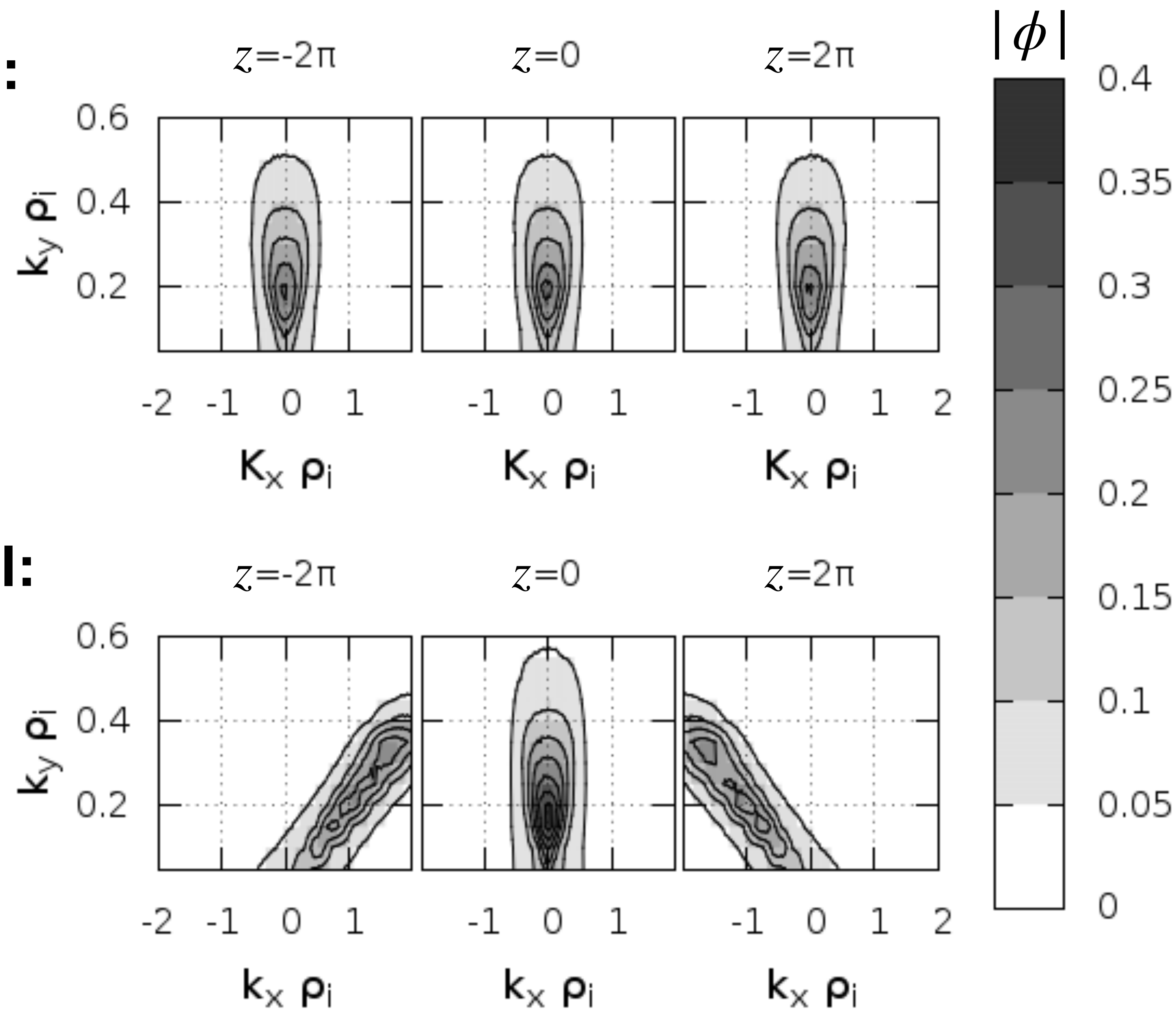
New choice of gridpoints can better fit turbulence

Dimits et al. *Phys. Plasmas* (2000).

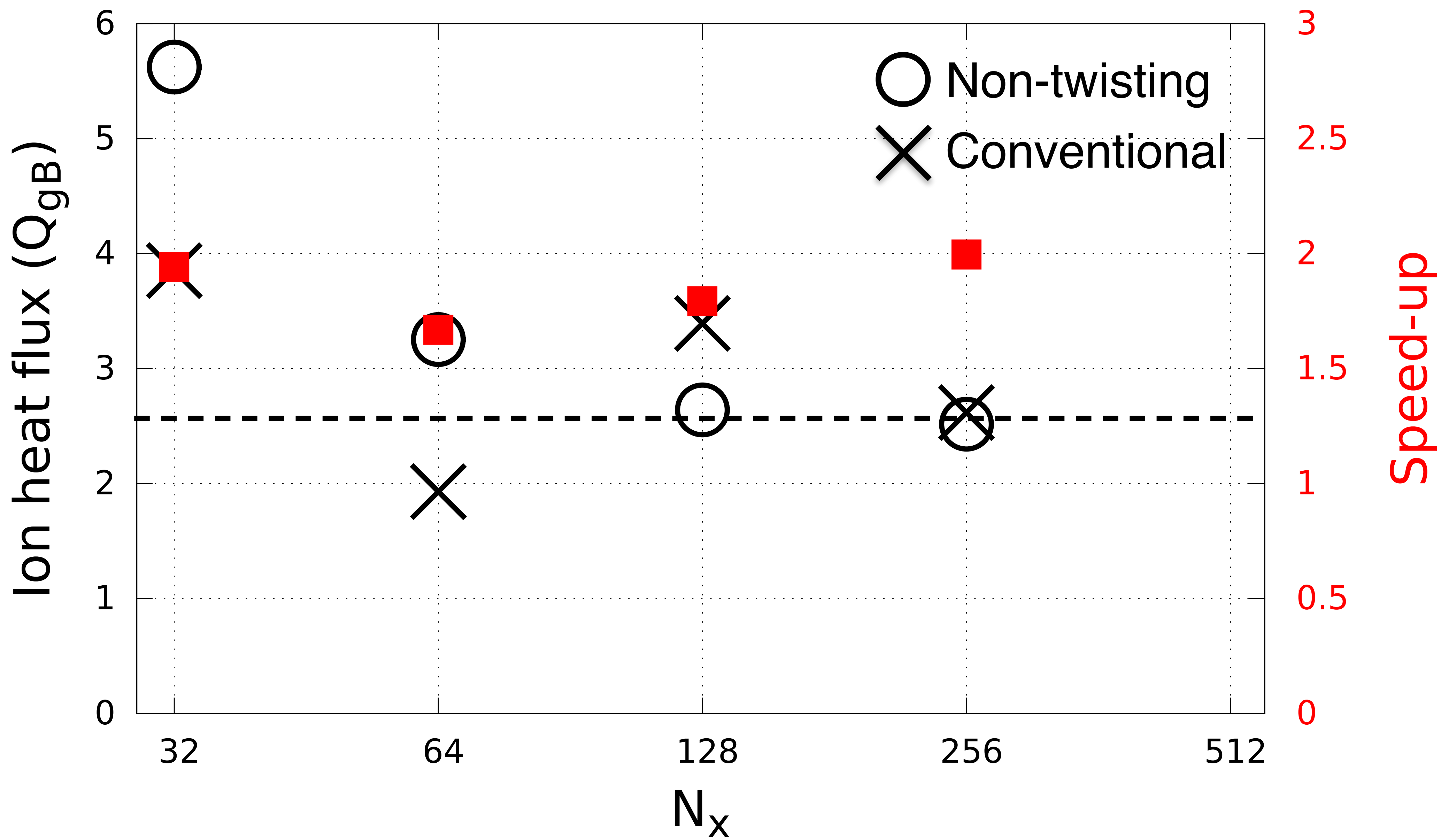
- Cyclone base case in a domain that is three poloidal turns long

Non-twisting:

Conventional:

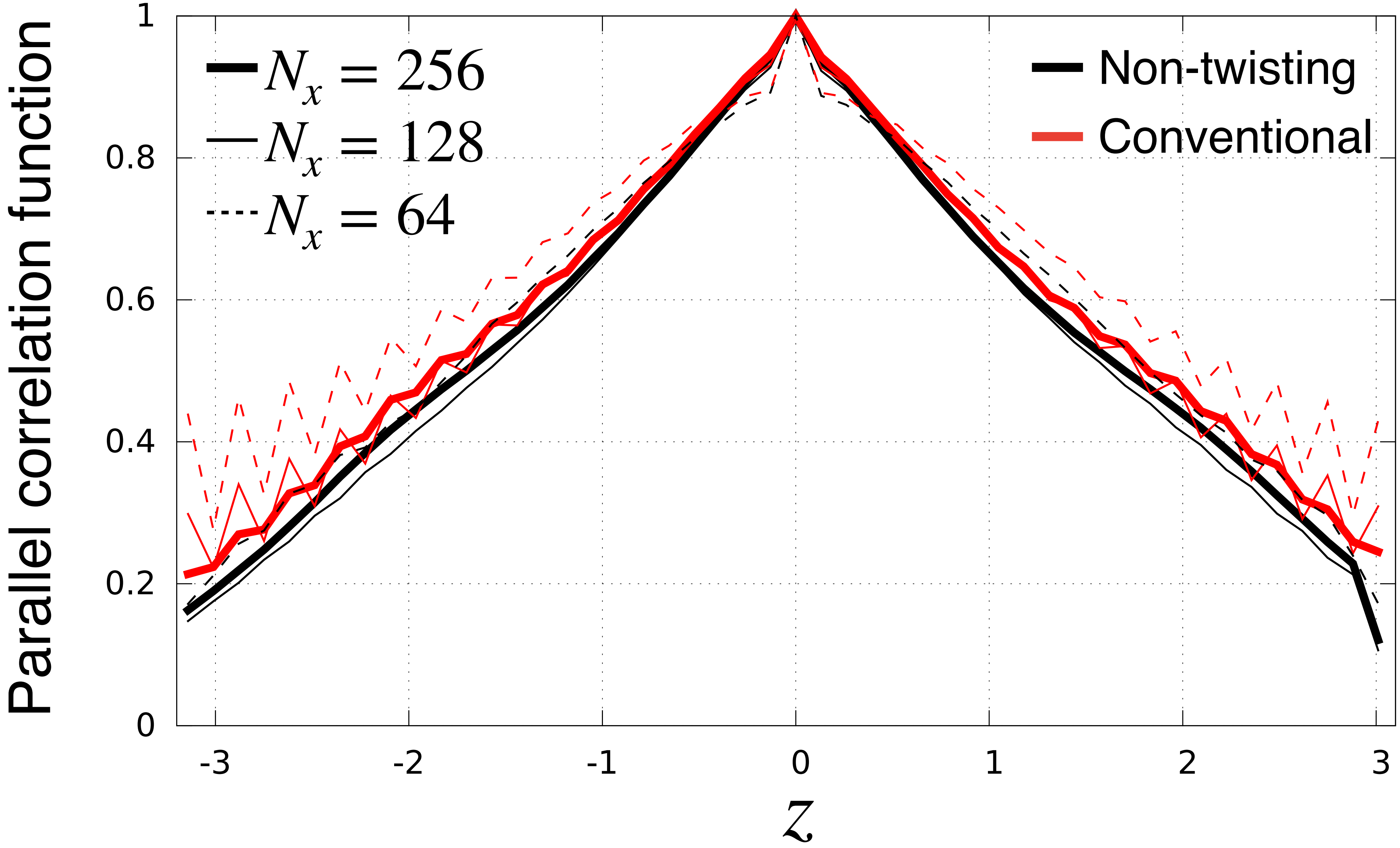


CBC with adiabatic electrons, $\hat{s} = 0.8$, $N_{\text{pol}} = 3$



- Non-twisting $N_x = 128$ sim. is **7x** faster than conventional $N_x = 256$

Shaped DEMO with kinetic elec., $\hat{s} = 2.4$, $N_{\text{pol}} = 1$



- Inboard is better resolved by non-twisting flux tube, but little flux there

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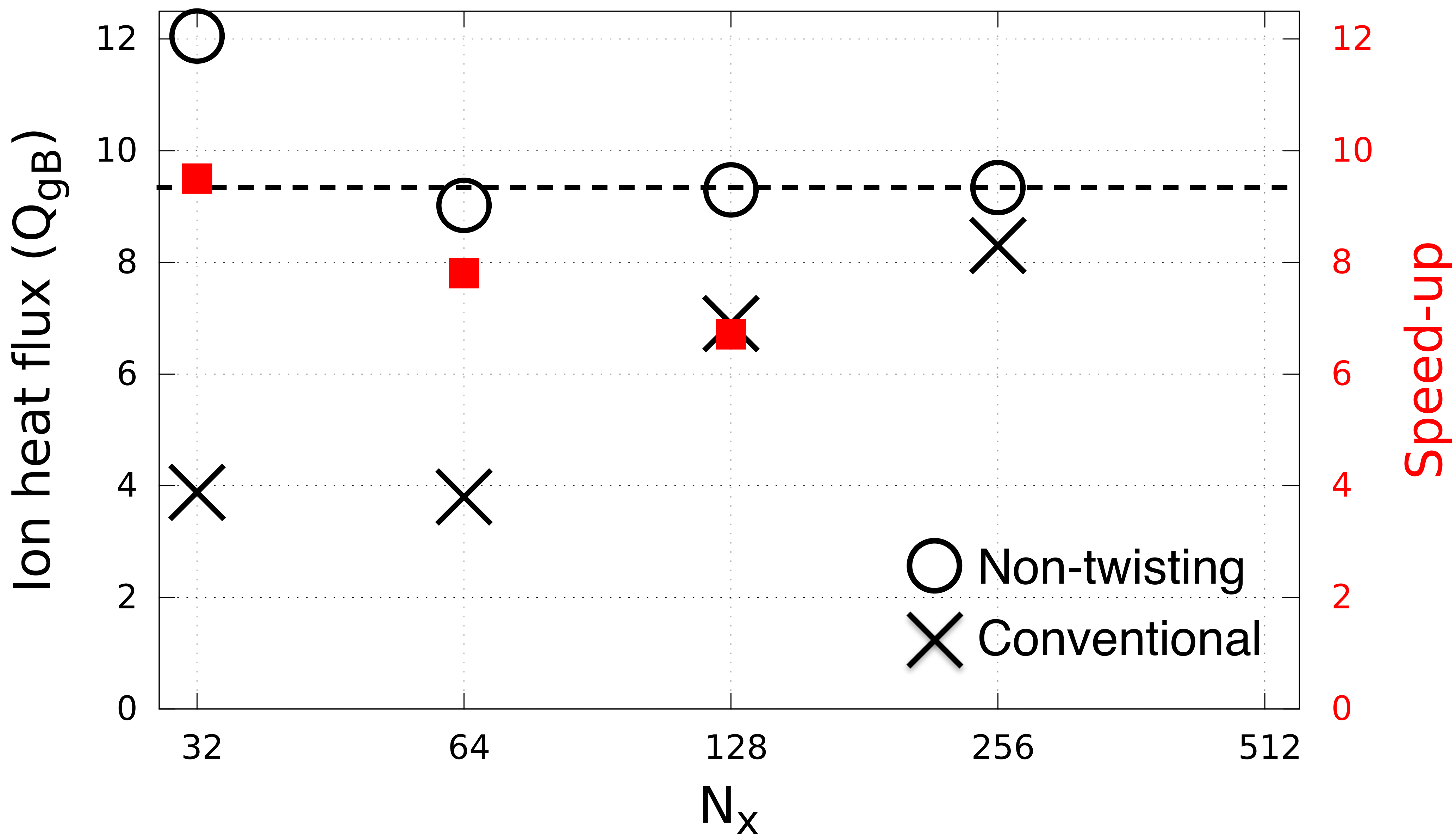
CBC with adiabatic electrons, $\hat{s} = 4.0$, $N_{\rm pol} = 3$



- Non-twisting flux tube is **>30x** faster than conventional

Transforming real space boundary conditions

Beer et al. *Phys. Plasmas* (1995).

- Using $Y\left(x, y, z\right) = y + \hat{s}zx$, the binormal boundary condition **stays boring**

$$\phi(x, Y(x, y + L_y, z), z) = \phi(x, Y(x, y, z), z), z) \quad \Rightarrow \quad \phi(x, Y + L_y, z) = \phi(x, Y, z)$$

- The radial boundary condition **becomes interesting**

$$\phi(x + L_x, Y(x + L_x, y, z), z) = \phi(x, Y(x, y, z), z) \quad \Rightarrow \quad \phi(x + L_x, Y + \hat{s}zL_x, z) = \phi(x, Y, z)$$

- Using $Y\left(x, \zeta, z\right) = C_y\zeta - C_y q_0 z$, the parallel "twist-and-shift" boundary condition **becomes boring**

$$\phi(x, Y(x, \zeta, z + 2\pi N_{pol}), z + 2\pi N_{pol}) = \phi(x, Y(x, \zeta, z), z)$$
$$\Rightarrow \quad \phi(x, Y, z + 2\pi N_{pol}) = \phi(x, Y, z)$$

Transforming radial Fourier-space boundary cond.

- Substituting $\overline{\phi}(x, Y, z) = \sum_{k_y} \hat{\phi}(x, k_y, z) e^{ik_y Y}$ from the binormal boundary condition,

$$\overline{\phi}(x + L_x, Y + \hat{s}zL_x, z) = \overline{\phi}(x, Y, z) \quad \Rightarrow \quad \hat{\phi}(x + L_x, k_y, z) e^{ik_y \hat{s}zL_x} = \hat{\phi}(x, k_y, z)$$

- By Floquet's theorem, substitute $\hat{\phi}(x, k_y, z) = P(x, k_y, z) e^{-ik_y \hat{s}zx}$ to find

$$P(x + L_x, k_y, z) = P(x, k_y, z)$$

- Thus, $P(x, k_y, z)$ has a standard Fourier form/discretization in x, implying that

$$K_x = \frac{2\pi}{L_x} m - k_y \hat{s}z \quad \text{where} \quad m \in \mathbb{Z} \quad \text{and} \quad \overline{\phi}(x, Y, z) = \sum_{K_x, k_y} \phi(K_x, k_y, z) e^{iK_x x + ik_y Y}$$

Transforming the gyrokinetic equation

- Fourier analyzing using $\overline{h}_s(x, Y, z) = \sum_{K_x, k_y} h_s(K_x, k_y, z) e^{iK_x x + ik_y Y}$ gives

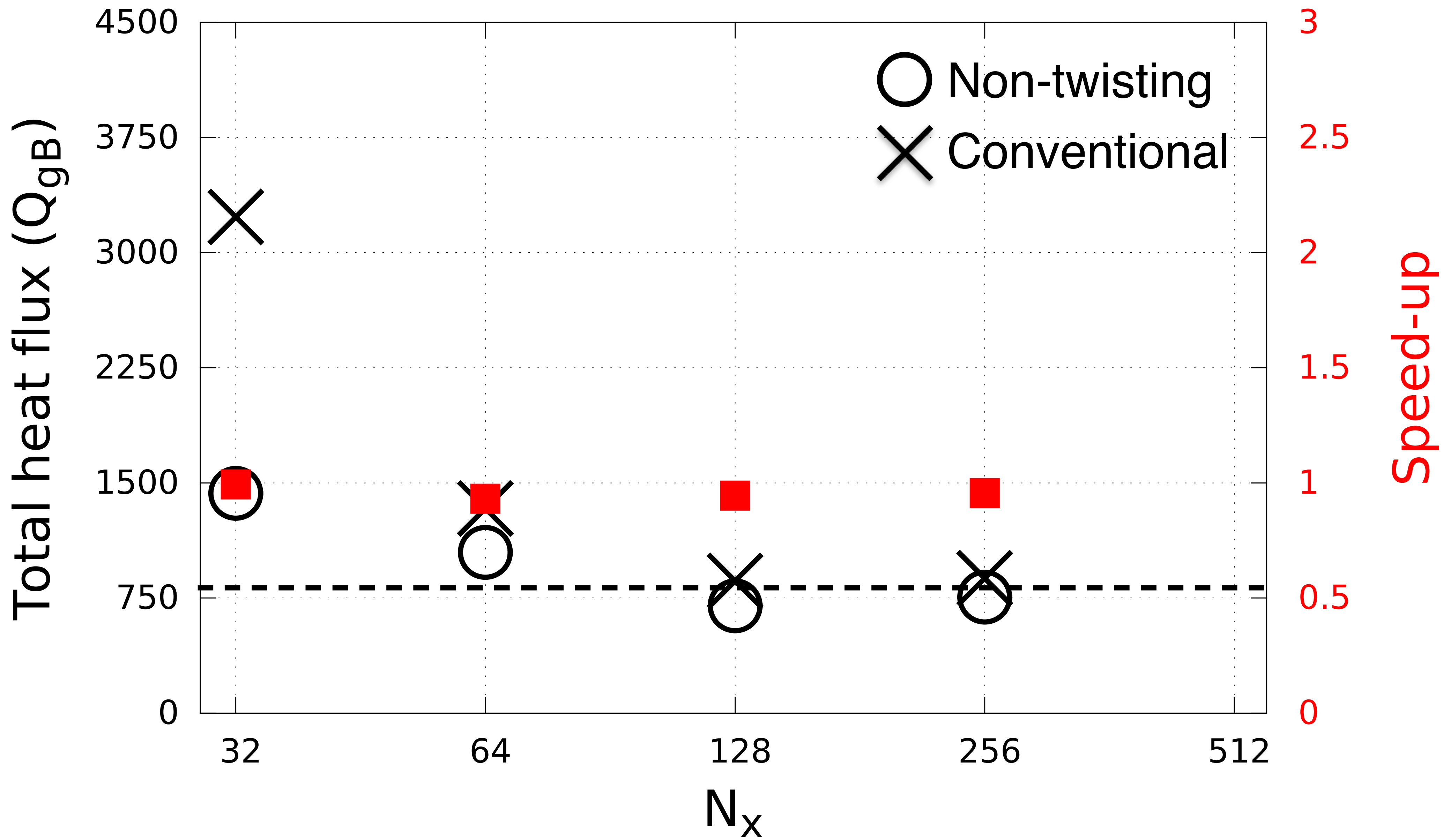
$$\frac{\partial h_s}{\partial t} + \frac{w_{||}\hat{b}\cdot\nabla z}{L_x L_y}\oint\oint dx dY e^{-iK_x x - ik_y Y}\frac{\partial}{\partial z}\bigg|_{x,y}\left(\sum_{K'_x, k'_y} h_s(K'_x, k'_y, z) e^{iK'_x x + ik'_y Y}\right) + i\vec{K}_\perp\cdot\vec{v}_{ds}h_s + \ldots$$

- Substituting $Y = y + \hat{s}zx$ allows you to Fourier analyze in y,

$$\frac{\partial h_s}{\partial t} + \frac{w_{||}\hat{b}\cdot\nabla z}{L_x}\oint dx e^{-i(K_x + k_y\hat{s}z)x}\frac{\partial}{\partial z}\bigg|_{x,k_y}\left(\sum_{K'_x} h_s(K'_x, k_y, z) e^{i(K'_x + k_y\hat{s}z)x}\right) + i\vec{K}_\perp\cdot\vec{v}_{ds}h_s + \ldots$$

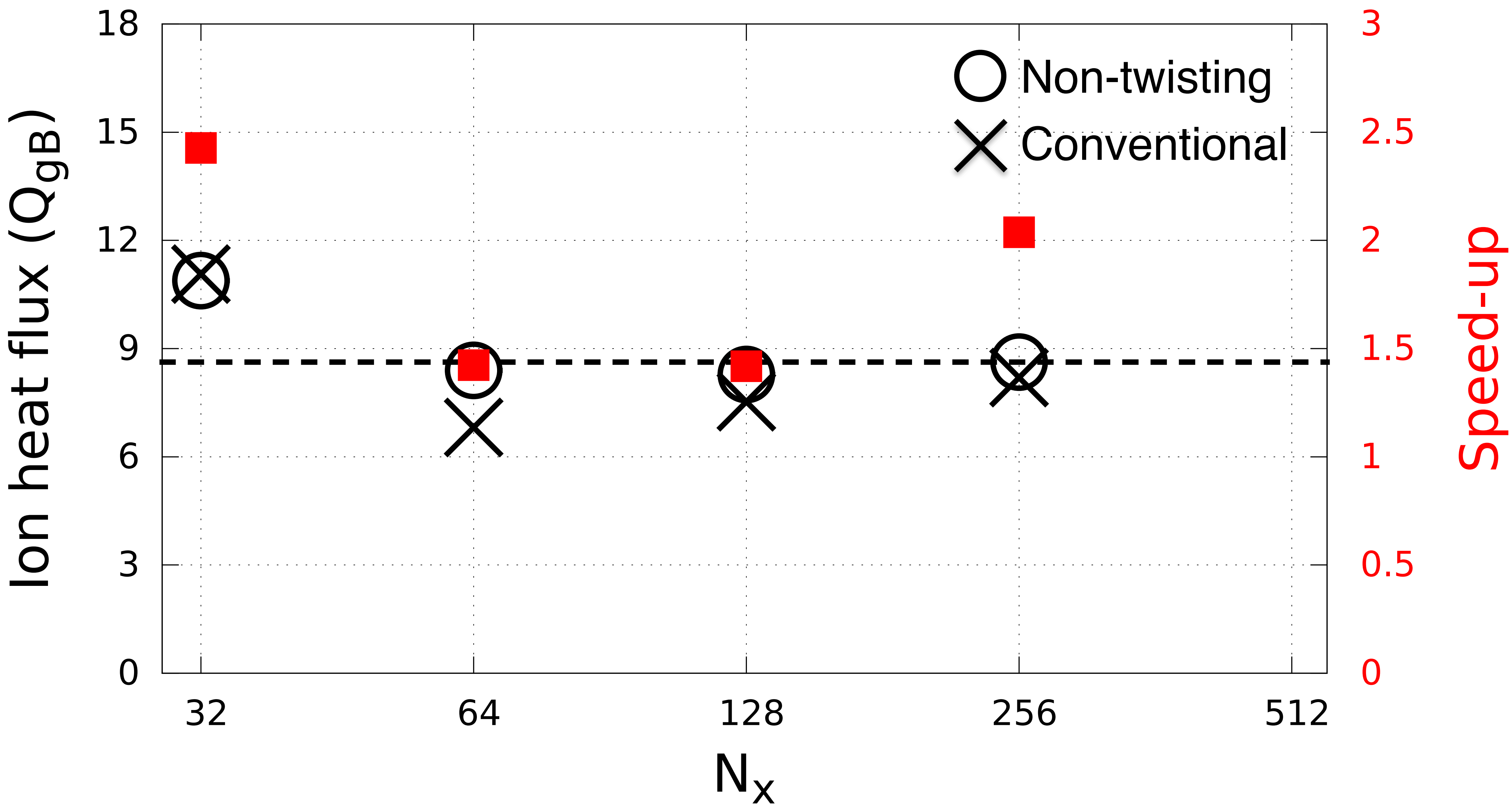
- Calculate parallel deriv. in (x, k_y) or by holding $k_x = K_x + k_y\hat{s}z$ constant

Shaped DEMO with kinetic elec., $\hat{s} = 2.4$, $N_{\text{pol}} = 1$



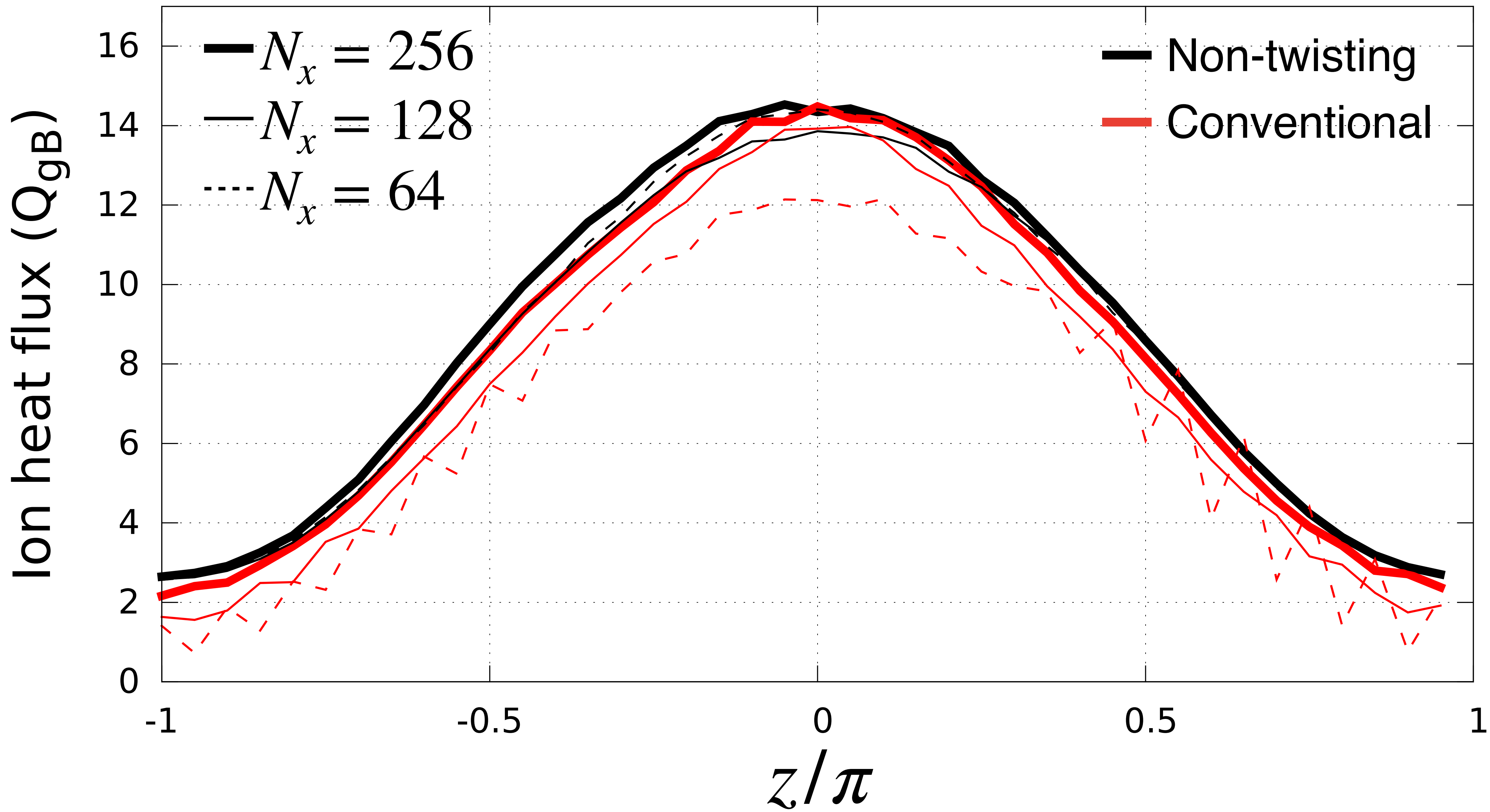
- Time step is now limited by shear Alfvén wave, even when $\beta \neq 0$

CBC with adiabatic electrons and $\hat{s} = 4.0$



- Significant, direct computational speed-up

CBC with adiabatic electrons and $\hat{s} = 4.0$



- Inboard is better resolved by non-twisting flux tube, but little flux there

Alternative twisting flux tubes

Watanabe et al. *Phys. Plasmas* (2015).

- It is straightforward to completely control the flux tube twist using

$$Y\left(x, y, z\right) = y + f_{tw}(z)x \qquad \text{and} \qquad K_x \equiv k_x - k_y f_{tw}(z)$$

where

Conventional:	$f_{tw}(z) = 0$
Globally non-twisting:	$f_{tw}(z) = \hat{s}z$
Non-twisting:	$f_{tw}(z) = -\nabla x \cdot \nabla y / \lvert \nabla x \rvert^2$
Flux tube train:	$f_{tw}(z) = 2\pi\hat{s}\text{Round}[z/(2\pi)]$

Alternative twisting flux tubes

- Potential applications
  - Coarsen grid at intermediate scales in multiscale simulations
  - Better adapt grids to local conditions in z
  - Prevent linear modes from being clustered around zero ballooning angle in conventional flux tube
  - **Better optimize the twist of the flux tube cross-section**