ENR monitoring 2021: “Operation limiting plasma instabilities in high performance tokamaks”

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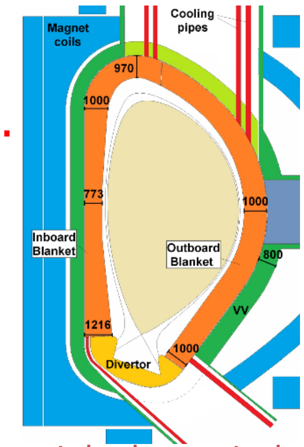
INTRODUCTION

STEP (spherical)

DEMO

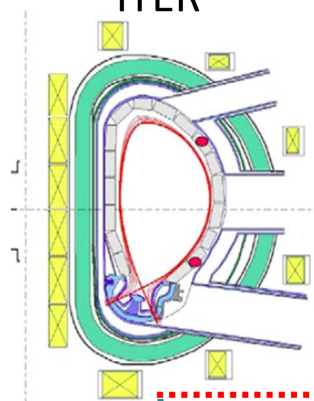


Decisions based on deep model, flight simulator?

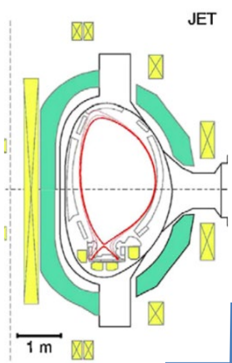
Extrapolation, mainly theoretical, models, new phenomena, sparse experimental data

MAST-U

ITER

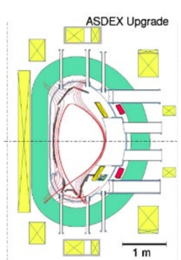


JET



Theory + semi-empirical models, scalings

AUG



Mainly experimental, confronted by theory

- DEMO design will be based on a plasma model e.g. Flight Simulator with full space and time dependence, control, initial and boundary conditions
- ITER contributes to DEMO via the ITER community, tools and studies. But not from ITER experimental data

INTRODUCTION

Why is fundamental theory important?

- EUROfusion can't afford to build tokamaks and technological equipment fast enough: so we have to rely on large codes

- Based on established models, these codes are becoming highly advanced. And they are being coupled to one another.

- Leaders of experiments, of laboratories and programmes, and even users and developers of codes, often do not have a broader understanding of the limitations and assumptions inherent in the models used in codes

- Our role in EnR is to challenge assumptions and approximations
- We can do this by addressing physics not covered by existing codes, sometimes at the cost of approximations and simplifications elsewhere
- It can be done with analytic treatment and with new codes
- Of course, we hope to uncover new interesting physics as we do this

Status of tasks June 1st – Dec 31st 2021

Core resistive infernal modes	Develop and solve equations that describe infernal modes with resistive effects in the region of low magnetic shear.	✓
Exfernal modes with finite magnetic shear	Develop and solve equations for exfernal modes (EHOs) with realistic q-profiles, including separatrix effects	✓
Near collisionless-ion infernal modes	Derive the equations for ideal interchange and infernal modes with nearly collisionless ions (for non resonant kink modes in hybrid)	✓
Non-linear ballooning in a slab	Develop a slab model of nonlinear ballooning mode to include an imposed shear flow.	✓
Ideal wall RWM model definition	Derive the RWM equations in the ideal wall limit including plasma beta effects and resistive plasma effects	✓
Ideal MHD n=0 modes and X-points	Develop a model for rigid-shift vertical displacement and the impact of X-point resonances for ideal MHD fluctuations	✓
Separatrix effects for kink-ballooning mode	Identify the essential physics for kink-ballooning that must be captured to describe MHD stability in full separatrix geometry.	✓

Status of deliverables June 1st – Dec 31st 2021

Long wavelength resistive MHD in a torus	The writing and of a numerical solver that treats pressure driven long wavelength MHD instabilities in a torus (resistive infernal modes)	✓
Ideal driven EHOs in a reactor relevant plasma	Writing of a code on the effects of realistic magnetic shear effects on EHOs.	✓
Non-linear ballooning in a slab	Report on a slab model of nonlinear ballooning mode to Include an imposed shear flow in a slab model.	✓
Ideal wall RWM model definition	Report on the RWM equations in the ideal wall limit including plasma beta effects and resistive plasma effects	✓
Ideal MHD n=0 modes and X-points	Publication on the validity of rigid-shift vertical displacement and the impact of X-point resonances for ideal MHD fluctuations	✓
Separatrix effects for kink-ballooning mode	Make recommendations for improvements to linear ideal MHD stability codes to capture the effects of full separatrix geometry	✓

Everything is forecast to be achieved in 2021

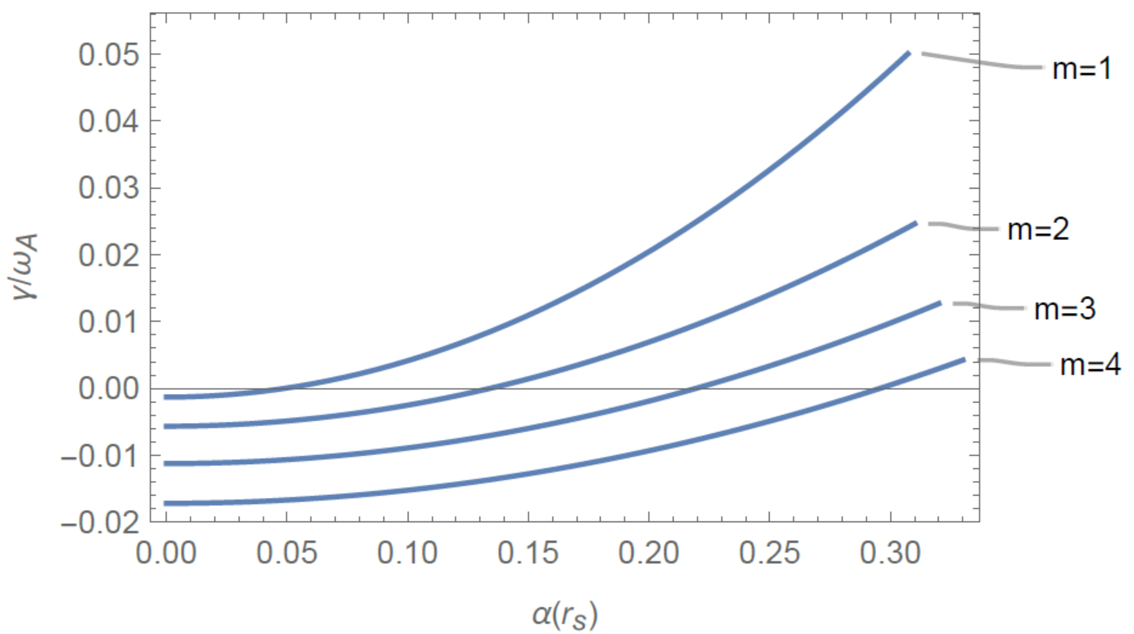
No modifications to the planned project are required for 2022

Long wavelength resistive MHD in a torus

Global infernal mode approach for pressure driven long wavelength modes with resistivity.
Work accepted for publication in Oct 2021 [Graves, Coste-Sarguet, Wahlberg, PPCF 2021]

The work recovers internal kink mode and interchange (ideal and resistive)

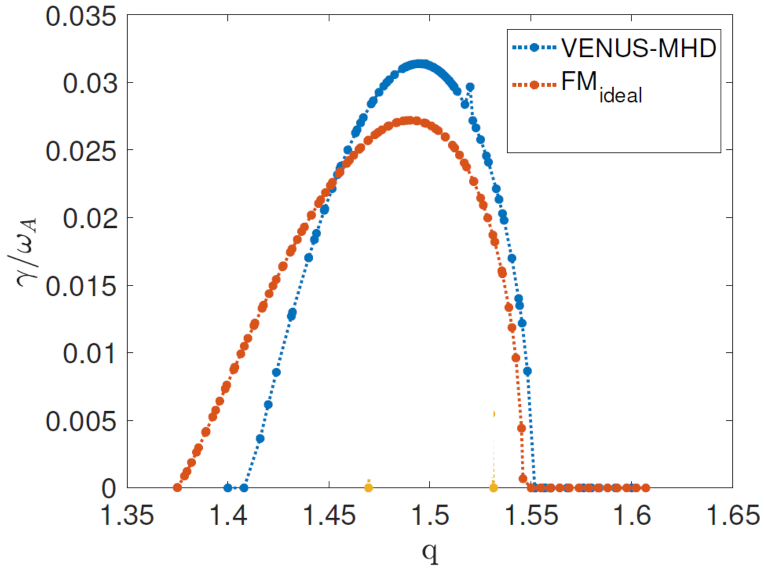
New equations describing global resistive infernal modes with multiple resistive surfaces have been derived. The equations form basis of new light code (next slide). New n=m>1 internal kink modes have been derived analytically (below are ideal modes):



Long wavelength resistive MHD in a torus

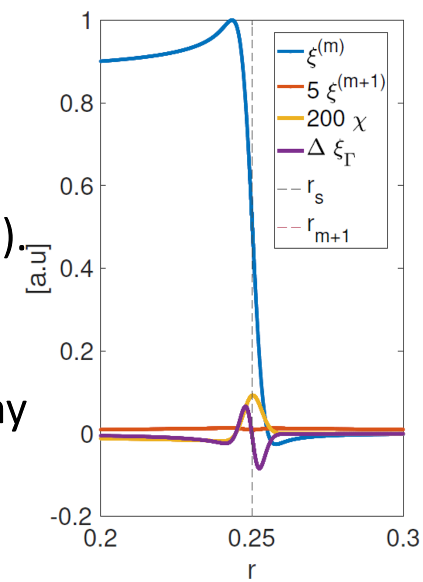
New light code has been written for global resistive instabilities.

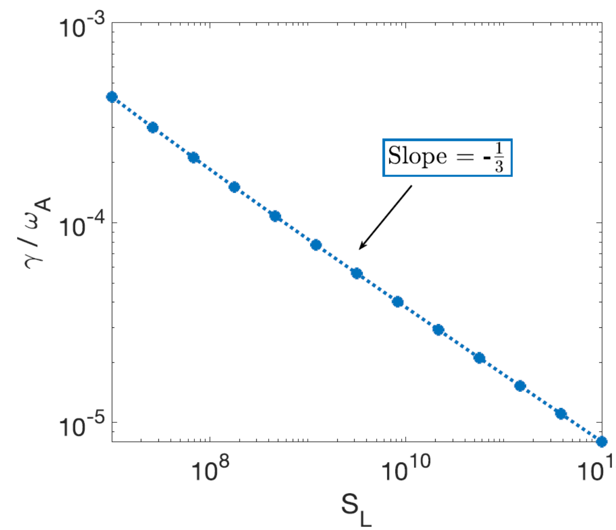
Excellent agreement with full ideal MHD eigenvalue codes (excellent given that light global code is based on large aspect ratio expansion and weak shear)



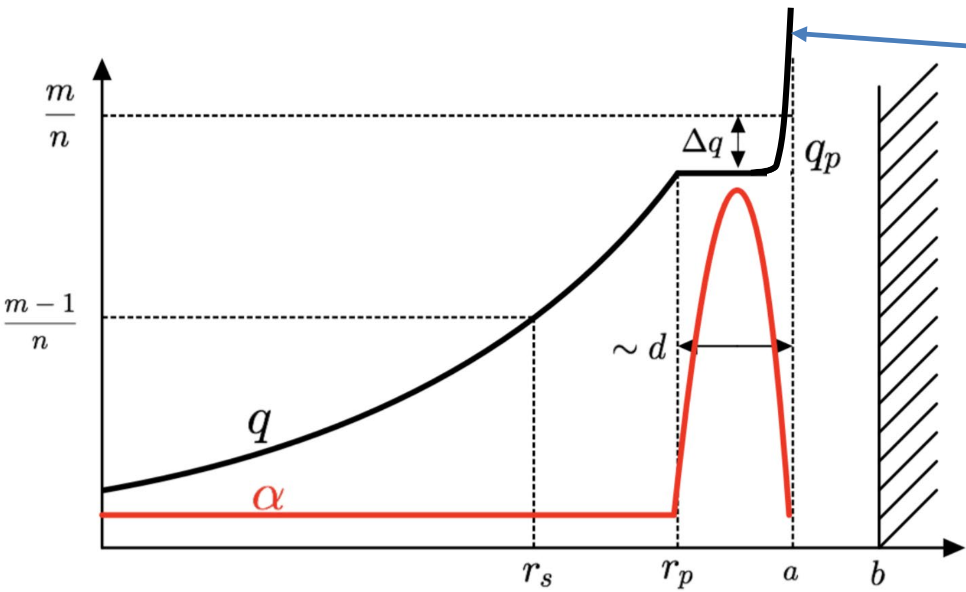
Global resistive eigenvalue solver correctly recovers resistive interchange and resistive internal kink modes for huge values of Lundquist number S_L (outside the range of most global codes).

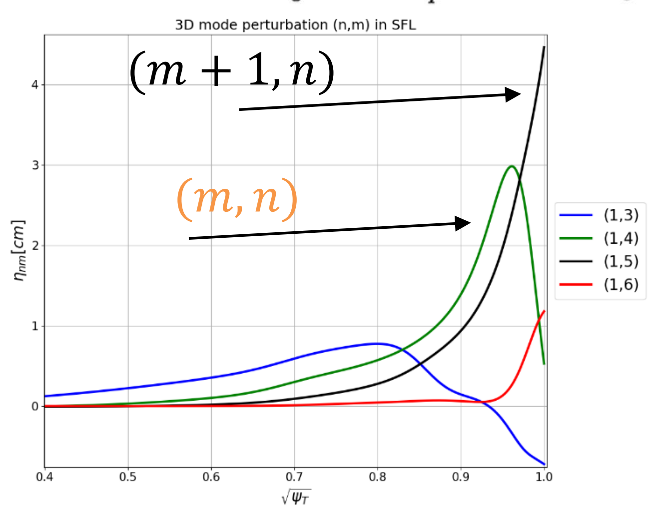
Work ongoing for resistive infernal modes and applications. Scope for many physics enhancements (e.g. flow shear)





Exfernal modes (EHOs) with finite magnetic shear





- Separatrix effect modelled here with fast rise in current. It suppresses current driven mode
- An infernal mode, driven by

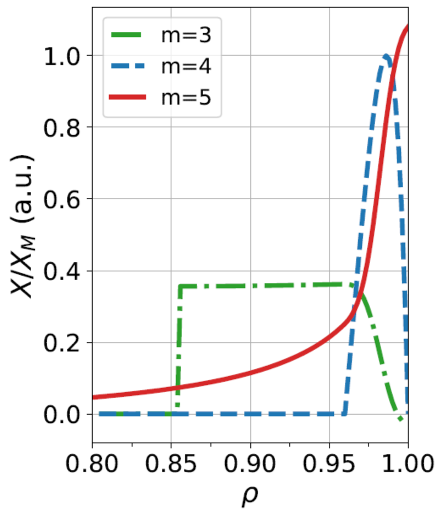
$$\alpha = -2q^2 \frac{R}{B^2} \frac{dP}{dr}$$

Continues to exist in region of low magnetic shear despite separatrix (current driven mode killed)

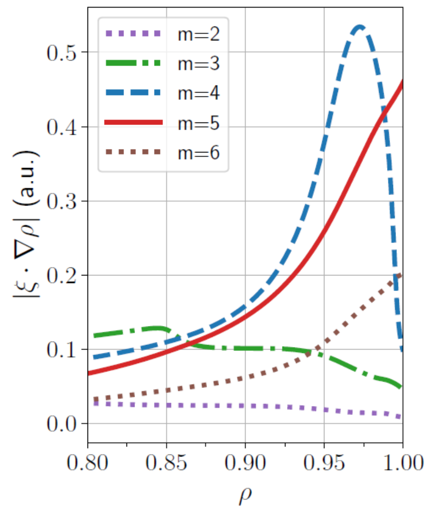
With conversion to SFL coordinates in VMEC, we can see if numerical simulations are current or pressure driven

Exfernal modes (EHOs) with finite magnetic shear

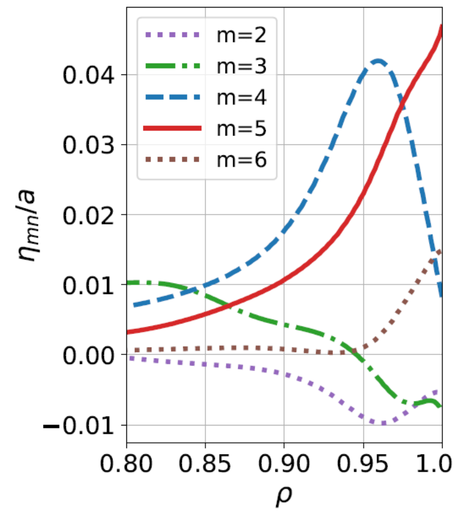
Analytic eigenfunctions



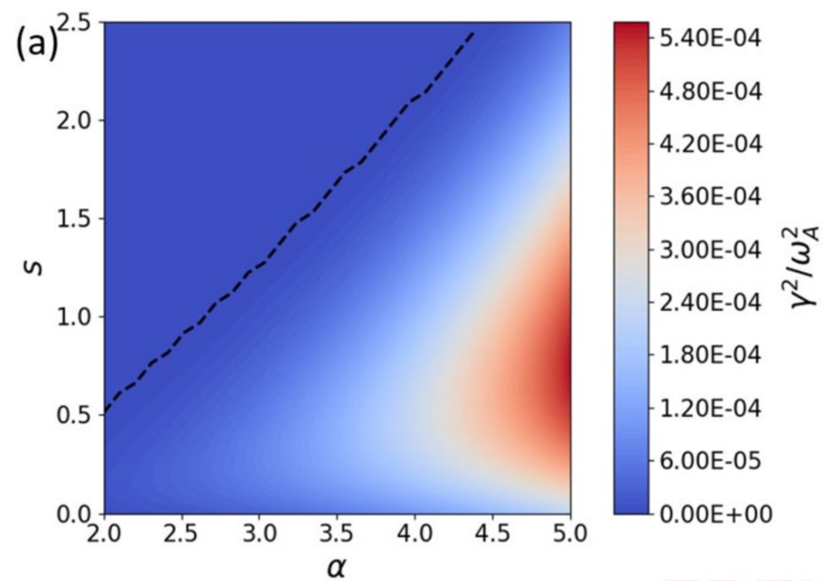
KINX numerical eigenfunctions



VMEC nonlinear displacements



- Analytic exfernal mode model extended to include magnetic shear effects everywhere. **Published**: [Bustos Ramirez, Graves, Brunetti, PPCF 2021]
- Eigenfunction agreement with KINX and VMEC modes
- We now see that these modes are robust by modelling separatrix, and with realistic shear. **Next look for effect of RMPs**



Analytical Kinetic MHD

Pressure moments of δf via the equilibrium distribution function $F = F(\mathcal{K}, \mu, P_\phi)$ and:

$$\delta f = \frac{Ze}{m} R^2 (\boldsymbol{\delta A} \cdot \boldsymbol{\nabla}\phi) \frac{\partial F}{\partial P_\phi} + \frac{Ze}{m} \delta\phi_L \frac{\partial F}{\partial \mathcal{K}} - \mu \frac{\delta B_\parallel}{B} \frac{\partial F}{\partial \mu} + \delta g$$

where the drift kinetic equation describes the kinetic contribution δg via

$$\frac{d}{dt}\delta g = -\frac{i}{m}\frac{\partial F}{\partial \mathcal{K}}(\omega - n\omega_*)\delta L,$$

and $\omega_* = (\partial F/\partial P_\phi)/(\partial F/\partial \mathcal{K})$. The perturbed Lagrangian is written

$$\delta L = Ze\boldsymbol{v}_d \cdot \boldsymbol{\delta A} - Ze\delta\phi_L - m\mu\delta B_\parallel,$$

Convenient to use the MHD gauge $\boldsymbol{\delta A} = \boldsymbol{\xi} \times \boldsymbol{B}$, parallel electric field thus defined via $\delta\phi_L$, $\boldsymbol{\delta E} = -\boldsymbol{\nabla}\delta\phi_L - \dot{\boldsymbol{\xi}} \times \boldsymbol{B}$. The Lagrangian can then be written:

$$\delta L = \delta L^\psi + \delta L^\theta, \quad \delta L^\psi = Ze\xi^\psi \omega_{d0}^\phi, \quad \delta L^\theta = qZe\xi^\theta \omega_d^\psi$$

with $$\omega_{d0}^\phi = -(\boldsymbol{\nabla}\phi - q\boldsymbol{\nabla}\theta) \cdot \boldsymbol{v}_d - \frac{\delta\phi_L}{\xi^\psi} - \frac{m\mu}{Ze}\frac{\delta B_\parallel}{\xi^\psi}, \qquad \omega_d^\psi = -\boldsymbol{v}_d \cdot \boldsymbol{\nabla}\psi.$$

Kinetic MHD continued

Standard kinetic corrections to low n modes arise from δL^{ψ}. Kinetic corrections to the inertia are found in contributuon of δL^{θ}. These effects have been identified for the cases of interchange and ballooning equations, and dispersion relations with these combined effects have been written down for those applications.

Kinetic-MHD equations have been derived consistently in the collisionless limit. Fluid FLR diamagnetic effects (very hard!), centrifugal corrections with wave-particle resonance physics:

$$\rho_0\left[\frac{\partial^2\vec{\xi}}{\partial t^2}+2(\vec{u}_0\cdot\nabla)\frac{\partial\vec{\xi}}{\partial t}+(\vec{u}_{0,*i}\cdot\nabla)\frac{\partial\vec{\xi}_\perp}{\partial t}\right]$$
$$+\rho_0\left[\frac{\partial}{\partial t}+(\{\vec{u}_0+\vec{u}_{0,*i}\}\cdot\nabla)\right]\left[\frac{\nabla\delta\phi_L\times\vec{B}}{B^2}\right]$$
$$+\nabla\cdot\delta\vec{P}^{\mathsf{CGL}}-\vec{j}\times\delta\vec{B}-\delta\vec{j}\times\vec{B}$$
$$=\nabla\otimes\left[\rho_0\vec{\xi}\otimes(\vec{u}_0\cdot\nabla)\vec{u}_0-\rho_0\vec{u}_0\otimes(\vec{u}_0\cdot\nabla)\vec{\xi}\right]$$

-For pressure driven instabilities we require full EM effects. The kinetic MHD approach was shown in [Lanthaler, Graves] to be more efficient than standard gyro-kinetic codes. Same physics in gyrokinetic code approach requires solving to one order higher in Larmor radius.
-**New Ph.D. student has started** work in order to initiate the numerical development project.

Non-linear ballooning in a slab

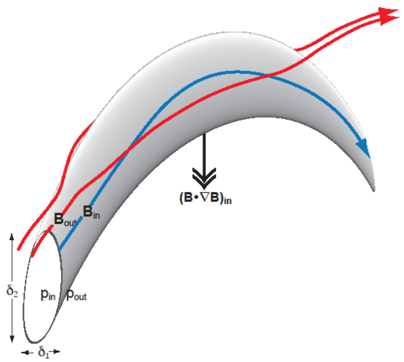
Force on a flux tube

- Task: Develop a slab model of nonlinear ballooning mode to include an imposed shear flow
- Deliverable: Report on a slab model of nonlinear ballooning mode to include an imposed shear flow in a slab model
- We calculate the nonlinear behaviour of erupting flux tubes by consideration of the two components of the MHD force equation perpendicular to the magnetic field

$$\mathbf{F} = \frac{1}{\mu_0}\left[\mathbf{B}\cdot\nabla\mathbf{B} - \nabla\left(\frac{B^2}{2} + \mu_0 p\right)\right]$$

- This implies total pressure inside must equal total pressure outside

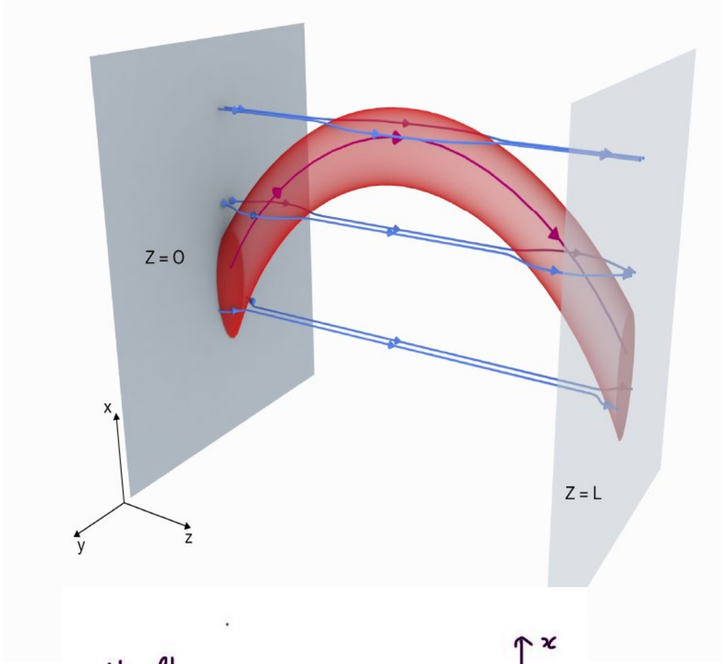
$$\left[\frac{B_{in}{}^2}{2} + \mu_0 p_0(r)\right]_{in} = \left[\frac{B_0{}^2}{2} + \mu_0 p_0(r_0)\right]_{out}$$

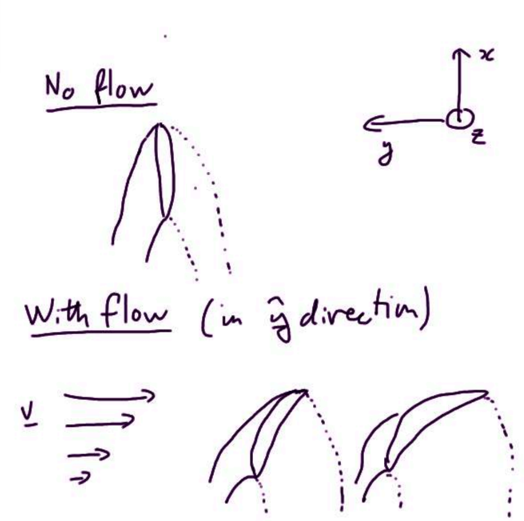


Non-linear ballooning in a slab

Adding flow

- We have assumed that the flow only acts in the y direction and on a timescale slower that the fast equilibration which sets the dimensions of the flux tube.
- We have derived the equations of motion for the flux tube assuming that a flow force acts on the plasma
- We have solved these time dependent equations assuming that the flux tube motion is viscous limited
- Preliminary results show that the flow is destabilizing but this needs to be confirmed.
- We will also calculate the equilibrium flux tubes assuming constant flow force and varying with height (x)

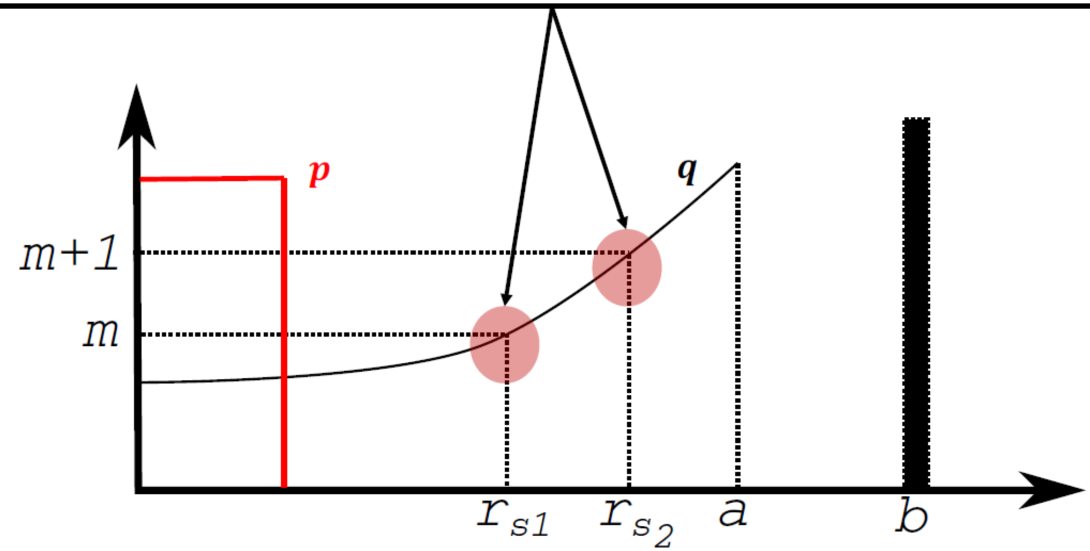




Cowley *et al.* Proc. R. Soc A **471** 20140913 (2015)

Ideal wall RWM definition

- Resistivity allowed the the resonances of $m, m+1$ modes
- WKB approach [Hegna PoP 1994] for the fluid displacement solution
  - Allows sufficiently generic q with constraint of tearing stability



The physical model for an appropriate description of the resistive wall mode (RWM) instability has been established.

- based on the resistive description of the plasma in a proper toroidal geometry, which accounts for mode coupling similarly to other types of internal MHD instabilities.
- Choosing carefully the form of the perturbation allows the constraints imposed by the presence of a separatrix in a diverted plasma to be automatically satisfied.
- Simple profiles for the relevant physical quantities, i.e. pressure and q, permit easier algebraic manipulations while keeping all the required physical information.

Ideal MHD n=0 modes and X-points

Task

The ideal-MHD theory of axisymmetric modes with toroidal mode number n=0 in tokamak plasmas has been developed. These modes are resonant at the magnetic X-points of the tokamak divertor separatrix. Consequently, current sheets form along the separatrix, which profoundly affect the stability of vertical plasma displacements. These current sheets lead to stabilization of n=0 modes, at least on the ideal-MHD time scale, adding an important ingredient to the mechanism of passive feedback stabilization.

Deliverable

Two articles on the validity of rigid-shift vertical displacements and the impact of X-point resonances for ideal MHD fluctuations has been published:

-A. Yolbarsop, F. Porcelli, R. Fitzpatrick, 2021 *Nucl. Fusion Letters* **61** 114003.

-F. Porcelli, A. Yolbarsop, T. Barberis, R. Fitzpatrick 2021 *J. Phys.: Conf. Ser.* **1785** 012004

Conference presentations:

-F. Porcelli, *Impact of divertor X-points on axisymmetric modes in Tokamaks,* oral presentation, 19th EFTC, October 2021.

Separatrix effects for kink-ballooning modes

Task (identify physics effect to be captured for full separatrix geometry)

- Magnetic shear becomes locally high along a field line due to vanishing poloidal magnetic field in vicinity of X-point

- Local poloidal variation of perturbations near X-point competes with ordinarily dominant radial variations, breaking eikonal structure of high-*n* ballooning theory, even though separation between rational surfaces shrinks to zero on the separatrix.

We may compare these observations with Ref.

[C M Bishop, P Kirby, J W Connor, R J Hastie & J B Taylor, "Ideal MHD stability in the vicinity of a separatrix," Nucl. Fusion **24** 1579 (1984)]

where a 'staircase of steps' near the periodically repeating X-points develops.

Separatrix effects for kink-ballooning modes

Deliverable (identify the physics approaches codes could take)

1. Adapt the approach of [Mattor and Cohen, "How fluctuations continue through an X point," Phys Plasmas **2** 4042 (1995)], who show that in a high-*n* eikonal approach the corresponding 'rays' depart from flux surfaces in the vicinity of an X-point, unlike in conventional ballooning theory.
   - this mitigates the stabilising effect arising from development of the highly radially sheared perturbations near the X-point.
   - challenge is to identify how to impose a periodicity constraint to provide an eigenvalue condition.

2. Because all the action occurs near the X-point one can consider separating the 2D region near the separatrix into two parts:
   - A poloidally narrow region near the X-point where radial and poloidal variations of the perturbations compete, though the equilibrium poloidal variation is small due to the narrow localisation.
   - The remaining poloidal region where radial equilibrium variations, including the *local* magnetic shear, can be considered small due to the high global shear arising from the X-point shrinking the distance between resonant surfaces: radial variation in this region is then driven by the matching to that in the X-point region.
   - Impose periodicity and matching conditions between the `solved' regions.

Journal Publications

A. Yolbarsop, F. Porcelli, and R. Fitzpatrick, "Impact of magnetic X-points on the vertical stability of tokamak plasmas," accepted NF 2021

G. Bustos Ramirez, J. P. Graves and D. Brunetti "Effect of edge magnetic shear on Edge Harmonic Oscillations in plasmas with separatrix," published PPCF 2021

E. Neto, J. P. Graves, M. Raghunathan, C. Sommariva and D. Pfefferle "Heavy impurity transport in tokamaks subject to plasma rotation, NTV and the influence of saturated ideal MHD perturbations," accepted PPCF 2021

J. P. Graves, M. Coste-Sarguet, C. Wahlberg "Pressure driven long wavelength MHD instabilities in an axisymmetric toroidal resistive plasma," accepted PPCF 2021

D. Brunetti, C. J. Ham, J. P. Graves, et al "Understanding JET-C quiescent phases with edge harmonic magnetohydrodynamic activity and comparison with behaviour under ITER-like wall conditioning," subm. PPCF 2021]

D. Brunetti, C. J. Ham, S. Saarelma, J. P. Graves et al "Finite magnetic well effects on resistive and drift-resistive ballooning modes in a shaped tokamak," subm Nucl. Fusion 2021

Conference contributions

F. Porcelli, "Impact of divertor X-points on axisymmetric modes in tokamaks," Abstract for 19th European Fusion Theory Conference

E. Neto, J. P. Graves, "Heavy impurity transport in the presence of 3D MHD ideal saturated modes, rotation and ambipolar electric field," Abstract and Invited oral for 10th International workshop on Stochasticity in Fusion Plasmas

T Barberis, F. Porcelli, "Fast particles resonance with axisymmetric modes in shaped plasmas," Abstract for 19th European Fusion Theory Conference

T Barberis, F. Porcelli, "Resonant interaction between feedback stabilized axisymmetric modes and energetic ions," Abstract for 17th IAEA Technical Meeting on Energetic Particles and Theory of Plasma Instabilities in Magnetic Confinement Fusion

G Bustos Ramirez, J. P. Graves, "The operating space of Edge Harmonic Oscillations in static plasmas," oral and abstract for 10th International workshop on Stochasticity in Fusion Plasmas (SFP)