RABBIT: A high-fidelity code to simulate the NBI fast-ion distribution in real-time

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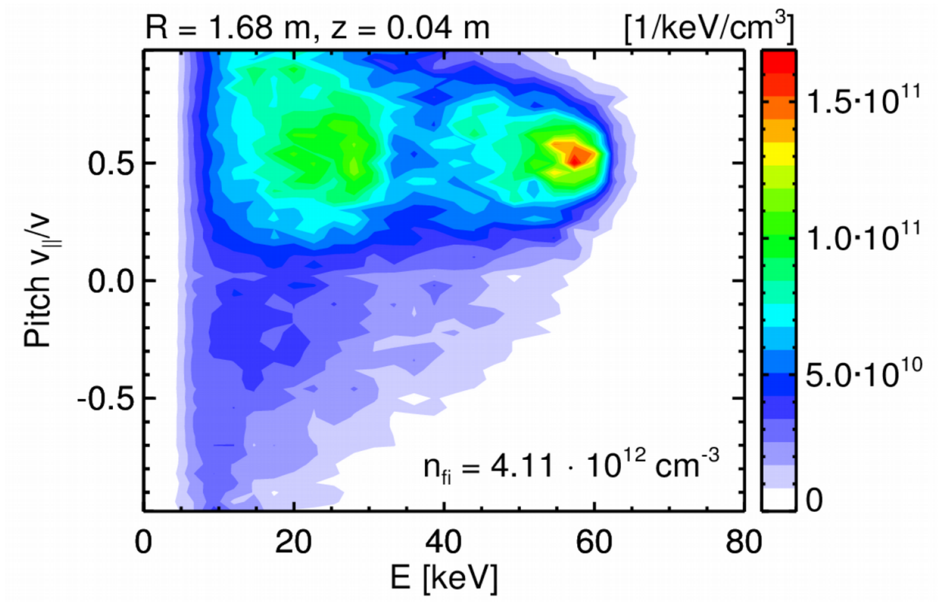
RABBIT: Real-time model for the NBI fast-ion distribution

Motivation:

- Fast ion distribution function is required for instance:
  - Heating profiles for transport calculations
  - pressure and current-drive for equilibrium reconstructions
- Sophisticated simulation codes exist (e.g. TRANSP/NUBEAM based on Monte Carlo), but long computation time (~ 30 s per time-step)
- → Too slow for real-time applications (e.g. discharge control systems, real-time transport solvers like RAPTOR)
- → Develop fast model Rapid Analytical Based Beam Injection Tool – RABBIT [M. Weiland, NF 2018] (~ 20 ms per time-step)

TRANSP fast-ion distribution function



Kinetic equation

Kinetic equation for distribution function f(x, v, t)

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}} f + \boldsymbol{a} \cdot \nabla_{\boldsymbol{v}} f = \nabla_{\boldsymbol{v}} \cdot \boldsymbol{\Gamma}_c(f) \quad + \text{Source}$$

Orbit effects

$$\boldsymbol{a} = \frac{q}{m}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right)$$

Time dependence (=0 for steady state solution)

collisions (e.g. slowing down, pitch angle scattering)

Source = NBI deposition

Kinetic equation - outline

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3. Orbit effects

$$\boldsymbol{a} = \frac{q}{m}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right)$$

4. Time dependence (=0 for steady state solution)

2. collisions (e.g. slowing down, pitch angle scattering)

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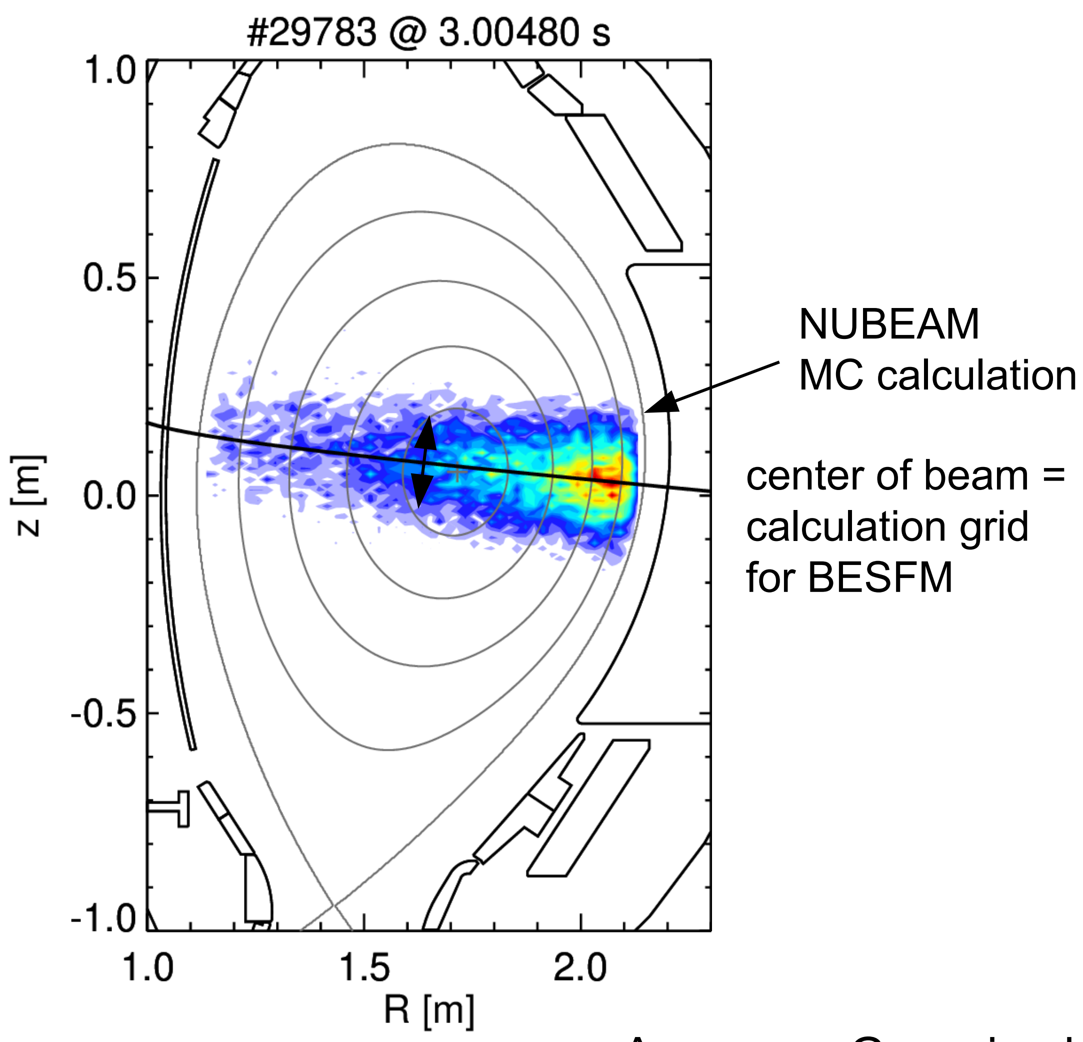
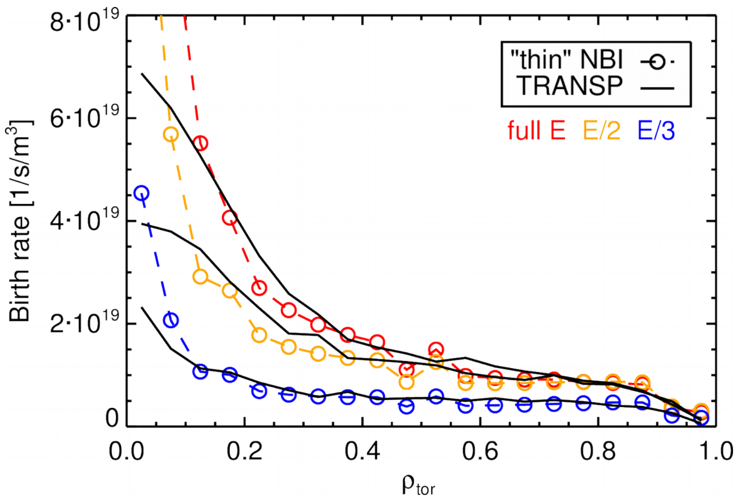
Beam deposition (birth profile)

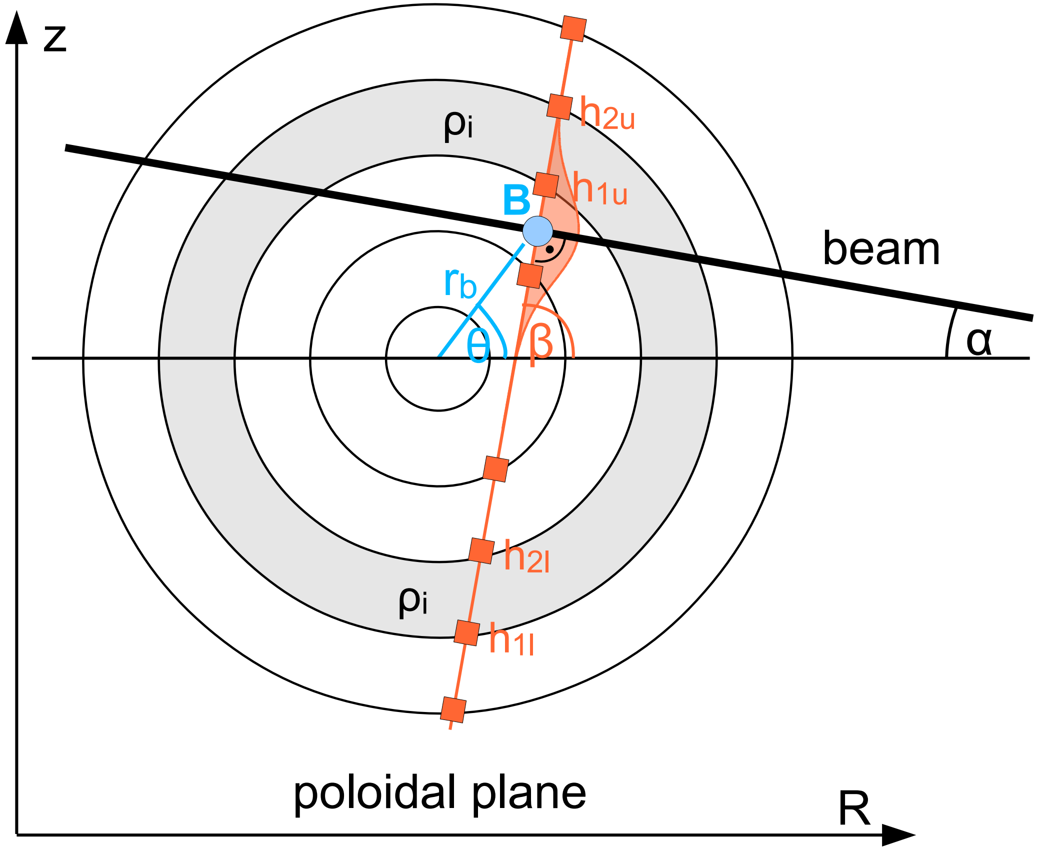


- Injection of fast neutrals into the plasma → Ionization → newly "born" fast ion
- Fast-ion birth rate = - (beam attenuation rate)
- Calculation of beam attenuation:
  BESFM Code by A. Lebschy, R. Dux, IPP
- We use the simplest geometry: NBI as thin line
- Good approximation for attenuation – for birth profile, we need to take into account the beam width:



- Assume a Gaussian broadening with standard deviation σ(l), l = coordinate along beam, σ(l) defined by NBI parameters (e.g. divergence)

Analytic model for the poloidal beam width



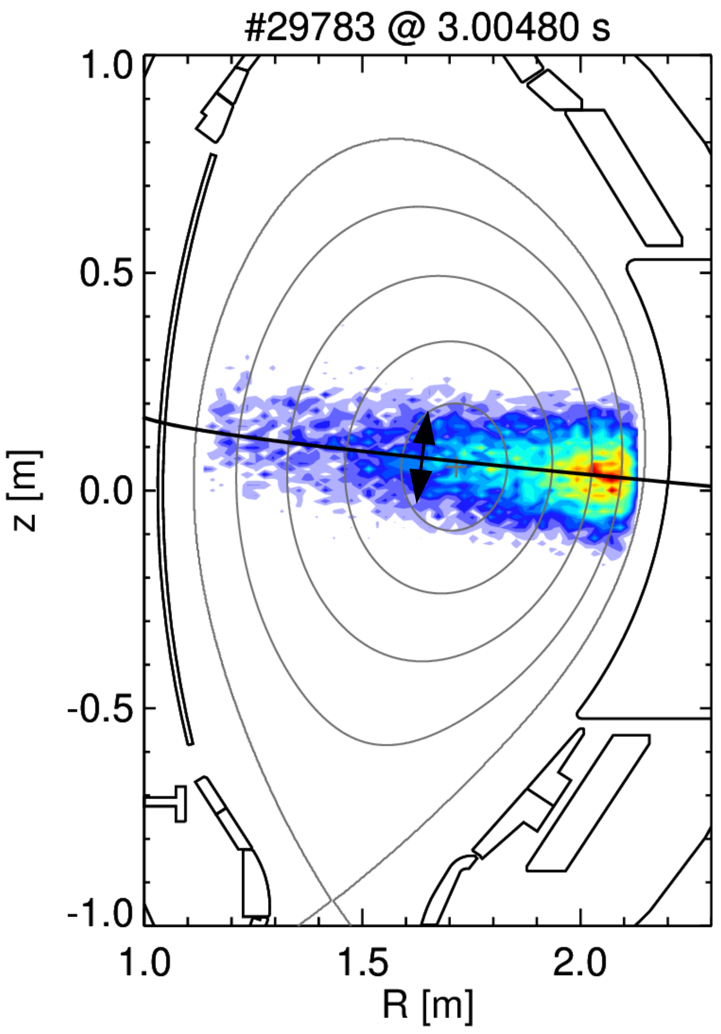
- We assume Gaussian spreading along orange line (standard deviation σ)
- Assume circular concentric flux surfaces
- Transformation between flux coordinate ρ and geometric radius r based on ratio at B: $r(\rho) = \rho * (r_b / \rho_b)$
- → Crossings ■ with ρ-cells can be calculated analytically
- → Contribution into i-th cell ρ_i:

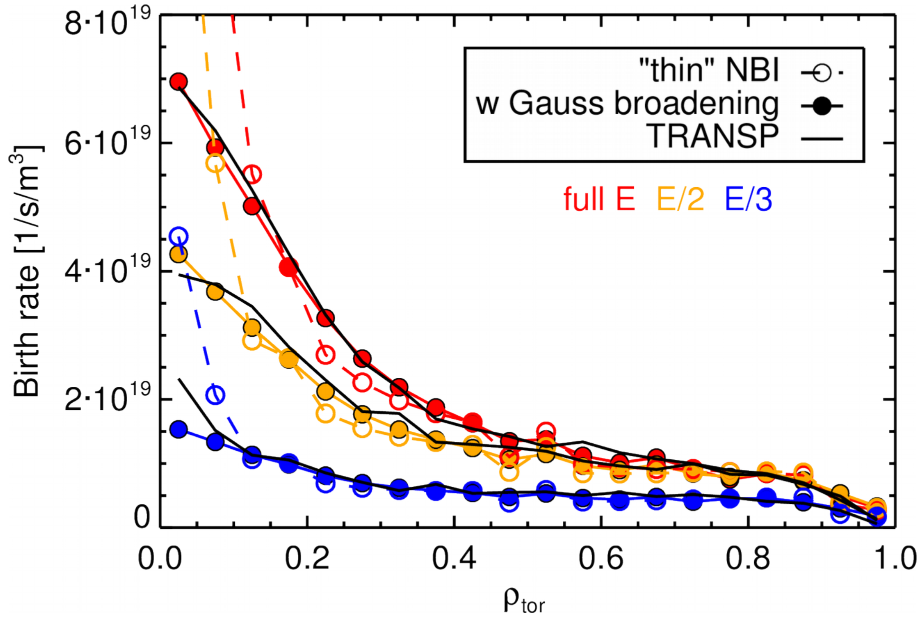
$$\frac{1}{2}\left(\mathrm{erf}\,\frac{h_{2u}}{\sqrt{2}\sigma} - \mathrm{erf}\,\frac{h_{1u}}{\sqrt{2}\sigma}\right) + \frac{1}{2}\left(\mathrm{erf}\,\frac{h_{2l}}{\sqrt{2}\sigma} - \mathrm{erf}\,\frac{h_{1l}}{\sqrt{2}\sigma}\right)$$

- Correction for plasma elongation: Scale beam width σ according to elongation b/a at B.

$$\sigma = \sigma_0 \cdot \sqrt{\frac{(a\cos\theta)^2 + (b\sin\theta)^2}{(a\cos\beta)^2 + (b\sin\beta)^2}}$$

Beam deposition (birth profile) with beam-width correction





- Taking into account a Gaussian broadening of the beam leads to good agreement with TRANSP/NUBEAM

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3. Orbit effects

$$\boldsymbol{a} = \frac{q}{m}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right)$$

4. Time dependence (=0 for steady state solution)

2. collisions (e.g. slowing down, pitch angle scattering)

1. Source = NBI deposition

Analytic solution of the Fokker-Planck equation

$$\frac{1}{\tau_s v^2}\frac{\partial}{\partial v}[(v^3+v_c^3)f] + \frac{\beta}{\tau_s}\frac{v_c^3}{v^3}\frac{\partial}{\partial \xi}(1-\xi^2)\frac{\partial f}{\partial \xi} + \frac{1}{\tau_s v^2}\frac{\partial}{\partial v}\left[\left(\frac{T_e}{m_{fi}}v^2+\frac{T_i}{m_{fi}}\frac{v_c^3}{v}\right)\frac{\partial f}{\partial v}\right] = \frac{\partial f}{\partial t} - \frac{S}{2\pi v^2}\delta(v-v_0)K(\xi)$$

slowing down | pitch angle scattering | speed diffusion | source term

S: deposition, v0: injection velocity (mono-energetic)
K(ξ): broad pitch distribution

- Solution [Cordey/Core 74]:

Legendre polynomials

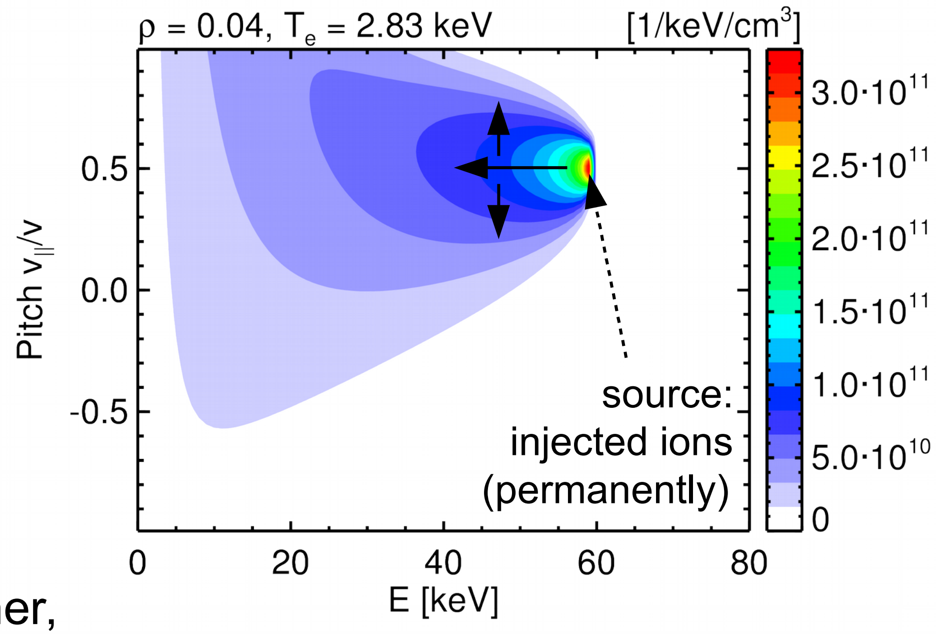
$$f(v,\xi) = \frac{1}{2\pi}\frac{S\cdot\tau_S}{v^3+v_C^3}\cdot\sum_{l=0}^{\infty}\left(l+\tfrac{1}{2}\right)u^{l(l+1)}P_l(\xi)K_l\cdot H(v_0-v)$$

v_C, v_0 : critical, injection velocity

$$K_l = \int K(\xi)P_l(\xi)\,d\xi$$

$$u = \left(\frac{v_0^3+v_C^3}{v^3+v_C^3}\frac{v^3}{v_0^3}\right)^{\pm\beta/3} \qquad \beta = \frac{Z_{eff}}{2}\frac{m_i}{m_{beam}}$$

$$\tau_S = 6.27\cdot 10^8\cdot\frac{A_b}{Z_b^2\ln\Lambda_e}\cdot\frac{(T_e\,[\mathrm{eV}])^{3/2}}{n_e\,[\mathrm{cm}^{-3}]}\ \mathrm{s} \qquad v_C^3 = 1.51\cdot 10^{14}\cdot(T_e\,[\mathrm{eV}])^{3/2}\cdot\frac{\sum_i n_i Z_i^2\ln\Lambda_i/A_i}{n_e\ln\Lambda_e}$$



- Uniform plasma solution, i.e. each radial cell is independent of each other, no particle trapping etc.
- A correction for speed diffusion is applied above injection energy.

Adding the effect of speed diffusion (above injection energy)

$$\frac{1}{\tau_s v^2}\frac{\partial}{\partial v}[(v^3+v_c^3)f] + \frac{\beta}{\tau_s}\frac{v_c^3}{v^3}\frac{\partial}{\partial \xi}(1-\xi^2)\frac{\partial f}{\partial \xi} + \frac{1}{\tau_s v^2}\frac{\partial}{\partial v}\left[\left(\frac{T_e}{m_{fi}}v^2 + \frac{T_i}{m_{fi}}\frac{v_c^3}{v}\right)\frac{\partial f}{\partial v}\right] = \frac{\partial f}{\partial t} - \frac{S}{2\pi v^2}\delta(v-v_0)K(\xi)$$

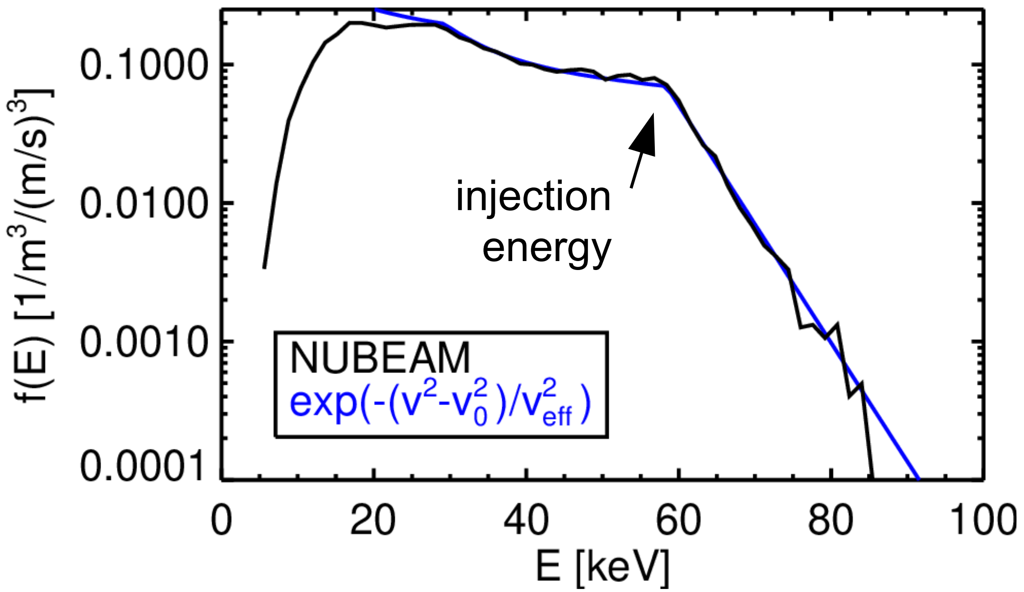
slowing down — pitch angle scattering — speed diffusion — source term

S: deposition, v0: injection velocity (mono-energetic)
K(ξ): broad pitch distribution

- Speed diffusion creates high energy tail above the injection energy
- assuming v ≈ v_0 :

$$f(v) = \frac{1}{2\pi}\frac{S\tau_s}{v_0^3+v_c^3}\exp\frac{-(v^2-v_0^2)}{v_{eff}^2}$$

$$v_{eff}^2 = \frac{2}{m_{fi}}\frac{T_e v_0^3 + T_i v_c^3}{v_0^3+v_c^3} \equiv \frac{2T_{eff}}{m_{fi}}$$

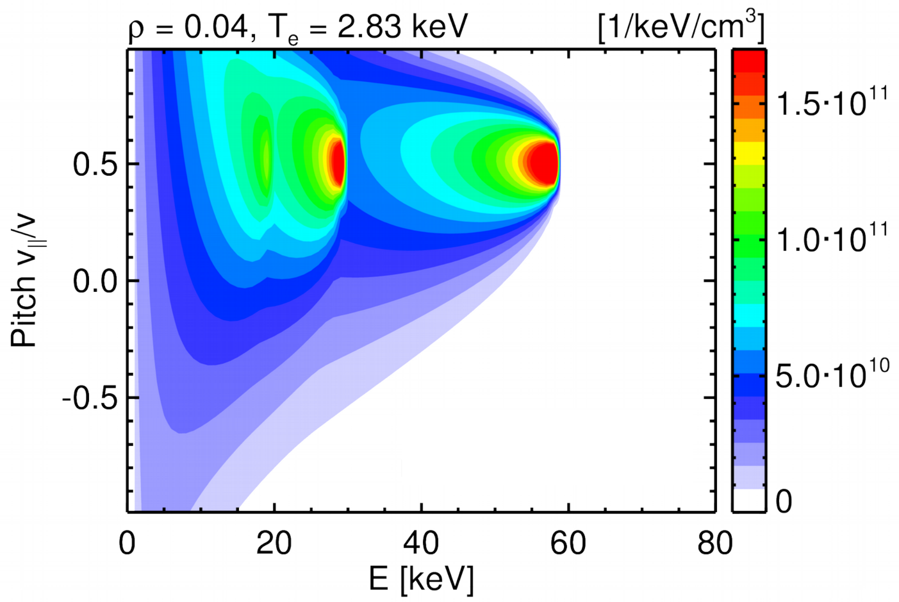


- Only few particles, but high energies → relevant for e.g. fast-ion pressure and neutron rates

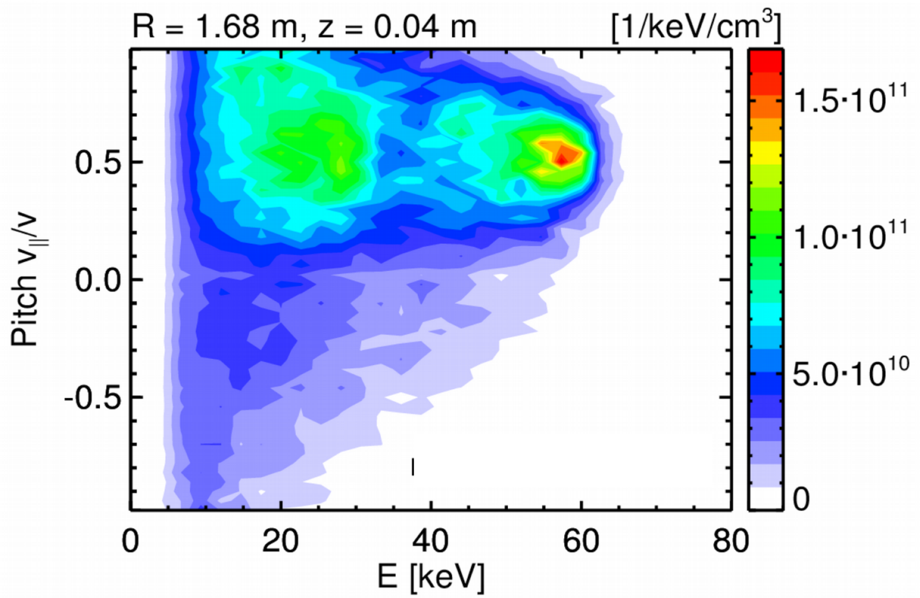
The full distribution function in the plasma center

- For full distribution function, add up all 3 energy components of NBI
- Comparison to TRANSP shows good agreement!

Model (w/o speed diffusion)



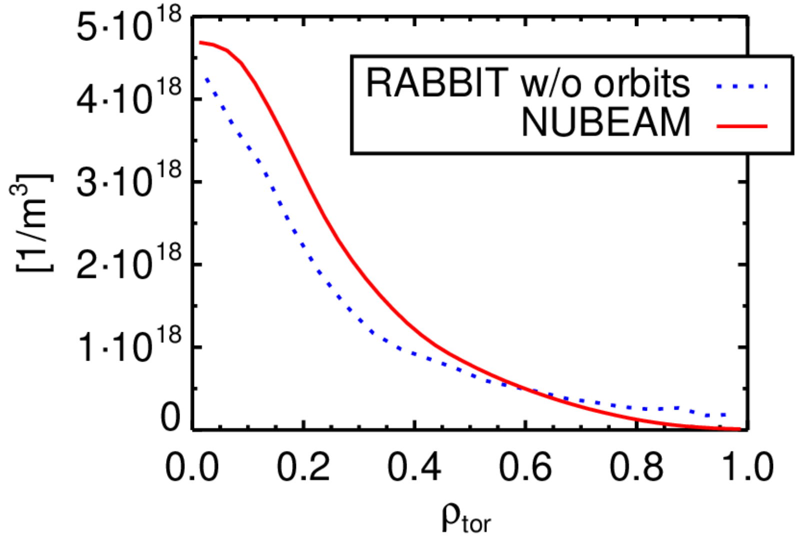
TRANSP

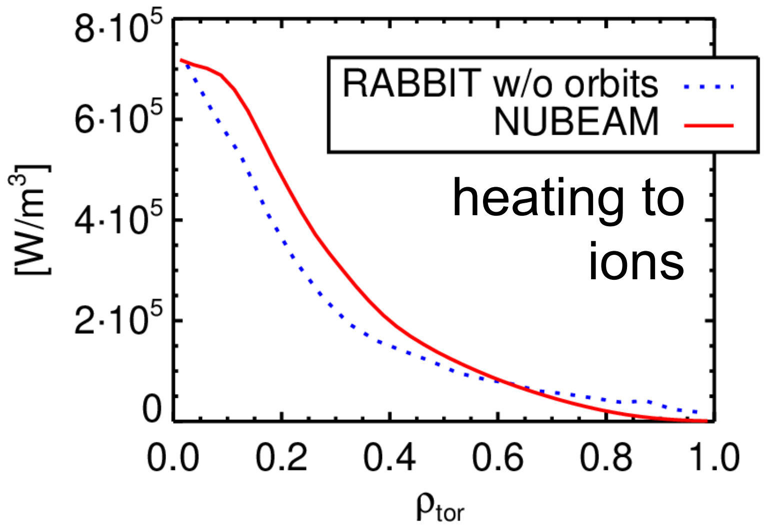


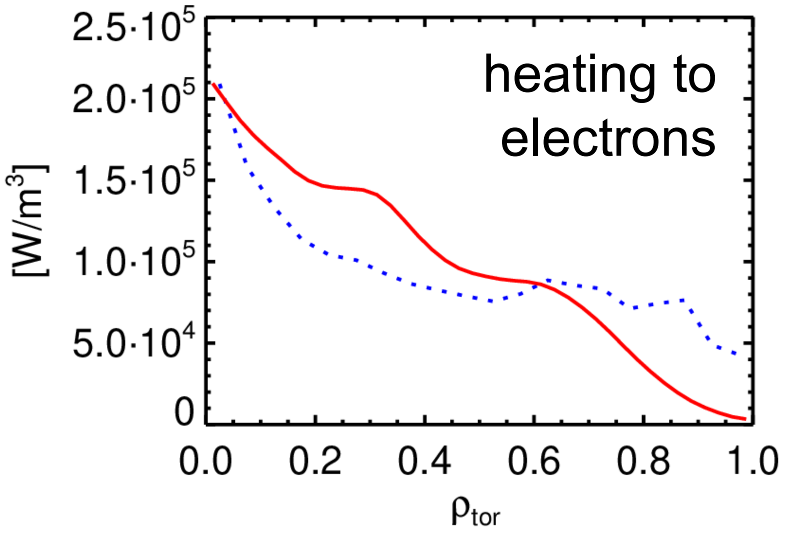
Density and heating profiles

$$f(v,\xi) = \frac{1}{2\pi}\frac{S\cdot\tau_S}{v^3+v_C^3}\cdot\sum_{l=0}^{\infty}\left(l+\tfrac{1}{2}\right)u^{l(l+1)}P_l(\xi)K_l\cdot H(v_0-v)$$

- In the end, we are interested in integrals of f, e.g.:
  Heating power (to electrons and ions), fast-ion pressure and current drive
- These integrals can also be solved analytically. Due to orthogonality of the Legendre polynomials, only first few moments are necessary (l=0, 1)
- E.g. fast-ion density: $n_{fi} = \iint 2\pi v^2\cdot f(v,\xi)\,dv\,d\xi = \frac{S\tau_S}{3}\ln\left(\frac{v_0^3+v_C^3}{v_C^3}\right)$







- Profile shapes do not (yet) agree well, due to missing orbit-effects

Kinetic equation - outline

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4. Time dependence (=0 for steady state solution)

2. collisions (e.g. slowing down, pitch angle scattering)

1. Source = NBI deposition

- In MC codes (e.g. NUBEAM)
  - MC representation of source
  - Calculate orbits for each MC marker
  - Apply collision operator during orbit steps
- For real-time: Only ad-hoc treatment of orbit effects possible

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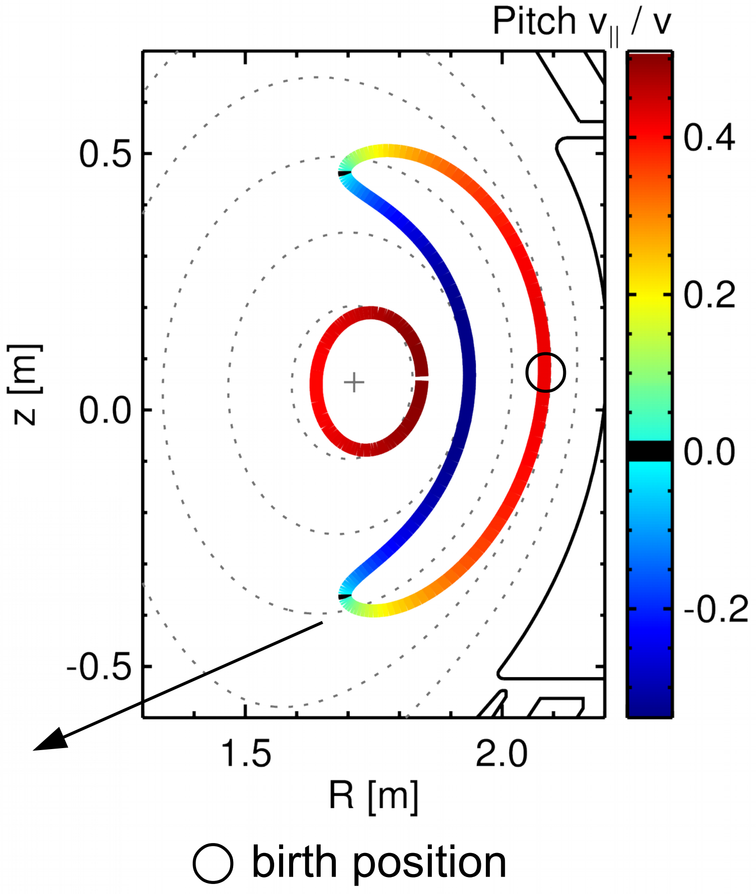
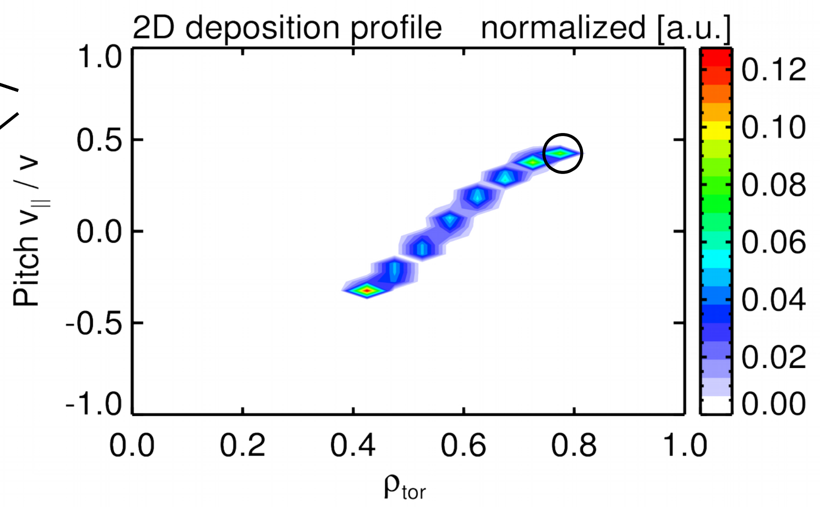
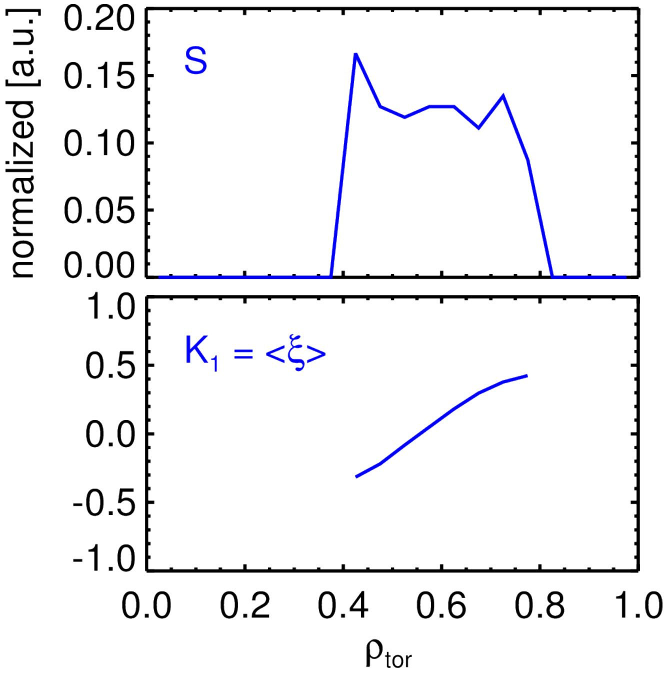
How to include the effect of first fast-ion orbit

- Orbit effects lead to a broadened deposition (towards the plasma center) and to changes of the pitch-distribution in the velocity space
- They can be taken into account, by averaging the deposition over the first orbit:
  - ➔ assume that slowing-down process starts on random position of first orbit
  - ➔ neglect orbit effects during slowing down



example for banana orbit:

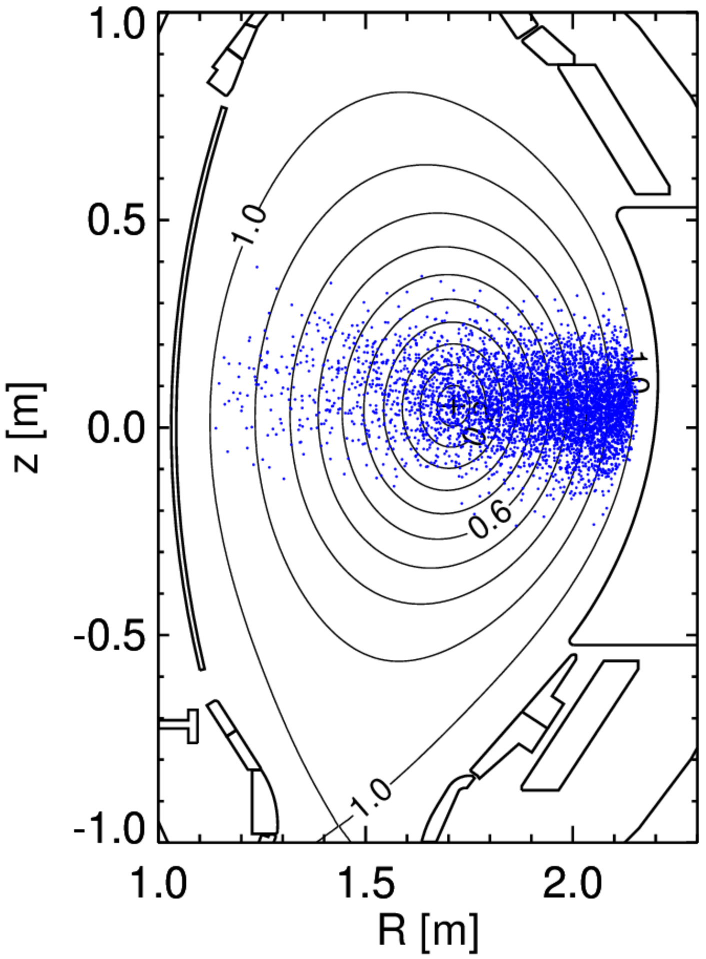




Monte-Carlo orbit-average is too slow for real-time applications

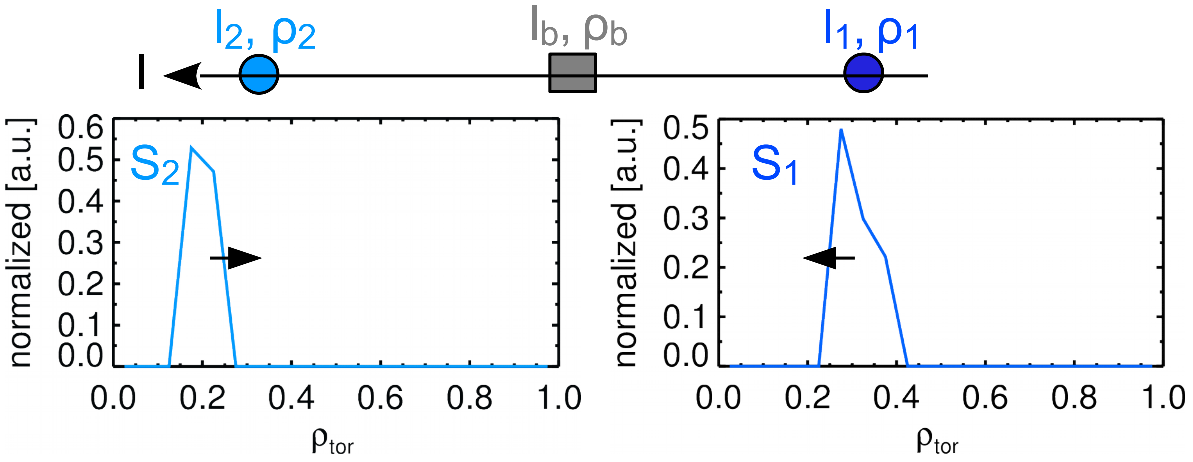
- Monte-Carlo orbit-average:
  - Take MC representation of birth distribution
  - Calculate orbit for each MC marker (e.g. ~5000)
  - → too slow for real-time purposes (takes ~1s)

- Possible solutions:
  - Either: Use approximation formulas for the orbits
  - Or: Reduce number of orbits (strongly)

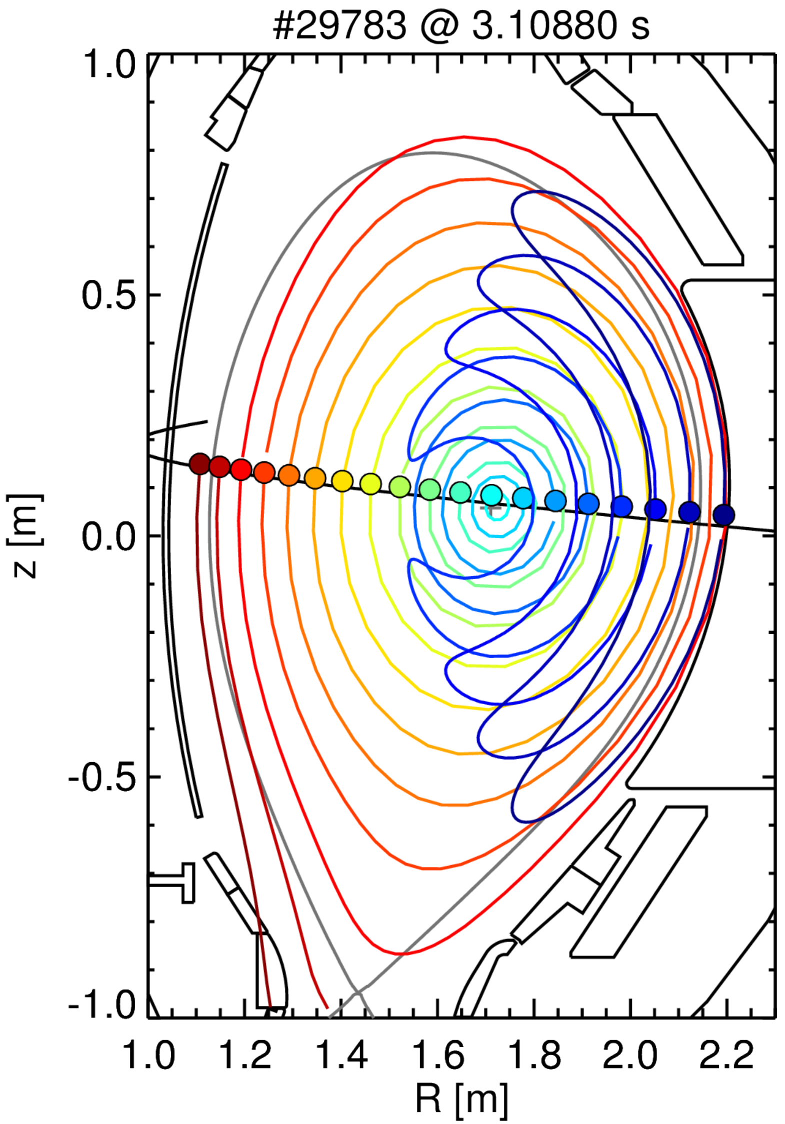


Orbit average in RABBIT

- Calculate orbit only every n-th grid point
  (4^{th} order Runge-Kutta guiding center integrator)
  Right: All calculated orbits for full energy component
- Here: 19 orbits x 3 energy components
  → possible within ~10 ms.
- In between: Shift neighboring profiles and interpolate linearly

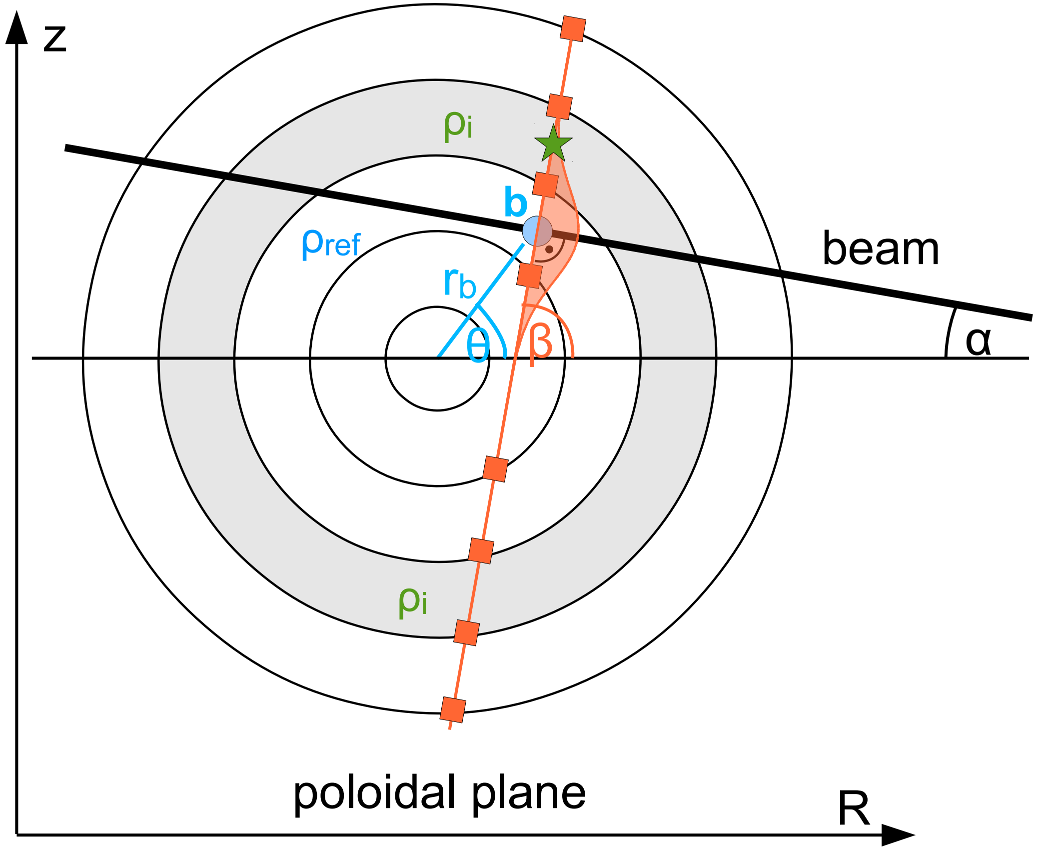


$$S_b(\rho) = S_1(\rho - (\rho_b - \rho_1)) \cdot \frac{l_2 - l_b}{l_2 - l_1} + S_2(\rho - (\rho_b - \rho_2)) \cdot \frac{l_b - l_1}{l_2 - l_1}$$



[Weiland et al., NF, 2018]

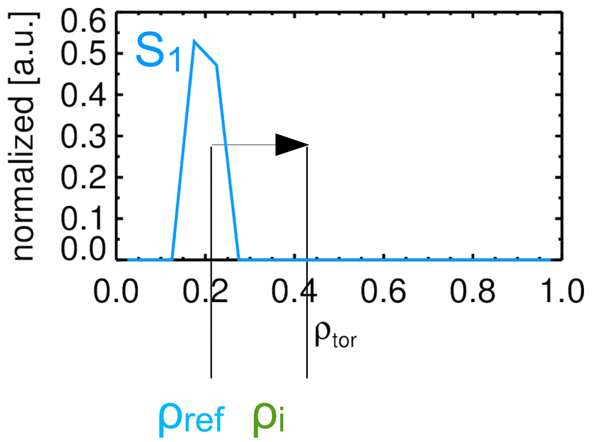
Orbit-average: compatible with beam-width model



- Up to now, we have calculated the orbit-average along the beam (at **b**).
- For the beam-width correction, we need to extrapolate from the ρ-cell containing **b** (ρ_{ref}) along the orange line to other radial cells
- E.g. from ρ_{ref} to ★(ρ_i):

$$S_i(\rho) = S_b(\rho - (\rho_i - \rho_{ref}))$$

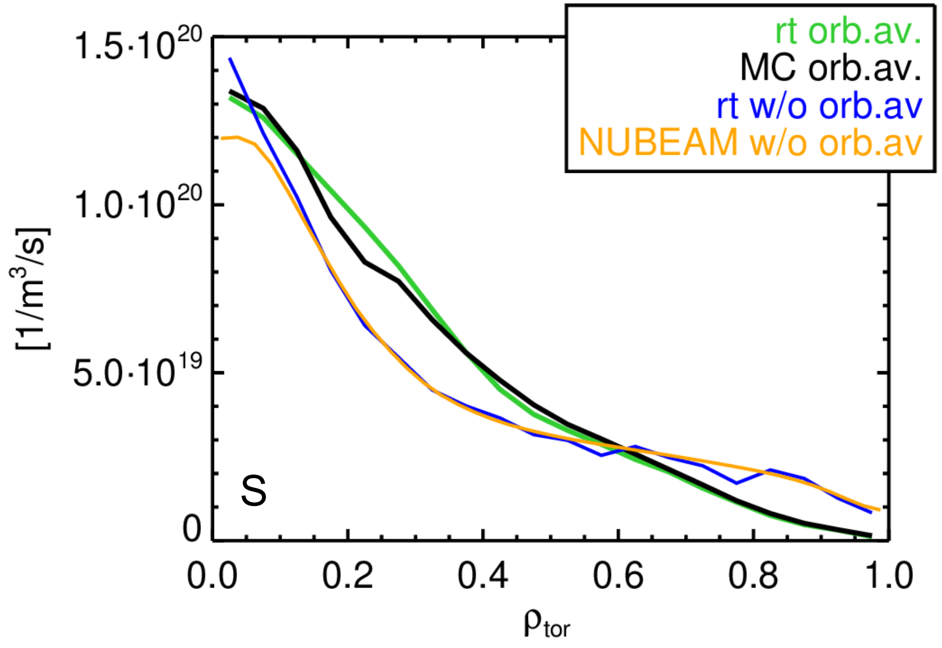
(similar to the interpolation method)



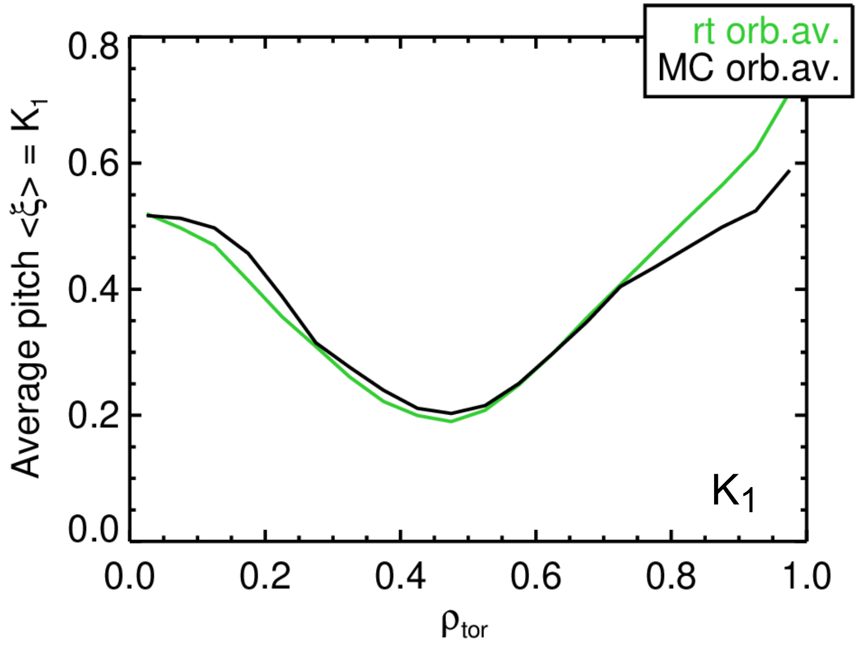
Results of „RABBIT orbit average“ in good agreement with MC orbit average

- Test accuracy of the RABBIT rt orbit average:
  Compare it to Monte-Carlo orbit average (including fully realistic NBI geometry)
- Very good agreement is found, despite orders of magnitude difference in calculation time (~5000 orbits vs. ~60 orbits)

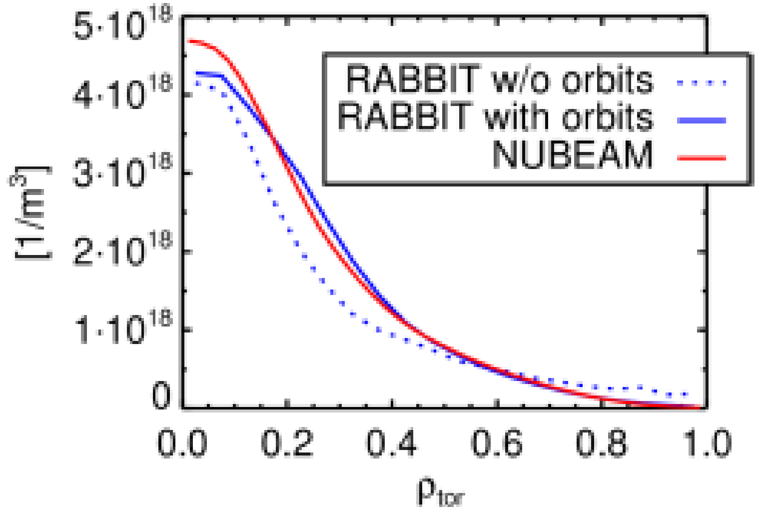
Birth profile (sum over all E components):



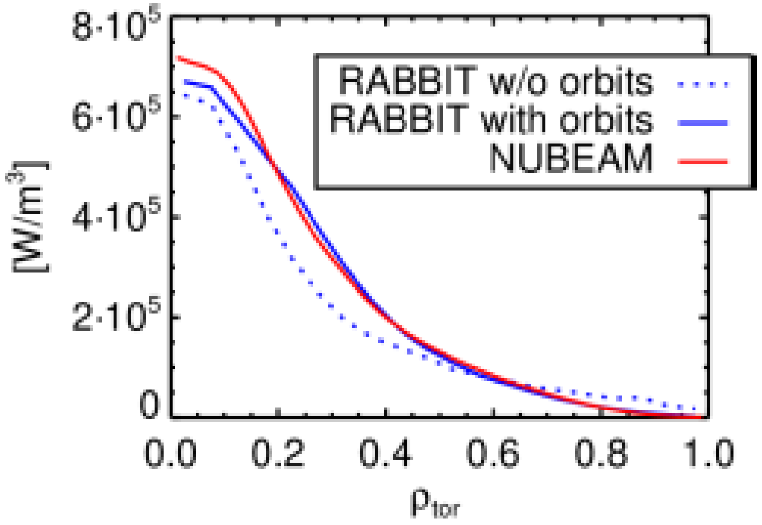
Average pitch $v_{||}/v$ of full-E component:



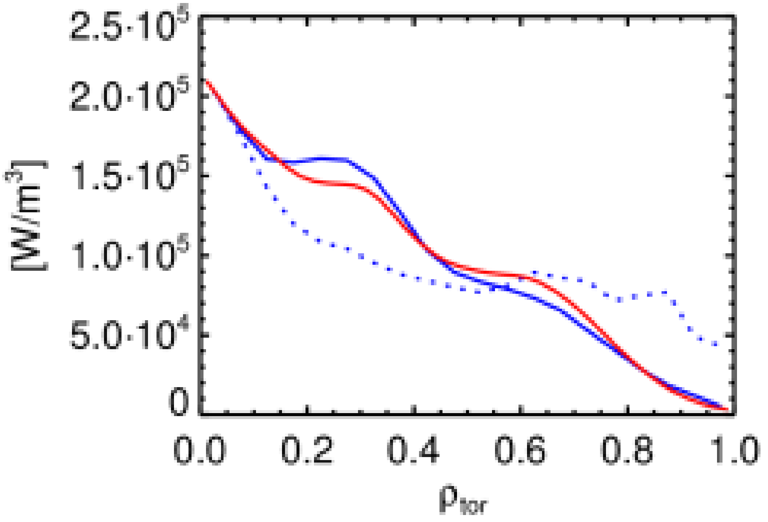
Comparison to NUBEAM



Fast-ion density



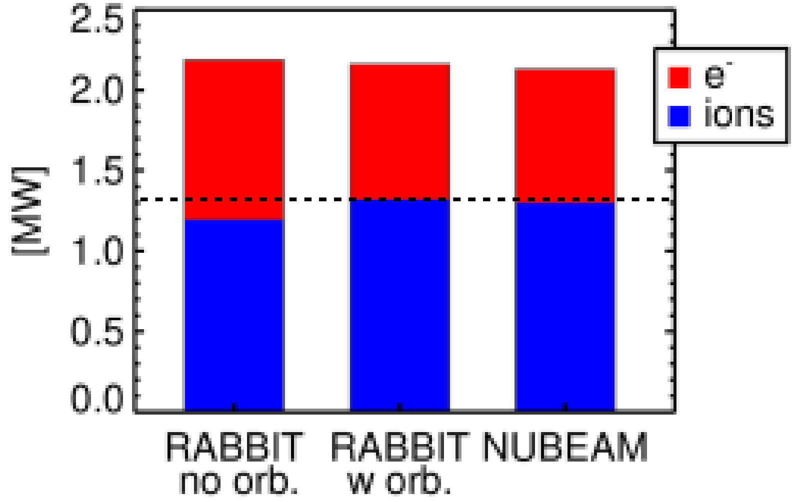
Heating to ions



Heating to electrons

- Orbit-average leads to good agreement in profile shape

- Orbit-average has also an impact on volume-integrated heating distribution to electrons/ions and improves agreement



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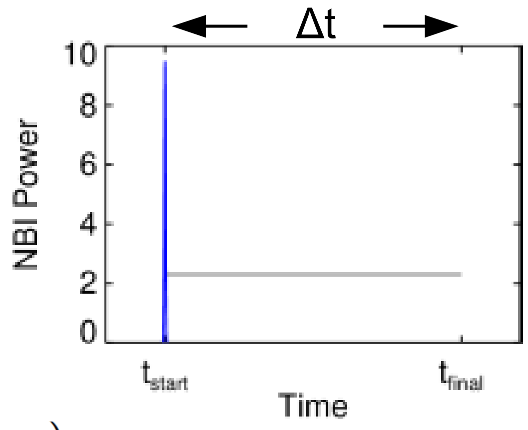
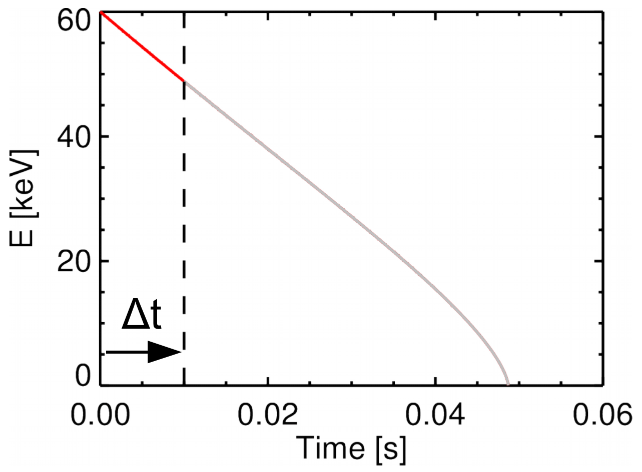
Time dependence

- Up to now: steady-state solution of Fokker-Planck equation f_{SS}
- For time-dependent simulation: Discrete time steps Δt, assume inputs are constant during each time-step.
- Model NBI with a δ-function-like pulse at the beginning of each time step →



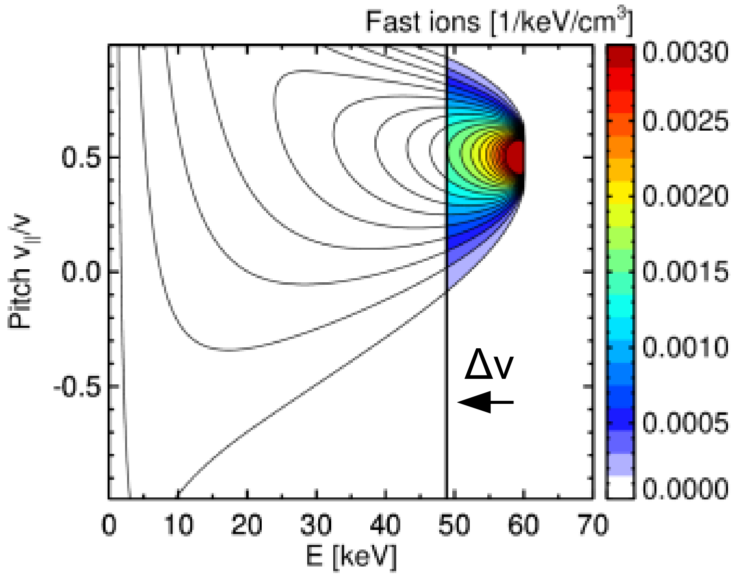
- Calulate how far the fast-ion pulse slowes down during time-step:

$$v_{final}^3 = (v_{start}^3 + v_c^3) \cdot \exp \frac{-3 \cdot \Delta t}{\tau_s} - v_c^3$$

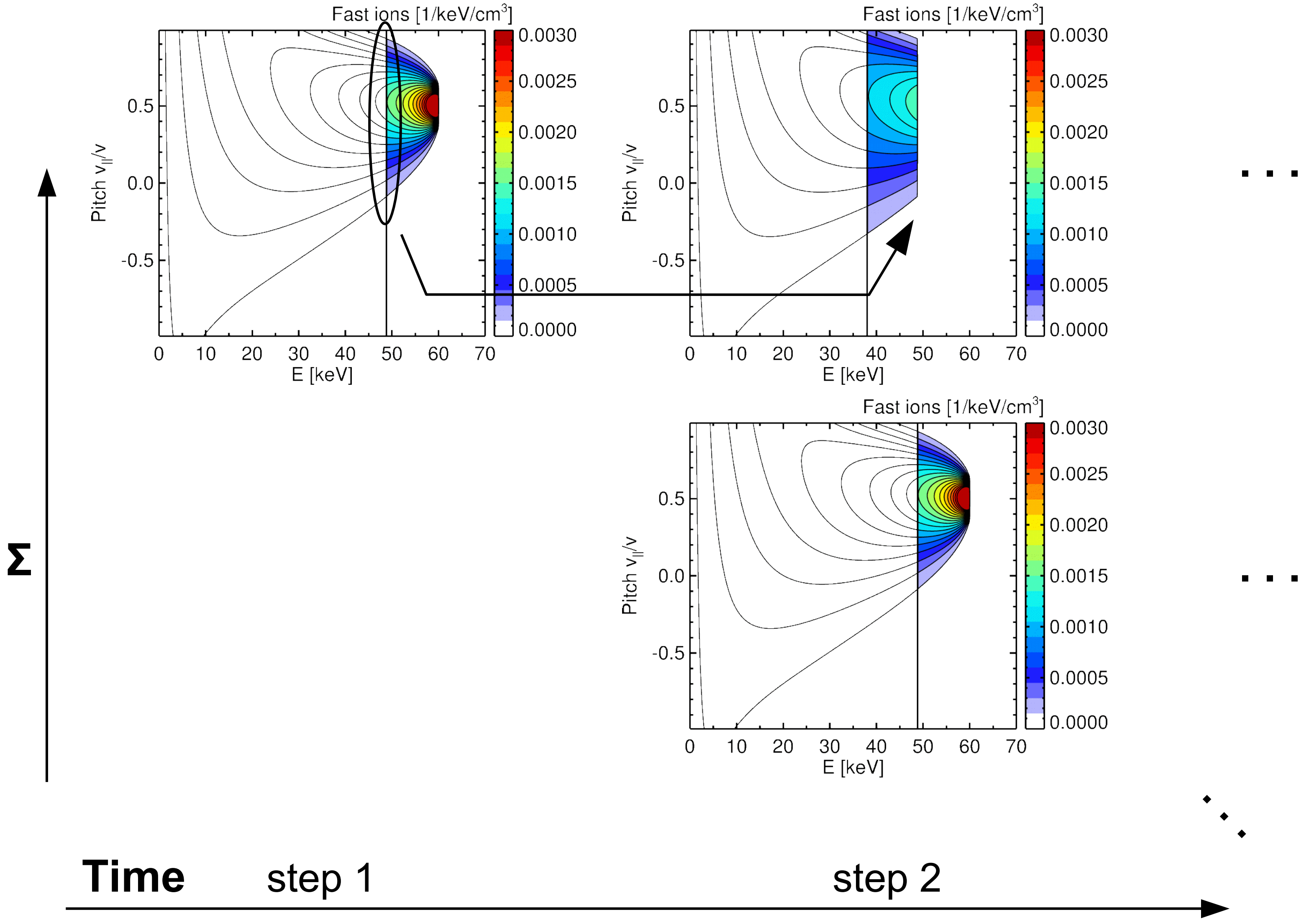


→ multiply f_{SS} with box function

$$\int_{t_{start}}^{t_{final}} dt \rightarrow f(v, \xi) = f_{ss}(v, \xi) \cdot H(v - v_{final})$$

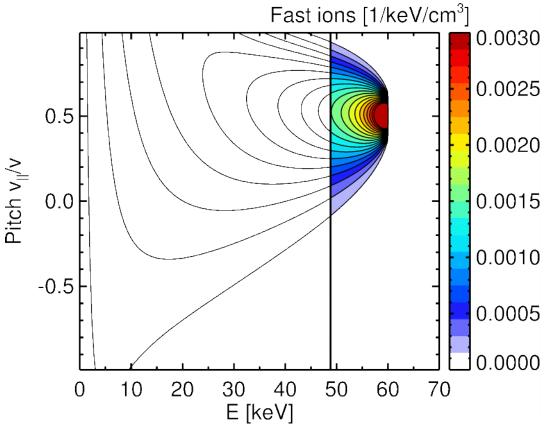


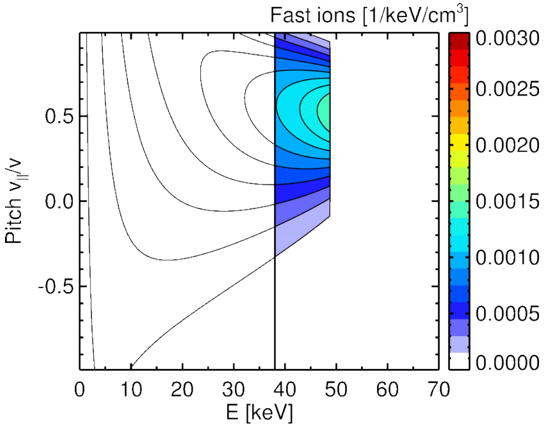
Time dependence via train of fast-ion pulses



- Final state of „step 1“ is starting point of „step 2“
- If beam is still turned on in „step 2“, add a new pulse at nominal injection energy
- continue …
  (add new rows each time-step, sum over rows)

Time dependence





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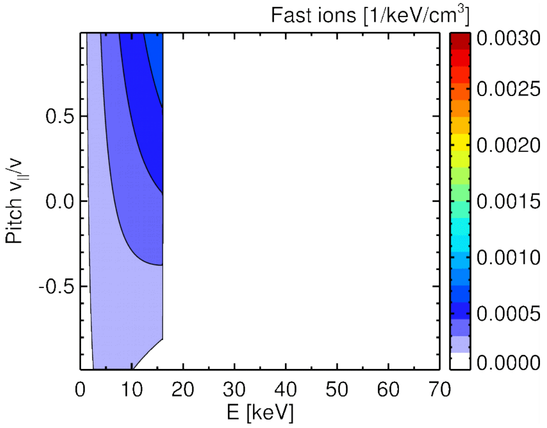

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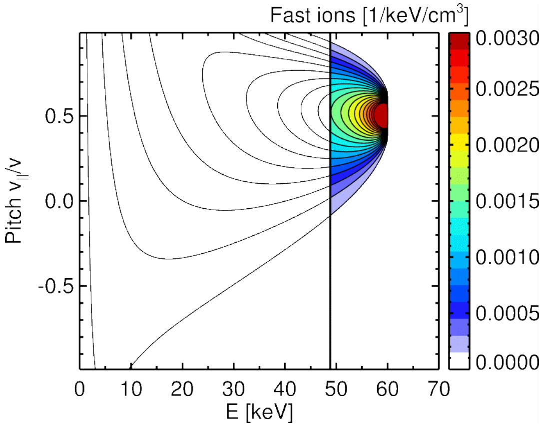

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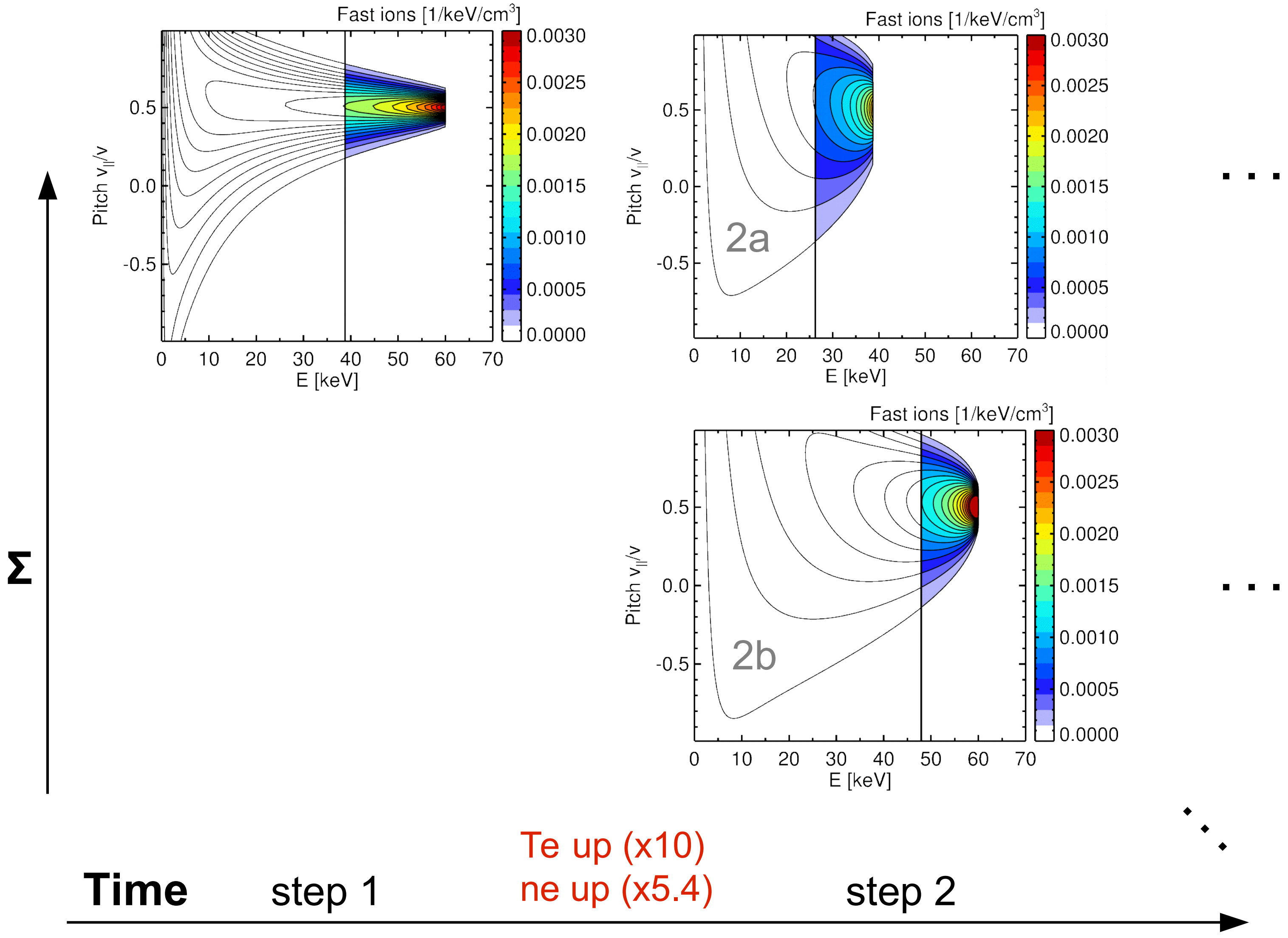
- A row is terminated once its fast-ion pulse has slowed down to zero.
- → If plasma parameters are constant, we eventually reach the steady state solution ✓

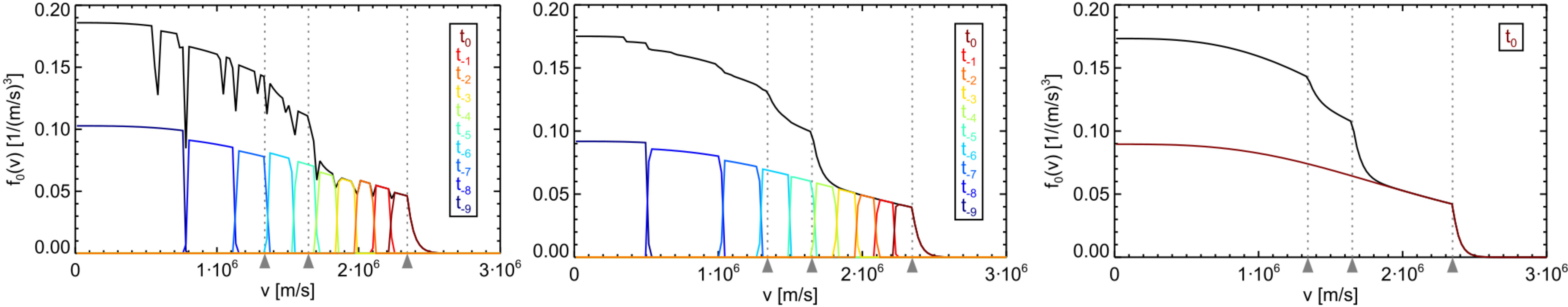
Time step 1 step 2

Time dependence



- Changes of plasma parameters are treated consistently
- E.g. different f_{SS} in 2a and 2b, because the fast-ions in 2a have had a different „past history“ - they had different plasma parameters in „step 1“

Steady-state solution procudes well-behaved gradients



Time-dependent solution, background plasma changes over time

Time-dependent solution, constant background plasma

→ still some discontinuities between individual pulses: due to (weak) v-dependence of FP-coefficents

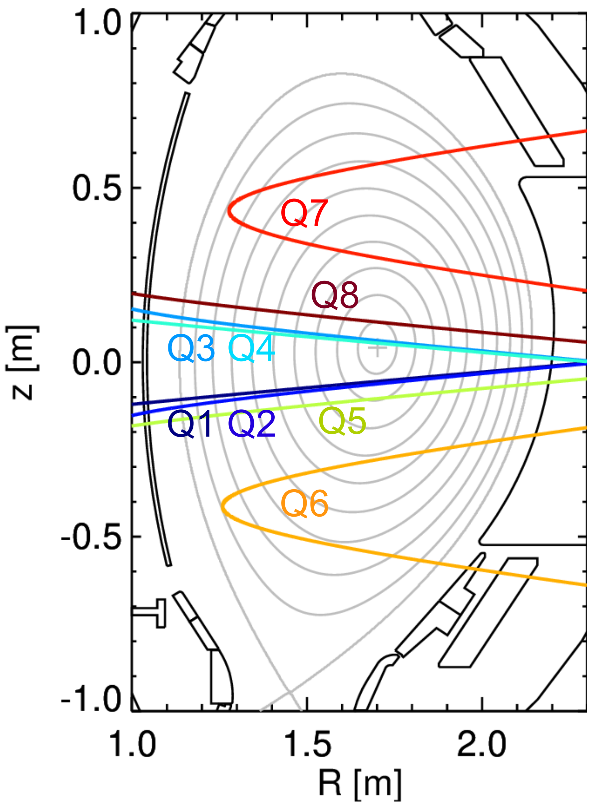
Steady-state solution, well-behaved gradients

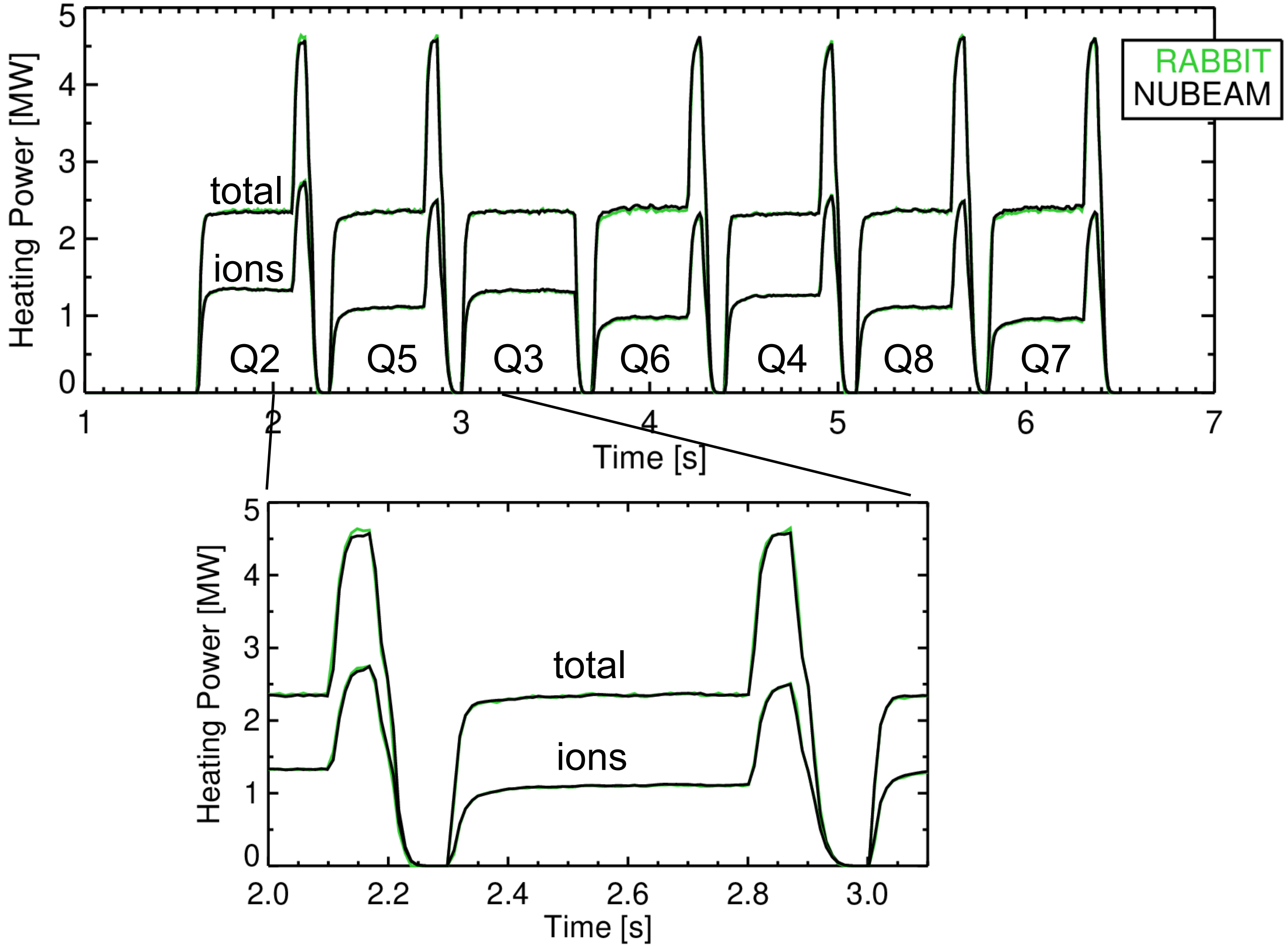
Technically this is done by setting the RABBIT time-step to a value larger than the slowing down time.

[Weiland et al., NF 2019]

Comparison of time evolution with NUBEAM

- Analyze discharge where different NBI sources (Q#) are interchanged
- Good agreement of temporal evolution

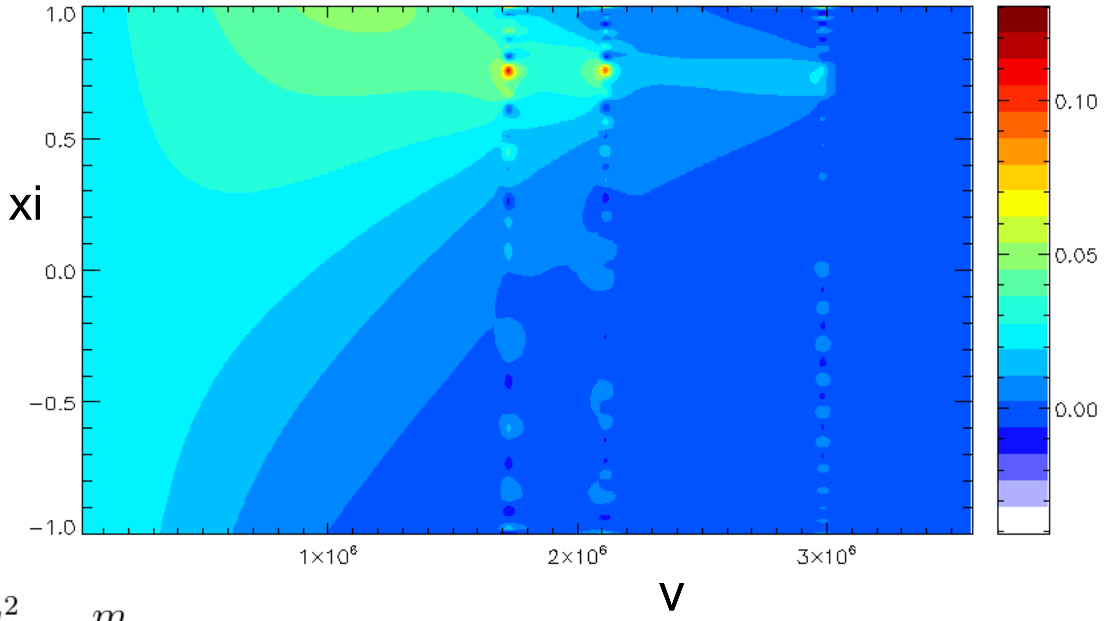




Summary & Conclusion

- For f, Rabbit solves $\frac{\partial f}{\partial t} = -\nabla_{\vec{v}} \cdot \vec{\Gamma}_c(f) + \langle\sigma\rangle(\rho, v, \xi)$, where <σ> is birth profile averaged over first orbit
- f can be interpreted as flux-surface averaged fast ion (NBI) distribution function, f(rho_tor, v, xi)
- For pitch xi, Legendre decomposition is used, such that the actual output is:
  f_l (rho_tor, v) with f(rho_tor, v, xi) = Sum_l (f_l (rho_tor, v) P_l(xi))

- Potential issues: (?)
- Numerical problems with Legendre Polynomials:
  - When NBI birth distribution is narrow in pitch xi, the Legendre series shows oscillations (that get worse with higher L's).
  - Also: negative values of f



- Mapping of f(rho_tor, v, xi) to constants of motion (COM)
  - No unique map to $P_\varphi = mR\frac{B_{tor}}{B}v_\parallel + q\Psi$ and mu ($\frac{m}{q} \cdot \frac{1}{2}\frac{mv_\perp^2}{B} \equiv \frac{m}{q} \cdot \mu$)