Max-Planck-Institut für Plasmaphysik

Anisotropic analytical and numerical distribution functions in the global gyrokinetic particle-in-cell code ORB5

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EUROfusion

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Outline

Energetic particles

The ORB5 Code

Distribution functions

- Numerical distribution functions
- Examples

Equilibrium distribution functions

Energetic Particles

Energetic Particles (EPs) are **supra**-thermal plasma species
Low density, low collisionality, moderate pressure
Sources:

- Alpha particles: born isotropically with E=3.5 MeV
- NBI particles: born **an**isotropically with E_{birth}
  - "PINI" (most present day machines): components E_{birth}, $E_{\mathrm{birth}}/2$, $E_{\mathrm{birth}}/3$
  - "NINI" (e.g. ITER): only E_{birth}
- Alpha + NBI: slow down from birth to thermal (collisions with electrons)
- ICRH, energy pumped into cyclotron resonance $\rightarrow$ energy into $v_\perp$ (μ)

Our interest is mostly dictated by resonant interaction with Alfvén waves (MHD-like Alfvén eigenmodes (AEs) & non-perturbative modes (EPMs)). Also Energetic particle driven Geodesic Acoustic Modes (EGAMs).
In general, trans-Alfvénic: Alfvénic physics typically depends on $v_{||}$ resonances (v_{A}, $v_{\mathrm{A}}/3$, ...)

Energetic Particles & instabilities

- AEs (e.g. Toroidal AE (TAE)) are a global problem, existence comes from profile variation and $k_{||}$ matching of Alfvén dispersion relation.

TAE drive comes from radial pressure gradient of EPs
TAE damping is typically non-local (radiative, continuum, and electron Landau damping)

EGAMs, (also GAE), can exist with n=0.
Drive is from the **anistropy** of the EP distribution
Sources of anisotropy/velocity space gradients:

- Turn on a beam, initially “bump-on-tail”. This effect ends after $t_{\mathrm{slow.down}}$
  - after this, dynamic equilibrium reached
- NBI beams have preferred pitch (geometric), injection anisotropic
- ICRH “pulls out” tails of distribution
- isotropic $\rightarrow$ anisotropic possible due to losses

ORB5

"ORB5: a global electromagnetic gyrokinetic code using the PIC approach in toroidal geometry" [for details, see Lanti 2020]

- delta-f modified distribution function discretized with PIC
- Fields solved using finite elements
  - Filter applied in toroidal and poloidal mode numbers
- Global electromagnetic (EM) simulations a difficult problem: small $k_\perp$ (e.g. MHD) particularly challenging for high beta.
  - Effectively mitigates with the so-called cancellation problem using the pullback scheme [Mishchenko 2019] (leads to an order of magn. increase of time step)
- ES: adiabatic, hybrid, or kinetic electrons, EM drift-kinetic electrons (or fluid)
- Previously used for turbulence studies as well as EP physics (separately, and interaction)

ORB5

ORB5's Vlasov equation (shown electrostatically, absence of collisions/sources)

$$\frac{d\delta f}{dt} = -\frac{df_0}{dt}$$

Total (Lagrangian) derivative (for plasma species s)[1],

$$\frac{df_s}{dt} = \frac{\partial f_s}{\partial t} + \frac{\partial f_s}{\partial \mathbf{R}} \cdot \dot{\mathbf{R}} + \frac{\partial f_s}{\partial v_\parallel} \dot{v}_\parallel = 0$$

full-f Vlasov equation, requires evaluation of background f_0 and its gradients at particle positions

Note: No assumption (here) made about smallness of δf to f_0

[1] We'll come back to this later

Distribution functions

- Maxwellian: local, shifted, canonical
- Bump-on-tail
- Isotropic slowing down
- Anisotropic slowing down
- Constant of Motion distribution functions[2]
- Numerical distribution functions

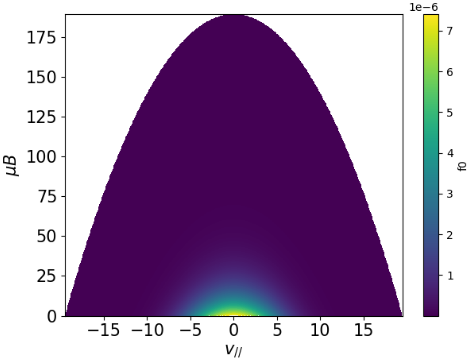
Analytical expressions, semi-analytical expressions, fully-numerical

[2] C. di Troia+, difficult to use for realistic experiments in practice

Maxwellian

$$F_{0,\mathrm{f,Max.}} = \frac{n_\mathrm{f}(r)}{(2\pi v_\mathrm{th}^2(r))^{3/2}} \exp\left(-E/v_\mathrm{th}^2\right) \exp\left(-\frac{u_\parallel}{2}\,(u_\parallel - 2v_\parallel)/\ v_\mathrm{th}^2\right)$$

in absence of shift ($u_\parallel \to 0$), reduces to function of Energy, radius



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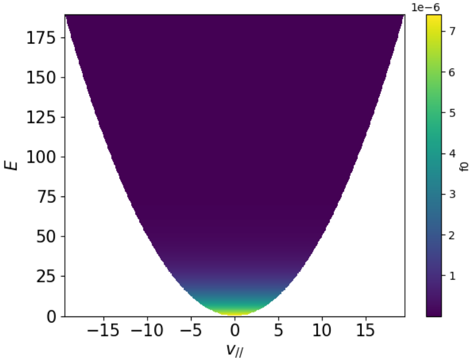
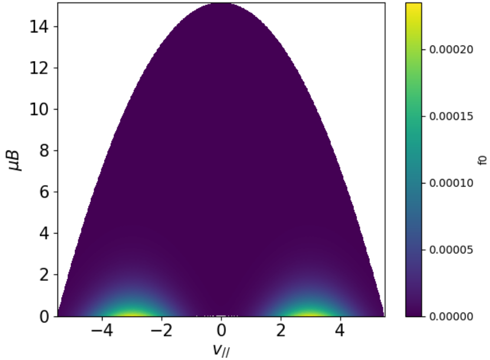


Bump-on-tail

$$F_{0,\mathrm{f,BoT}} = C \cdot n_\mathrm{f}(r) \exp(-E \cdot m_\mathrm{f} / T_\mathrm{f}) \exp(-v_{\|,f}^2 / (2 T_\mathrm{f})) \cosh(v_\| v_{\|,f} / T_\mathrm{f})$$

function of energy, radius, $v_\|$

- "Toy" distribution function with strong anisotropy (ideal to study EGAMs)
- Original version implemented for [Zarzoso+, NF, 2014]
  - Based on previous GYSELA work
- Originally zero radial dependence, since extended to include $n(r)$



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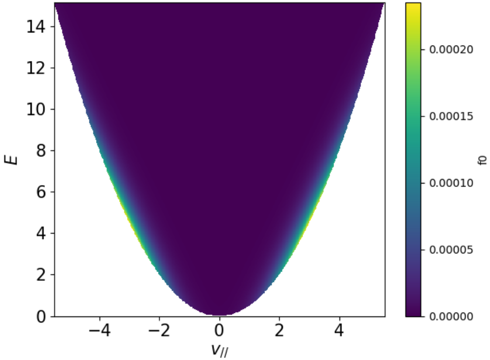
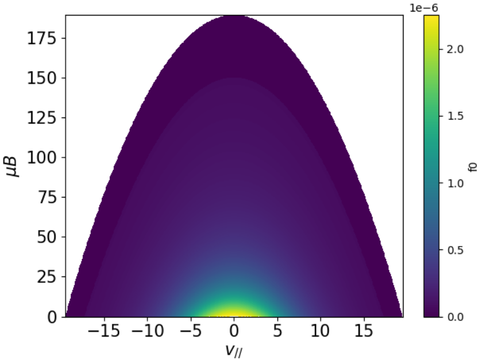


Isotropic Slowing down[3]

$$F_{0,\mathrm{f},\mathrm{SD}} = \frac{3 n_\mathrm{f}(r)}{4\pi} \frac{\Theta(v_0 - |v|)}{(v_\mathrm{c}(r)^3 + |v|^3)\ln(1 + v_0/v_\mathrm{c}(r))}$$

also function of Energy ($|v|$), radius

- Decent approximation for alpha particles



[3]Vannini+, thesis+paper 2021+

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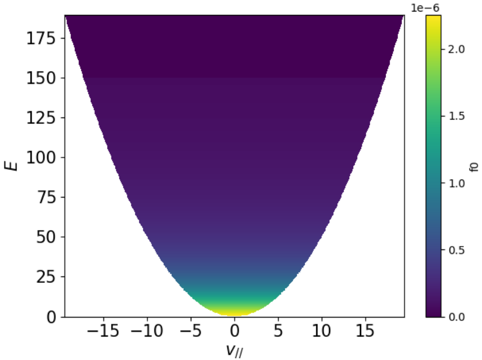


[3] Vannini+, thesis+paper 2021+

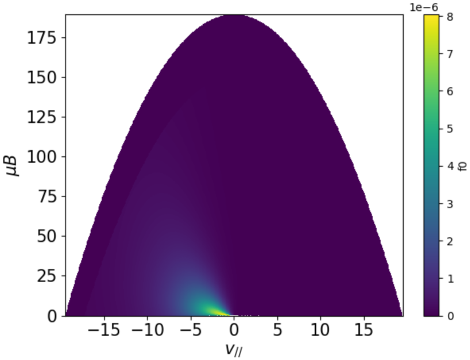
Anisotropic Slowing down[4]

$$F_{0,\mathrm{f,ASD}} = F_{0,\mathrm{f,SD}}(r, E) \cdot \; C \exp\left(-(\xi - \xi_0)^2/(2\Delta\xi^2)\right)$$

where $\xi = v_{||}/|v|$, $\rightarrow$ function of energy, radius, and parallel velocity

'C' is a messy term of error functions, ...
derivatives 'tricky' $\rightarrow$ semi-analytical
F_0 analytic, but compute $\partial F_0/\partial X$ numerically

- Reasonable approximation for NBI



[4] Rettino+, paper 2021+

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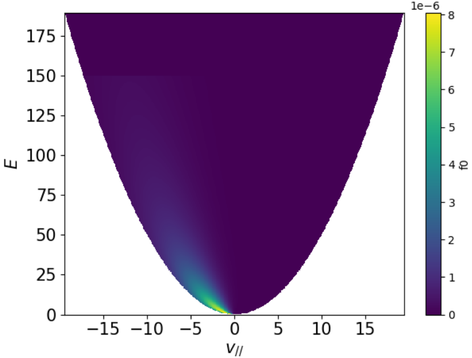

[4]Rettino+, paper 2021+

Numerical validation for analytic distributions

When implementing purely analytical distribution functions:

- Need analytical derivatives of F_0 w.r.t. E, $v_{\|}$, $\psi = r^2$
  - ... consistent with quirks of ORB5
- Validate analytical expressions by comparing to numerical derivatives

Validating F_0 numerically

- Output F_0 on mesh $(E, v_{\|}, \psi)$
- Implement finite differences evaluation of $\frac{\partial F_0}{\partial X}$
- Output $\frac{\partial F_0}{\partial X}$ on mesh
- Output $\frac{\partial F_0}{\partial X}_{\mathrm{FD}}$ on mesh

Validating F_0 numerically

- Output F_0 on mesh $(E, v_{\|}, \psi)$
- Implement finite differences evaluation of $\frac{\partial F_0}{\partial X}$
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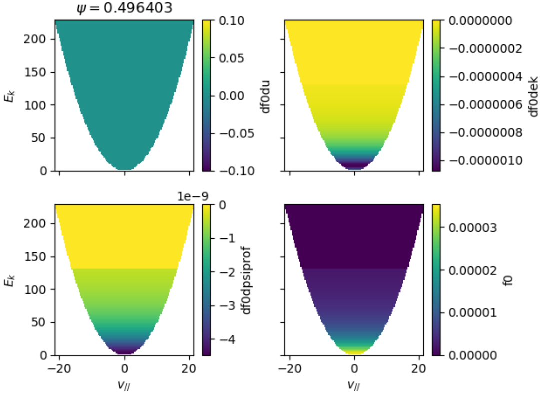
Analytical Derivatives



Validating F_0 numerically

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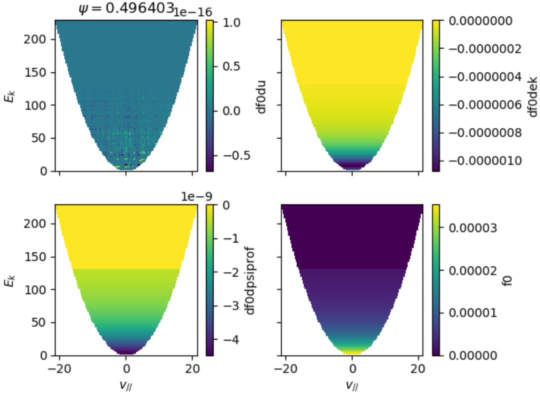
Numerical Derivatives



Validating F_0 numerically

- Output F_0 on mesh $(E, v_{\|}, \psi)$
- Implement finite differences evaluation of $\frac{\partial F_0}{\partial X}$
- Output $\frac{\partial F_0}{\partial X}$ on mesh
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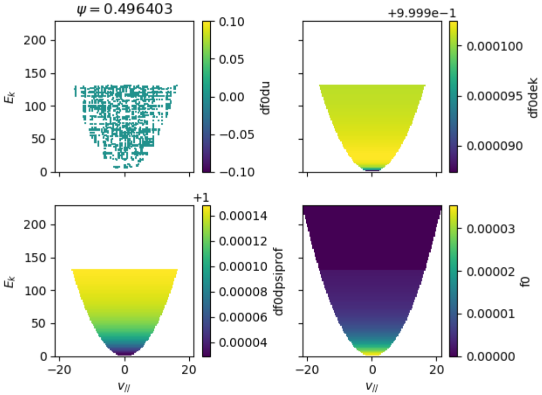
Ratio analytical/numerical

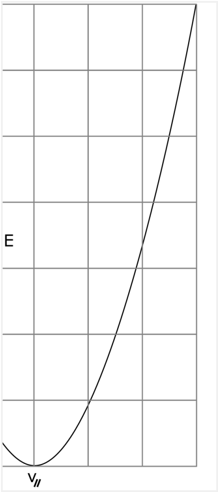


Validating F_0 numerically

- Output F_0 on mesh $(E, v_\parallel, \psi)$
- Implement finite differences evaluation of $\frac{\partial F_0}{\partial X}$
- Output $\frac{\partial F_0}{\partial X}$ on mesh
- Output $\frac{\partial F_0}{\partial X}_{\mathrm{FD}}$ on mesh
- We can probably reverse this?

Numerical F_0

- Assume we have F_0 on mesh (vertices) $(E, v_{||}, \psi)$
  - Can test with the output on previous slide
- F_0 for markers can be interpolated on the mesh



Numerical F_0

- Assume we have F_0 on mesh (vertices) $(E, v_{\|}, \psi)$
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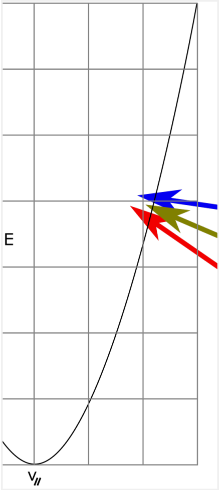
- F_0 for markers can be interpolated on the mesh
- Consider 3 example markers



Numerical F_0

- Assume we have F_0 on mesh (vertices) $(E, v_\parallel, \psi)$
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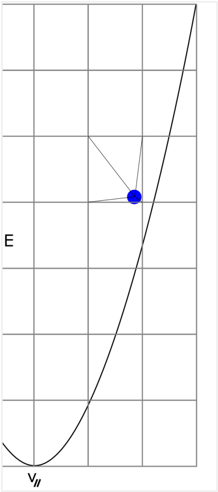
- F_0 for markers can be interpolated on the mesh
- Consider 3 example markers
  - Blue is well behaved



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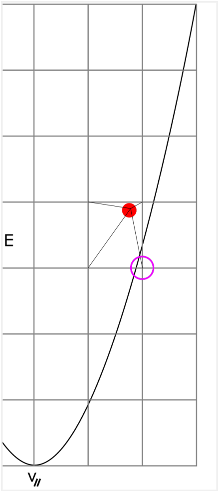
  - Blue is well behaved
  - Red requires point with $\mu < 0$ for interpolation



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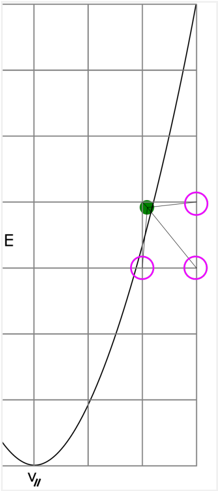
  - Green even worse



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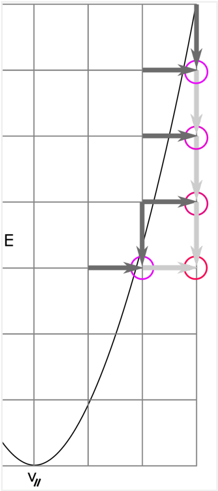
- Consider 3 example markers
  - Blue is well behaved
  - Red requires point with $\mu < 0$ for interpolation
  - Green even worse
  - Approach: smoothly continue F_0 across $\mu = 0$



RABBIT

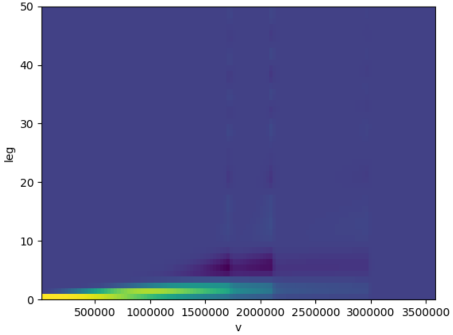
- RABBIT [Weiland+, NF, 2018+19]
  - real-time capable NBI code
- Describes NBI distribution function in experiment
- Non-Monte-Carlo method gives smooth function, good for derivatives
- We use RABBIT for ASDEX Upgrade (AUG) NBI F_0 (e.g. shot #31213 (NLED-AUG)) in the time-independent mode

$$f_-(v,\xi) = \frac{1}{2\pi}\frac{\tau_s}{v^3+v_c^3}\cdot \sum_{l=0}^{\infty}\left(l+\frac{1}{2}\right)P_l(\xi)S_l\cdot \left(\frac{v_0^3+v_c^3}{v^3+v_c^3}\frac{v^3}{v_0^3}\right)^{\frac{\beta}{3}l(l+1)}$$

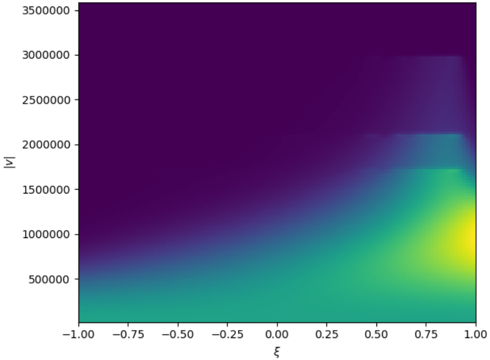
$$\xi = v_{||}/v$$

RABBIT AUG example

AUG shot #31213 at t=0.84 s ("NLED-AUG" case[5])



$F_0(|v|, P_l(\xi))_{r=r_{\mathrm{ref}}}$

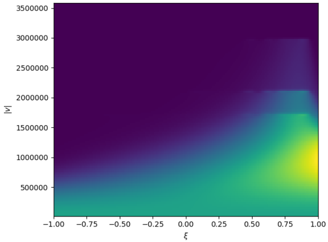


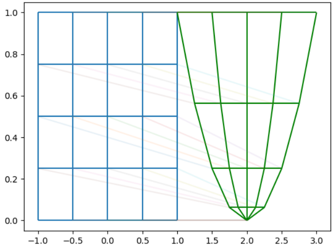
$F_0(\xi, |v|)_{r=r_{\mathrm{ref}}}$

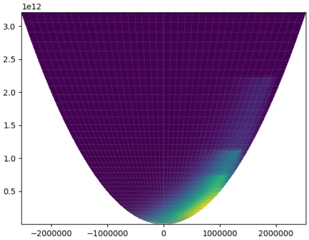
[5] Lauber+, IAEA FEC 2018

Mapping from RABBIT to ORB5

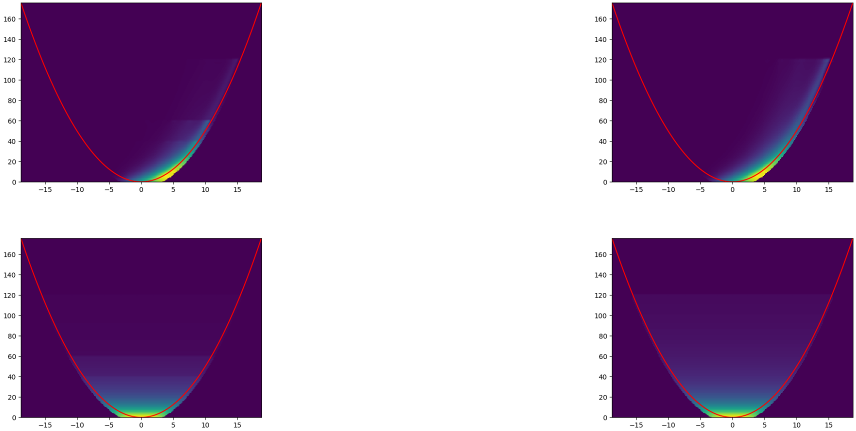
- Coordinate mapping from $(|v|, \xi, \rho_t) \to (E, v_\parallel, \psi)$
- Some additional details (interpolation objects, etc.)
- Fill in $F_0(\mu \lesssim 0)$







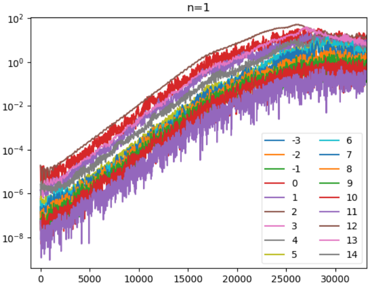
RABBIT AUG example



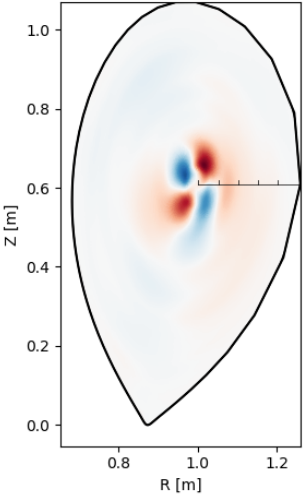
Clockwise (from TL): triple [nominal], single, single-no-pitch, triple-no-pitch.

ORB5 RABBIT AUG example[6]

n=1 EPM

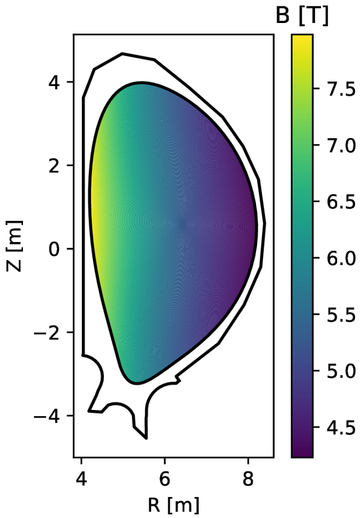


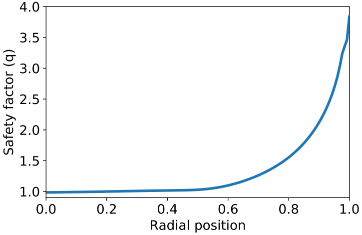
amplitude of poloidal harmonics ES potential vs time



[6]detailed EGAM study in Rettino+ 2021+

ITER 15 MA Scenario





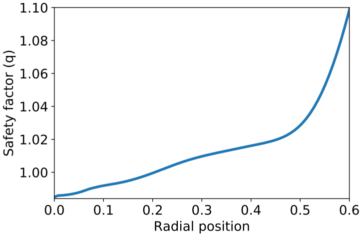
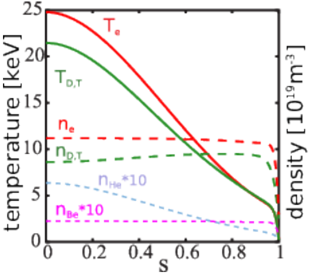
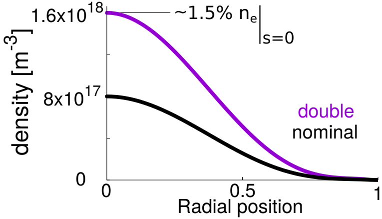


Fig. Lauber 2015



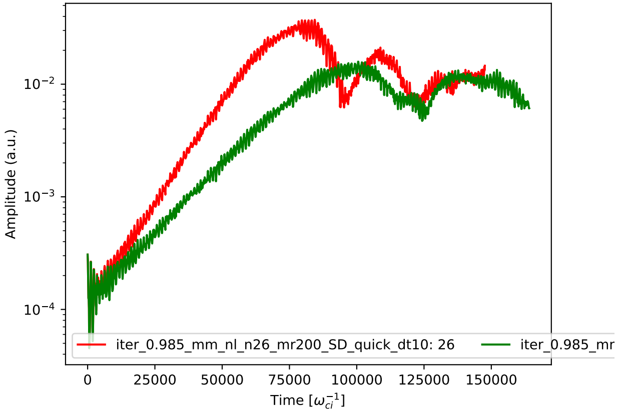


Slowing down

Maxwellian (900keV) studied in [Hayward-Schneider+ NF 2021]

With realistic 3.5 MeV isotropic slowing down:

$\rightarrow$ mode drive increased

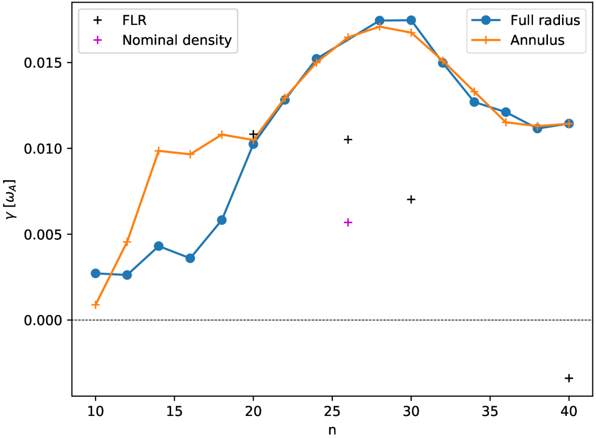


(low resolution run)

Slowing down

$\gamma = 0.0218\ \omega_{\mathrm{A}}$ (high resolution run)
c.f. $\approx 0.16\ \omega_{\mathrm{A}}$ for Maxwellian

$\rightarrow$ Ready to study nominal parameters
(e.g. EP FLR at nominal density)



Bulk FLR always kept

Back to theory – equilibrium distribution functions

Footnote 1 promised we'd come back to the total derivative (Vlasov equation)

$$\frac{df_s}{dt} = \frac{\partial f_s}{\partial t} + \frac{\partial f_s}{\partial \mathbf{R}} \cdot \dot{\mathbf{R}} + \frac{\partial f_s}{\partial v_\parallel} \dot{v}_\parallel = 0$$

All distribution functions mentioned (except canonical Maxwellian, C. di Troia constants of motion distribution function, or local Maxwellian in homogeneous plasma) are not actually equilibrium distribution functions since they **do not** depend **only** on constants of motion.

Back to theory – equilibrium distribution functions

In general true for anisotropic functions $\left(\frac{\partial f_0^{EP}}{\partial v_\parallel} \neq 0\right)$, even with homogeneous profiles.

$$\frac{\partial f_0^{EP}}{\partial v_\parallel} \dot{v}_\parallel = \frac{\partial f_0^{EP}}{\partial v_\parallel} \left.\dot{v}_\parallel\right|_0 + \frac{\partial f_0^{EP}}{\partial v_\parallel} \left.\dot{v}_\parallel\right|_1$$

At least for linear simulations, we need this term to disappear.
We can use a "trick":

$$\dot{v}_\parallel = \left.\dot{v}_\parallel\right|_1 = -\frac{e}{m} \frac{\mathbf{B}^*}{B_\parallel^*} \cdot \nabla \Phi$$

Circa 2014, EGAMs with bump-on-tail paper, solved in [Zarzoso+ NF 2014] with the trick[7]:

$$\dot{v}_\parallel = \dot{E} \frac{1}{v_\parallel}$$

[7] Reference also includes a nice justification of why this should be fine

Back to theory – equilibrium distribution functions

Comparing these, we find different EGAM excitation.

Why?

Total derivative can alternatively be written in other coordinates, e.g. μ if we keep explicit poloidal dependence (r, θ, μ, E). Different models like keeping/neglecting poloidal dependence.

Equivalent to setting (in F_0), $v_\parallel \rightarrow$

$$v_{\parallel H} = \mathrm{sign}(\mathrm{v}_\parallel)\sqrt{2|\mathrm{E} - \mu \mathrm{B}_{\mathrm{ref}}|} \quad \text{“deeply passing” model}$$

$$v_{\parallel H} = \mathrm{sign}(\mathrm{v}_\parallel)\sqrt{2(\mathrm{E} - \mu \mathrm{B})} \quad \text{“full orbit” model}$$

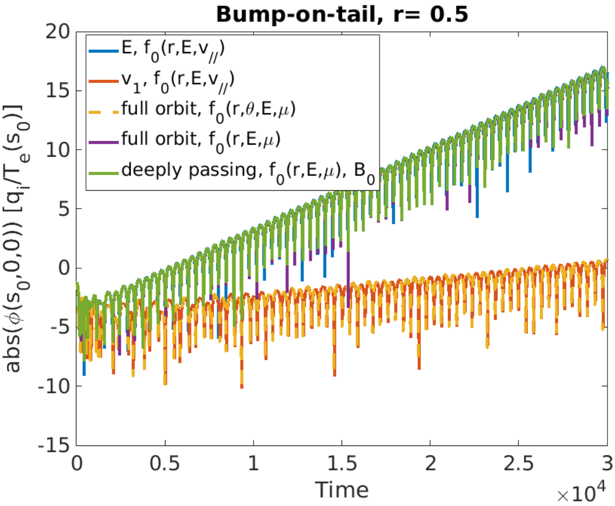
Back to theory – equilibrium distribution functions

Put it all together, we see:

'neglecting θ' $=$ 'replacing B with B_0' $=$ '$\dot{v}_\parallel = \dot{E}\frac{1}{v_\parallel}$'

and

'$\dot{v}_\parallel = \dot{v}_\parallel\Big|_1$' $=$ 'full theta dependence'



Summary

Implementation:

- Additional analytical distribution functions, for use in Energetic Particle modelling have been added to the global gyrokinetic code ORB5
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