

# Global fluid simulations of plasma turbulence in diverted stellarators

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## Introduction

- Recent **W7-X experiments** showed significant differences with respect to tokamaks:
  - Filaments bound to their flux surface [Killer, 2021]
  - Fluctuations normally distributed (local origin)
- Stellarator turbulence simulations still in its infancy:
  - Gyrokinetic  $\delta f$  codes (GENE-3D, Stella, XGC-S, ...) – study the core
  - Fluid code BOUT++ simulated edge filaments in a rotating ellipse [Shanahan, 2019]

# GBS solves the drift-reduced Braginskii equations

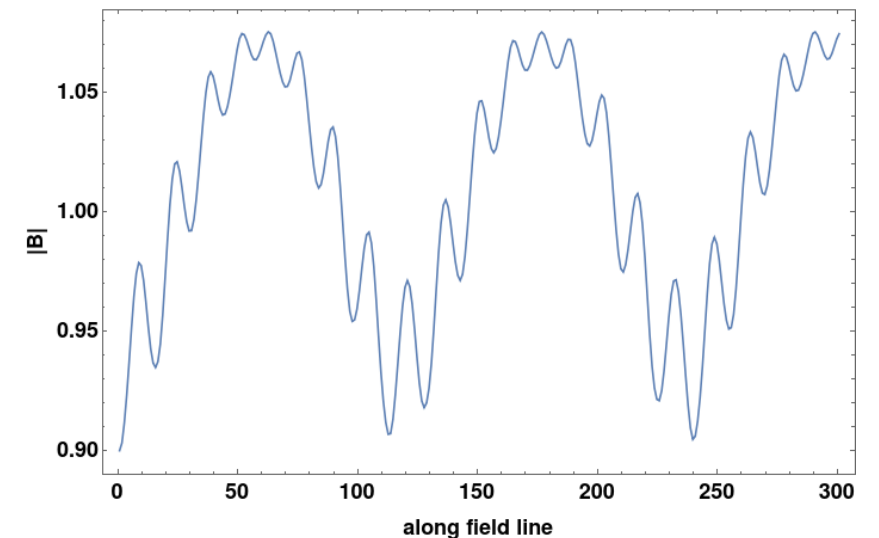
- Set of equations for  $n, T_e, T_i, V_{\parallel e}, V_{\parallel i}, \omega, \phi$ 
  - Electrostatic simulations
  - Boussinesq approximation
  - No neutrals

# GBS solves the drift-reduced Braginskii equations

- Geometrical operators:

$$[\boldsymbol{\phi}, \mathbf{u}] = \mathbf{b} \cdot [\nabla \phi \times \nabla \mathbf{u}] \quad \nabla_{\parallel} \mathbf{u} = \mathbf{b} \cdot \nabla \mathbf{u} \quad C(\mathbf{u}) = \frac{B}{2} \left[ \nabla \times \frac{\mathbf{b}}{B} \right] \cdot \nabla \mathbf{u} \quad \nabla_{\perp}^2 u = \nabla \cdot [(\mathbf{b} \times \nabla \mathbf{u}) \times \mathbf{b}]$$

Expansion parameters:  $\delta = \frac{B_p}{B}$  ;  $\sigma = \frac{l_{\perp}}{l_{\parallel}}$  ;  $\Delta = \frac{B_{max} - B_{min}}{B}$



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$$[\phi, u] = \frac{\partial\phi}{\partial Z} \frac{\partial u}{\partial R} - \frac{\partial\phi}{\partial R} \frac{\partial u}{\partial Z}$$

$$\nabla_{\perp}^2 u = \frac{\partial^2 u}{\partial R^2} + \frac{\partial^2 u}{\partial Z^2}$$

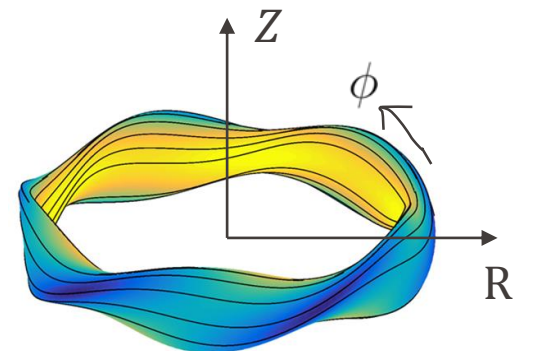
## Stellarator with an island divertor

$$\nabla \times \mathbf{B} = 0 \rightarrow \mathbf{B} = \nabla V$$

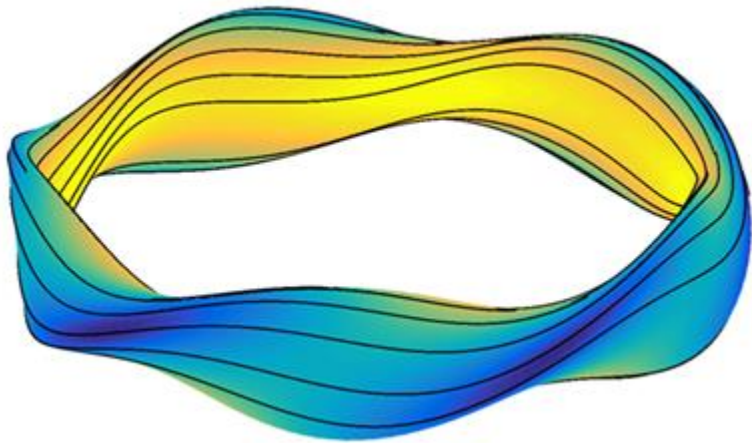
$$\nabla \cdot \mathbf{B} = 0 \rightarrow \nabla^2 V = 0$$

- Dommaschk potentials [Dommaschk, CPC 1986] are a solution of Laplace's equation in a torus:

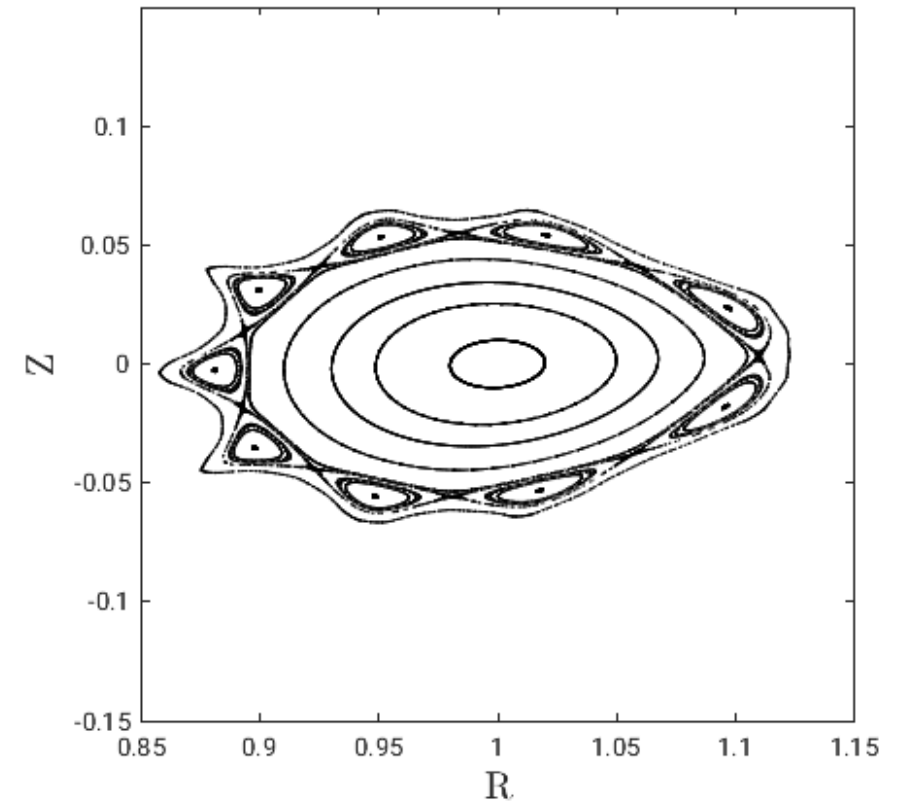
$$V(R, \phi, Z) = \phi + \sum_{m,l} V_{m,l}(R, \phi, Z)$$



**We simulate a 5-field period stellarator...**



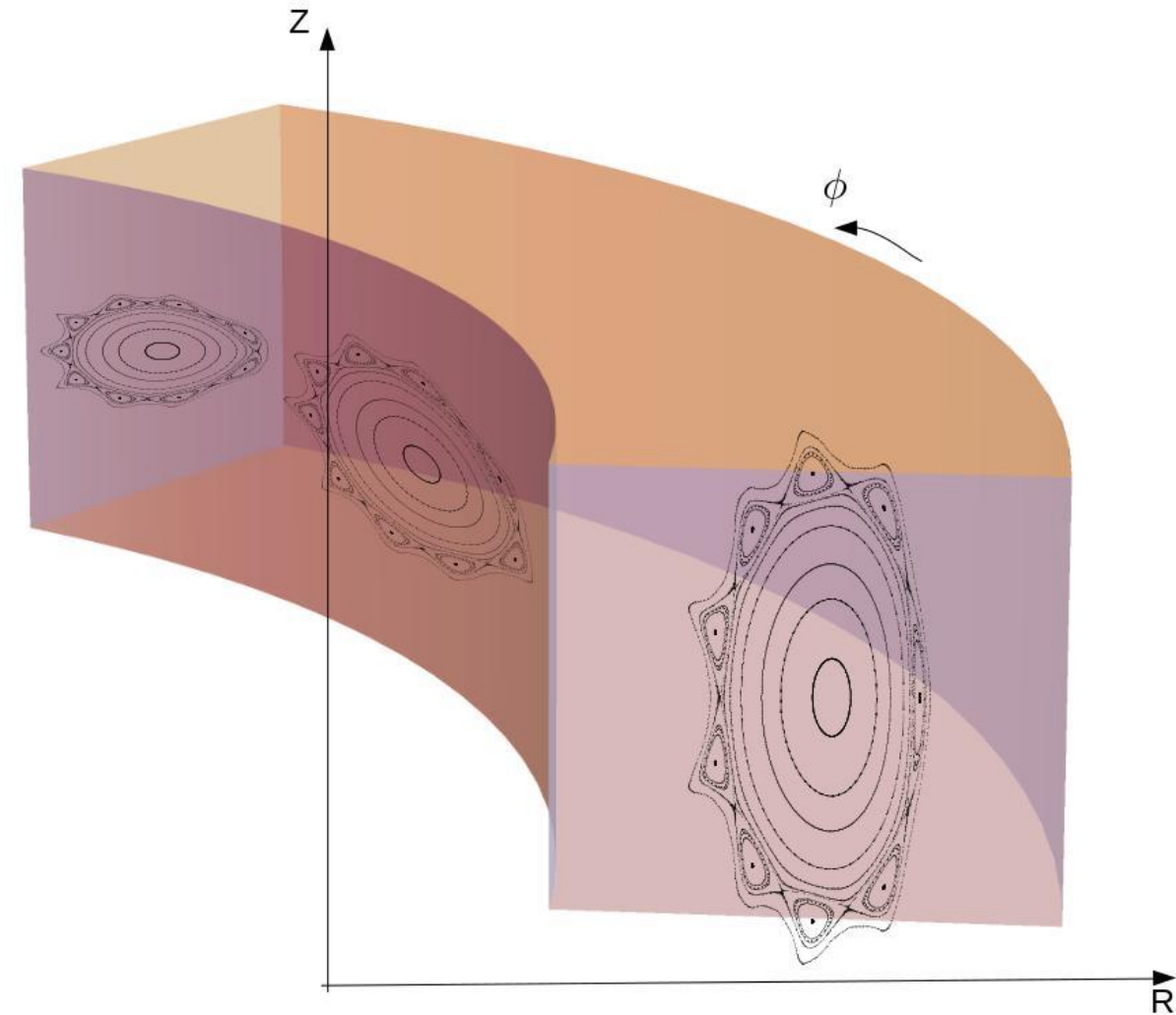
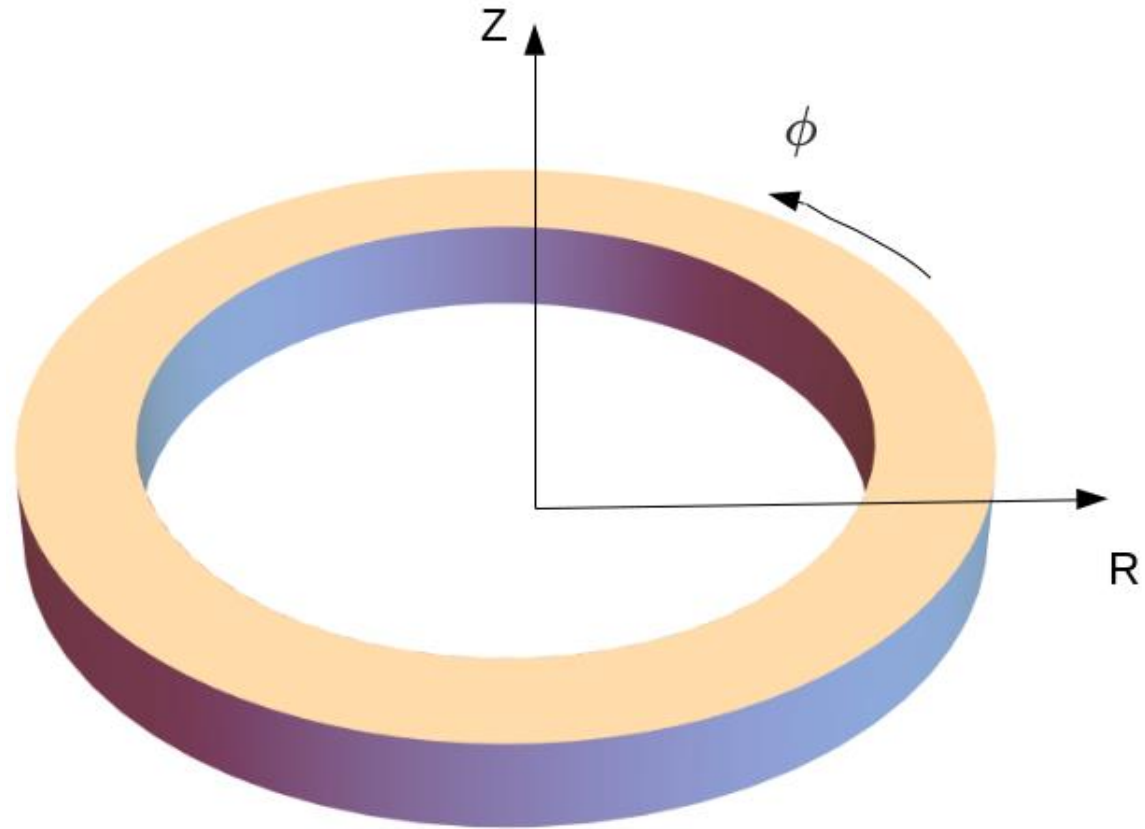
# We simulate a 5-field period stellarator... with a 5/9 chain of islands



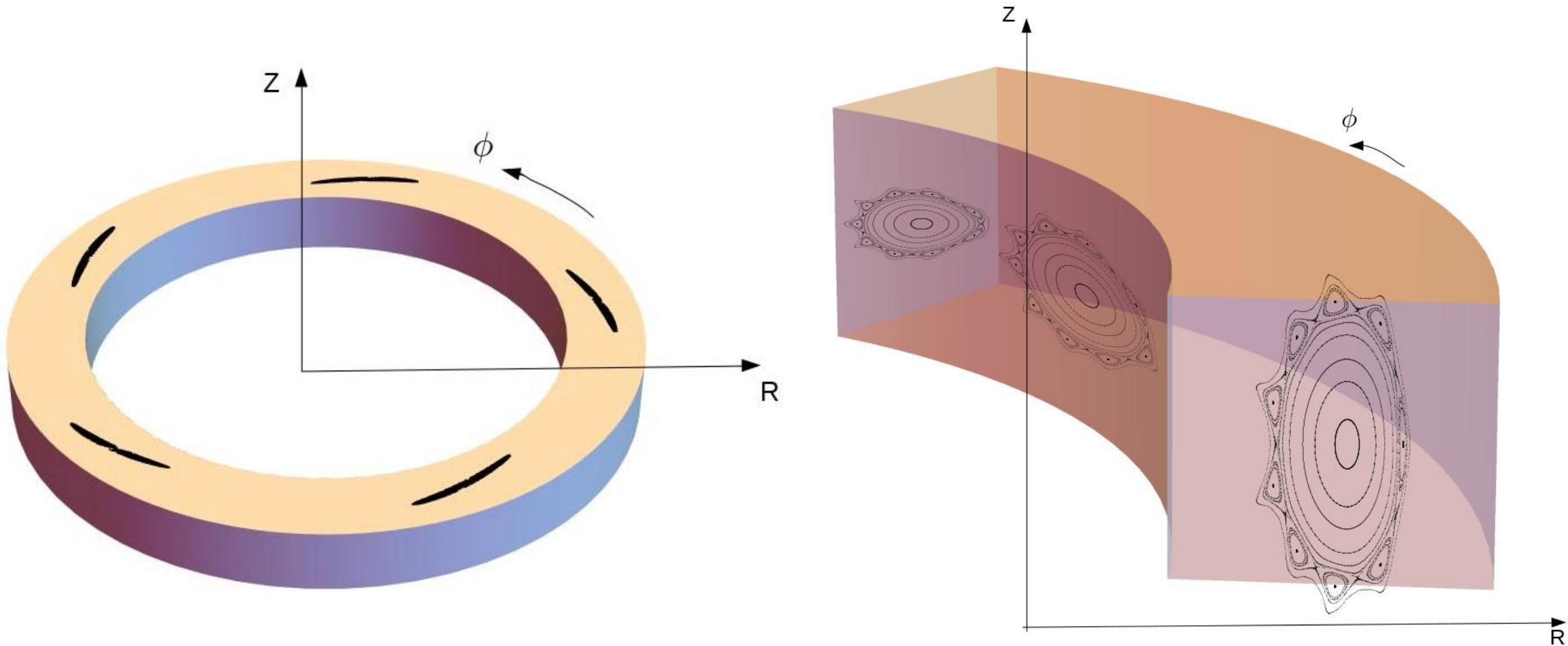
- All rotational transform from rotation of the ellipses



# GBS domain boundary intersects divertor islands

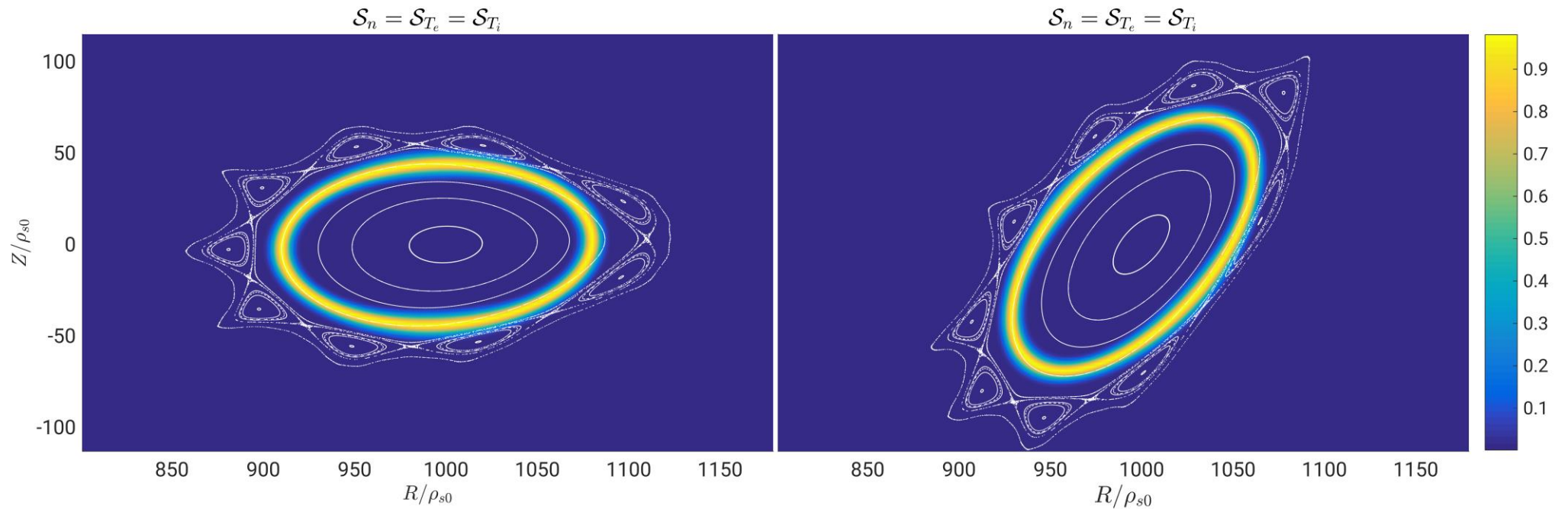


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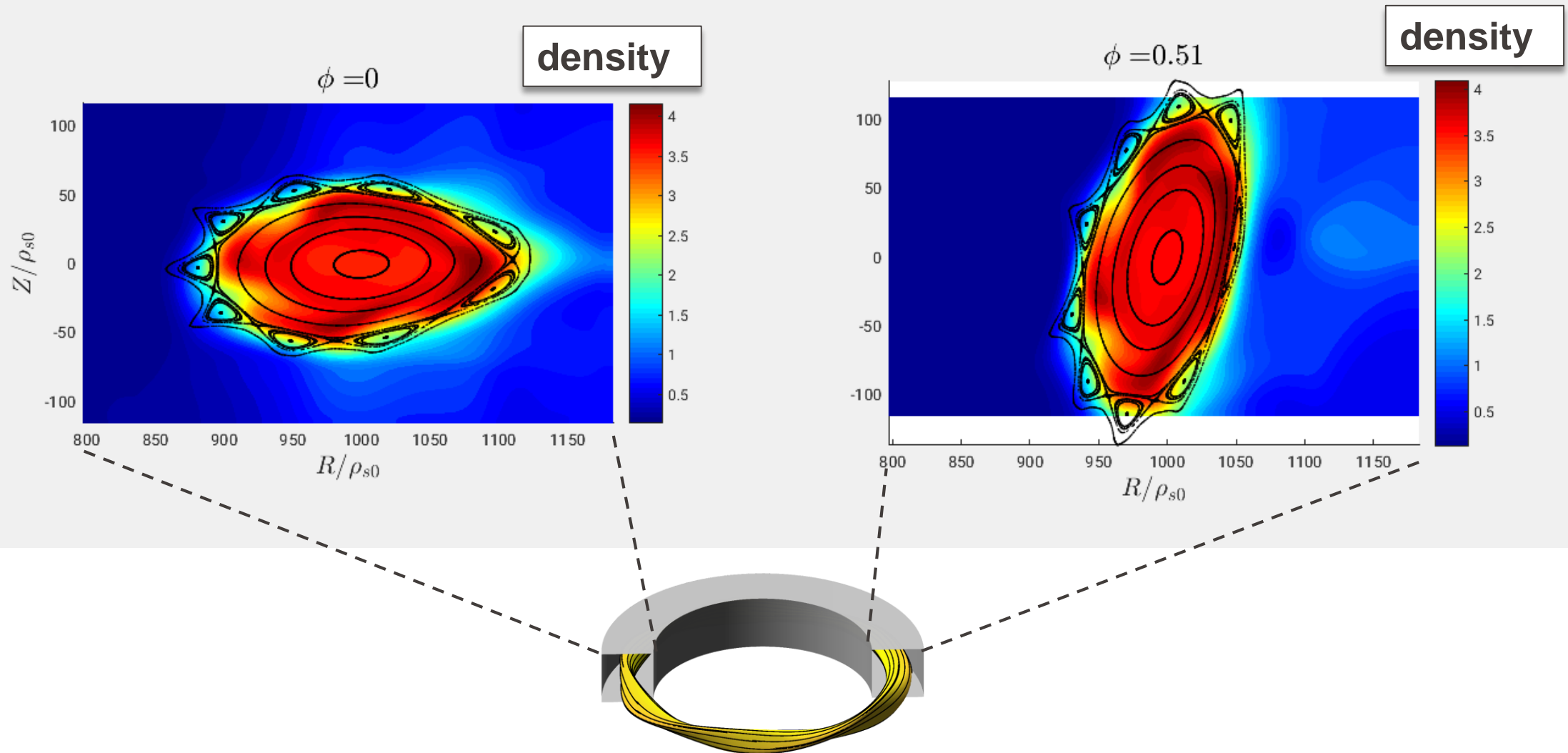


# Source for density and temperature localized around a magnetic surface

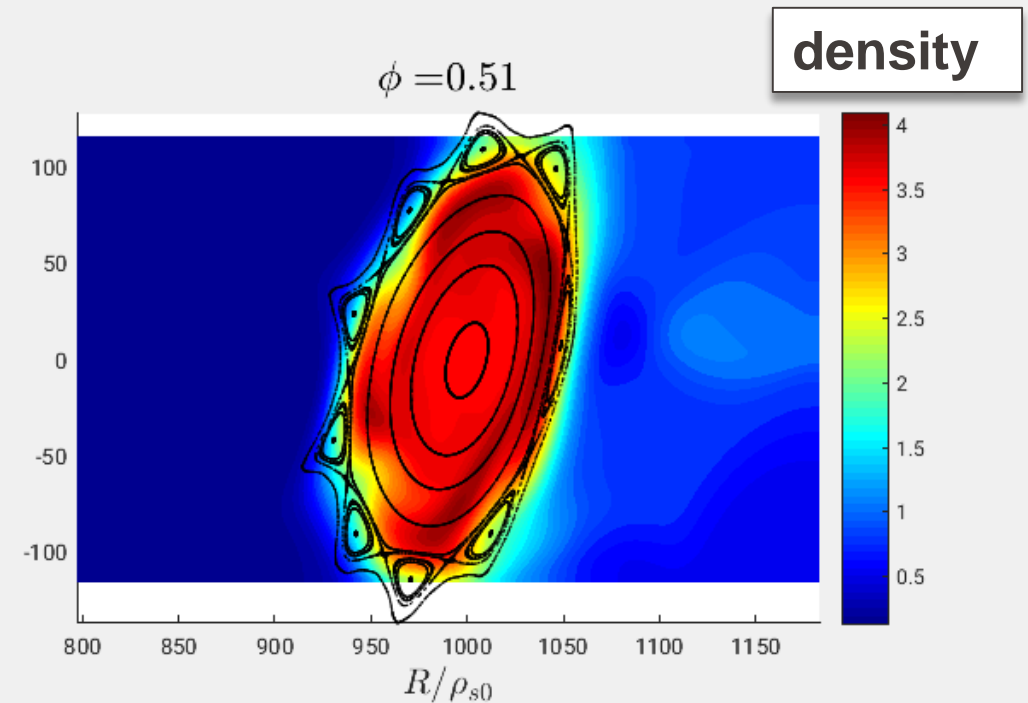
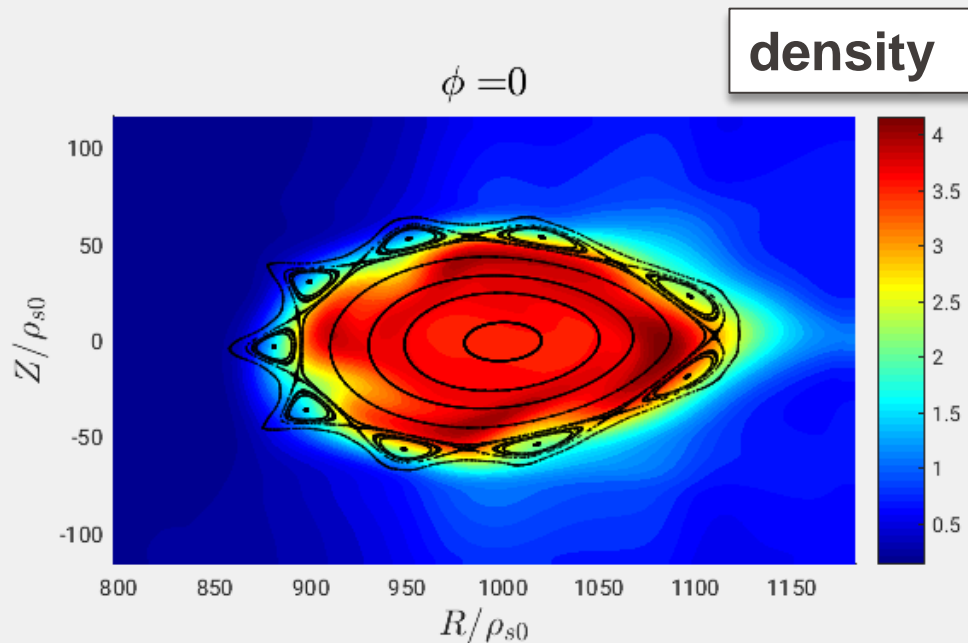
- Simulation doesn't strongly depend on the sources' profile



# Steady-state of simulation dominated by coherent mode

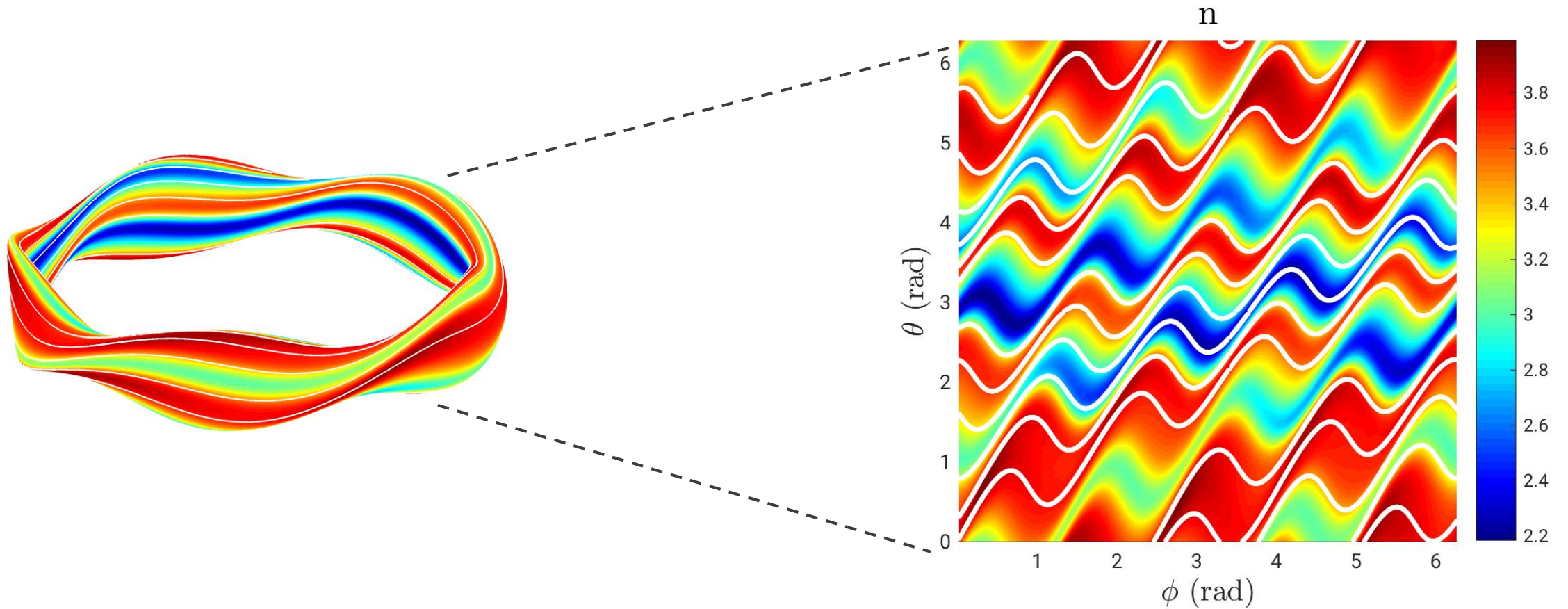


# Steady-state of simulation dominated by coherent mode

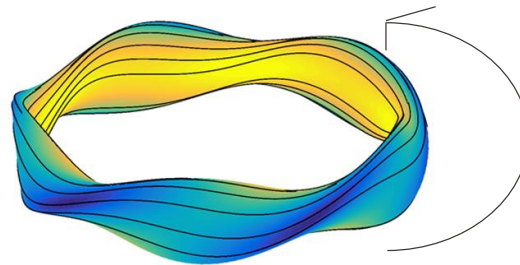
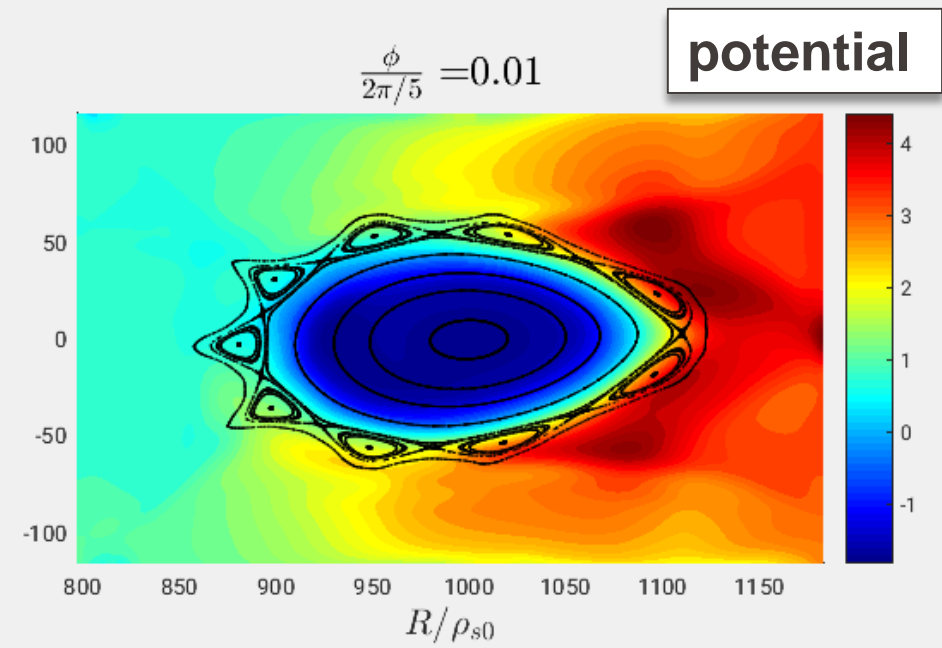
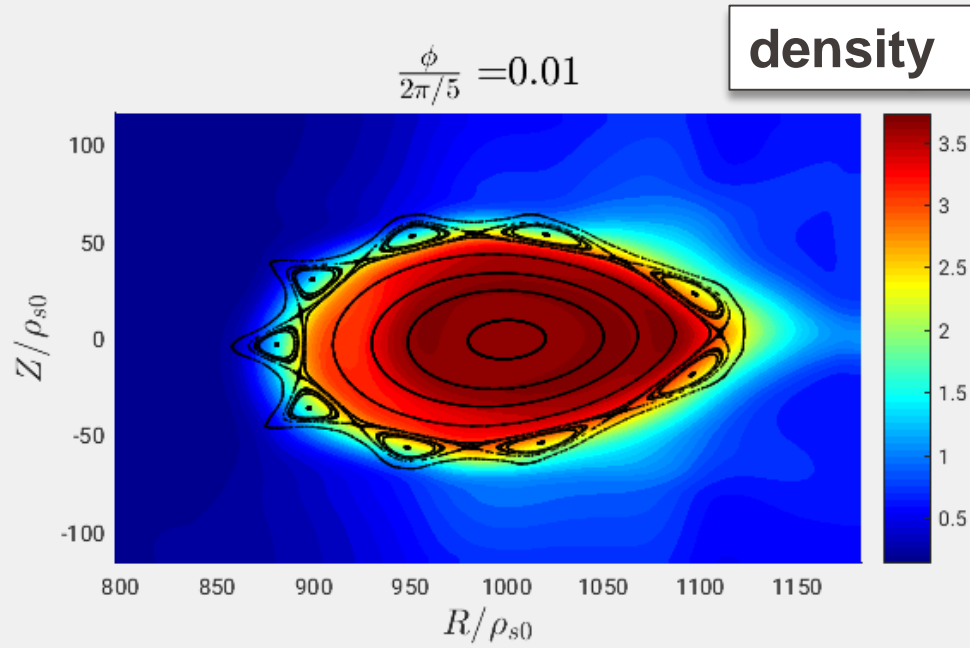


- An  $m=4$  mode dominates the global dynamics
- Mode rotates with  $\sim$  ion diamagnetic frequency
- No broad-band turbulence
- Radial turbulent transport due to  $\langle \tilde{\Gamma}_{\text{ExB}} \rangle_t = \langle \tilde{n} \tilde{v}_{\text{ExB}} \rangle_t$  balances source

# Mode is field-aligned

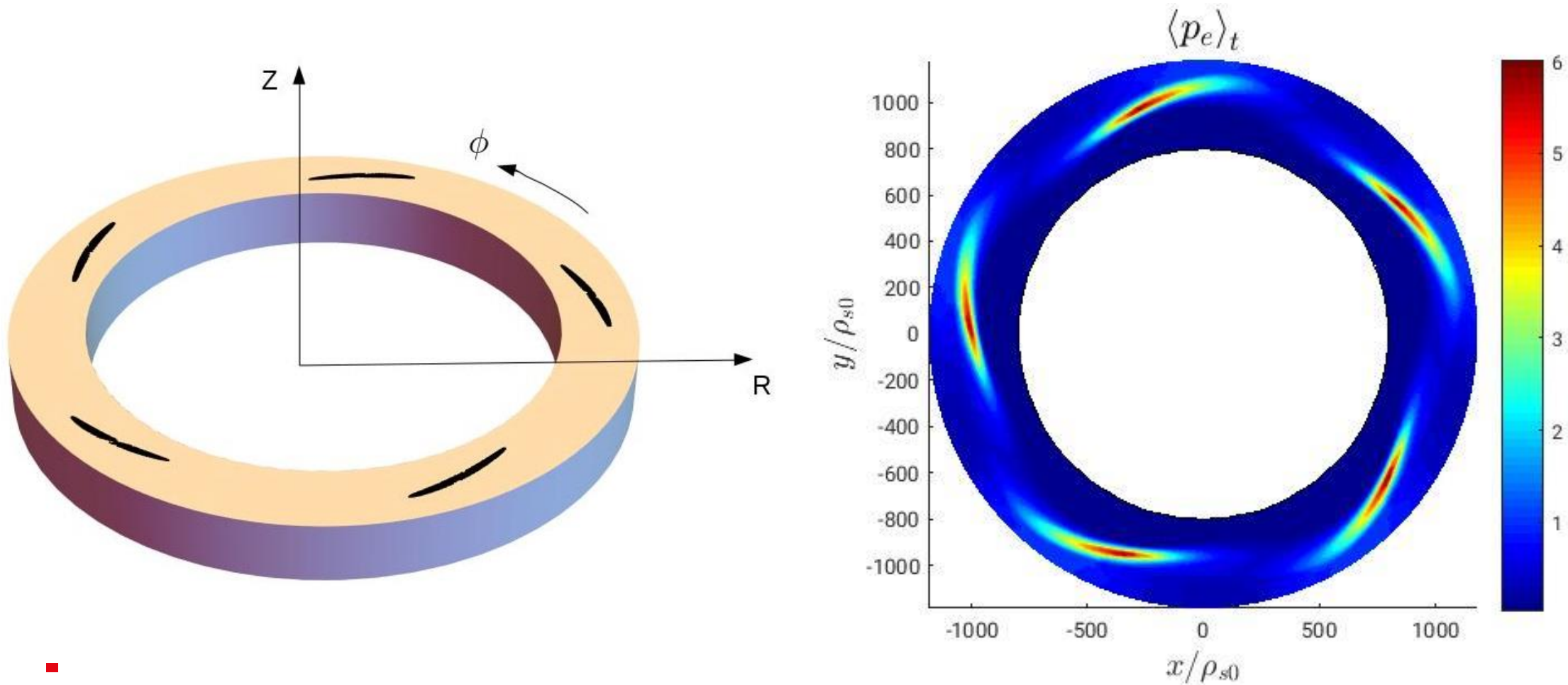


# Equilibrium profiles



## Effectiveness of the island divertor

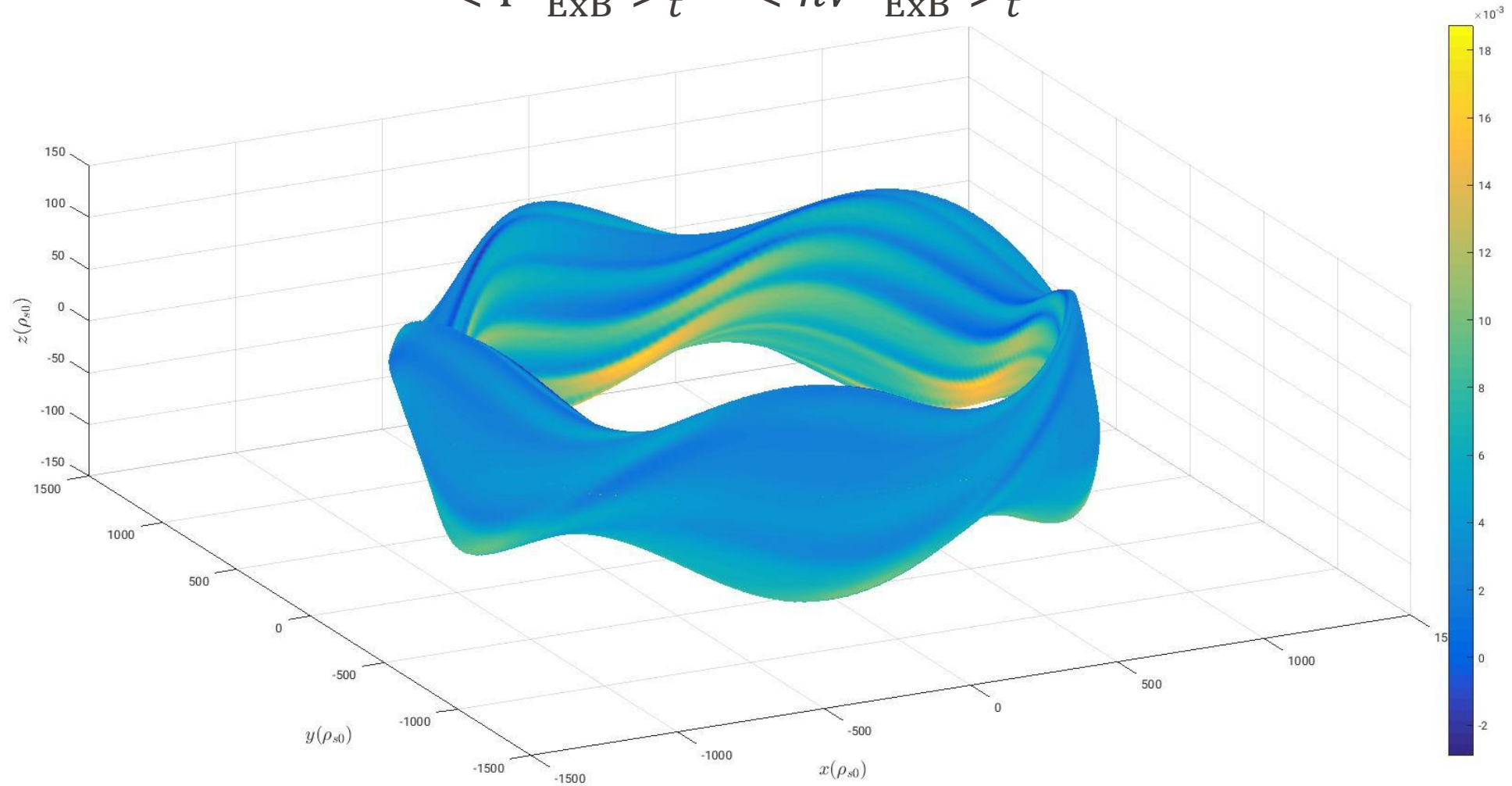
- On the **TOP** of the simulation box, pressure is maximum where field lines strike:





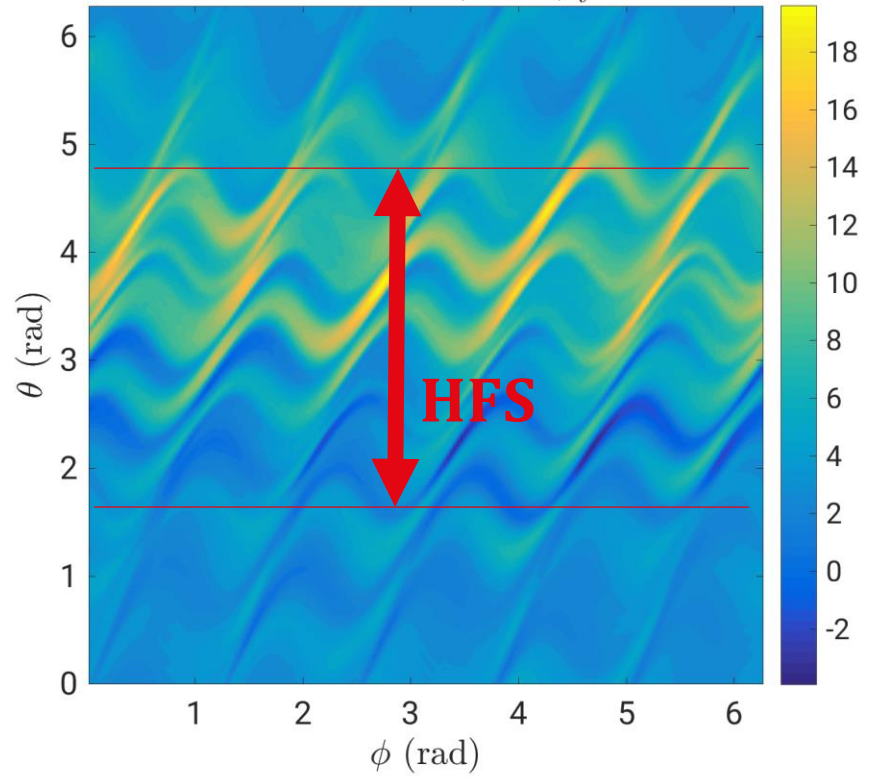
# Asymmetry of ExB-flux between HFS/LFS

$$\langle \widetilde{\Gamma}^r_{\text{ExB}} \rangle_t = \langle \tilde{n} \widetilde{V}^r_{\text{ExB}} \rangle_t$$

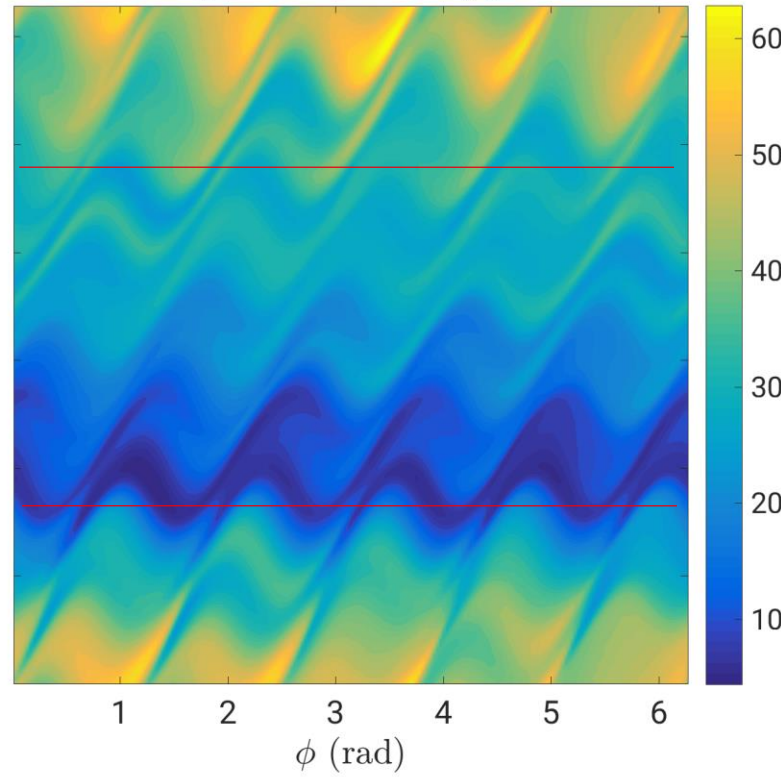


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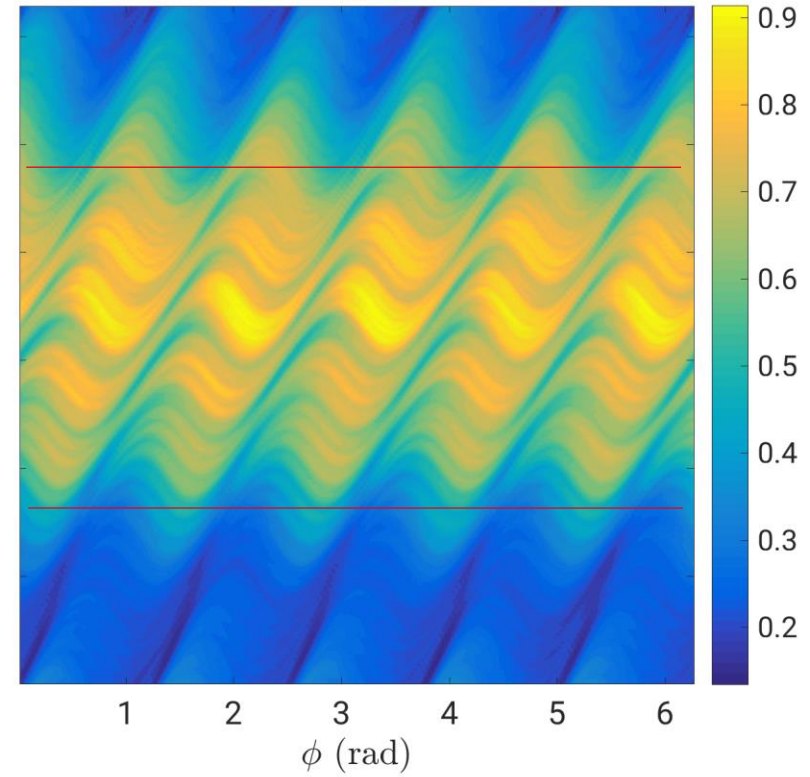
simulation  $\langle \tilde{\Gamma}_{E \times B}^r \rangle_t$



phase difference ( $^\circ$ )



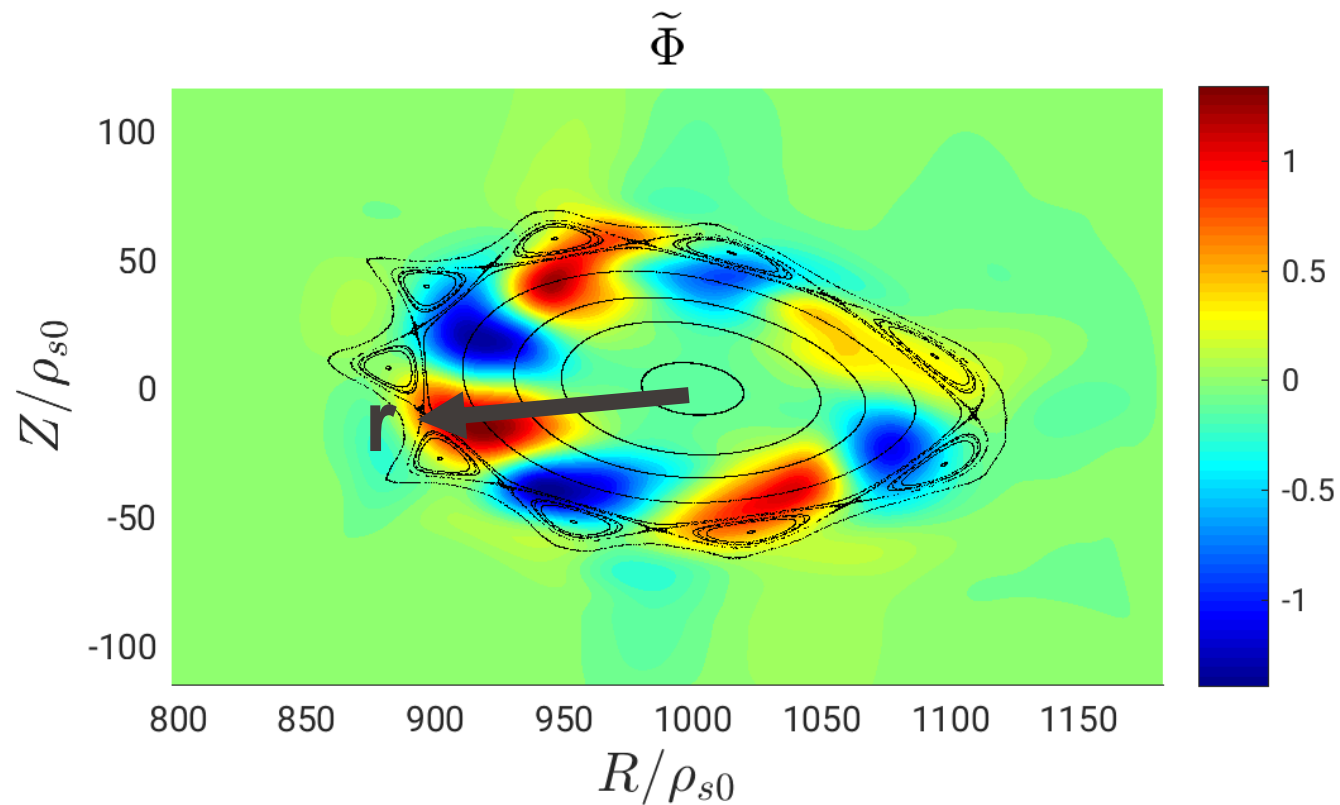
$|\tilde{n}||\tilde{\Phi}|$



# Understand the mode with non-local linear theory

- Linearize GBS equations by assuming quantities vary as:

$$n = n_0(r) + \tilde{n}(r) e^{j(m\theta + n\phi)} e^{\gamma t}$$

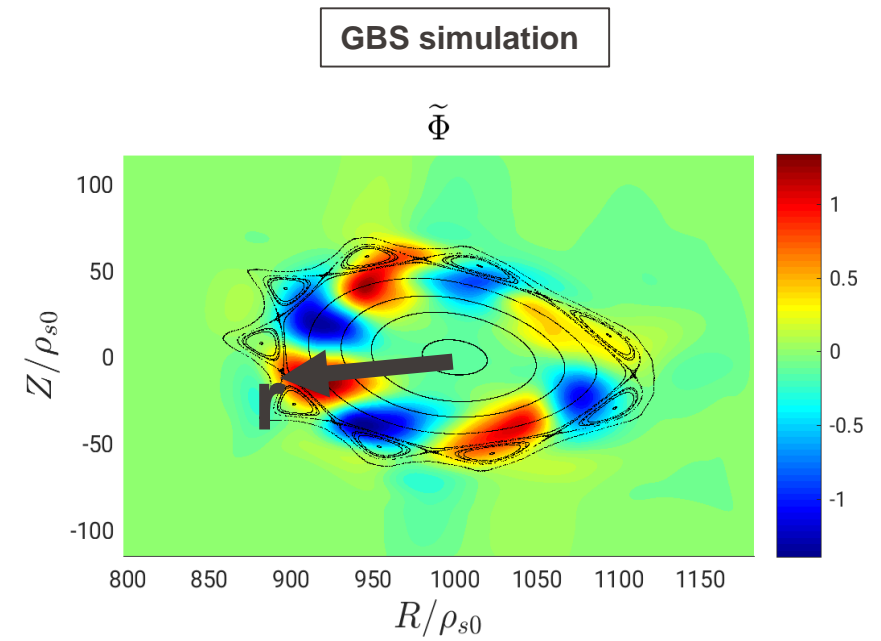
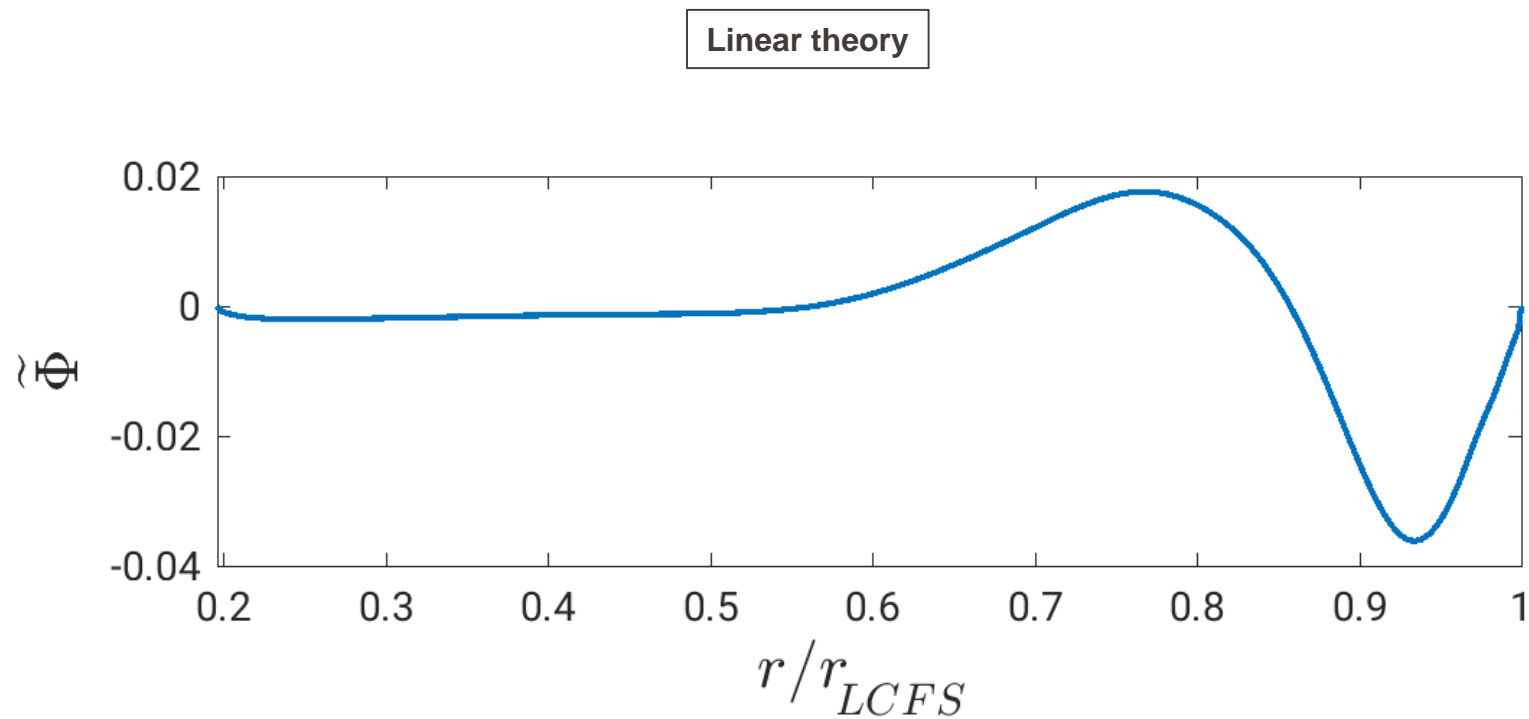


GBS simulation

$m = 4$

$n = 5$

# Linear theory predicts the observed mode



Is the linear mode able to transport the same  $\Gamma_{\text{ExB}}$ ?

$$\Gamma^c = \frac{k_y}{2B} |\tilde{n}\tilde{\phi}| \sin(\delta_{\Phi-n})$$

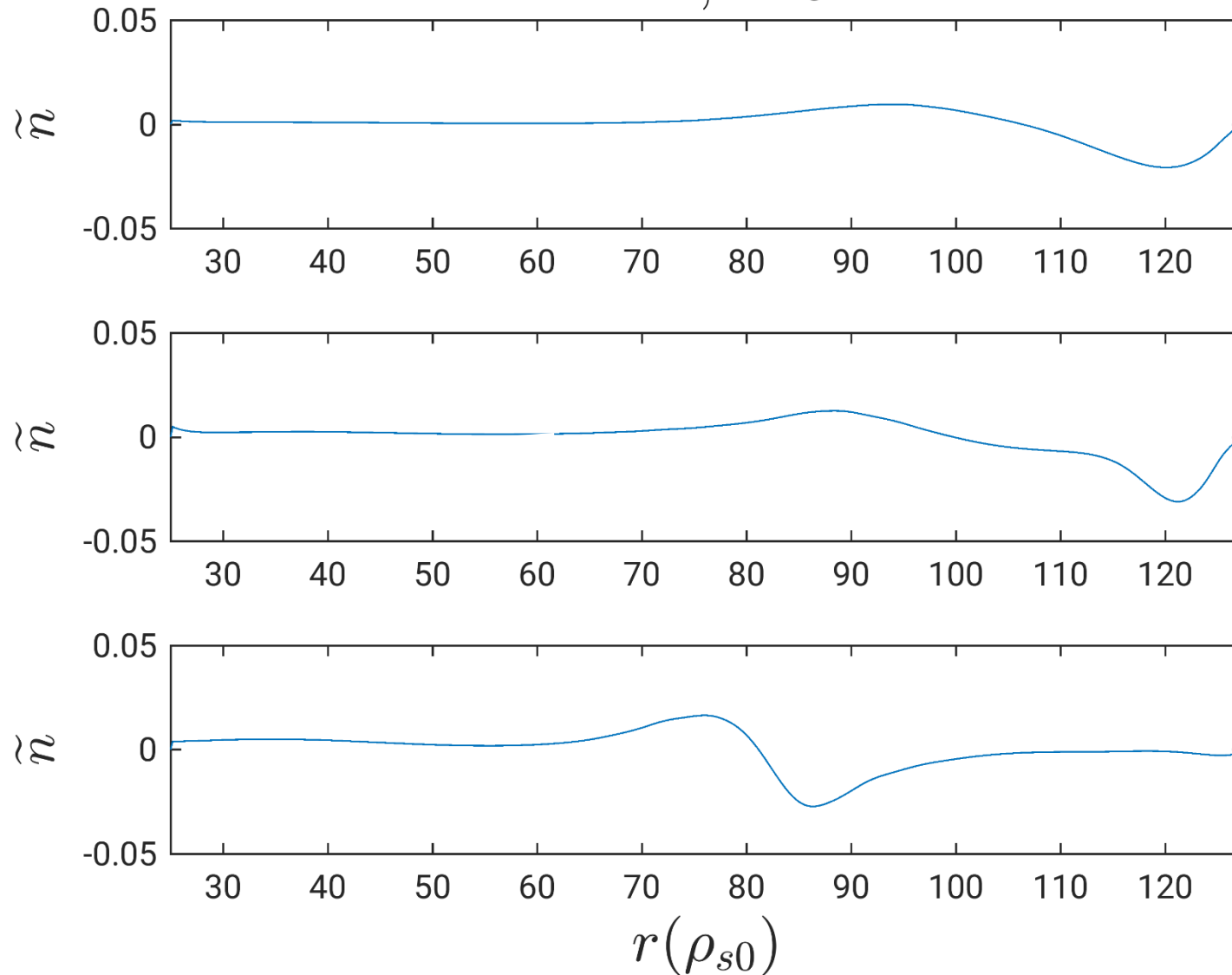
$$\int_{\partial\Omega} \Gamma^c dS = \int_{\Omega} \mathcal{S}_n \rho_* dV$$

$$|\tilde{n}\tilde{\phi}| \sim \frac{2B}{k_y} \frac{\int_{\Omega} \mathcal{S}_n \rho_* dV}{\int_{\partial\Omega} dS} \frac{1}{\sin(\delta_{\Phi-n})}$$



## Nature of the linear mode: ballooning

$$m=4, n=5$$



**No drift-waves drive**

( $\nabla_{\parallel} p_e = 0$  in  $V_{\parallel e}$  eq.)

**No ballooning drive**  
(curvature(p)=0 in vorticity eq.)

## Conclusions

- First global fluid simulations of a **stellarator** have been performed with **GBS code**
- Unlike tokamak experiments/simulations, **no broad-band turbulence nor blobs** were observed. Instead, a low **poloidal mode (m=4) dominates transport**
- Linear theory points to **ballooning mode**
- Is this coherent mode a property of the configuration used?



## Technical difficulties

- Number of grid points in toroidal direction ( $N_{\phi}=200$ )  $\rightarrow dt \sim 3e-6$
- Broad parameter space in the magnetic field configuration
- Difficult to predict what happens in the stochastic region. Regions of very small connection length. Density may decrease quickly
- Boundary conditions. Difficult to run with magnetic BC on density and vorticity. Instead:

$$\partial_s n = 0 \quad \text{and} \quad \omega = 0$$