3D MHD equilibrium and stability with SPEC

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in collaboration with

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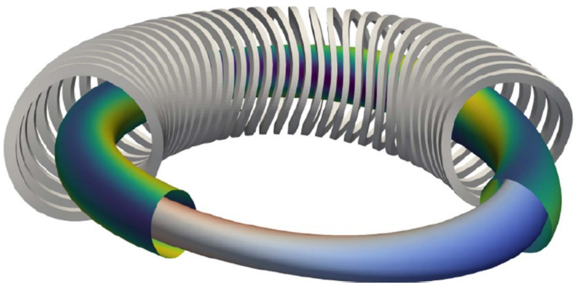


FSD Science meeting - 3D effects, February 1st 2022

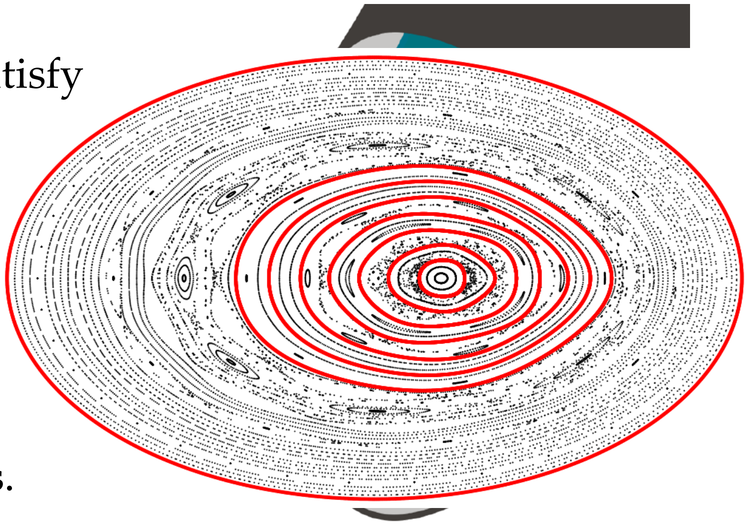
Outline

- The **SPEC code** as a tool to find equilibria with islands and chaos.
- Calculation of stellarator **equilibrium $\boldsymbol{\beta}$-limits**.
- Stellarator **optimization** for magnetic surfaces at finite β.

SPEC finds MHD equilibria with islands & chaos

The SPEC code [Hudson et al, PoP 2012] finds equilibria that satisfy

$$\boxed{\begin{array}{c} \nabla \times \mathbf{B} = \mu_l \mathbf{B} \\ [[p + \dfrac{B^2}{2\mu_0}]] = 0 \end{array}}$$



in a number of volumes **N**, each with constant pressure,

and separated by topologically robust magnetic surfaces.

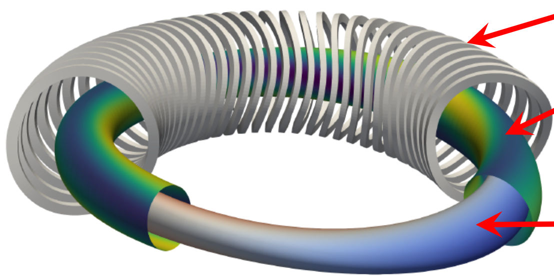
- The equations extremize the MHD energy allowing non-ideal variations. [Hole et al, JPP 2006]
- The magnetic field in each volume can form islands and chaos. [Loizu et al, JPP 2017]
- Required constraints in each volume: $\{\, \mathbf{p} \,,\, \mathbf{\Psi}_{\mathrm{tor}} \,,\, \iota_{\mathrm{in}} \,,\, \iota_{\mathrm{out}} \}$ or $\{\, \mathbf{p} \,,\, \mathbf{\Psi}_{\mathrm{tor}} \,,\, \mathbf{I}_{\phi}{}^{\mathrm{v}} \,,\, \mathbf{I}_{\phi}{}^{\mathrm{s}} \,\}$
- SPEC can also calculate linear stability. [Kumar et al PPCF 2021]
- SPEC was able to predict tearing mode saturation. [Loizu et al PoP 2020]

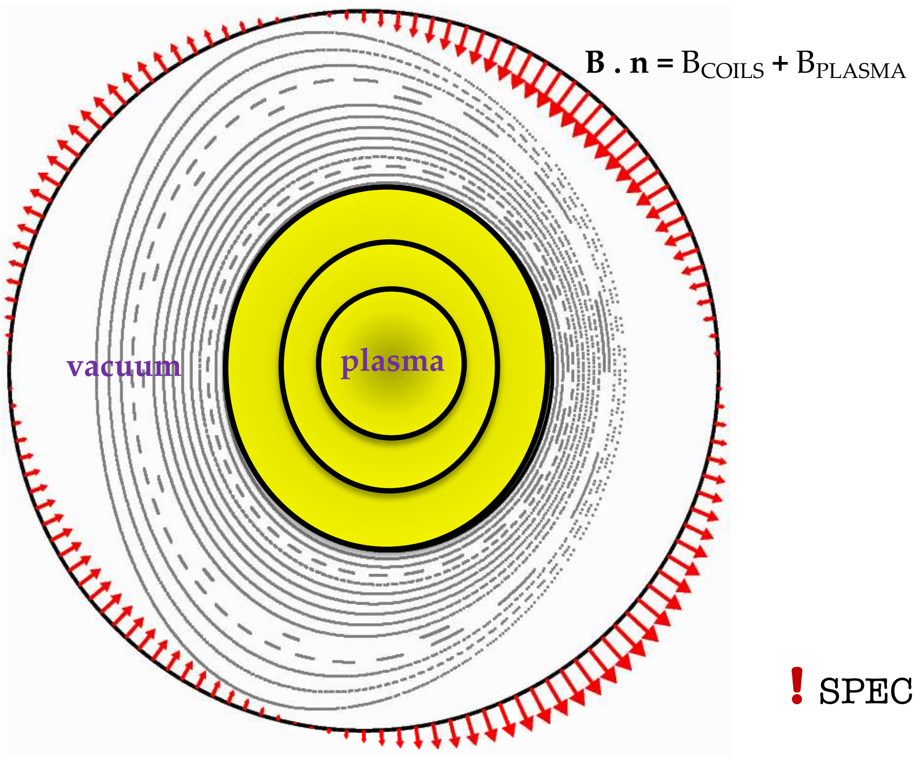
rotational transform

volume and surface currents
[Baillod et al, JPP 2021]

SPEC has been extended to free-boundary

- Free-boundary calculations recently achieved with SPEC. [Hudson et al PPCF 2020]





INPUT

- $\mathbf{B}_{COILS}$ on a computational boundary
- Profile of pressure and rotational transform
- Linking currents

OUTPUT

- Equilibrium total **B**
- Geometry of plasma interfaces

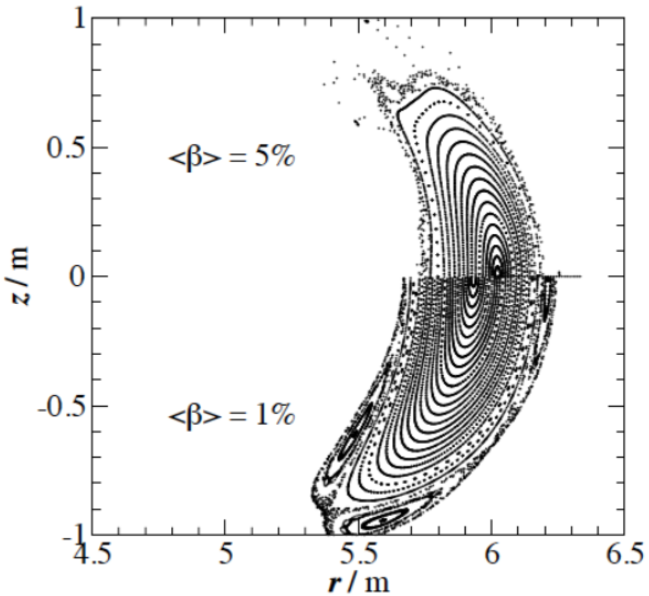
! SPEC still fragile in strongly shaped configurations.

Need to explain stellarator equilibrium β-limits

- The vacuum field can be designed to possess magnetic surfaces. [Pedersen et al, Nature Comm 7, 2016]

- But, inevitable **pressure-induced plasma currents** potentially degrade magnetic surfaces, or harmfully modify the topology of the scrape-off-layer.

- This is similar to **RMPs in tokamaks**, except that here the source of the perturbations is the plasma.



[Helander et al, PPCF, 2012]

Questions

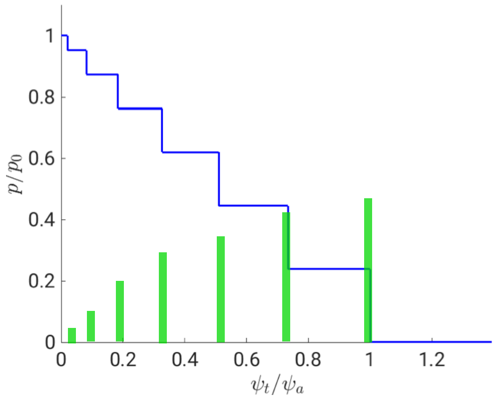
At what β magnetic surfaces start to degrade?

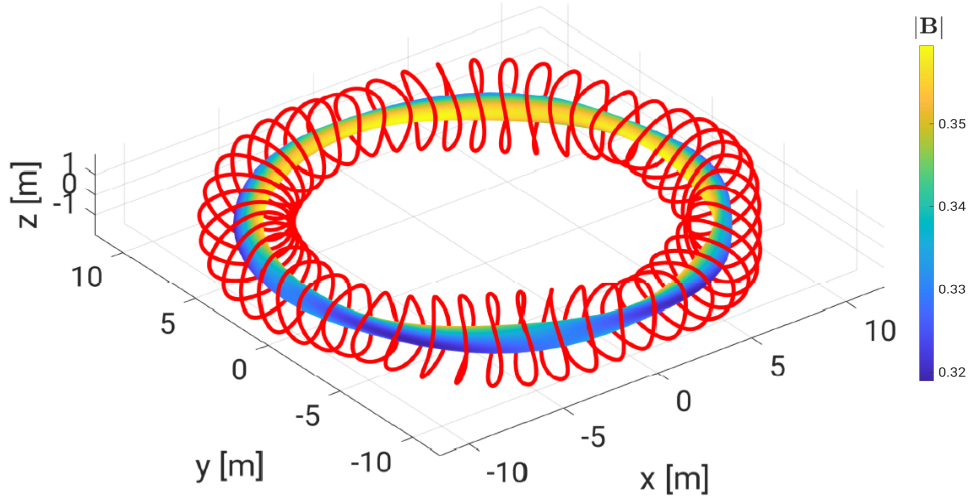
How does β_{lim} depend on design parameters?

β-limits studied in a classical stellarator

- 1st study: classical stellarator ~ W7-A
- Scalable **pressure profile**:

$$p = p_0 \left(1 - \frac{\psi_t}{\psi_a}\right)$$





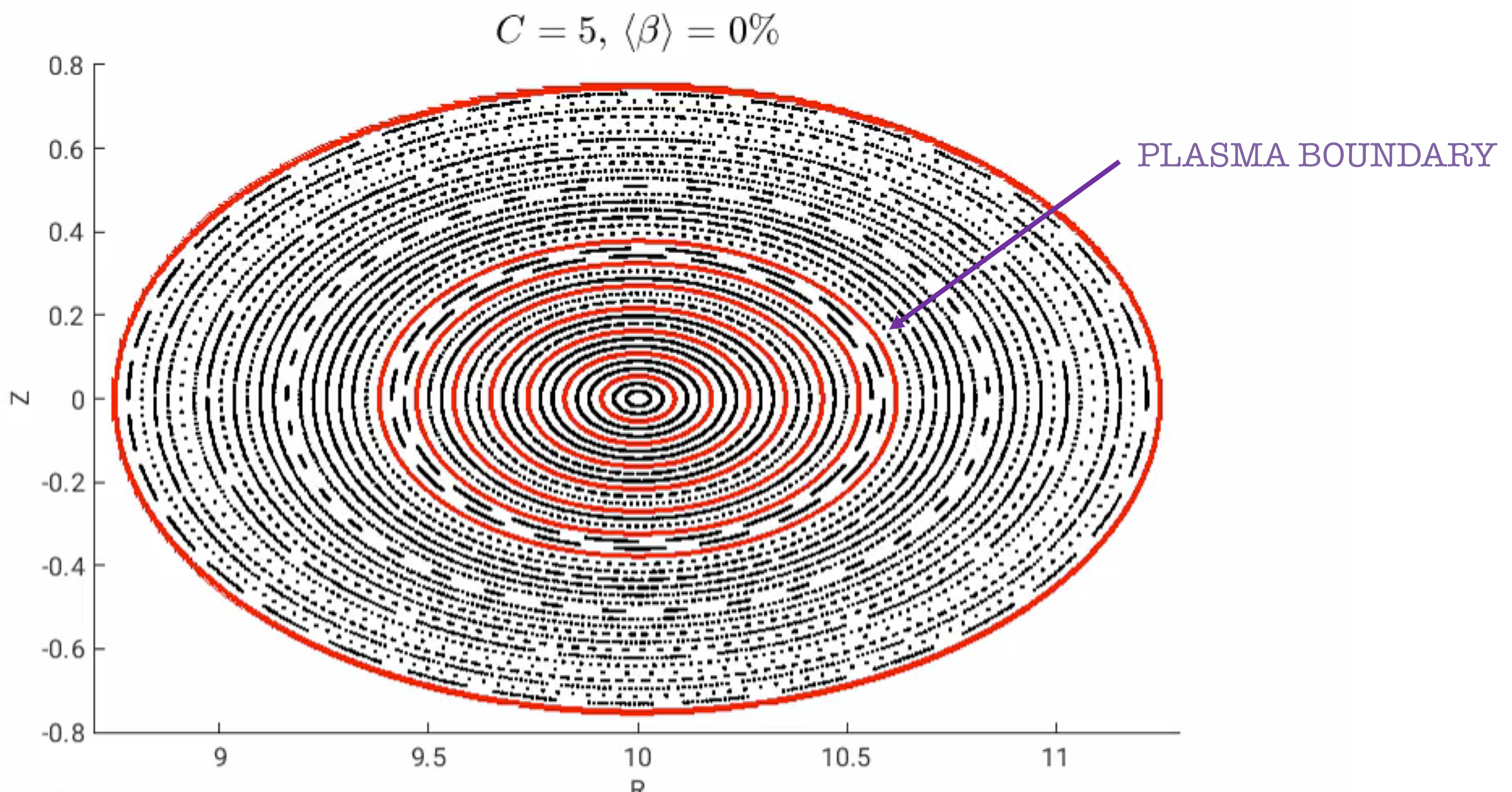
Parameters of interest: {C, p_0}

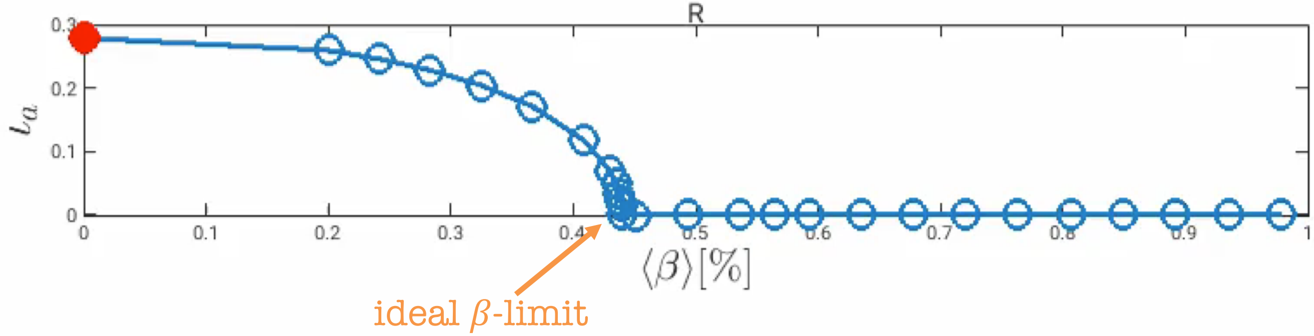
- **Toroidal current** adjusts according to pressure (bootstrap):

$$I_\phi^s = -C \left(\frac{\Psi_{t,l}}{\Psi_a}\right)^{1/4} [[p]]_l$$

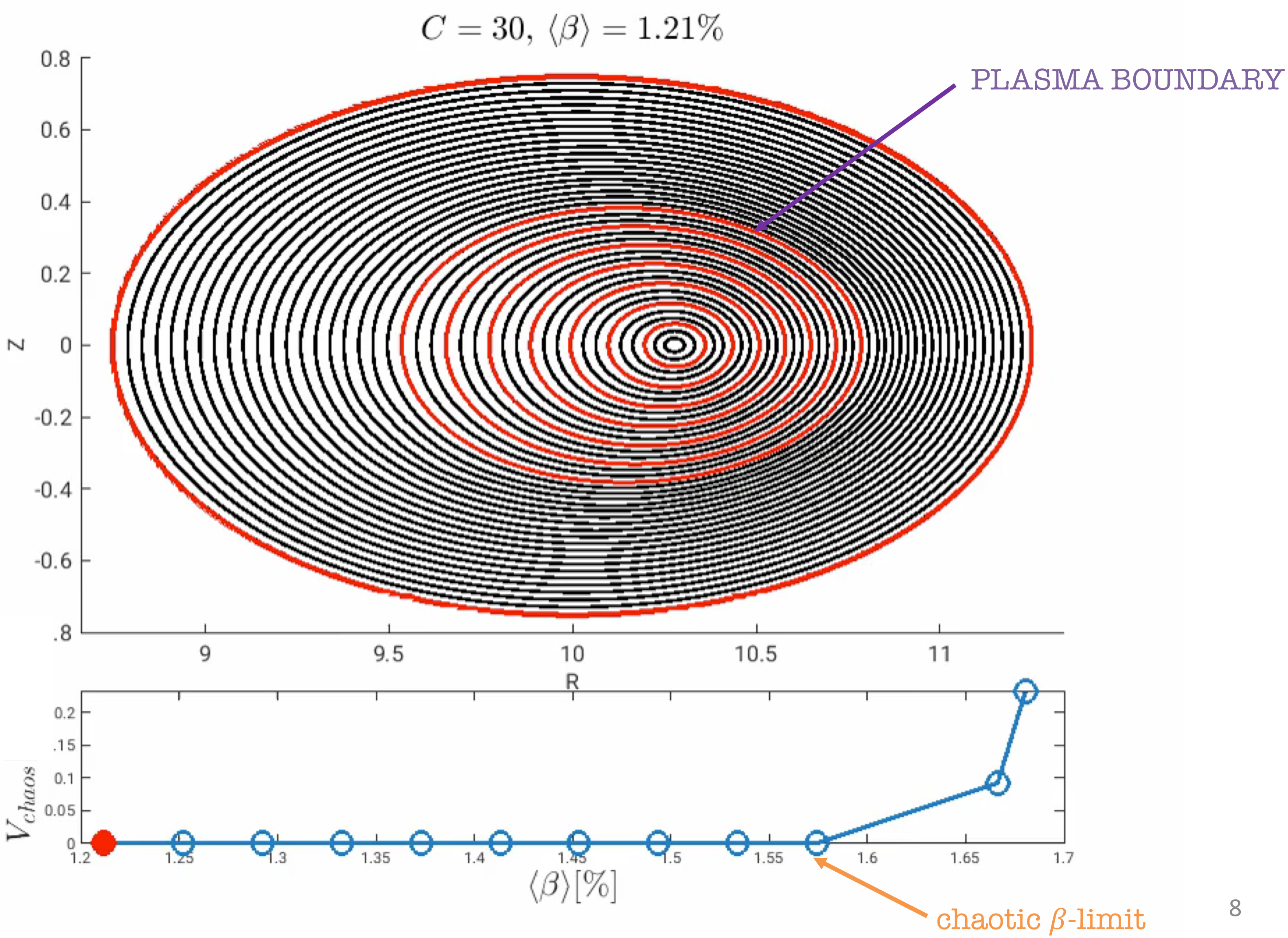
C : “coupling constant” (reference large aspect ratio circular tokamak $C_0 = \dfrac{R_0}{\iota B_0}$)

At low C, β-limit given by separatrix formation

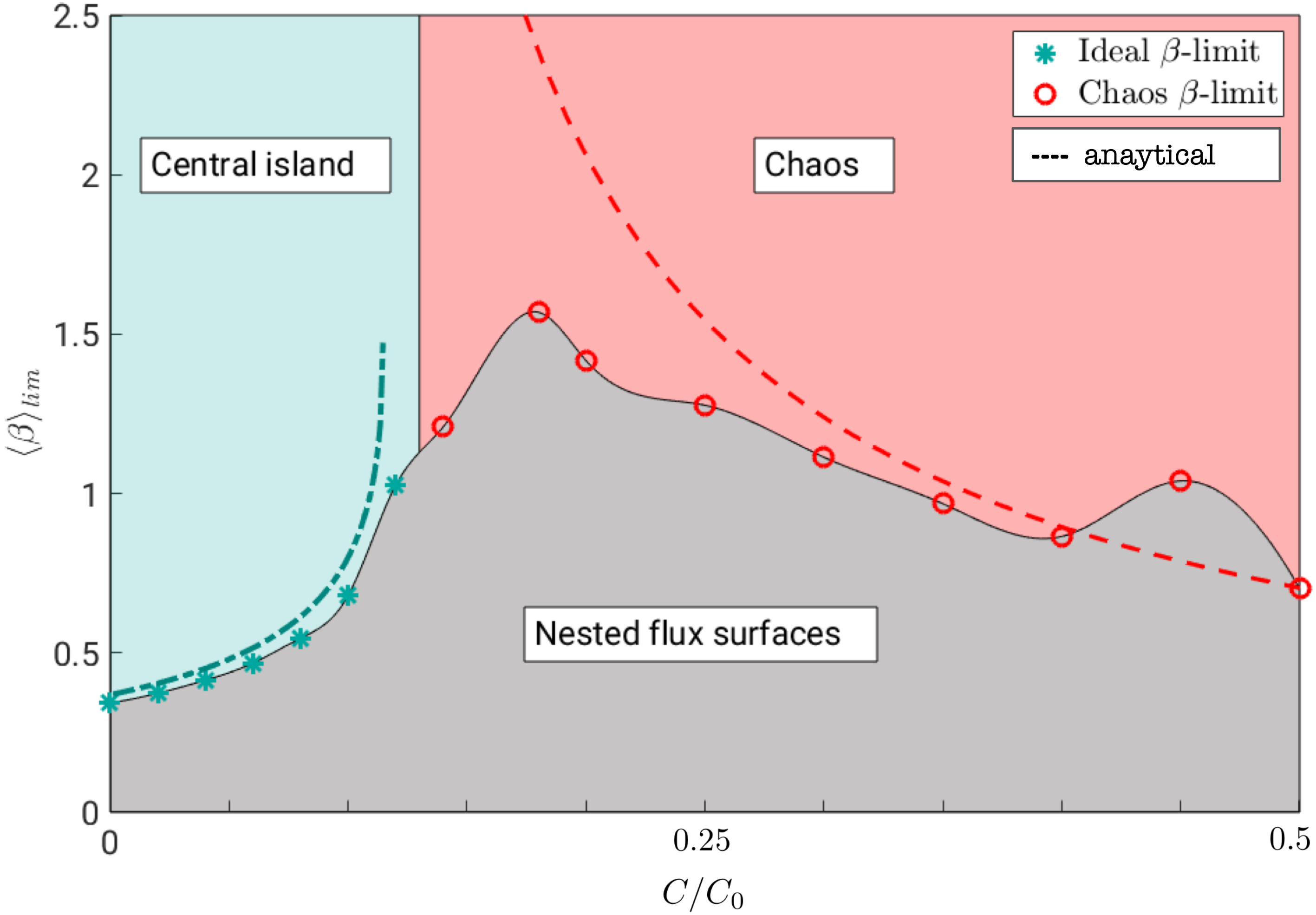




At high C, β-limit given by emergence of chaos

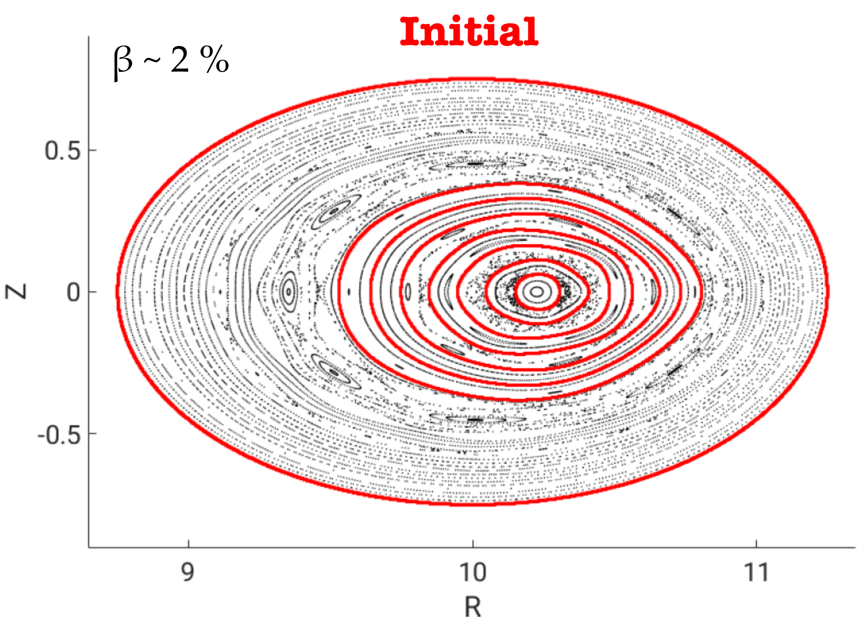


We can establish a diagram for the β-limit



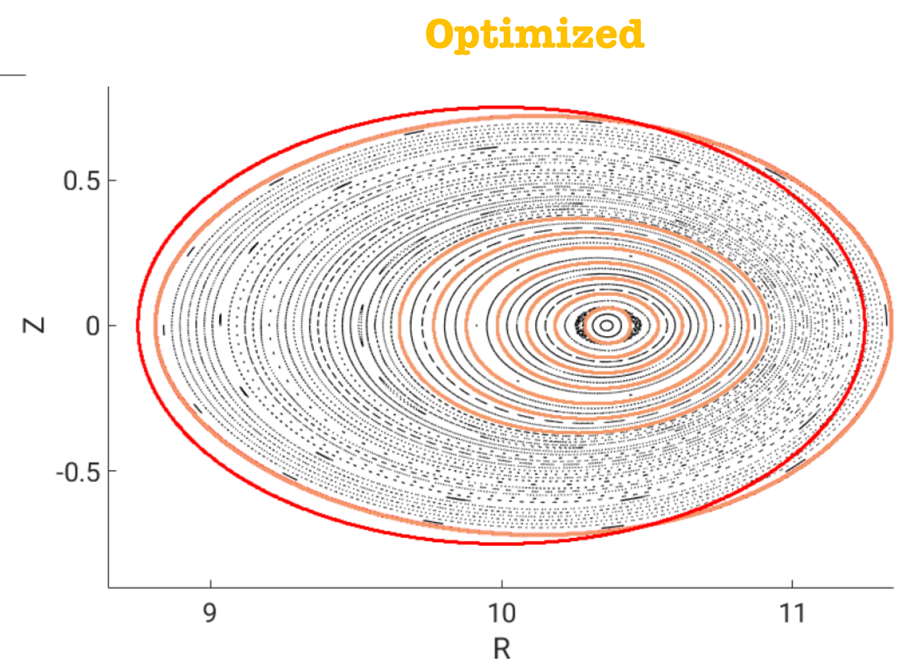
~ **500** equilibria were computed with SPEC

We can optimize the boundary for integrability



Objective: minimize Greene's residue

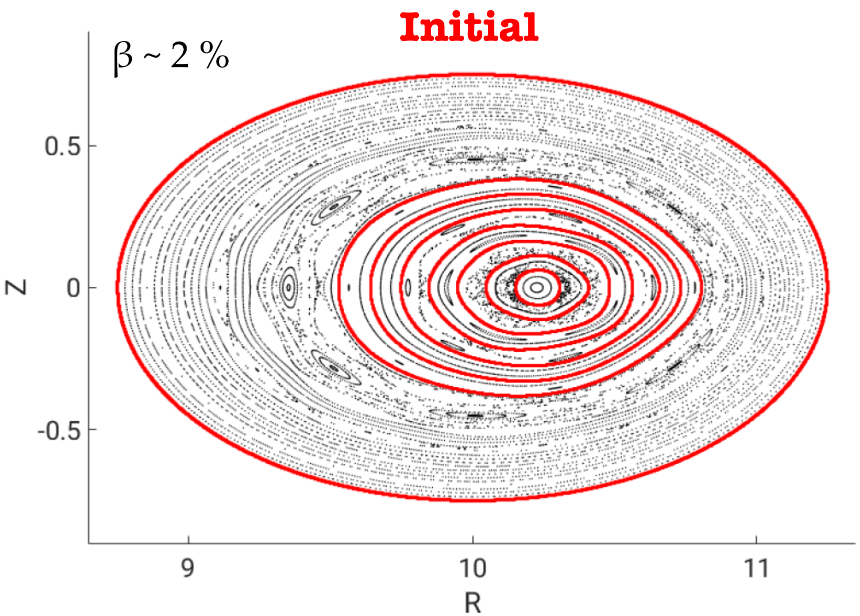
Degrees of freedom: shape of conducting wall



SPEC was coupled to SIMSOPT, a stellarator optimization framework

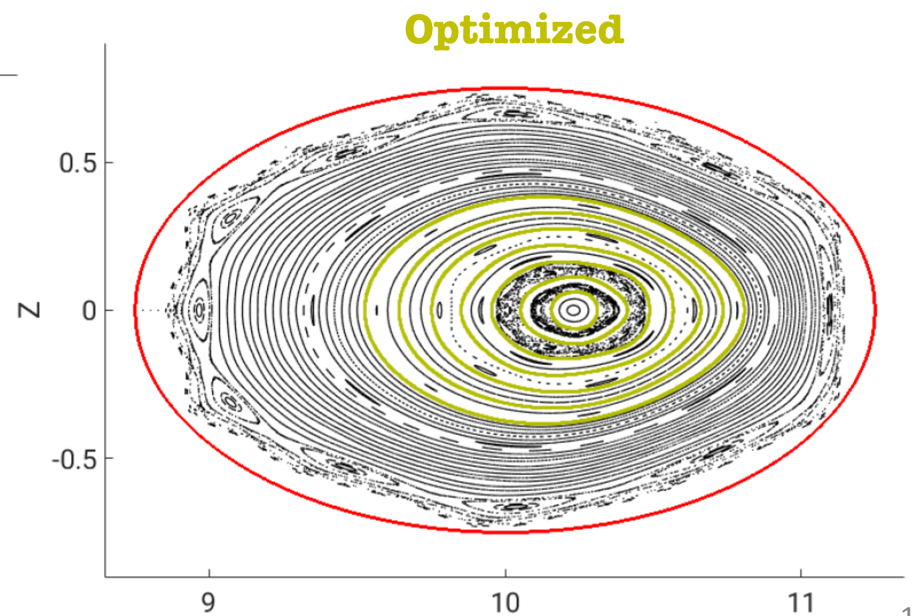
[Landreman et al, JOSS, 2021]

We can optimize the coils for integrability



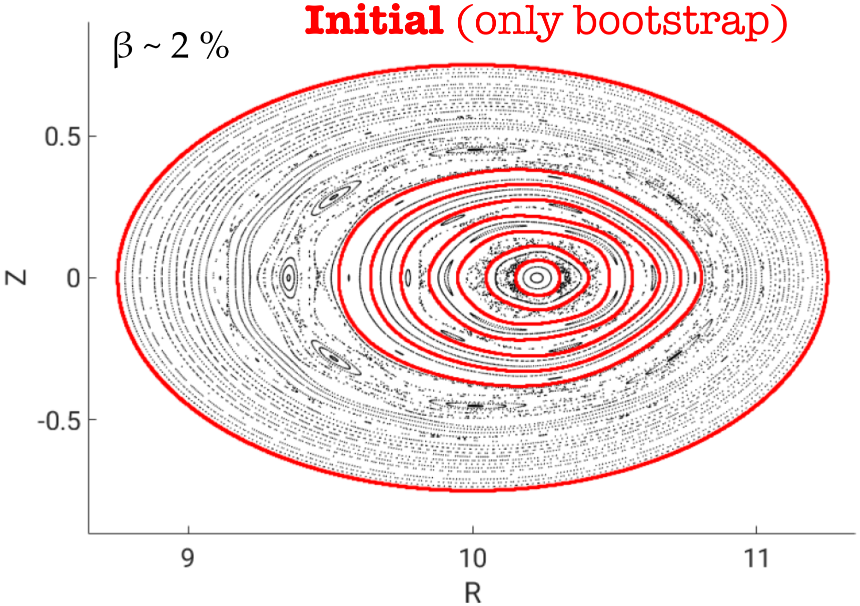
Objective: minimize Greene's residue

Degrees of freedom: external fields



We can optimize ECCD for integrability

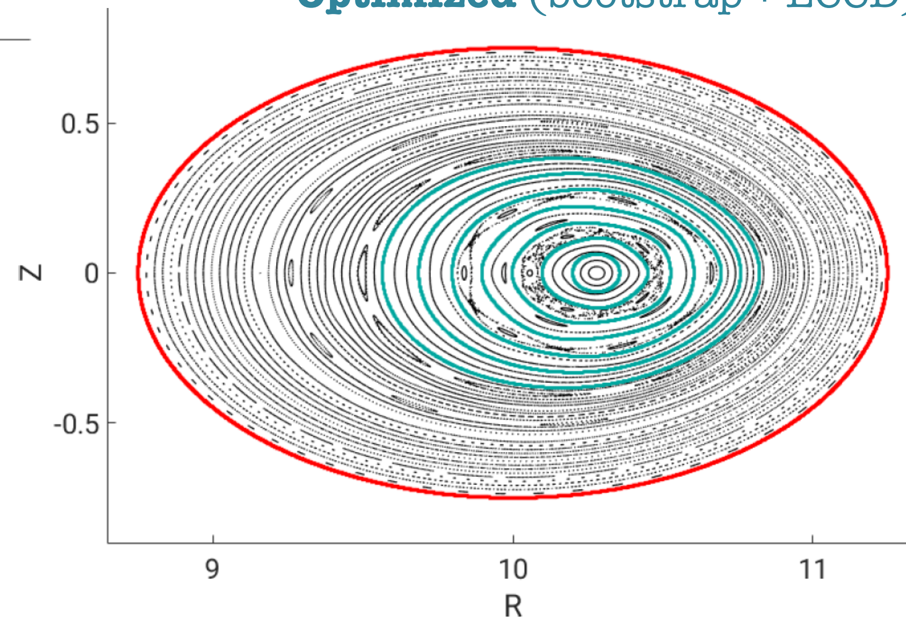
Initial (only bootstrap)



Objective: minimize Greene's residue

Degrees of freedom: current deposition

Optimized (bootstrap + ECCD)



[Baillod et al, submitted to PoP]

https://arxiv.org/abs/2111.15564

Summary

- SPEC allows fast free-boundary 3D equilibrium calculations with finite β and current, and with islands and chaos.

- We are using SPEC to **investigate and optimize the equilibrium β-limits** in different classes of stellarators: QA, QH, QI, …

- A further advantage of SPEC is that it can evaluate ideal and resistive MHD **stability**.

- Perhaps SPEC could assist in the study of tokamak equilibria with RMPs?