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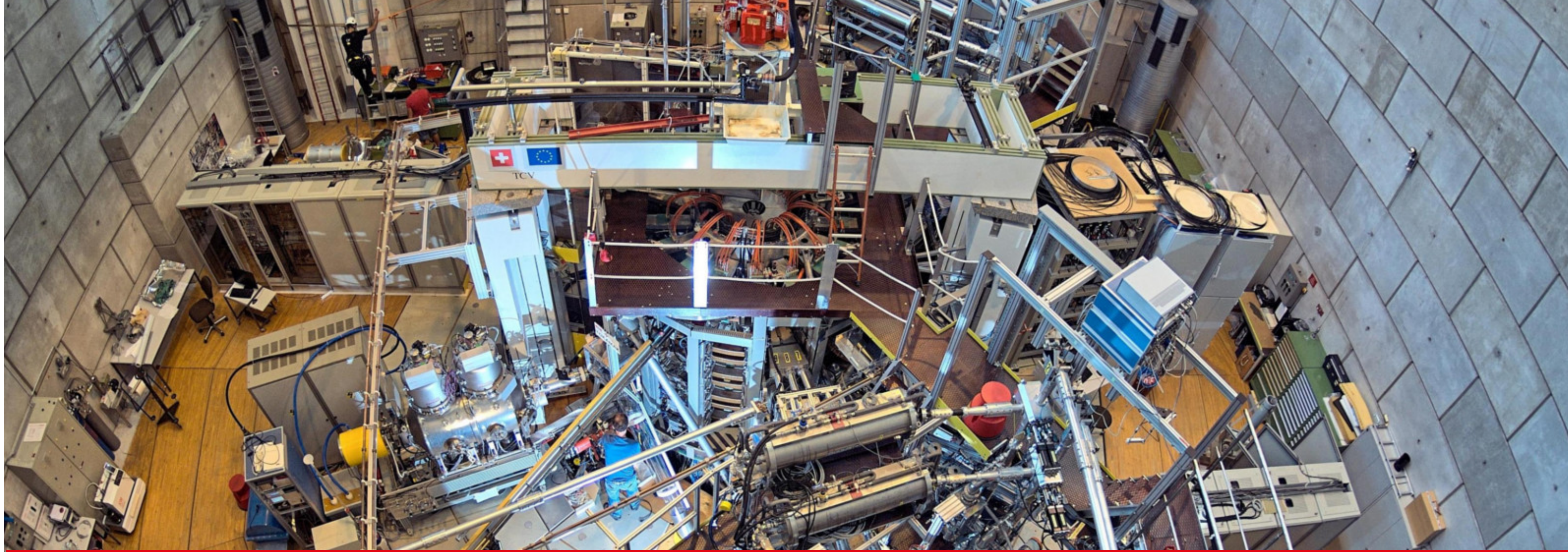

# Recent Advancements in the Gyrokinetic Moment-Based Approach for Boundary Plasmas

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# Overview

- Motivations and The GyroKinetic (GK) Moment-Based Approach
- Numerical studies and benchmarks with GENE code
- Conclusions

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# Challenges of Simulating the Turbulent Boundary Plasmas

- **Boundary region** (edge and scrape-off-layer) controls the performance of fusion devices (H-L transition, pedestal, ELMs, …)
- **Boundary region** is characterised by **different plasma collisionality regimes**:

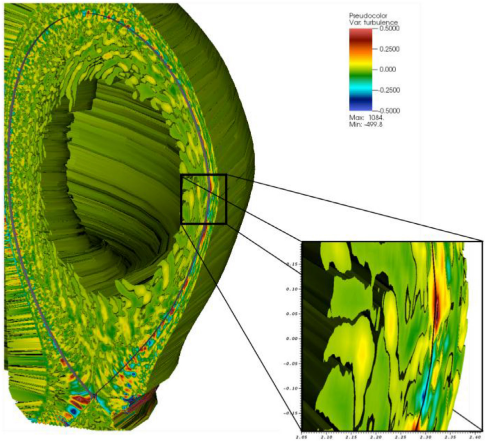
**ITER:** $T_e \sim 10 - 10^4$ eV and $n \sim 10^{18} - 10^{20}$ m$^{-3}$ $\Rightarrow \lambda_{mpf}/R_0 \sim \dfrac{T_e^2}{n_e R_0} \sim 10^{-1} - 10^3$

- Two currently-used turbulent modelling approaches:
  - **Drift-Kinetic (DK) fluid modelling** (lowest-order moments, less expensive, $k_\perp \rho_i \ll 1$ and $\lambda_{mfp} k_\parallel \ll 1$, no kinetic effects, Full-F)
  - **GyroKinetic (GK) modelling** (expensive, $k_\perp \rho_i \sim 1$, kinetic effects, also suitable for $\lambda_{mfp} k_\parallel \gtrsim 1$)

**GK Moment Approach**

$$k_\parallel \lambda_{mfp} \ll 1 \quad \& \quad k_\parallel \lambda_{mfp} \gtrsim 1$$

Giacomin M. *et al.* JPP, **2020**



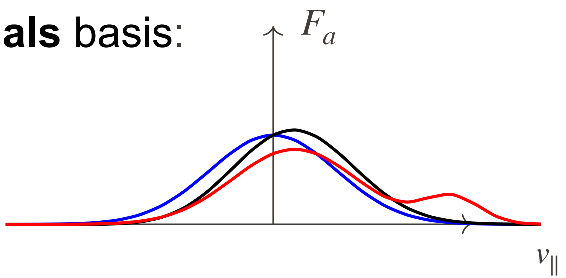

Chang C. S. *et al.* Nucl. Fusion, **2017**

# The GK Moment Approach to the GK Boundary Model

- Expansion of $F_a$ (Full-F) on a **Hermite-Laguerre polynomials** basis:

**Gyro-Moment of $F_a$**

$$F_a = \sum_p \sum_j \boxed{N_a^{pj}(\boldsymbol{R}, t)} \frac{H_p(v_\parallel) L_j(\mu)}{\sqrt{2^p p!}} F_{Ma}$$




- **Equation for $N_a^{pj}$** from the projection **onto the Hermite-Laguerre basis** of GK Equ.

$$||\ldots||^{pj} = \int d\theta \int d\mu dv_\parallel H_p(v_\parallel) L_j(\mu) \ldots \qquad ||(\text{GK Equ.})||^{pj}$$

- **Gyro-Moment Hierarchy Equation** and **GK Field Equations**

Projection of GK collision operator

$$\frac{\partial}{\partial t} N_a^{pj} + \nabla \cdot ||\dot{\boldsymbol{R}}||^{pj} - \frac{\sqrt{2l}}{v_{th\parallel a}} ||\dot{v_\parallel}||^{p-1j} + \mathcal{F}_a^{pj} = \sum_b C_{ab}^{pj} \qquad C_{ab}^{pj} = || \langle C_{ab} \rangle ||^{pj}$$

B. J. Frei *et al.*, JPP **86**, 905860205 (2020)

# Nonlinear GK Coulomb in the Gyro-Moment Approach



- In $(\mathbf{x}, \mathbf{v})$ phase-space, **Coulomb ("Landau") collision** operator is defined by

$$C_{ab}[f_a, f_b] = C_{ab}(\boldsymbol{x}, \boldsymbol{v}) = \frac{\partial}{\partial \boldsymbol{v}} \cdot \left[ \boldsymbol{A}_{ab} f_a + \frac{\partial}{\partial \boldsymbol{v}} \cdot (\boldsymbol{D}_{ab} f_b) \right]$$

- **Gyroaveraged** of $C_{ab}$ performed in $(\mathbf{R}, \mu, v_\|, t)$ phase-space and project on Hermite-Laguerre, i.e.

$$C_{ab}^{pj} = \int d\boldsymbol{v} H_p(v_\|) L_j(\mu) \langle C_{ab} \rangle$$

## Full-F Nonlinear GK Coulomb Collision Operator

$$C_{ab}^{pj} = \nu_{ab} \sum_{\boldsymbol{k}, \boldsymbol{k}'} \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} \sum_{r,s,p,q,\dots} \nu_{ab\dots}^{lpj\dots} \left(T^{-1}\right)_{pq}^{rsm} \dots K_s(b_a) K_q(b_b') N_a^{rs}(\boldsymbol{k}) N_b^{pq}(\boldsymbol{k}'),$$

Numerical coefficients | FLR Kernels | Gyro-Moments

- Arbitrary $k_\perp$, and $m_a/m_b$ , $T_a/T_b$ ratios

Jorge R., B. J. Frei and P. Ricci, JPP **85**, 905850604 (2019)

# Linearized GK Coulomb in the Gyro-Moment Approach



- Assuming $F_a = F_{aM} + f_a \quad (f_a \ll F_{aM})$

Test — Field

$$C_{ab} \simeq C_{ab}^T + C_{ab}^F \qquad C_{ab}^T = C_{ab}[f_a, f_{bM}] \qquad C_{ab}^F = C_{ab}[f_{aM}, f_b]$$

- Linearized **GK Coulomb Collision Operator** in the gyro-moment approach

$$C_{ab}^{Tpj} = \sum_{l=0}^{\infty} \sum_{m=-l}^{m=l} \sum_{r,s,p,q,\dots} \nu_{ab\dots}^{Tlpj} \dots (T^{-1})_{pq}^{rsm} K_s(b_a) N_a^{rs}$$

Numerical coefficients — FLR Kernels — Gyro-Moments

- Gyro-moment approach applied to different **simplified/*ad hoc*** linearized collision operators (e.g. GK/DK Sugama, GK/DK pitch-angle, GK/DK Dougherty)

B. J. Frei *et al.*, JPP **87,** 905870501 (2021)

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# Linearized Gyro-Moment Hierarchy in a Flux-Tube Code

- Assuming $F_a = F_{aM} + f_a \quad (f_a \ll F_{aM})$

Landau Damping — Particle trapping — Magnetic Drifts

$$\frac{\partial}{\partial t}\mathbf{N}_a + \mathbf{H}_a \cdot \nabla_\parallel \mathbf{N}_a + (\mathbf{M}_a \cdot \mathbf{N}_a)\nabla_\parallel \ln B + \mathbf{D}_a \cdot \mathbf{N}_a$$

$$= \mathbf{S}_\phi \phi + \mathbf{S}_\psi \psi + \sum_b \left( \mathbf{C}_{ab}^T \cdot \mathbf{N}_a + \mathbf{C}_{ab}^F \cdot \mathbf{N}_b \right)$$

Background Gradient drive — **Linearized GK Collision Operators**

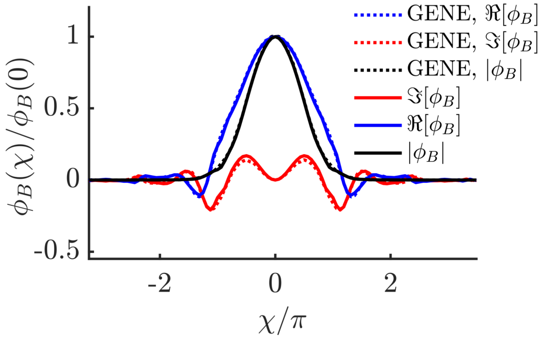
With $\delta \mathbf{E}_{es} = -\nabla \phi$, $\delta \mathbf{B}_\perp \simeq -\mathbf{b} \times \nabla \psi$ and $\mathbf{N}_a = [N_a^{00}, N_a^{01}, \dots N_a^{10}, N_a^{11} \dots N_a^{PJ}]^T$

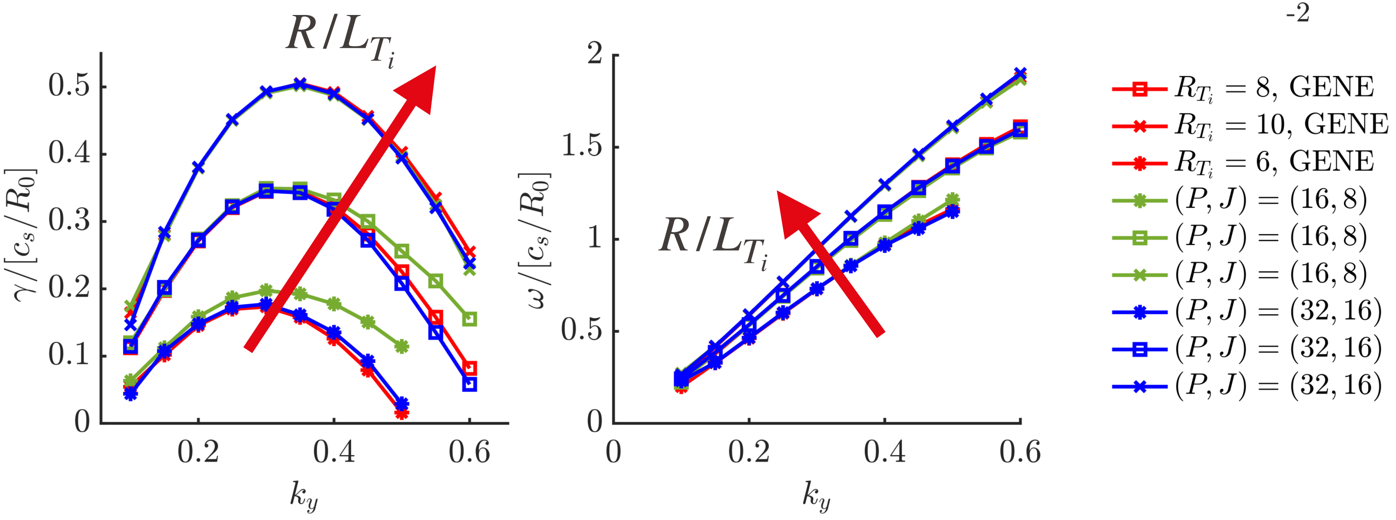
- Closure by truncation (arbitrary collisionality closure available): $P = P(\nu, k_\parallel)$, $J = J(\nu, k_\perp)$
- Implemented in a flux-tube code with concentric, circular flux surface: $\mathbf{B} = B_0 \nabla x \times \nabla y$, $\mathbf{k}_\perp = k_x \nabla x + k_y \nabla y$

# Benchmark ($\nu \ll 1$) with GENE: Cyclone Base Case

- **Collisionless** ITG with **adiabatic electron**:
  - $\mathbf{B} = B_0 \nabla x \times \nabla y$ and $\mathbf{k}_\perp = k_x \nabla x + k_y \nabla y$

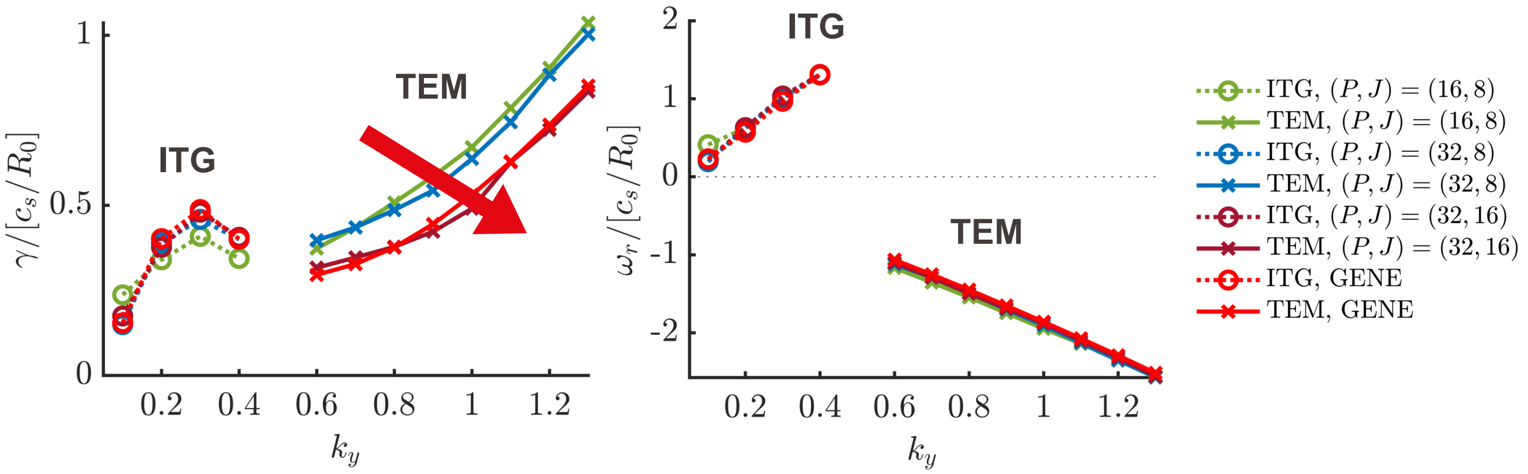
$$R_{T_i} = \frac{R}{L_{Ti}} \simeq -R\partial_x \ln T_i$$



# Benchmark ($\nu \ll 1$) with GENE: ITG to TEM mode

- Retrieve **collisionless ITG to TEM** transition (cyclone base case parameters) with $(P, J) \simeq (32{,}16)$ (GENE velocity-space resolution $(N_{v_\parallel}, N_\mu) \simeq (64{,}24)$)




- Good agreement with ballooning mode structures
- Strong kinetic physics (e.g. trapped/passing boundary) **require larger** $(P, J)$

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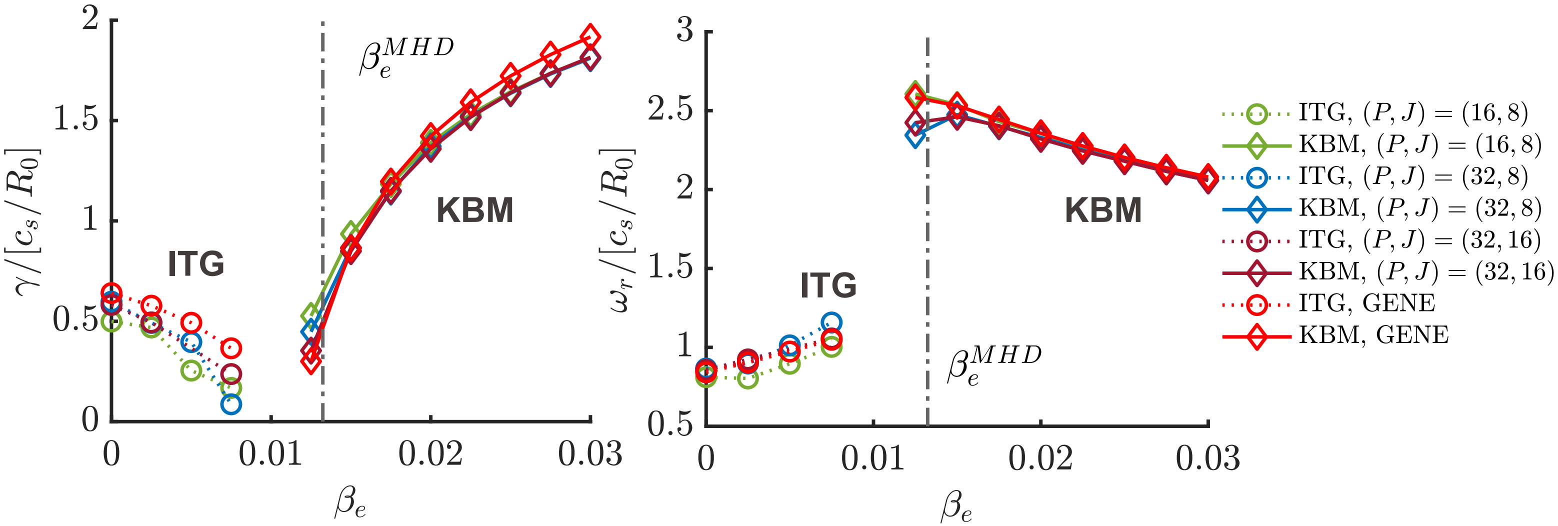
# Benchmark ($\nu \ll 1$) with GENE: ITG to KBM mode

- Retrieve **KBM** transition (cyclone base case parameters) at finite $\beta_e$, with $(P, J) \simeq (16{,}8)$




- Low number of gyro-moments (w.r.t to ITG) underlies the "fluid-like" properties of KBM

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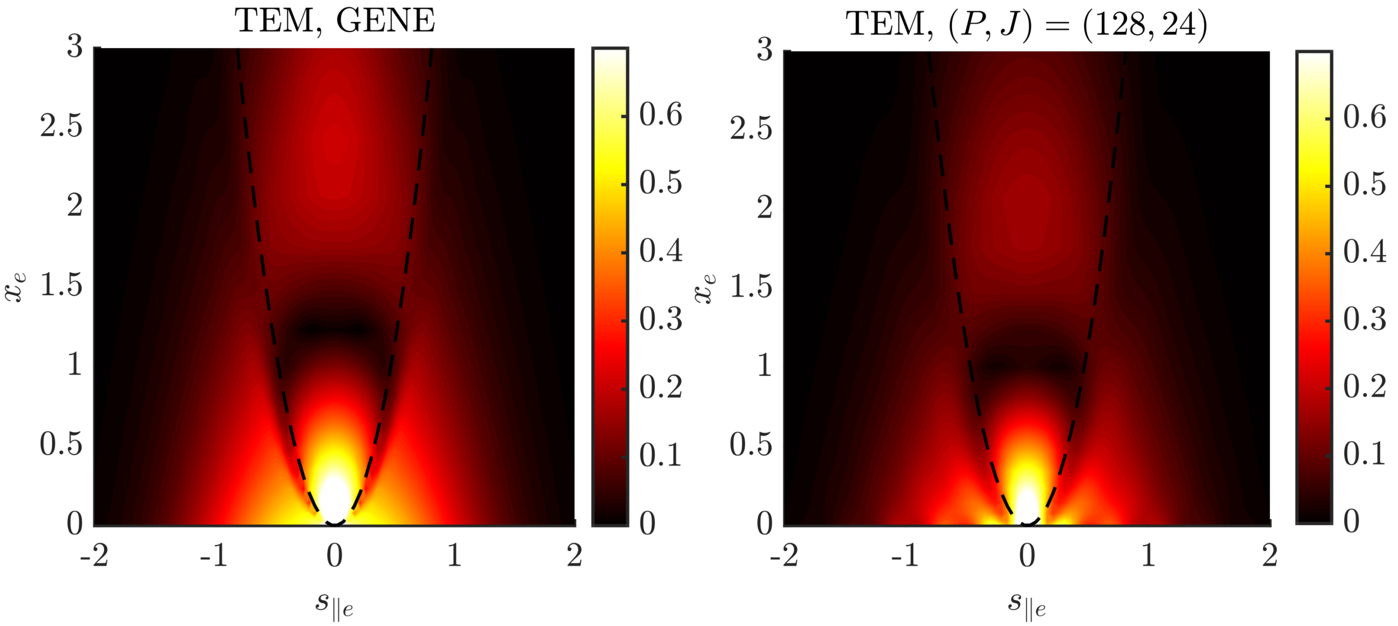
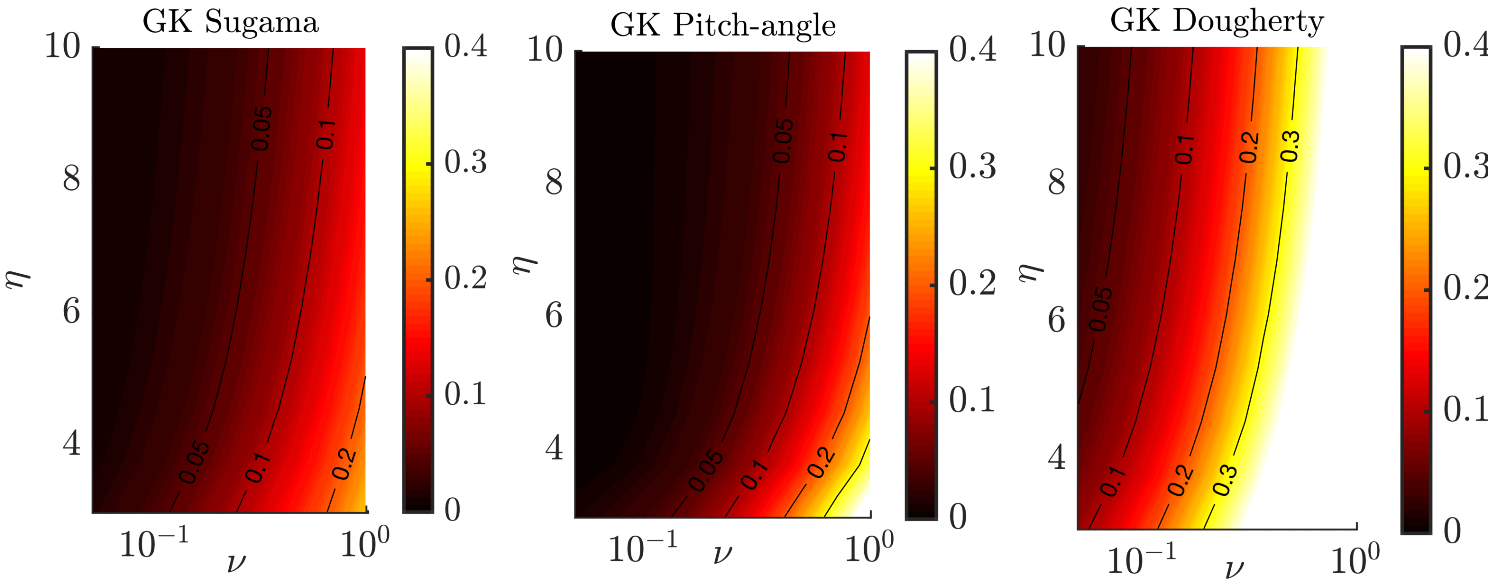
# Benchmark ($\nu \ll 1$) with GENE: ITG to TEM modes

- Fine scale structures near **passing/trapped boundary**
- With sufficient number of gyro-moments, good qualitative agreement with the modulus of $f_e$ at the outboard mid plane



# ITG mode with GK Coulomb and other GK Collision Models

- Relative deviation of **ITG growth rate peak w.r.t GK Coulomb** as a function of collisionality, $\nu = \nu_{ii}/[c_s/L_N]$, and normalized temperature gradient, $\eta = L_N/L_T$




- Deviation up to $20\,\%$ with GK Sugama (benchmarked with GENE)

# First Nonlinear Simulation using the Gyro-Moment Approach

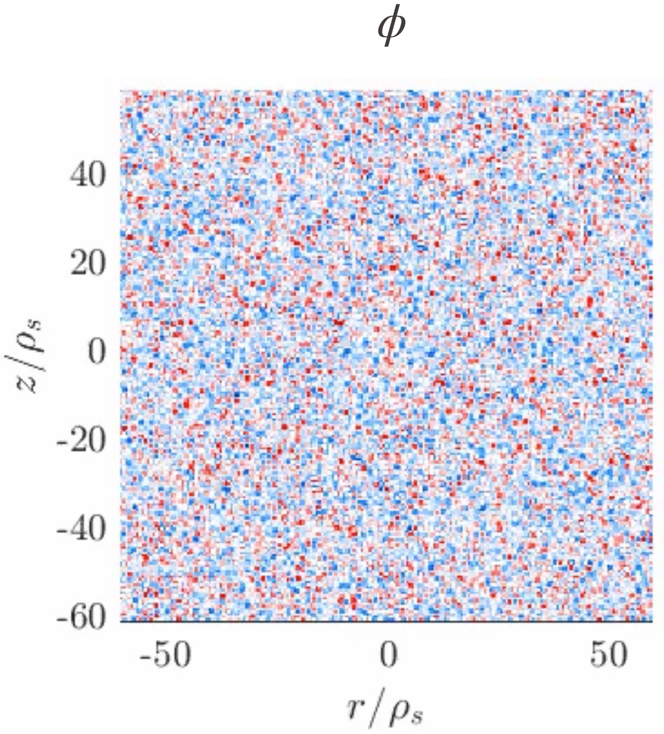
- **Z-Pinch** geometry ($k_{\|} = 0$), kinetic electrons

- Retain $\mathbf{E} \times \mathbf{B}$ nonlinearity

$$\sim \sum_{\boldsymbol{k},\boldsymbol{k}'} \frac{i\boldsymbol{E}(\boldsymbol{k}) \times \boldsymbol{B}}{B^2} \cdot \boldsymbol{k}' N_a^{pj}(\boldsymbol{k}') e^{i(\boldsymbol{k}+\boldsymbol{k}')\cdot\boldsymbol{R}}$$

- Collisional effects with **linearized GK collision operators** (GK Coulomb, GK Sugama, GK Dougherty)

- Cyclic, bursting behaviour of fluctuations, transport, and zonal flows



# Conclusions



- **A new GK model** for the tokamak boundary based on **a Hermite-Laguerre polynomials basis** (gyro-moment expansion)
- **A nonlinear GK Coulomb collision operator** (arbitrary mass and temperature ratios) is derived; a **linearized version derived and numerically implemented**; gyro-moment expansion of different **simplified/*ad hoc*** collision operator models (e.g., Sugama operator)
- Recover relevant **microinstability linear properties** (e.g.,TEM, ITG and KBM); in agreement with GENE
- Explore **deviations** between **GK Coulomb** and other GK collision operator models (e.g. ITG mode)
- **Number of gyro-moment decreases with collisionality (SOL)** $\Rightarrow$ optimal for edge and SOL modelling.
- Extend to flux-tube code to nonlinear and realistic 3D equilibrium and Full-F conditions

**Thank you for your attention**