Finite element representation of Phase Space Zonal Structures (PSZS) in ORB5.

A. Bottino

[1]Max Planck Institute for Plasma Physics,
Boltzmannstr. 2, 85748 Garching, Germany

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Sergio Briguglio, Matteo Falessi, Thomas Hayward-Schneider,
Alexey Mishchenko and Xin Wang

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Nonlinear PSZS describes long time scale transport

PSZS: Flux surface averaged distribution function, as a function of constant of motion on unperturbed trajectories and of the adiabatic invariant:

$$\hat{F}_0(P_\varphi, \mu, H_0) = \tau_b^{-1} \oint \frac{\mathrm{d}\theta}{\dot{\theta}} F_z \ , \quad \tau_b = \oint \frac{\mathrm{d}\theta}{\dot{\theta}} \ , \quad F_z = \frac{1}{2\pi} \int_0^{2\pi} F(P_\varphi, \mu, H_0, \theta, \varphi) \mathrm{d}\varphi$$

$$\dot{\theta} = -\frac{v_\parallel}{B_\parallel^* J} \frac{\partial P_\varphi}{\partial \psi} \ , \quad J^{-1} = \nabla\varphi \cdot (\nabla\psi \times \nabla\theta) \ , \quad P_\varphi = \psi + m_s v_\parallel \frac{F(\psi)}{eB}$$

Canonical toroidal angular momentum (conserved in Tokamaks):

$$P_\varphi = \psi + m_s v_\parallel \frac{F(\psi)}{eB}$$

Kinetic energy:

$$H_0 = \frac{1}{2} v_\parallel^2 + \mu B$$

$$\psi_0 = \psi + \frac{p_z c}{eB} F(\psi)$$

$$\dot{\psi}_0\Big|_0 = \dot{\mathbf{R}}\Big|_0 \cdot \nabla\psi_0 + \frac{\partial \psi_0}{\partial p_z} \dot{p_z}|_0$$

$$\nabla\psi_0 = \left(1 + \frac{p_z c}{eB} F'(\psi)\right) \nabla\psi - \frac{p_z c}{eB^2} F(\psi) \nabla B$$

$$\dot{\mathbf{R}}\Big|_0 = \frac{p_z}{m}\mathbf{b} - \left(\frac{p_z}{m}\right)^2 \frac{mc}{eB_\parallel^*}\mathbf{G} + \frac{\mu B}{m} \frac{mc}{eB_\parallel^*} \mathbf{b} \times \frac{\nabla B}{B}$$

$$\dot{p_z}|_0 = \mu B \nabla \cdot \mathbf{b} + \frac{\mu c}{eB_\parallel^*} p_z \mathbf{b} \times \left(\mathbf{b} \times \frac{\nabla \times \mathbf{B}}{B}\right) \cdot \nabla B$$

PSZS can improve the GK PIC algorithm.

Physics: evolution of the PSZS describes the evolution of plasma profiles (particle and energy transport) on long time scales [Falessi, Zonca 2019].

Numerics: allow for longer ORB5 turbulence simulations (noise reduction, cost reduction):

$$\frac{\mathrm{d}\delta \mathrm{F}}{\mathrm{dt}} = -\dot{\mathbf{R}} \cdot \nabla F_0 - \dot{v}_{\parallel} \frac{\partial F_0}{\partial v_{\parallel}},$$

- Run a "traditional" simulation (F_0: Maxwellian, Slowing-Down…) for a while.
- Construct the PSZS.
- Use PSZS to construct a new background: $\hat{F}_0(P_\varphi, \mu, H_0) \rightarrow F_0(\psi, v_\parallel, \mu)$
- Restart the simulation with the new background (adapt the weights).

Difficulties:

1) Grid projection of F in velocity space (absent in PIC).
2) Mapping the coordinate systems from $(P_\varphi, \mu, H_0) \rightarrow (\psi, v_\parallel, \mu)$ and back.

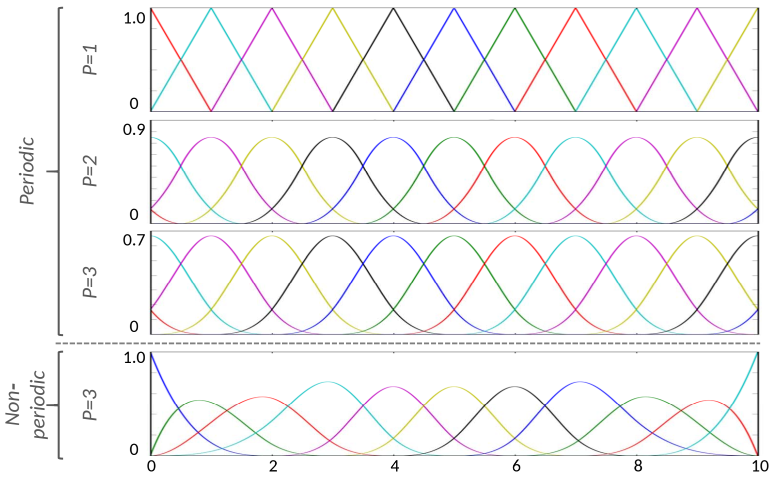
First step: F(r,v//,μ) using finite elements.

- **Define a new B-splines basis:**

$$\Lambda_\mu(\mathbf{x}) = \Lambda_{\mu 1}(r)\Lambda_{\mu 2}(v_\parallel)\Lambda_{\mu 3}(\mu)$$

- **Discrete f:**

$$f(r, v_\parallel, \mu, t) \simeq f_h(r, v_\parallel, \mu, t) = \sum_{\mu=1}^{N_g} f_\mu(t)\Lambda_\mu(\mathbf{x})$$



- **Equations to solve:**

$$\sum_\mu A_{\mu\nu} f_\mu = b_\nu$$

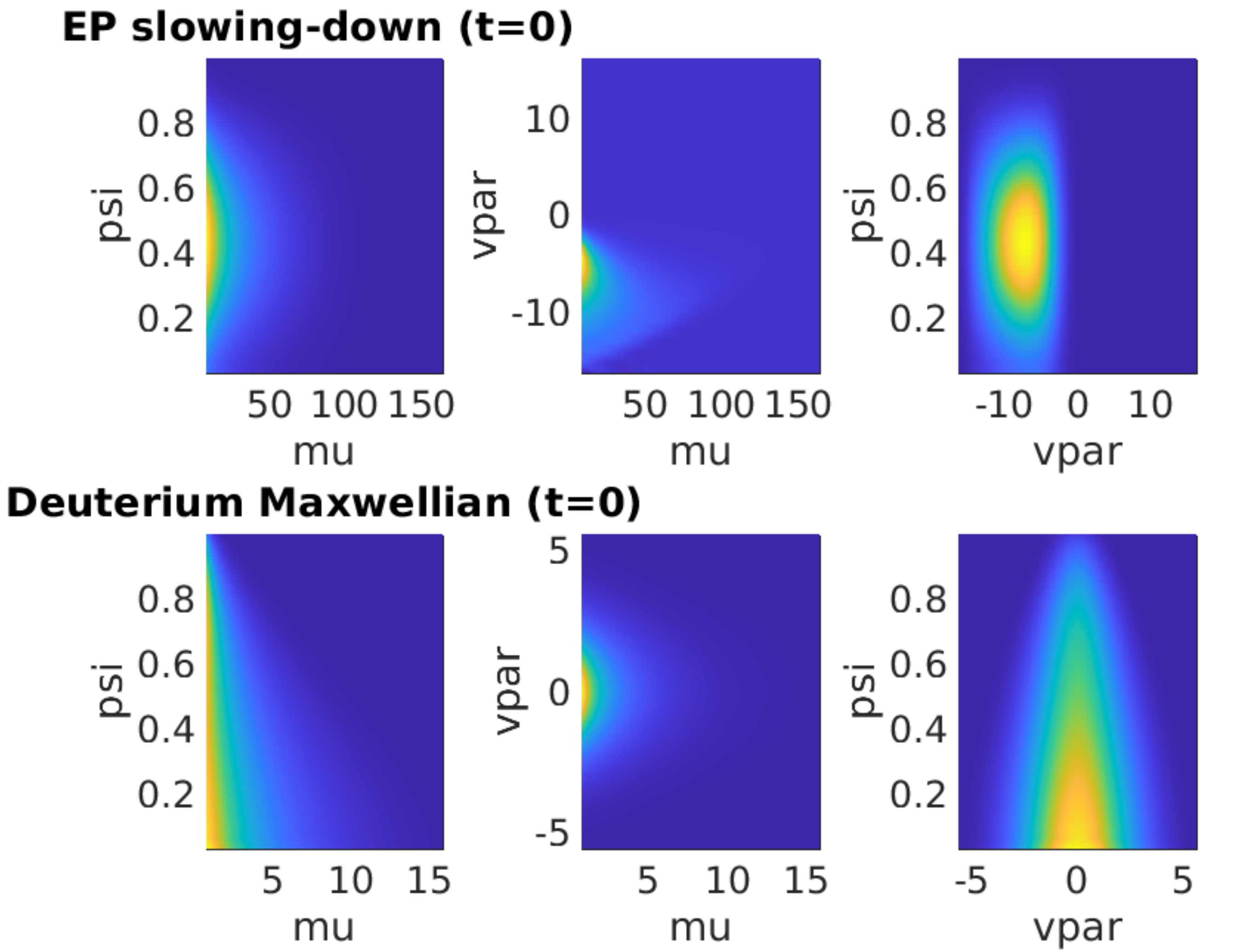
$$\mathrm{d}\Omega = \langle J(r,\chi) B_\parallel^*(r,\chi)\rangle \mathrm{d}r\mathrm{d}v_\parallel \mathrm{d}\mu$$

$$A_{\mu\nu} = \int \mathrm{d}\Omega \Lambda_\nu \Lambda_\mu \qquad b_\nu = \frac{1}{N_p}\sum_{k=1}^{N_p}\left(w_k + f_0(r_k, \theta_k, \varphi_k, v_{\parallel k}, \mu_k)V_k\right)\Lambda_\nu(r_k, v_{\parallel k}, \mu_k).$$

- **Caveat**: Matrix construction requires knowledge of the 3D metric (flux surface averaged **Jacobian**), B*// not needed (importance sampling).
  Note: requires consistency between particle distribution of phase-space and Jacobian.

- *Numerical and mathematical issues: see TSVV10 talk (or ask later).*

Test: NLED-AUG B-spline projection of equilibrium



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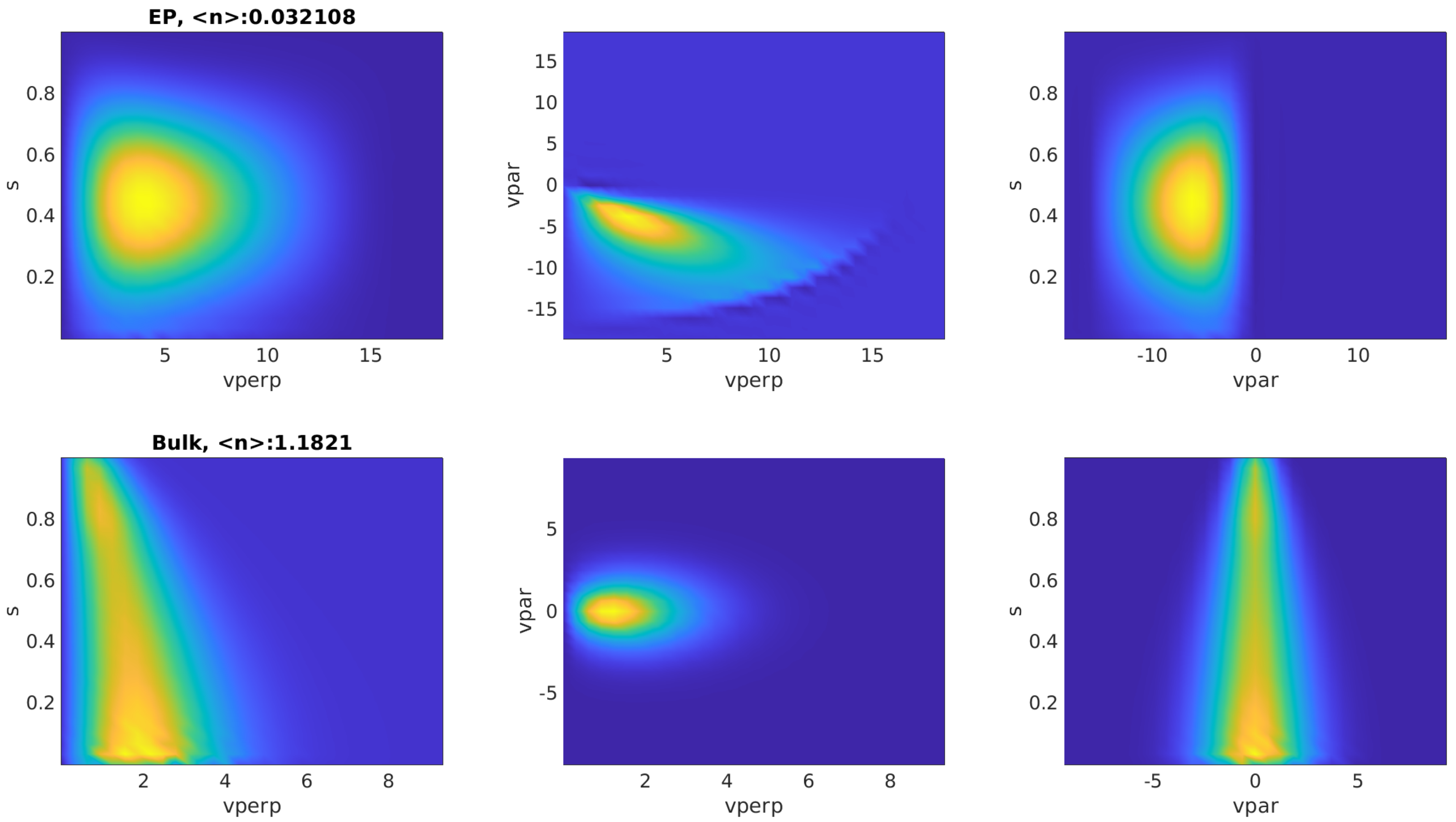

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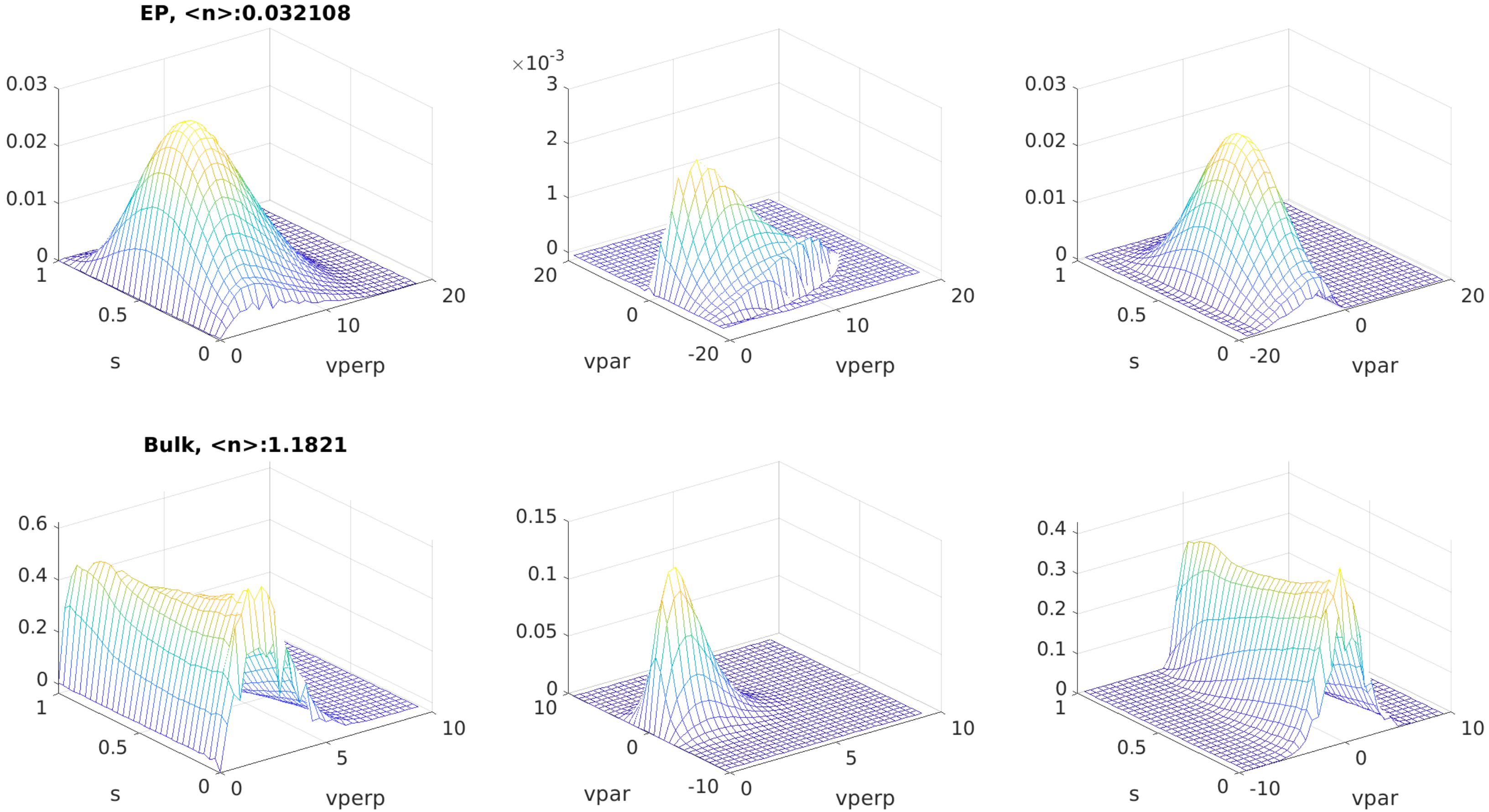

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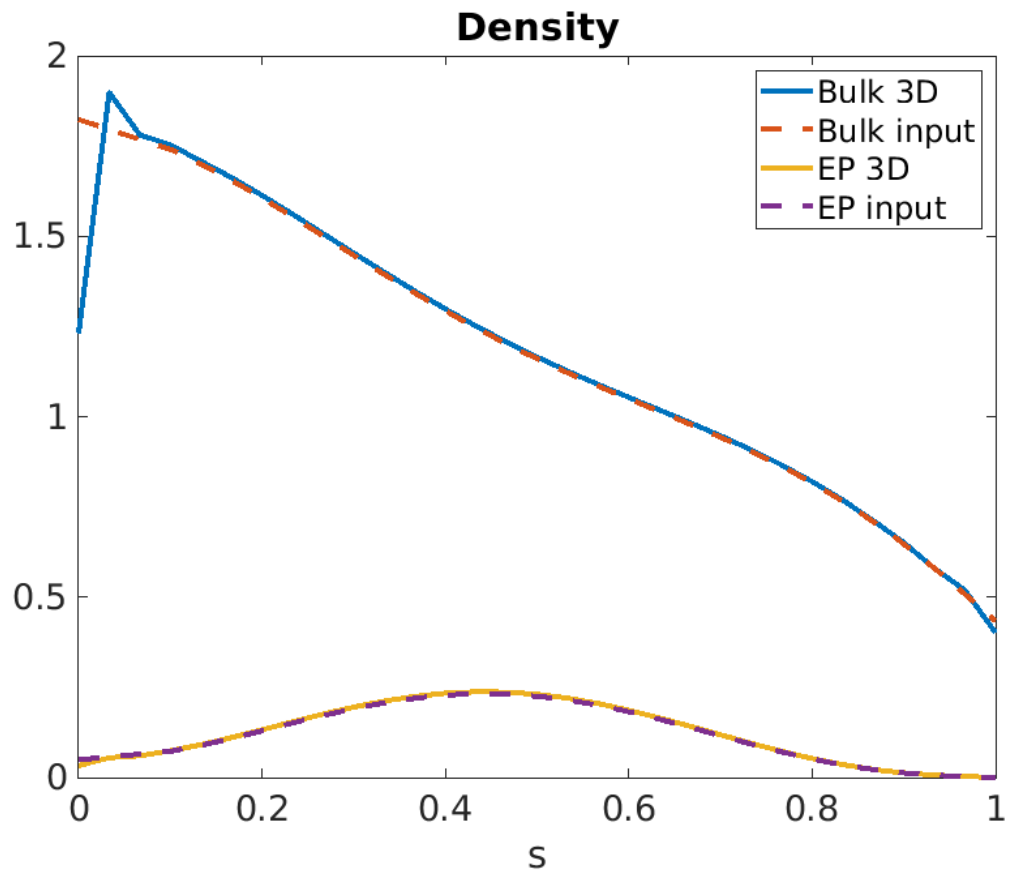


Change of coordinates, approximations

PSZS: Flux surface averaged distribution function, as a function of constant of motions on unperturbed trajectories and of the adiabatic invariant:

$$\hat{F}_0(P_\varphi,\mu,H_0) = -\frac{e}{2\pi\tau_b}\oint \mathrm{d}\theta \frac{B_\parallel^* J}{v_\parallel}\left(\frac{\partial P_\varphi}{\partial\psi}\right)^{-1}\int_0^{2\pi}\mathrm{d}\varphi F(P_\varphi,\mu,H_0,\theta,\varphi)$$

Canonical toroidal angular momentum (conserved in Tokamaks):

$$P_\varphi = \psi + m_s v_\parallel \frac{F(\psi)}{eB} \qquad\qquad B_\varphi = \frac{F(\psi)}{R}$$

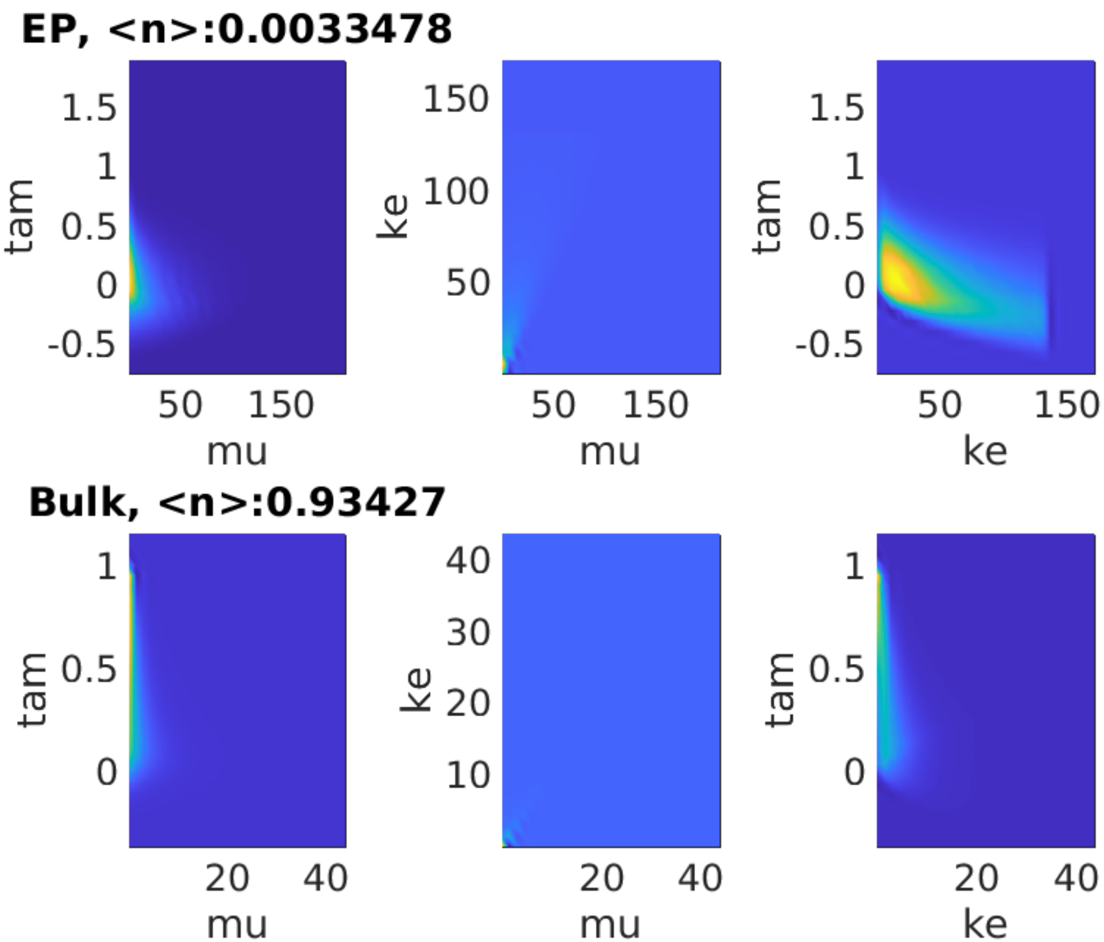
Replaced, in the matrix construction, by:

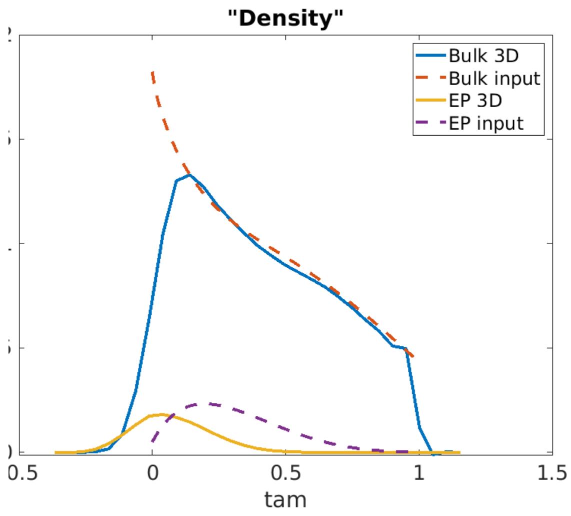
$$P_\varphi = \psi + m_s v_\parallel F(\psi)/\langle B\rangle(\psi)$$

$$\frac{\partial P_\varphi}{\partial\psi} = 1 + \frac{m_s v_\parallel}{\langle B\rangle}\frac{\partial F}{\partial\psi} - \frac{m_s v_\parallel F}{\sqrt{\langle B\rangle}}\frac{\partial\langle B\rangle}{\partial\psi}$$

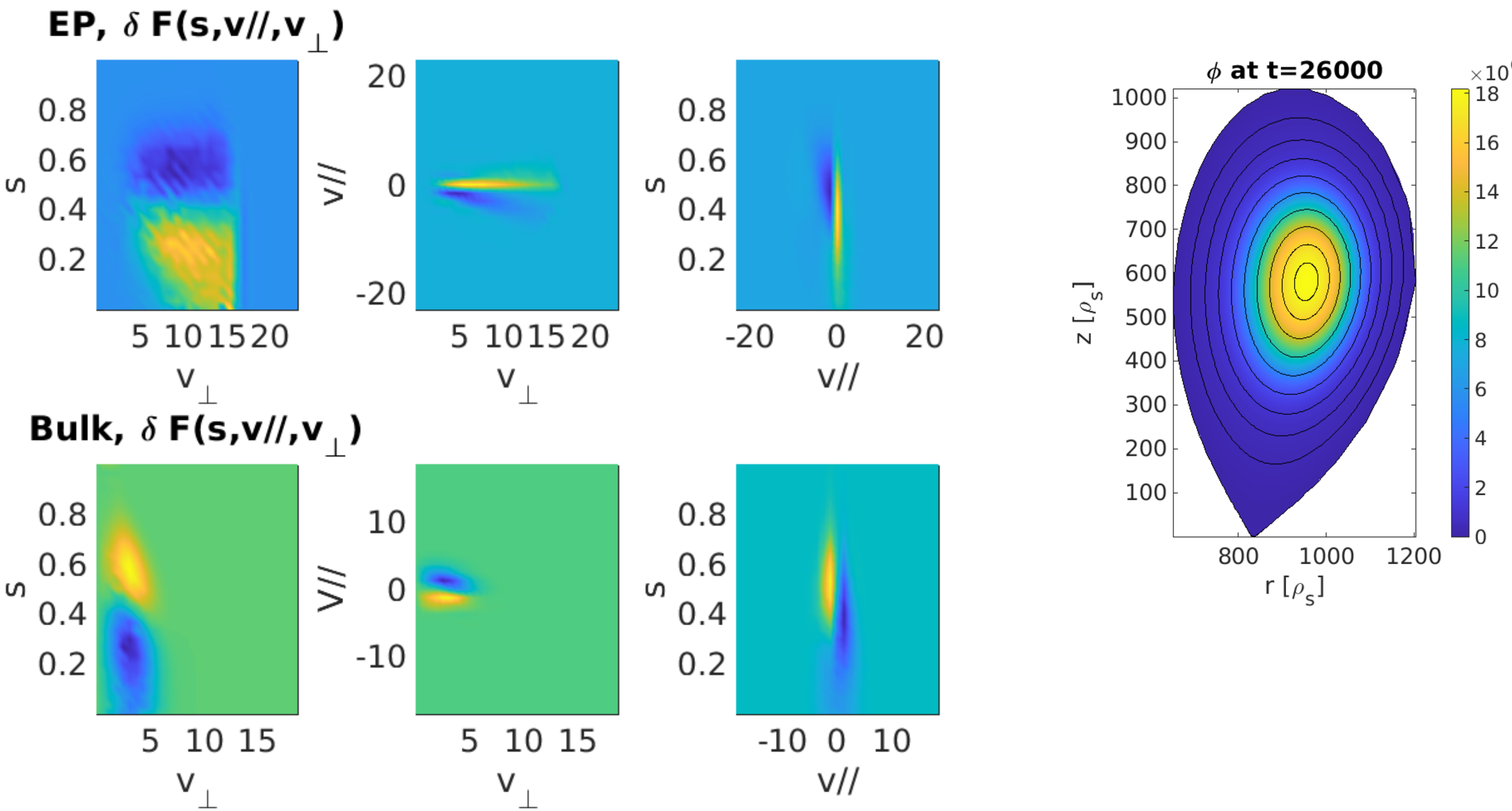
Note: the equivalence of the Monte-Carlo and Gauss integral is probably now questionable. The importance sampling in ORB5 relies on real and velocity space separation (still thinking about it…).

Test: NLED-AUG B-spline projection of equilibrium





Test: NLED-AUG, Linear, EGAM simulation [Rettino 21]



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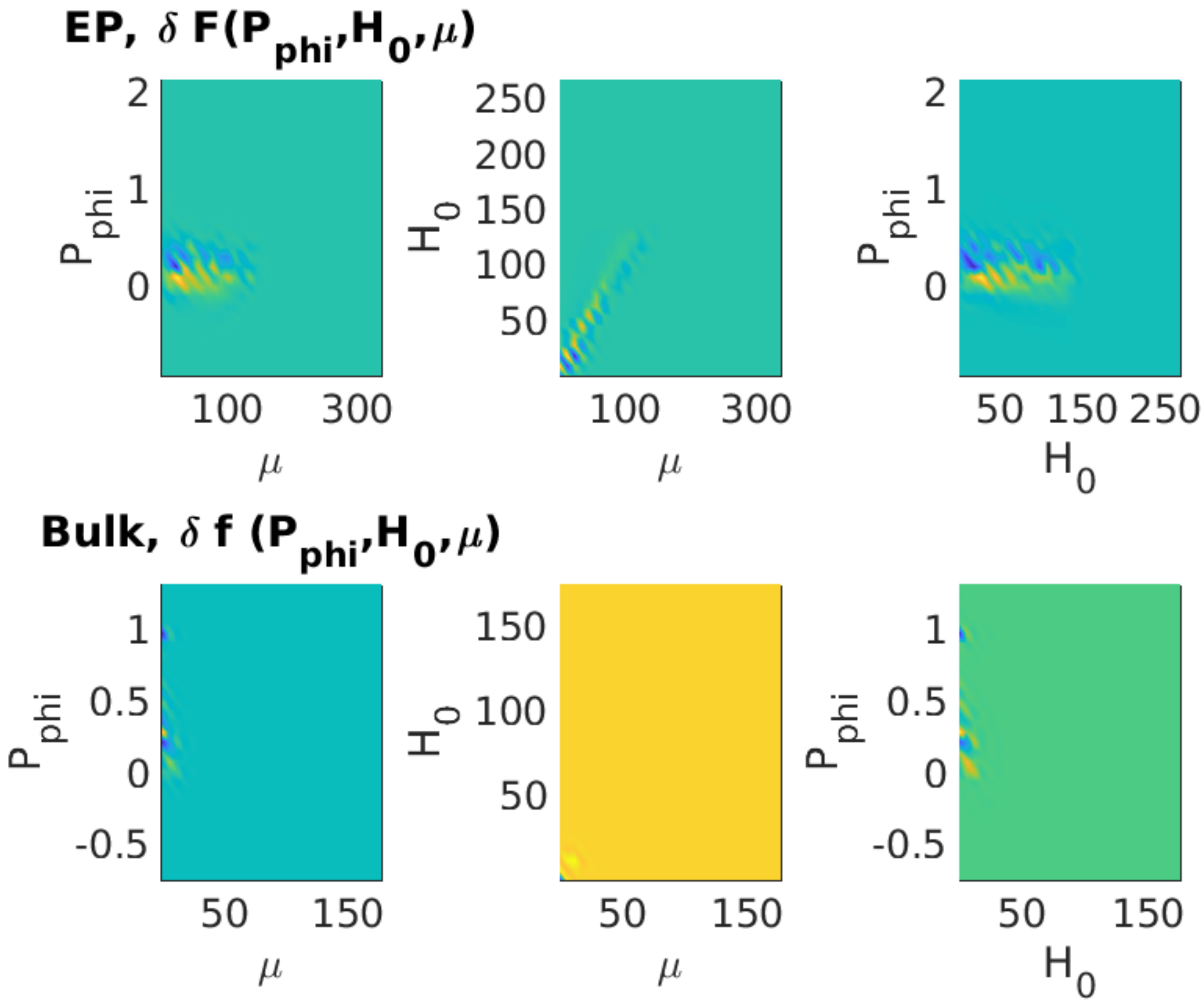

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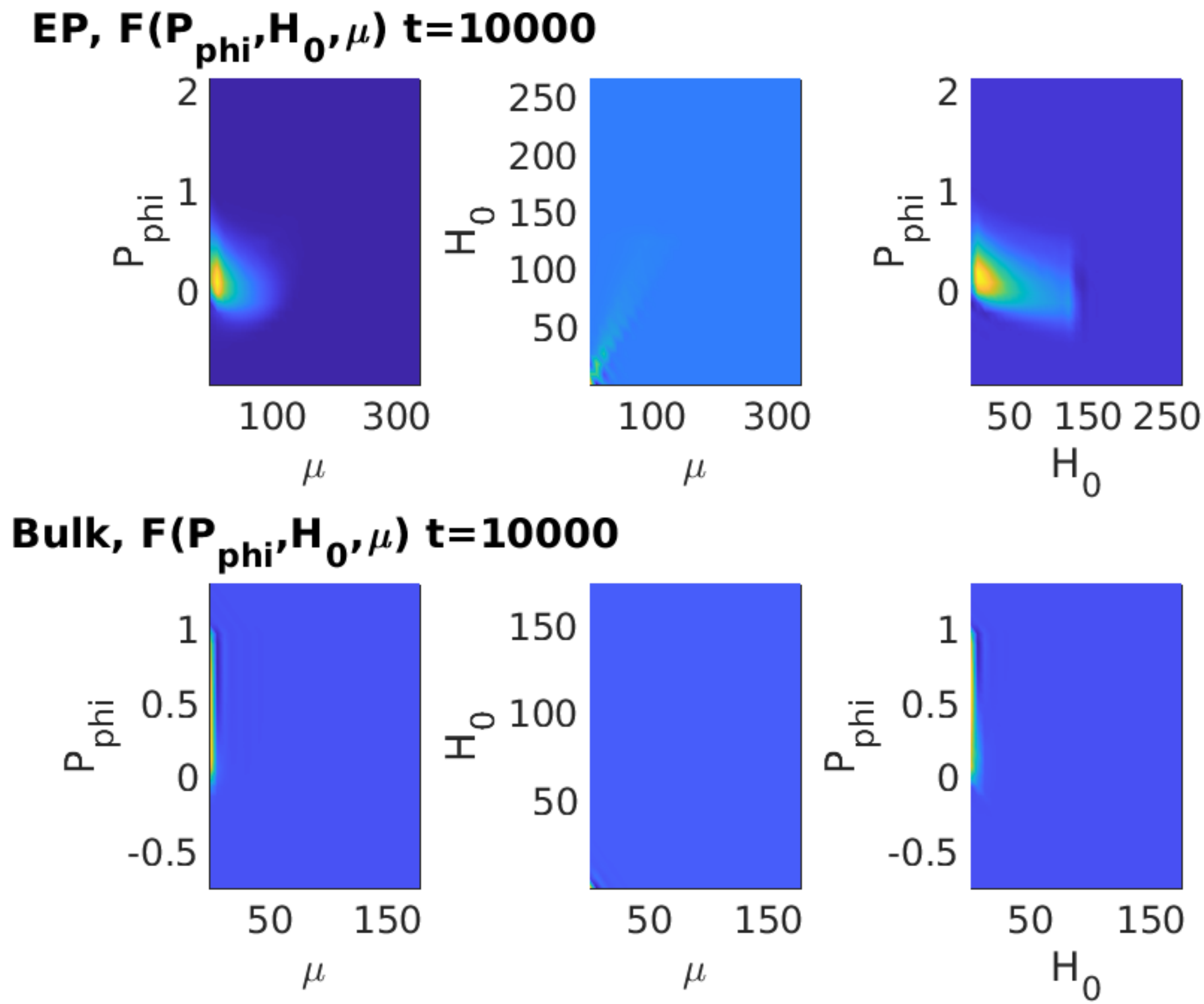

Outlook and todo list

- **Check importance sampling and Jacobian for toroidal canonical momentum.**
- **Still some work to do on ORB5 and pszs3d (restart, Jacobian offline, mixed variables…).**

- **ATEP: how to map back to GK coordinates? What about sign(v//)?**
- **TSVV10: Extend the diagnostic to other possibly useful (zonal) quantities as power balance.**
- **ATEP: Physics, nonlinear simulations (turbulence, chirping,…?).**
- **ATEP: Compare with XHMGC (Wang).**