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A self-consistent $\kappa_\perp$ model for anomalous transport due to electrostatic ExB drift turbulence in the scrape-off layer and implementation in SOLPS-ITER

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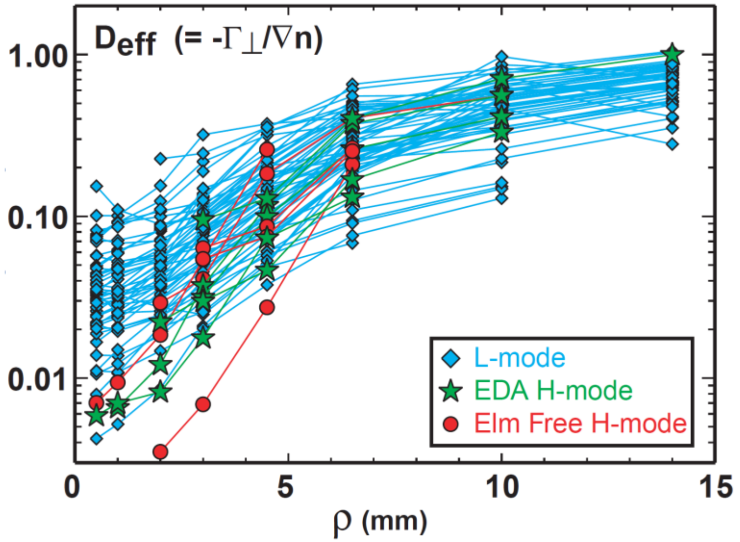
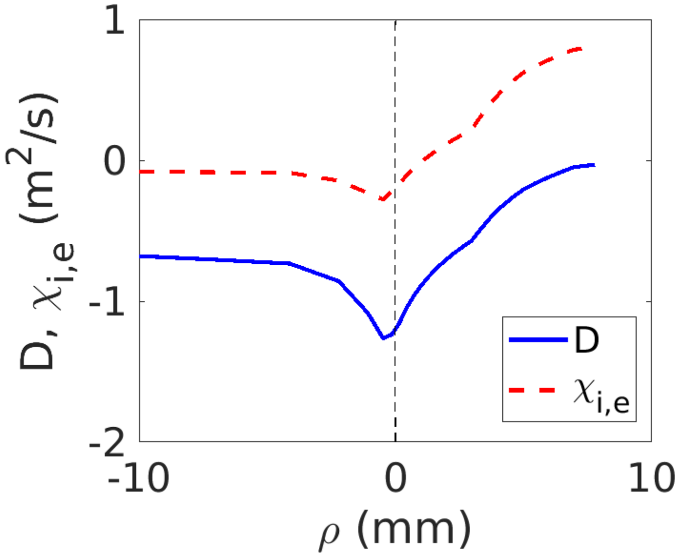
Motivation

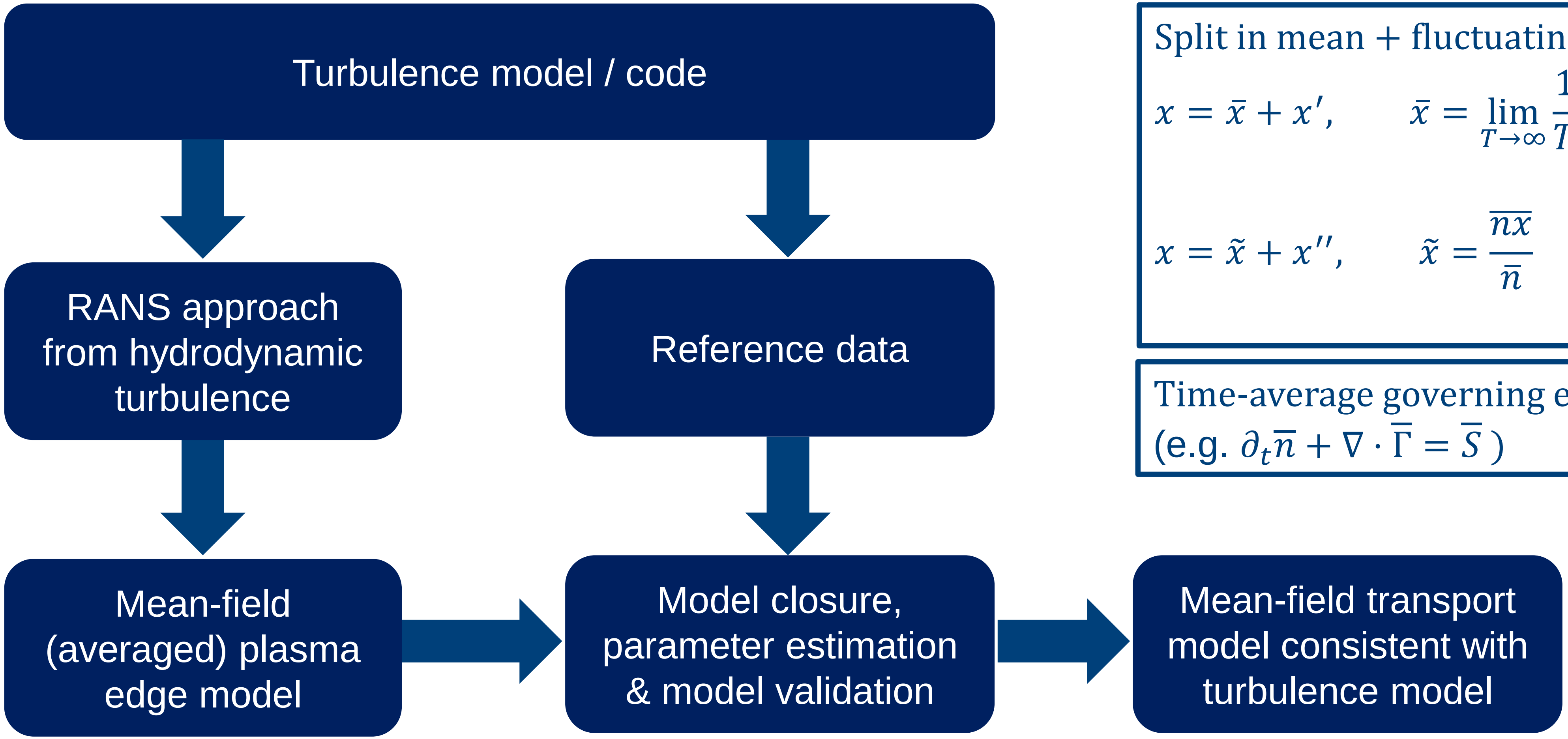
- Anomalous transport crudely approximated in existing mean-field codes (SOLPS-ITER, SOLEDGE2D,…)
  - Usually ad-hoc diffusive ansatz
  - Values of transport coefficients vary over wide range between devices, between regimes, and even within a single discharge
- Expensive, manual tuning procedures, usually limited to specifying radial profile at the outer mid plane (OMP)
- Strong impact on reliability plasma edge simulations:
  - Consistent analysis of competing transport mechanisms (turbulence, mean-field drifts)?
  - Variation during parameter scans?
  - Predictive value?



[B. LaBombard et al., Nucl. Fusion **40** (2000) 2041.]



Approach



Split in mean + fluctuating components

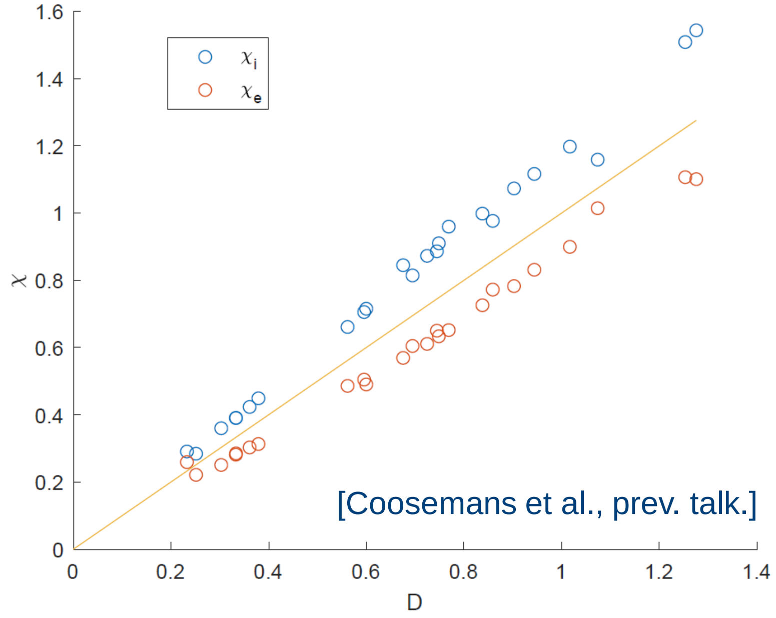
$$x = \bar{x} + x', \qquad \bar{x} = \lim_{T \to \infty} \frac{1}{T} \int_0^T x dt$$

$$x = \tilde{x} + x'', \qquad \tilde{x} = \frac{\overline{nx}}{\bar{n}}$$

Time-average governing equations (e.g. $\partial_t \bar{n} + \nabla \cdot \bar{\Gamma} = \bar{S}$)

Mean-field particle and heat fluxes

- Average fluxes, electrostatic turbulence (no B-fluctuations): fluctuating $E \times B$-terms need closure!
  - $\overline{\boldsymbol{\Gamma}}_i = \overline{n}_i \tilde{u}_{||}\boldsymbol{b} - \frac{\overline{n}_i}{B}\nabla\overline{\phi}\times\boldsymbol{b} - \overline{\frac{n_i'}{B}\nabla\phi'\times\boldsymbol{b}} - \frac{1}{eB}\nabla\overline{p}_i\times\boldsymbol{b} + \frac{m_i\boldsymbol{b}}{eB}\times\left(\frac{\partial\overline{n}_i\tilde{\boldsymbol{V}}}{\partial t} + \nabla\cdot\overline{n}_i\tilde{\boldsymbol{V}}\tilde{\boldsymbol{V}} + \nabla\cdot\overline{n_i\boldsymbol{V}''\boldsymbol{V}''}\right) + \cdots$
  - $\overline{\boldsymbol{Q}}_i = \left(\frac{5}{2}\overline{n}_i\tilde{u}_{||}\boldsymbol{b} - \frac{3}{2}\frac{\overline{n}_i}{B}\nabla\overline{\phi}\times\boldsymbol{b} - \overline{\frac{3}{2}\frac{n_i'}{B}\nabla\phi'\times\boldsymbol{b}}\right)\tilde{T}_i - \kappa_{||}\nabla_{||}\tilde{T}_i\boldsymbol{b} - \overline{\frac{3}{2}\frac{n_iT_i''}{B}\nabla\phi'\times\boldsymbol{b}} + \cdots$ (note: $\overline{p}_i = \overline{n}_i\tilde{T}_i$)



- Propose diffusive model
  - $\overline{\boldsymbol{\Gamma}}_{i,E\times B} = -\overline{\frac{n_i'}{B}\nabla\phi'\times\boldsymbol{b}} \sim -D_{E\times B}\nabla\overline{n}_i$
  - $\overline{\boldsymbol{Q}}_{i,E\times B} = -\overline{\frac{3}{2}\frac{n_iT_i''}{B}\nabla\phi'\times\boldsymbol{b}} \sim -\chi_{i,E\times B}\,\overline{n}_i\nabla_\perp\tilde{T}_i$

$$\chi_{e,E\times B} \sim \frac{3}{2}D_{E\times B}$$

- Link coefficients to turbulent kinetic energy:

$$D_{E\times B} \sim C_D\rho_L\sqrt{\frac{\kappa_\perp}{m_i}} \text{ with } \overline{n}\kappa_\perp = \frac{\overline{nm_iV_{E\times B}''^2}}{2}$$

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Transport equation for $\kappa_\perp$

- $\kappa_\perp$ equation derived analytically for 2D electrostatic interchange model

$$\phi\nabla\cdot\left(j_{||}\boldsymbol{b}+\boldsymbol{j}_\perp\right)=0 \Rightarrow \ldots \Rightarrow \frac{\partial}{\partial t}\bar{n}\kappa_\perp + \nabla\cdot\overline{\boldsymbol{\Gamma}}_{\kappa_\perp} = \overline{S}_{\kappa_\perp}$$

- Approximate model:
  - Total source:

$$\overline{S}_{\kappa_\perp} \approx \overline{S}_{IC} + \overline{S}_{||} + \overline{S}_{RS}$$

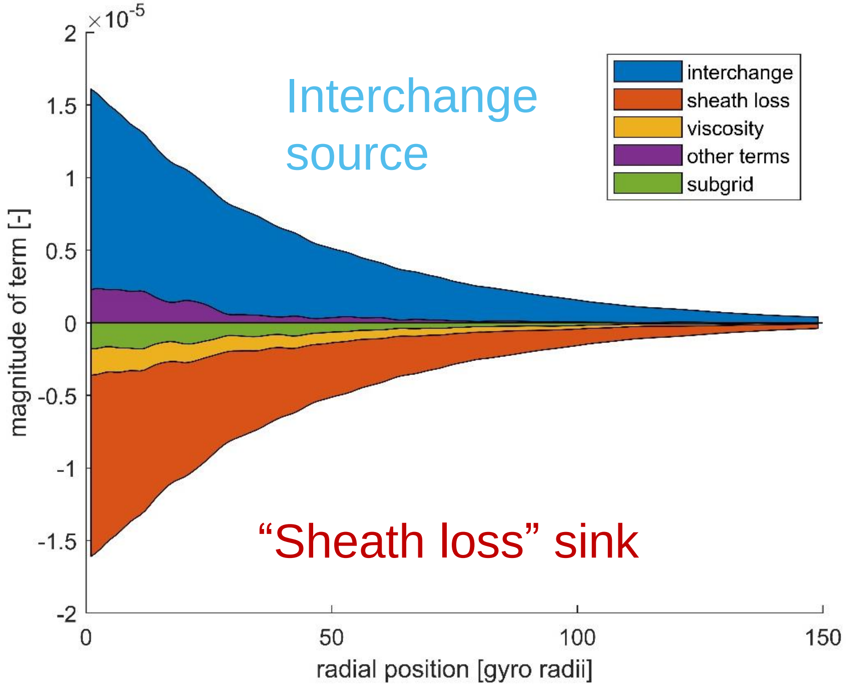
  - Transport:

$$\nabla\cdot\overline{\boldsymbol{\Gamma}}_{\kappa_\perp} \approx \nabla\cdot\left(\overline{\boldsymbol{\Gamma}}\kappa_\perp + \frac{1}{2}\overline{mn\boldsymbol{V}''V''^{2}_{E\times B}} + \overline{\phi'\boldsymbol{J}'_*} + \overline{p'\boldsymbol{V}'_{E\times B}} + \overline{\phi'\boldsymbol{J}'_{||}}\right)$$

Model as (small) diffusion term on $\kappa_\perp$

- (Small) viscous dissipation term can be linked to turbulent enstrophy $\zeta_\perp$: ongoing work [Coosemans et al., CPP **60** (2020) e201900156.]

Balance from T2D simulations



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The interchange source of $\kappa_\perp$

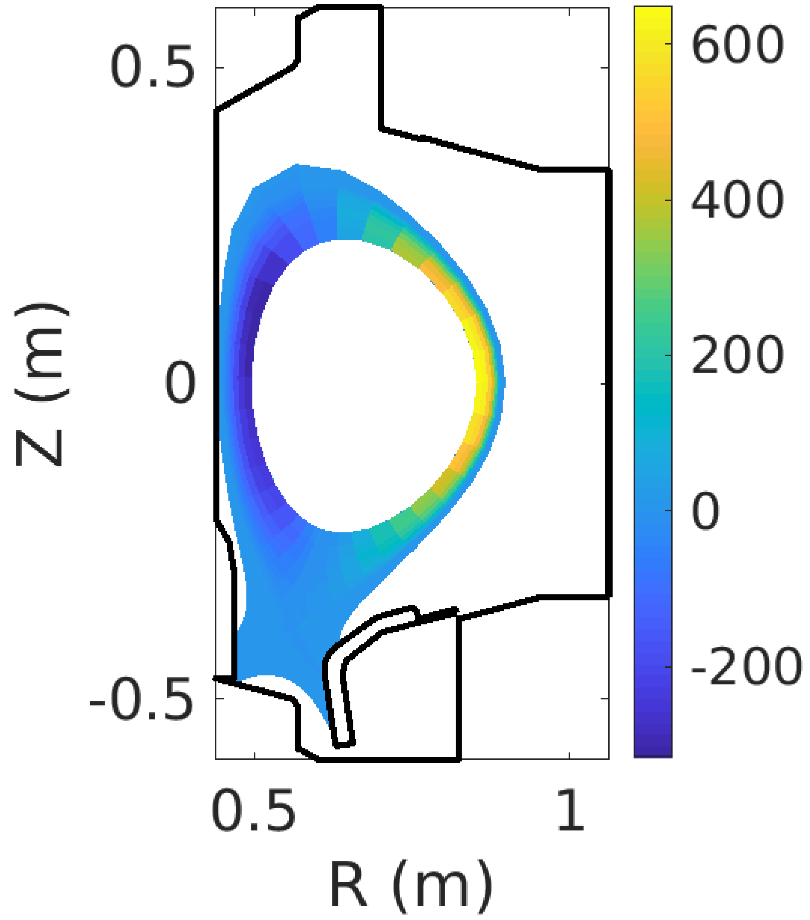
- Total heat flux due to $E \times B$ fluctuations drives production of $k_\perp$
  [Coosemans et al., prev. talk.]

$$\overline{S}_{IC} = -\frac{2}{3}\left(\overline{\boldsymbol{\Gamma}}_{i,E\times B}\tilde{T}_i + \overline{\boldsymbol{\Gamma}}_{e,E\times B}\tilde{T}_e + \overline{\boldsymbol{Q}}_{i,E\times B} + \overline{\boldsymbol{Q}}_{e,E\times B}\right)\cdot\nabla\ln B^2$$

  - Source in ‘bad-curvature’ regions
  - Sink (!) in ‘good-curvature’ regions
  - Internal saturation mechanism
  - Energy conservation: coupling with ion/electron internal energy equations

- Neglect transport contributions (cancel exactly in 1D)

$$\nabla\cdot\left(\overline{\phi' J'_*} + \overline{p' V'_{E\times B}}\right) \approx 0$$



Transport of $\kappa_\perp$ due to parallel current fluctuations

- Parallel current fluctuations:

$$j'_{||} \approx -\sigma_{||}\nabla_{||}\phi' + \frac{\sigma_{||}}{en_e}\nabla_{||}p'_e + \frac{0.71\sigma_{||}}{e}\nabla_{||}T'_e$$

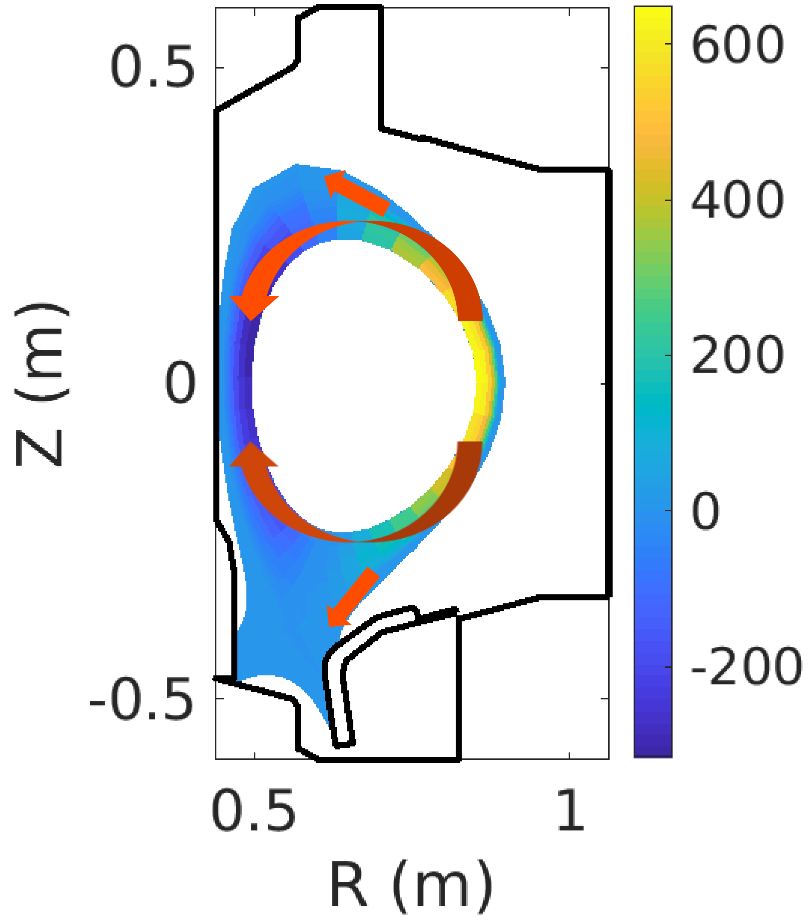
- Model for transport of $\kappa_\perp$:

$$\overline{\phi' j'_{||}} \sim -\sigma_{||}\nabla_{||}\frac{\overline{\phi'^2}}{2} \sim -C_{\sigma 1}\sigma_{||}\rho_L^2\nabla_{||}\kappa_\perp$$

Strongly exceeds parallel convection with $\tilde{u}_{||}$!

(‘ideal’ interchange: $\frac{\pi}{2}$ phase shift n'/T'_e and ϕ')

$\left(\kappa_\perp \sim (\nabla_\perp\phi')^2 \sim \frac{\phi'^2}{\rho_L^2}\right)$



- Model for (small) dissipation term for $\kappa_\perp$:

$$\overline{S}_{||} = \overline{j'_{||}\cdot\nabla_{||}\phi'} \sim -\sigma_{||}(\nabla_{||}\phi')^2 \sim -C_{\sigma 2}\sigma_{||}\left(\frac{\rho_L}{L_{||}}\right)^2 k_\perp$$

  - Energy balance: coupling with electron energy equation

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Impact of (mean) $E \times B$ flow shear

- Reynolds-stress tensor: negative-viscosity model

$$\Pi_{RS} = \overline{mnV''_{E\times B}V''_{E\times B}} \sim \frac{2}{3}\bar{n}\kappa_{\perp}\mathrm{I} - 2\eta_{E\times B}\left(\nabla\bar{V}_{E\times B} + \nabla\bar{V}_{E\times B} - \frac{1}{3}(\nabla\cdot\bar{V}_{E\times B})\mathrm{I}\right)$$

$$\eta_{E\times B} = -C_{\eta}m\bar{n}D_{E\times B}$$

- Turbulence suppression due to flow shear

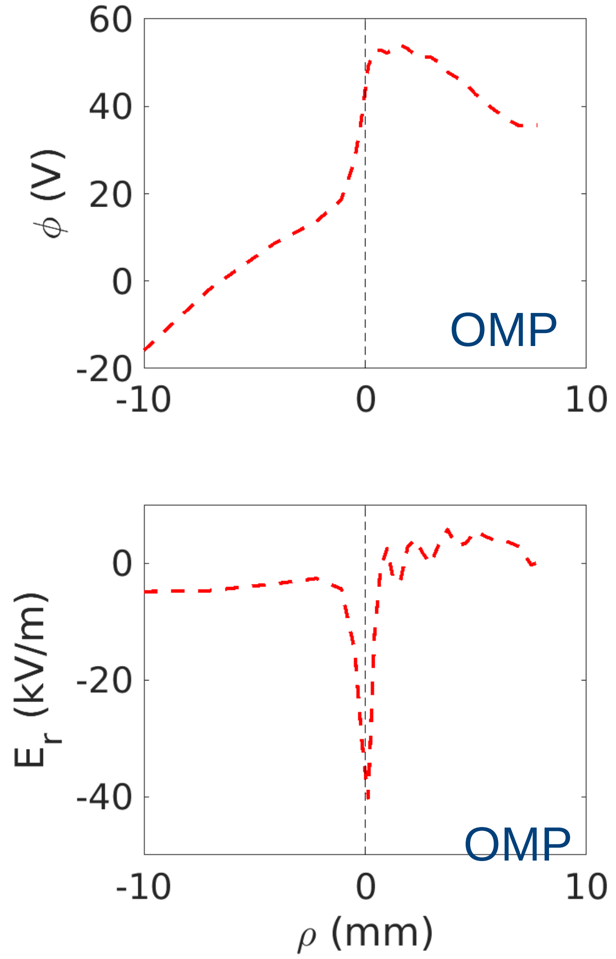
$$\bar{S}_{RS} = -\Pi_{RS}\colon\nabla\bar{V}_{E\times B} \sim \eta_{E\times B}\left(\frac{\partial\bar{V}_{E\times B,\theta}}{\partial r}\right)^2$$

- Energy conservation: corresponding ion drift/current

$$\mathbf{\Gamma}_{RS} = \frac{m\boldsymbol{b}}{eB}\times\left(\nabla\cdot\overline{nV''_{E\times B}V''_{E\times B}}\right) \approx \frac{\boldsymbol{b}}{B}\times\nabla\left(\frac{2}{3}\bar{n}\kappa_{\perp}\right) - \frac{\boldsymbol{e_r}}{B}\times\nabla\cdot\left(\eta_{E\times B}\nabla_r\bar{V}_{E\times B,\wedge}\,\boldsymbol{e_r}\right)$$

- Transport reduction due to flow shear: $D_{E\times B} \sim \dfrac{C_D\kappa_{\perp}}{\sqrt{\kappa_{\perp}/\mathrm{m}_i/\rho_L} + C_s\left|\nabla\bar{V}_{E\times B}\right|}$

[Coosemans et al., J. Phys.: Conf. Series **1785** (2021) 012001.]



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Model summary

- $\kappa_\perp$ equation for 2D electrostatic interchange turbulence

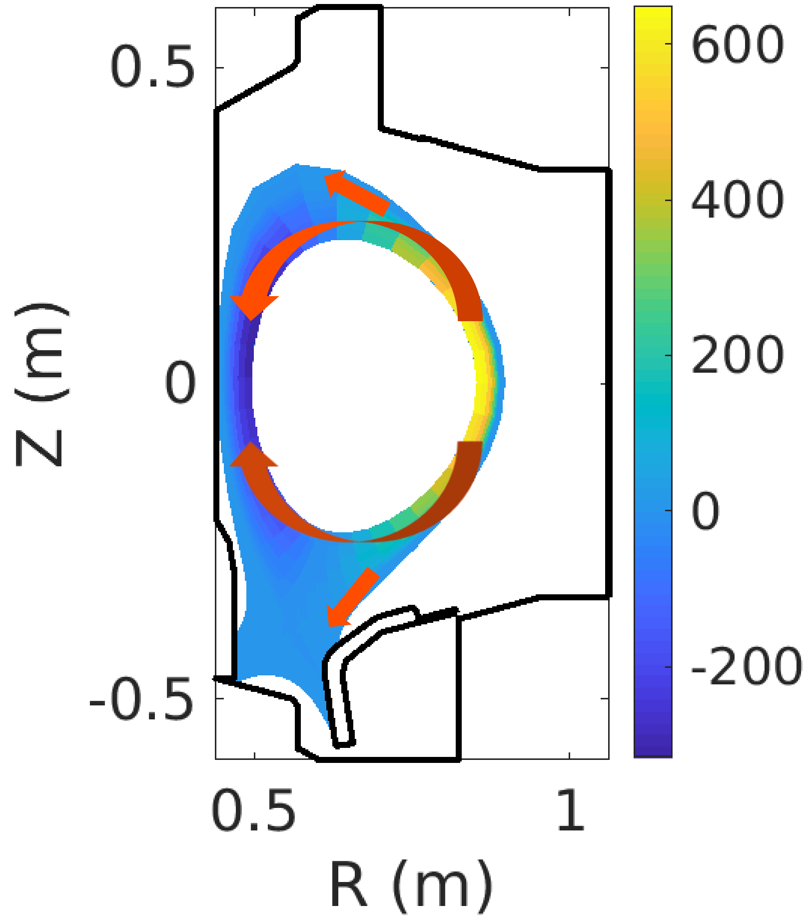
$$\frac{\partial}{\partial t}\bar{n}\kappa_\perp + \nabla\cdot\overline{\boldsymbol{\Gamma}}_{\kappa_\perp} = \overline{S}_{\kappa_\perp}$$

  - Source/sink of $\kappa_\perp$: $\overline{S}_{\kappa_\perp} \approx \overline{S}_{IC} + \overline{S}_{||} + \overline{S}_{RS}$
  - Transport: $\overline{\boldsymbol{\Gamma}}_{\kappa_\perp} \approx \nabla\cdot\left(\overline{\boldsymbol{\Gamma}}\kappa_\perp + \frac{1}{2}\overline{mn\boldsymbol{V}''V''^2_{E\times B}} + \overline{\phi'\boldsymbol{J}'_{||}}\right)$
- Couple to 'regular' mean field equations
  - Transport coefficients determined by local value of $\kappa_\perp$

$$D_{E\times B}\sim\frac{C_D\kappa_\perp}{\sqrt{\kappa_\perp/\mathrm{m}_i}/\rho_L + C_s\left|\nabla\overline{V}_{E\times B}\right|} \qquad \chi_{E\times B}\sim D_{E\times B}\sim\eta_{E\times B}$$

  - Energy conservation (mean field + turbulent + RS-drift)
- Implemented in new 'extended grids' version of SOLPS-ITER

[Dekeyser et al., NME **27** (2021) 100999.]



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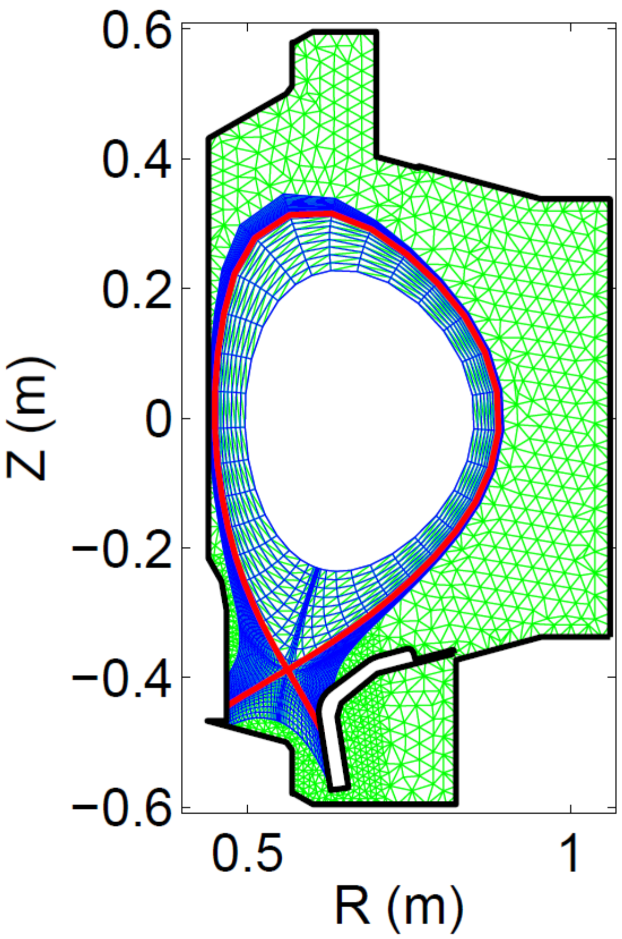
Test case based on C-Mod shot #1070627009

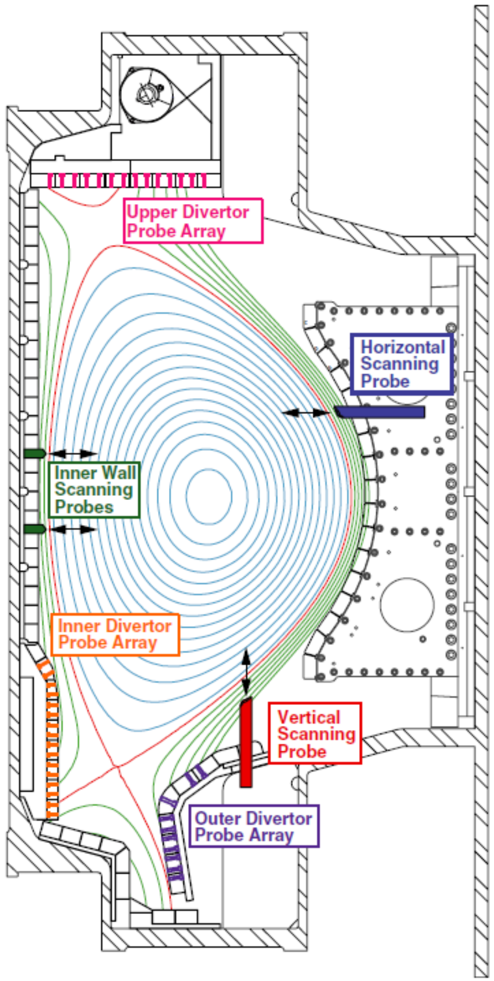
[Dekeyser et al., NME **12** (2017) 899.]



Model
- Single species deuterium plasma
- SOLPS-ITER drifts model incl. (mean-field) ExB and diamagnetic drifts
- Complete kinetic neutral model (atoms + molecules), including n-n collisions
- Newly developed $\kappa_\perp$ model for anomalous transport

Setup and boundary conditions
- Lower Single Null (LSN), ion $B\times\nabla B$ drift towards divertor ("normal" field direction)
- Core: fixed density, power $P_{OH} - P_{rad,core}$ ~ 0.8 MW
- Targets: standard sheath conditions
- Radial boundaries: leakage BCs

Experimental data
- Focus on midplane and target probes



[Smick et al., Nucl. Fusion **53** (2013) 023011.]

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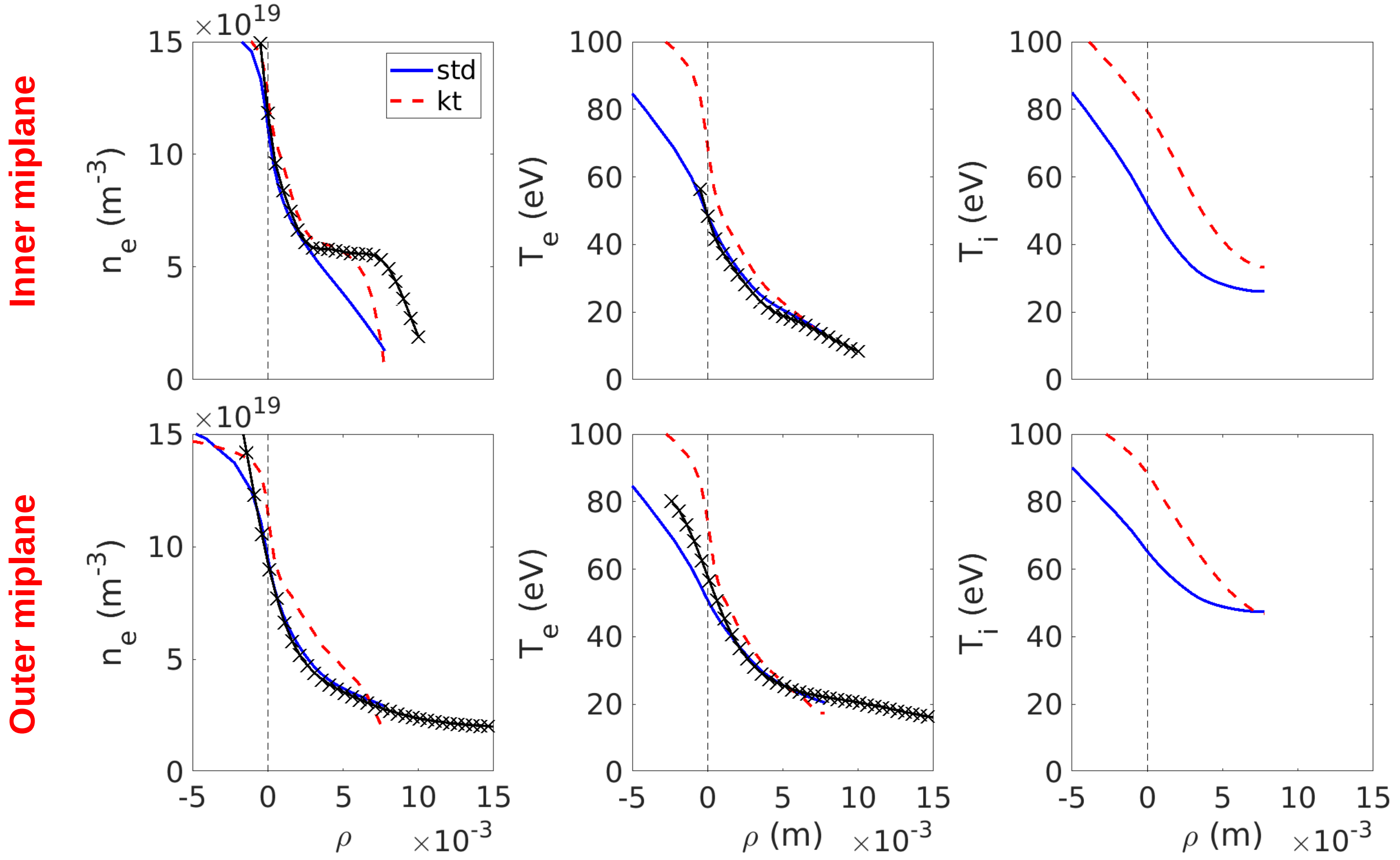
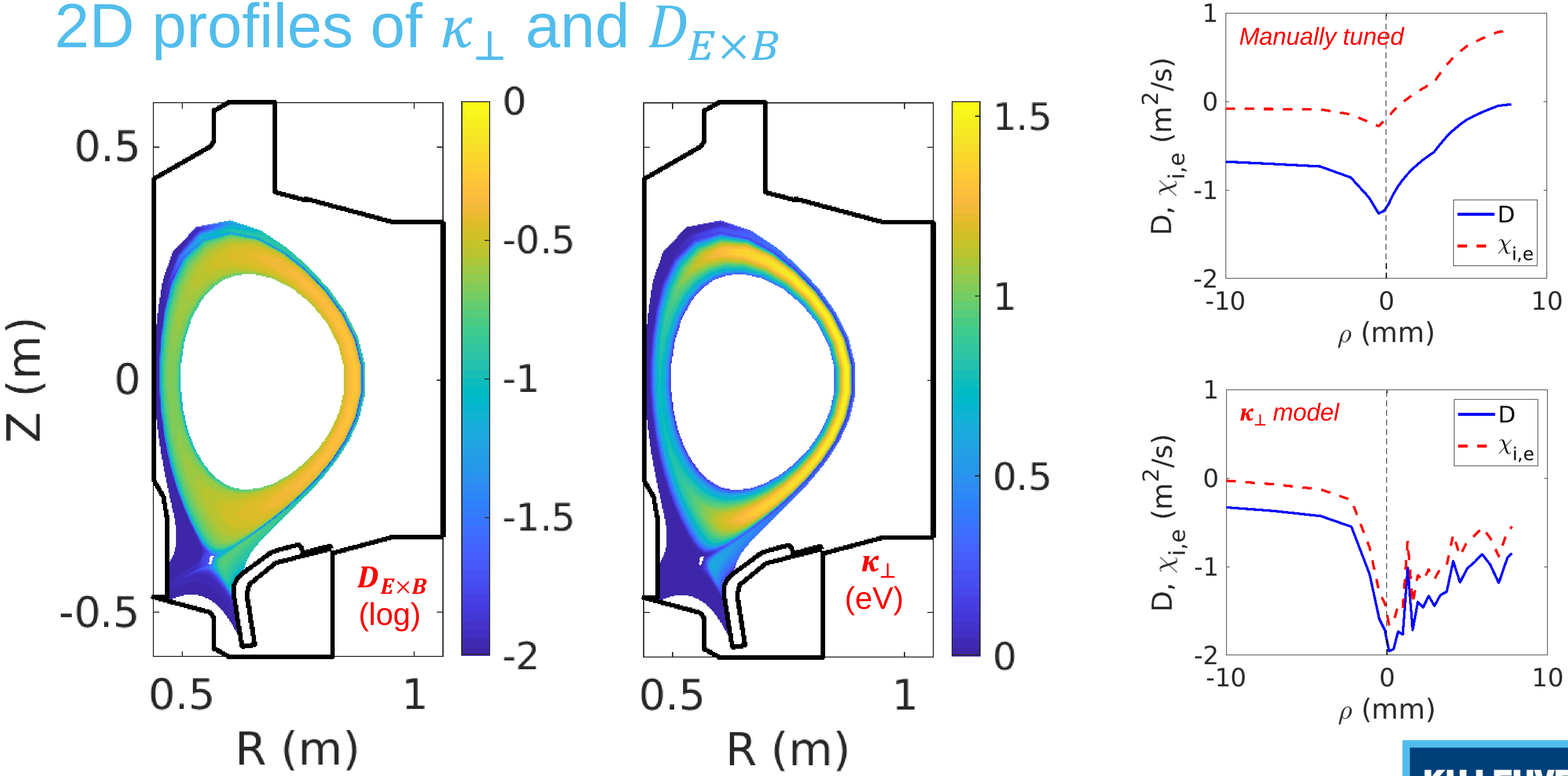
Midplane profiles compared to ‘standard’ approach



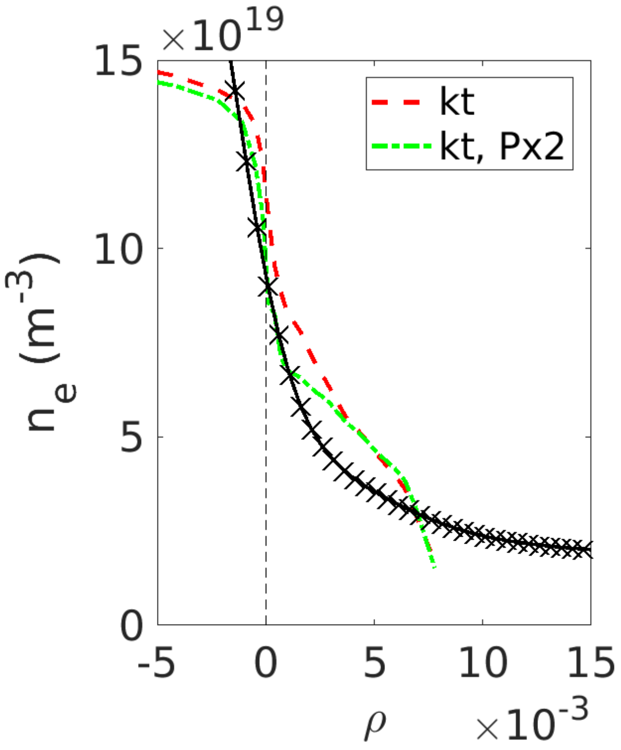
2D profiles of $\kappa_\perp$ and $D_{E\times B}$

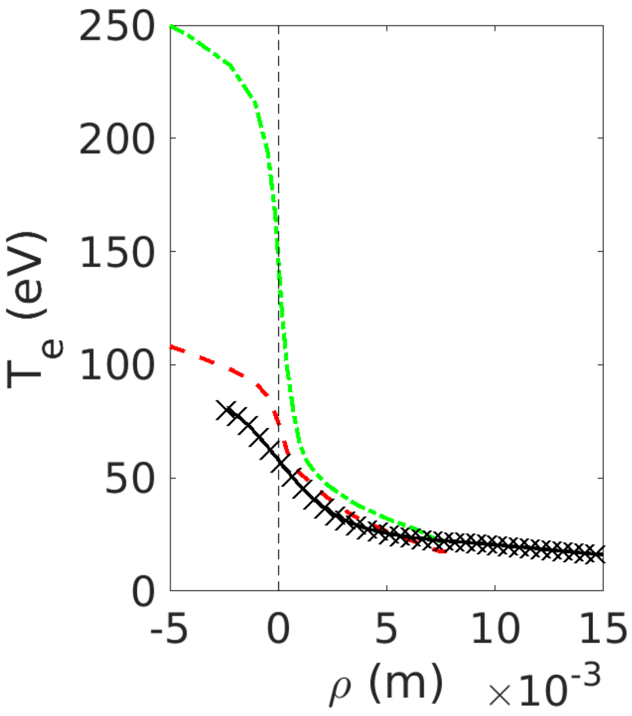


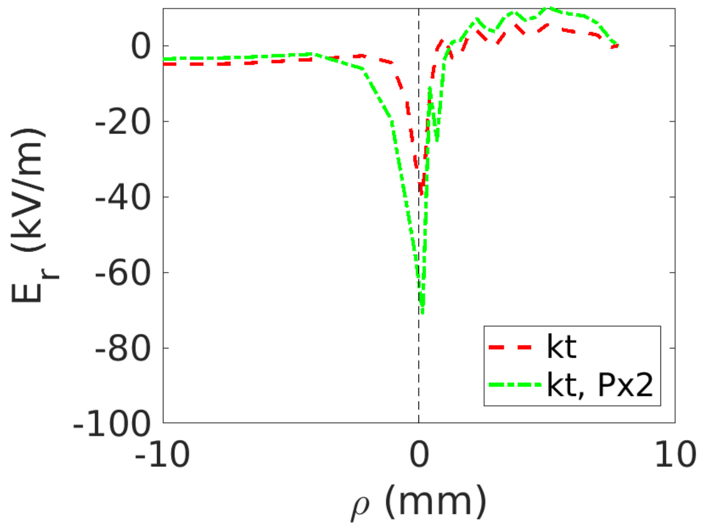
Assessment power dependency

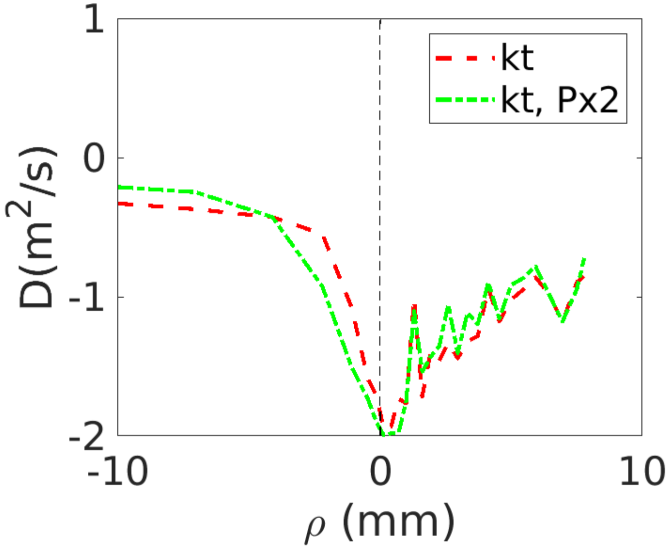
Double power at fixed density

Outer miplane









Similar effect when reducing density at fixed power

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Conclusions and perspectives

- Anomalous transport model for electrostatic interchange turbulence proposed based on RANS approach. Key features:
    - $E \times B$ heat flux determines production/dissipation of $\kappa_\perp$
    - Parallel current fluctuations determine parallel transport of $\kappa_\perp$
    - Negative-viscosity model to account for impact (mean) $E \times B$ shear

- Self-consistent simulations of mean-field-drift and anomalous transport in the edge

- Further model enhancement and parameter calibration based on turbulence simulations and/or experiment needed

Questions?