Excitation of TAE modes by an electromagnetic antenna using the global gyrokinetic code ORB5

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ORB5 eqs. of motion

Consider evolution of reduced velocity distribution function $f(\boldsymbol{R}, v_{||}, \mu; t)$

$$\frac{\partial \delta f_s}{\partial t} + \dot{\boldsymbol{R}} \cdot \frac{\partial \delta f_s}{\partial \boldsymbol{R}}\Big|_{v_{||}} + \dot{v}_{||} \frac{\partial \delta f_s}{\partial v_{||}} = -\dot{\boldsymbol{R}}^{(1)} \cdot \frac{\partial F_{0s}}{\partial \boldsymbol{R}}\Big|_{\epsilon} - \dot{\epsilon}^{(1)} \frac{\partial F_{0s}}{\partial \epsilon} \ . \qquad (1.1)$$

The 0th/1st-order gyrocenter characteristics follow

$$\dot{\boldsymbol{R}}^{(0)} = v_{||} \boldsymbol{b}^* + \frac{1}{q B_{||}^*} \boldsymbol{b} \times \mu \nabla B,$$

$$\dot{v}_{||}^{(0)} = -\frac{\mu}{m} \boldsymbol{b}^* \cdot \nabla B \ ,$$

$$\dot{\boldsymbol{R}}^{(1)} = \frac{\boldsymbol{b}}{B_{||}^*} \times \nabla \langle \phi - v_{||} A_{||}^{(s)} - v_{||} A_{||}^{(h)} \rangle - \frac{q_s}{m_s} \langle A_{||}^{(h)} \rangle \boldsymbol{b}^*,$$

$$\dot{v}_{||}^{(1)} = -\frac{q_s}{m_s} \left[\boldsymbol{b}^* \cdot \nabla \langle \phi - v_{||} A_{||}^{(h)} \rangle + \frac{\partial \langle A_{||}^{(s)} \rangle}{\partial t} \right] - \mu \frac{\boldsymbol{b} \times \nabla B}{B_{||}^*} \cdot \nabla \langle A_{||}^{(s)} \rangle,$$

and particle energy $\dot{\epsilon}^{(1)} = v_{||} \dot{v}_{||}^{(1)} + \mu \dot{\boldsymbol{R}}^{(1)} \cdot \nabla B,$

where $\mu = v_\perp^2/(2B)$, $B_{||}^* = \boldsymbol{b} \cdot \nabla \boldsymbol{A}^*$, $\boldsymbol{b}^* = \nabla \times \boldsymbol{A}^*/B_{||}^*$, and $\boldsymbol{A}^* = \boldsymbol{A} + \frac{m_s v_{||}}{q_s} \boldsymbol{b}$.

ϕ is computed from the quasineutrality equation

$$-\nabla \cdot \left[\left(\sum_{s=i,f} \frac{q_s^2 n_s}{T_s} \rho_s^2 \right) \nabla_\perp \phi \right] = \sum_{s=i,e,f} q_s n_{1,s} \tag{1.2}$$

where $n_{1,s} = \int \delta f_s \delta(\boldsymbol{R} + \boldsymbol{\rho} - \boldsymbol{x}) d^6 Z$, $\rho_s = \sqrt{m_s T}/(q_s B)$, and $d^6 Z = B_{||}^* d\boldsymbol{R}\ dv_{||}\ d\mu\ d\alpha$.

Given the decomposition $A_{||} = A_{||}^{(s)} + A_{||}^{(h)}$, symplectic part is computed from Ohm's law

$$\frac{\partial}{\partial t} A_{||}^{(s)} + \boldsymbol{b} \cdot \nabla \phi = 0, \tag{1.3}$$

and the Hamiltonian part from Ampere's law

$$\left(\sum_{s=i,e,f} \frac{\beta_s}{\rho_s^2} - \nabla_\perp^2 \right) A_{||}^{(h)} = \mu_0 \sum_{s=i,e,f} j_{||,1s} + \nabla_\perp^2 A_{||}^{(s)}, \tag{1.4}$$

$$\text{and} \quad j_{||,1s} = \int v_{||} \delta f_s \delta(\boldsymbol{R} + \boldsymbol{\rho} - \boldsymbol{x}) d^6 Z. \tag{1.5}$$

Pullback scheme

Initialize markers in the phase space;

while $t < t_{\text{final}}$ **do**

  for $k = 1, ..., 4$ *step of Runge-Kutta scheme* **do**

    - Compute ϕ, $A_{||}^{(s)}$ and $A_{||}^{(h)}$;
    - Push particles according to equations of motion;
    - Apply boundary conditions;

  end

  - Update mixed-variable δf with $\delta f_s^{(m)} \leftarrow \delta f_s^{(m)} + \frac{q_s \langle A_{||}^{(h)} \rangle}{m_s} \frac{\partial F_{0s}}{\partial v_{||}}$;
  - Update $A_{||}$ decomposition, i.e, $A_{||}^{(s)} \leftarrow A_{||}^{(s)} + A_{||}^{(h)}$ and $A_{||}^{(h)} \leftarrow 0$;
  - $t = t + \Delta t$;

end

Algorithm 1: The δf solution algorithm used in ORB5 within the pull-back scheme framework in linear setting

Antenna

We devise antenna as an electrostatic potential with profile

$$\phi_{\mathrm{ant}}(s, \theta, \varphi; t) = \mathrm{Re}\left[\sum_{l \in \mathcal{T}} h_l(s) A_l e^{\hat{i}(l_1\theta + l_2\varphi + \Phi_l)} (c_1 + c_2 e^{\hat{i} w_{\mathrm{ant}} t})\right] \quad (1.6)$$

where the set $\mathcal{T} = \{(m_1, n_1), ...\}$ includes the target mode numbers. Electromagnetic potential of antenna from Ohm's law is computed

$$\frac{\partial}{\partial t} A^{(s)}_{\mathrm{ant},||} + \boldsymbol{b} \cdot \nabla \phi_{\mathrm{ant}} = 0. \quad (1.7)$$

Eqs. of motion are adapted by setting $\phi_{\mathrm{plasma}} \rightarrow \phi_{\mathrm{plasma}} + \phi_{\mathrm{ant}}$.

Integrating electrostatic antenna in the eqs. of motion

$$\dot{\boldsymbol{R}}^{(1)} = \dot{\boldsymbol{R}}^{(1)}_{\mathrm{plasma}} + \frac{\boldsymbol{b}}{B^*_{||}} \times \nabla \langle \phi_{\mathsf{ant}} \rangle, \tag{1.8}$$

$$\dot{v}^{(1)}_{||} = \dot{v}^{(1)}_{||,\mathrm{plasma}} - \frac{q_s}{m_s} \boldsymbol{b}^* \cdot \nabla \langle \phi_{\mathsf{ant}} \rangle, \tag{1.9}$$

$$\dot{\epsilon}^{(1)} = \dot{\epsilon}^{(1)}_{\mathrm{plasma}} - \frac{q_s}{m_s} \left[\frac{v_{||} \boldsymbol{B}}{B^*_{||}} + m_s \mu \frac{\boldsymbol{b} \times \nabla B}{q_s B^*_{||}} + \frac{m_s v^2_{||}}{q_s B^*_{||}} \nabla \times \boldsymbol{b} \right] \cdot \nabla \langle \phi_{\mathsf{ant}} \rangle \ . \tag{1.10}$$

Integrating electromagnetic antenna in the eqs. of motion

$$\dot{\boldsymbol{R}}^{(1)} = \dot{\boldsymbol{R}}^{(1)}_{\text{plasma}} + \frac{\boldsymbol{b}}{B^*_{||}} \times \nabla \langle \phi_{\text{ant}} \rangle - \frac{\boldsymbol{b}}{B^*_{||}} \times \nabla \langle v_{||} A^{(s)}_{||,\text{ant}} \rangle, \tag{1.11}$$

$$\dot{v}^{(1)}_{||} = \dot{v}^{(1)}_{||,\text{plasma}} - \frac{q_s}{m_s} (\boldsymbol{b}^* - \boldsymbol{b}) \cdot \nabla \langle \phi_{\text{ant}} \rangle - \frac{\mu}{B^*_{||}} \boldsymbol{b} \times \nabla B \cdot \nabla \langle A^{(s)}_{||,\text{ant}} \rangle, \tag{1.12}$$

and

$$\dot{\epsilon}^{(1)} = \dot{\epsilon}^{(1)}_{\text{plasma}} - \frac{q_s}{m_s} \left[m_s \mu \frac{\boldsymbol{b} \times \nabla B}{q_s B^*_{||}} + \frac{m_s v^2_{||}}{q_s B^*_{||}} \nabla \times \boldsymbol{b} \right] \cdot \nabla \langle \phi_{\text{ant}} \rangle. \tag{1.13}$$

ITPA-TAE test case

Uniform profile of density/temperature for ion and electron.
Ad hoc magnetic field.
$q \in [1.71, 1.87]$, and $\beta = 0.000448$.
kinetic ion/electron.

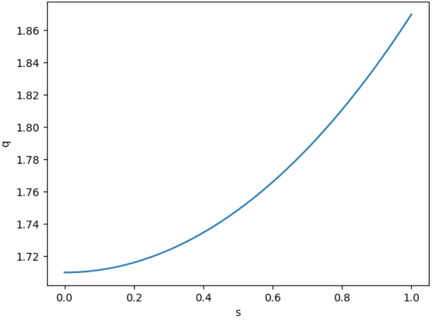
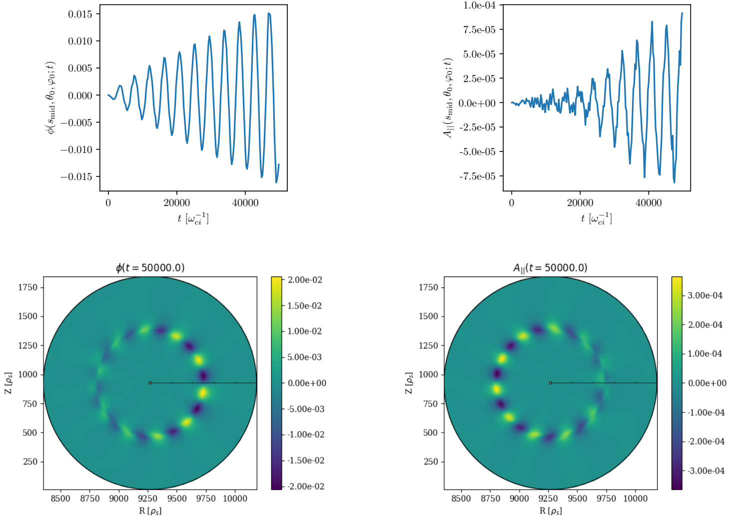
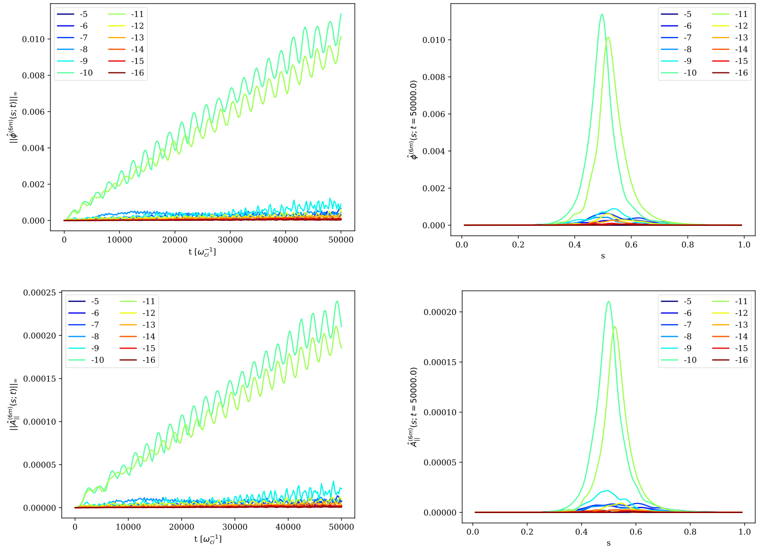


Figure: q profile

Linear simulations: n=6, m=-11,-10



Linear simulations: n=6, m=-10,-11



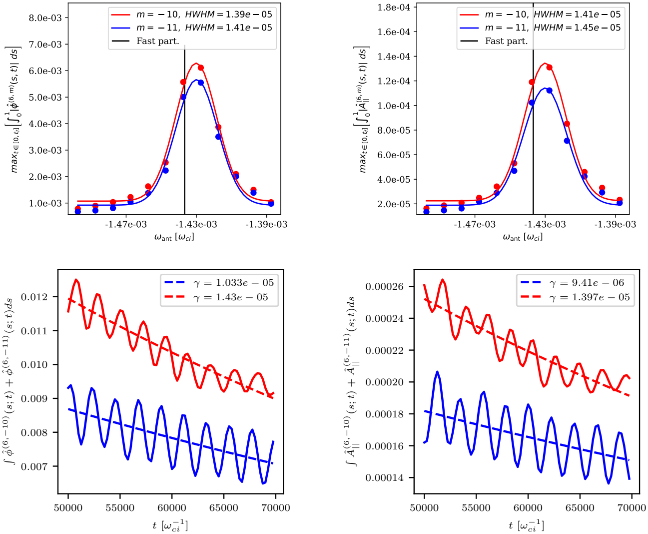
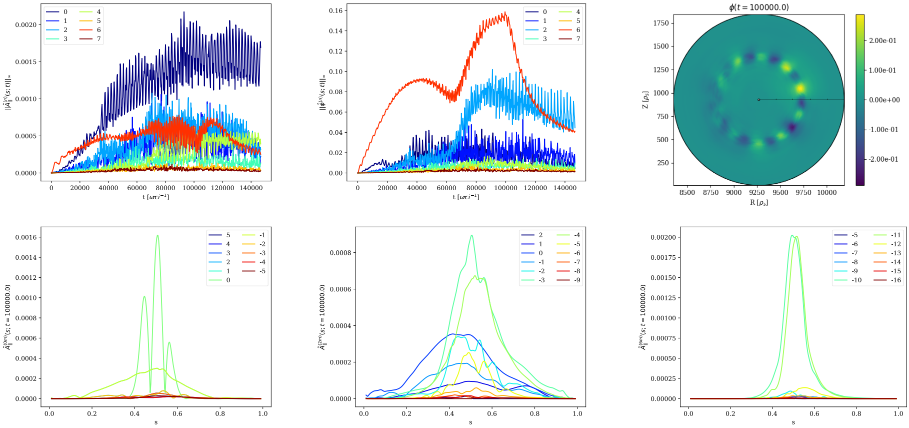
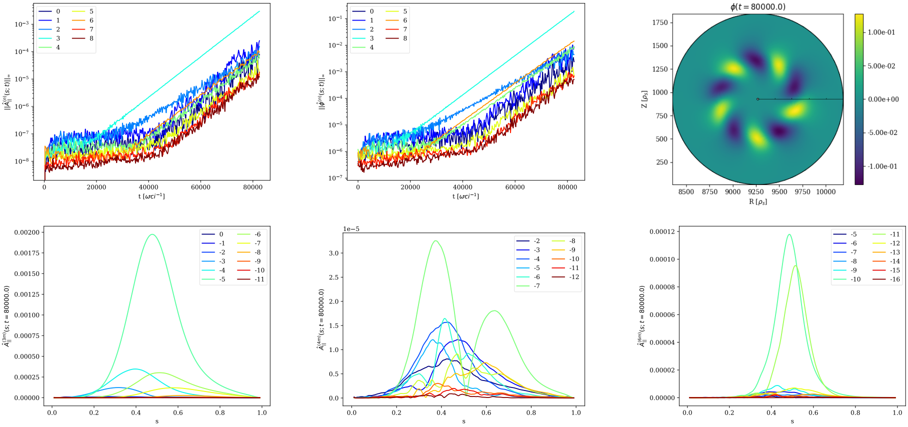


Figure: Damping rates for $\omega_{\rm ant} = -0.00144$ (blue) and 0.00144 (red).

Nonlinear simulation ($n = 0, ..., 18$), exciting $n = 6, m = -10, -11$ with antenna



Nonlinear simulation ($n = 0, \ldots, 18$) with fast particle, no antenna



Conclusion and outlook

- An electromagnetic antenna was devised to excite TAE modes.
- Reasonable agreement with fast particle simulations in linear simulations.
- Damping rate can be measured by switching off antenna.
- First nonlinear simulations are shown.

Acknowledgements

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