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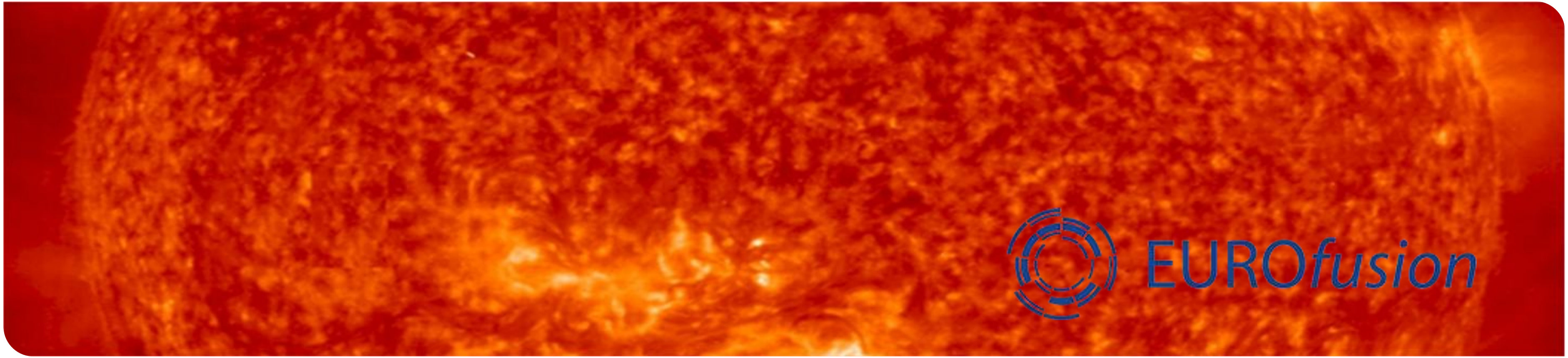
EUROfusion Science Meeting on Status of Enabling Research Projects 2022

New Generation of Megawatt-Class Fusion Gyrotron Systems Based on Highly Efficient Operation at the Second Harmonic of the Cyclotron Frequency (ENR-TEC.01.KIT)

Stefan Illy on behalf of the ENR Project Team



Outline

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- Introduction / Main Objective and Approach
- Studies on MW-class 2nd Harmonic Operation (w/o Injection Locking)
- Mode Converting Outer Corrugations
- Studies on MW-class 2nd Harmonic Operation with Injection Locking
- Research on Coupling System for Injection Locking
- Depressed Collector for 2nd Harmonic Gyrotron Operation
- Conclusion and Outlook

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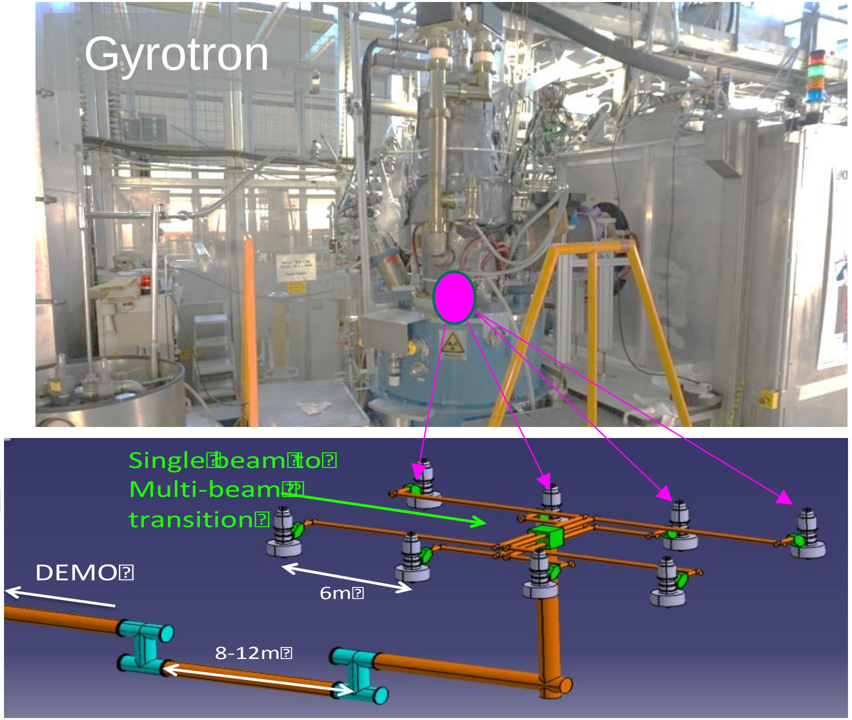
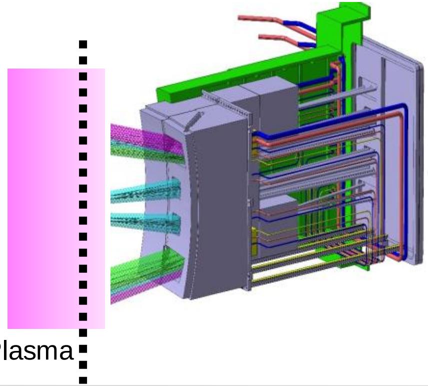
Introduction

Key Components of the DEMO EC System



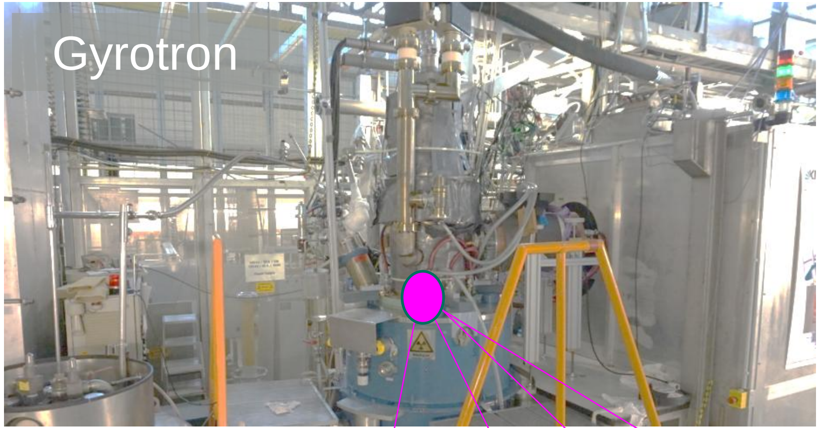
Quasi-optical transmission line

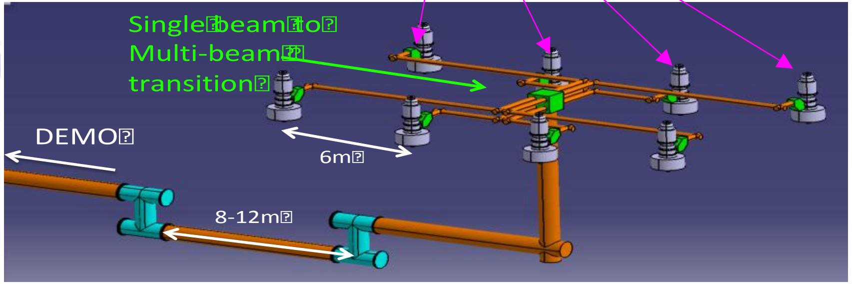
EC equatorial launcher



Plasma

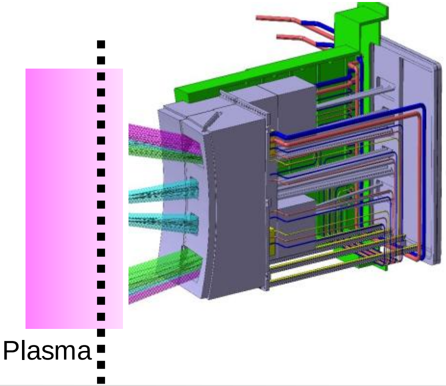
Key Components of the DEMO EC System

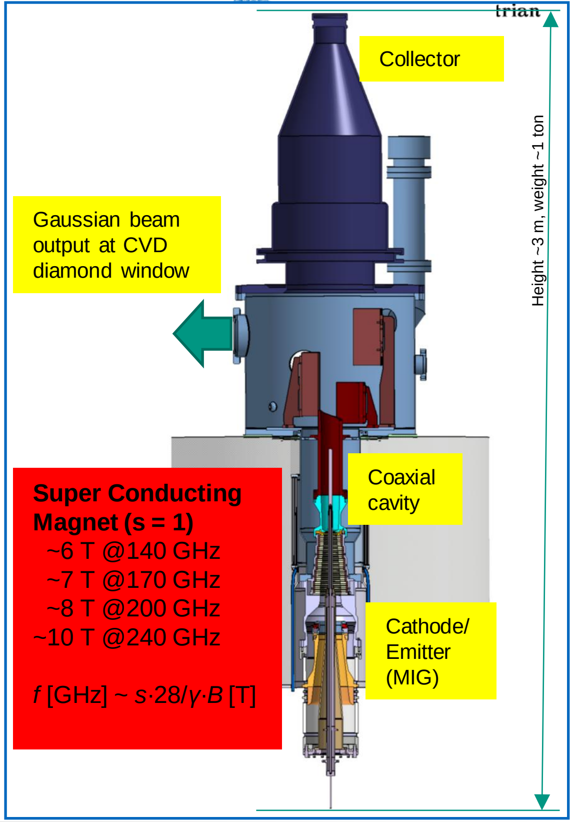




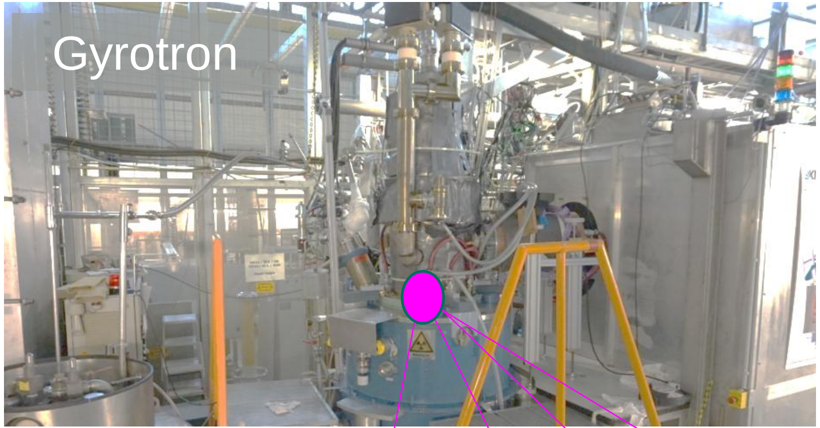
Quasi-optical transmission line

EC equatorial launcher

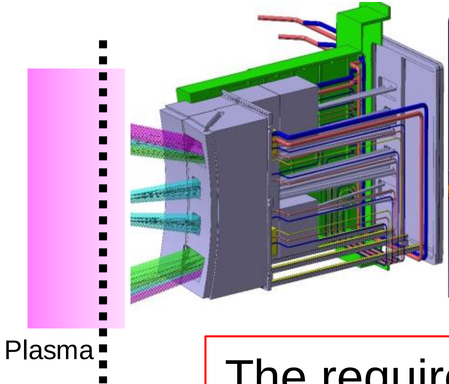


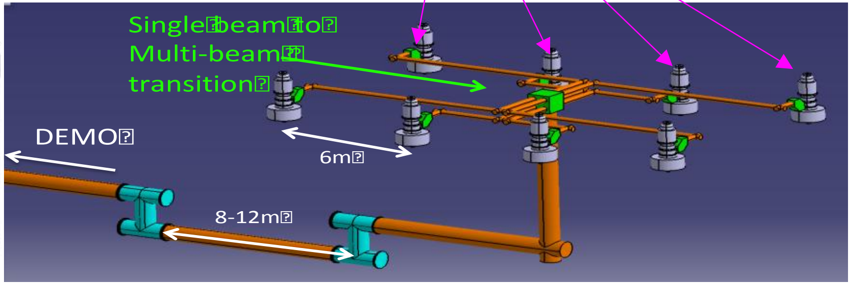


Key Components of the DEMO EC System

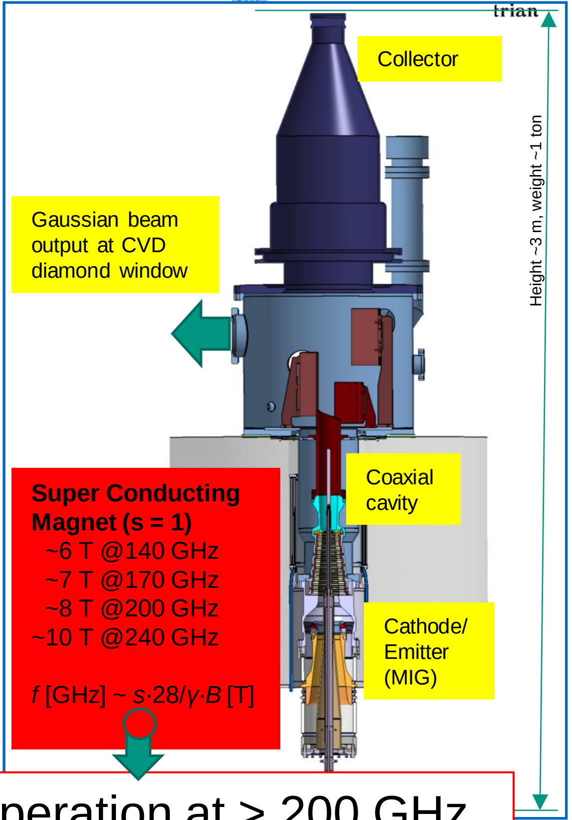


EC equatorial launcher





Quasi-optical transmission line



The required magnetic field becomes an issue for operation at > 200 GHz

→ **Gyrotrons operating at 2nd Harmonic (*s*=2) might be the solution!**

Main Objective and Approach

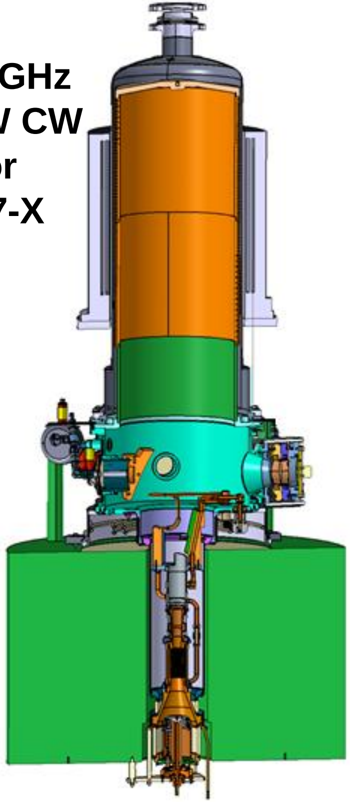
We aim for a completely new generation of highly efficient, megawatt-class fusion gyrotrons that will operate at the **2nd harmonic (*s* = 2)** of the electron cyclotron frequency.

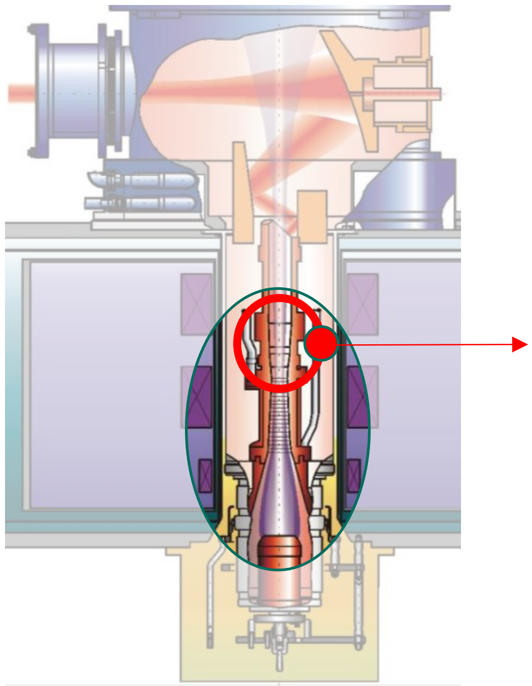
Three enabling technologies will be addressed:

1. The coaxial-type cavity technology that is specifically adapted to the operation at 2nd harmonics in combination with advanced inner and outer corrugations.
2. Injection locking using an external driving source.
3. The Multi-stage Depressed Collector (MDC) technology based on $E \times B$ drift concept.

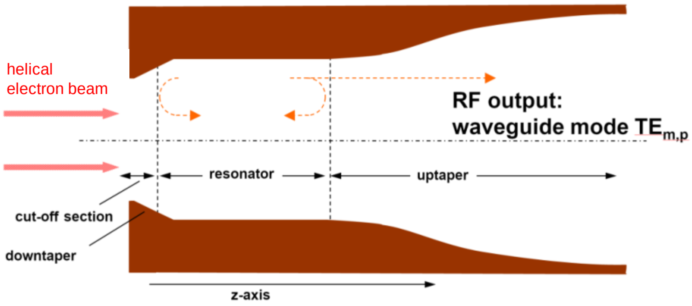
Basics: The Classic Hollow-Cavity Type Gyrotron

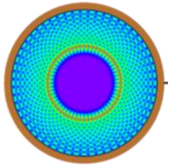
140 GHz
1 MW CW
for
W7-X





Hollow-type cavity

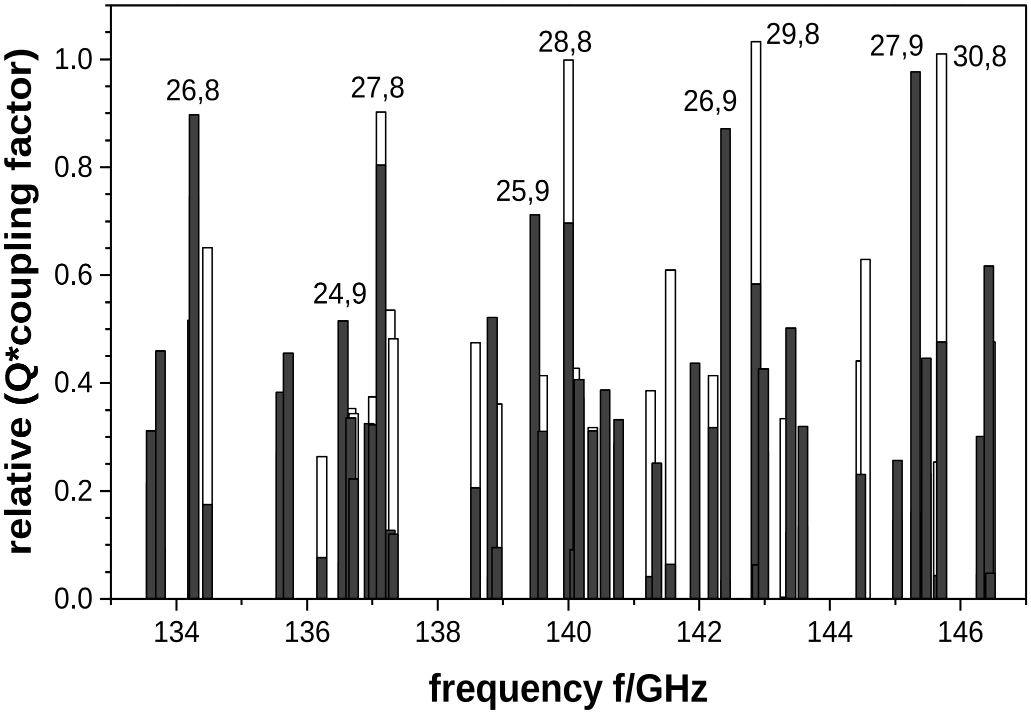




$TE_{28,8}$-mode

Fundamental Challenge

Typical mode spectrum at 140 GHz



Very dense mode spectrum even for 140 GHz $TE_{28,8}$-mode gyrotron.

For higher frequency:

→ Advance to coaxial-cavity design

→ Introduce inner-/outer corrugations

→ Move to injection locking

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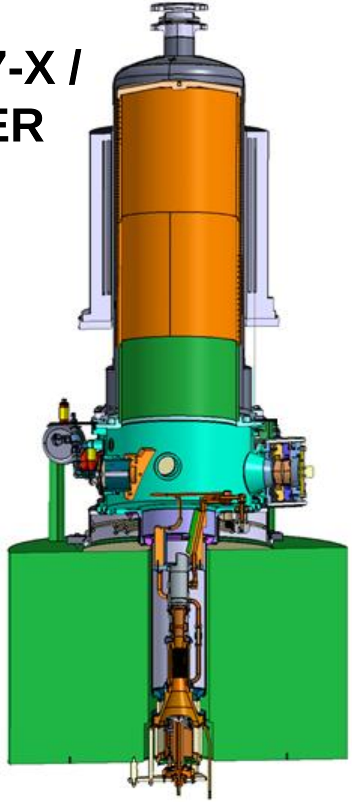
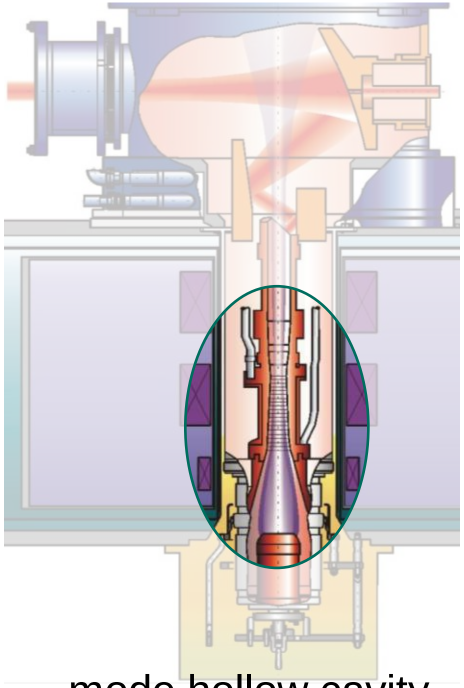
Towards Coaxial Cavity Gyrotron

W7-X / ITER



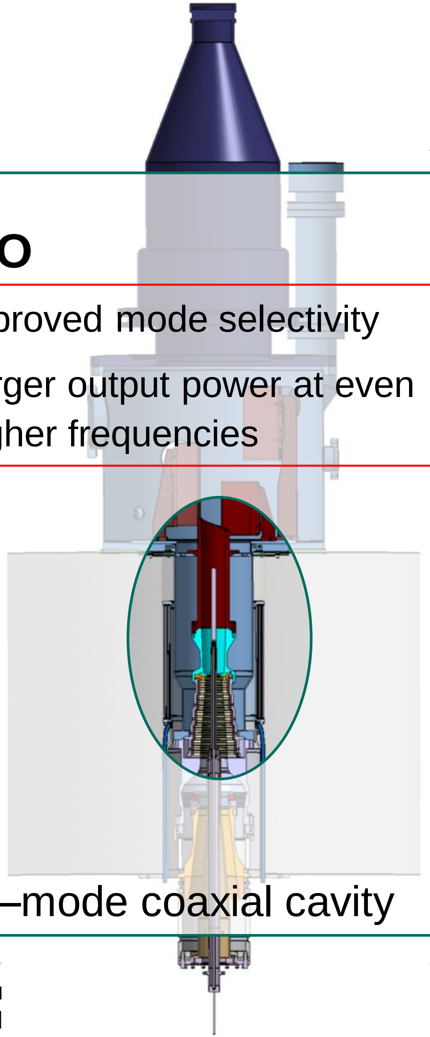


TE$_{28,8}$–mode hollow cavity

DEMO

→ Improved mode selectivity

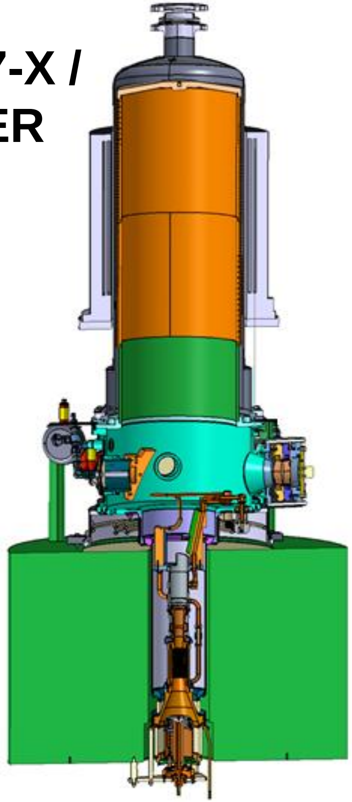
→ Larger output power at even higher frequencies

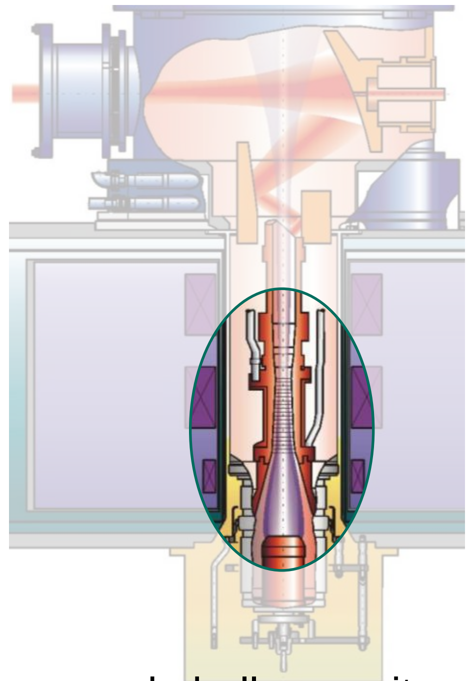


TE$_{34,19}$–mode coaxial cavity

Towards Coaxial Cavity Gyrotron

W7-X / ITER





$TE_{28,8}$–mode hollow cavity

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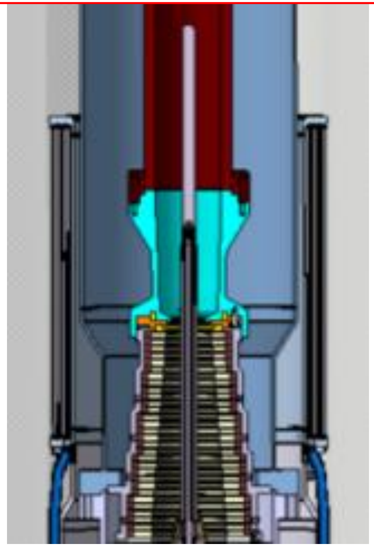

T1.2.1:
Enabling research on corrugations for second harmonic operation

T1.2.1 Enabling research on corrugations for second harmonic operation

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- *"Continuation of the studies of the eigenvalues and azimuthal indices of operating modes appropriate for efficient MW-class second-harmonic operation at* ***170 GHz****,* ***204 GHz****, and >280 GHz"*

Inner and Outer Corrugations

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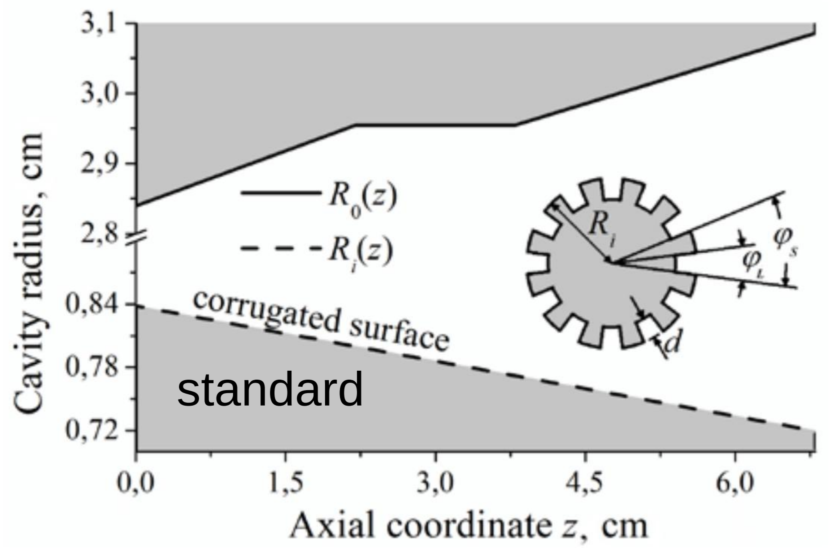
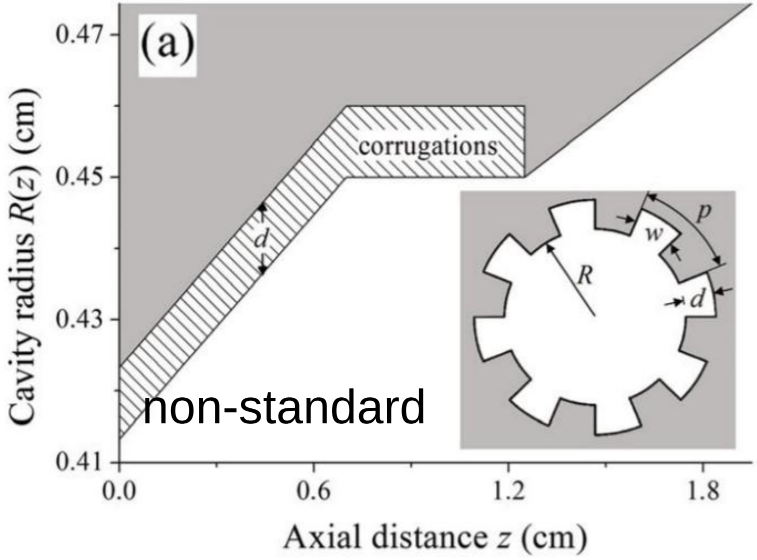
Inner corrugations (insert)



Effect due to impedance corrugation
→ Attenuates unwanted modes

Outer (wall) corrugations



Effect due to mode conversion
→ Converts energy to other modes

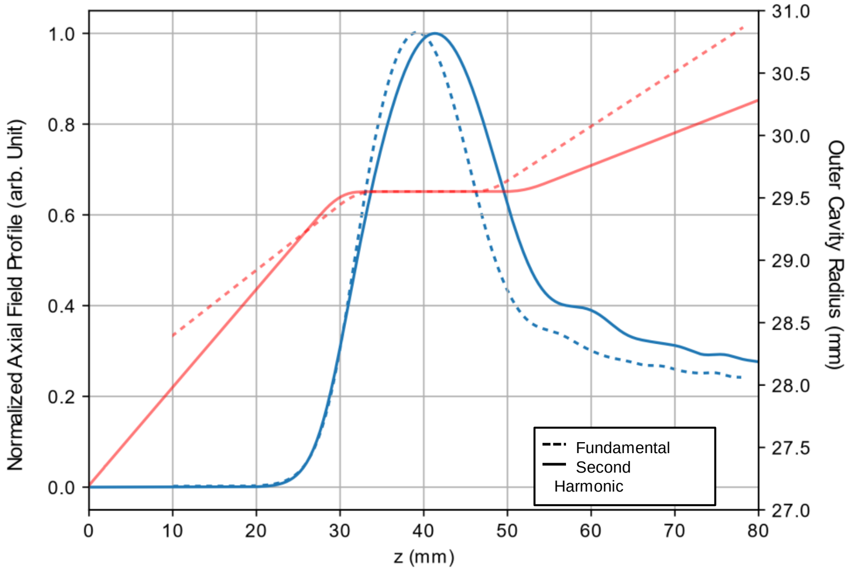
Fundamental KIT Coaxial 2 MW Gyrotron

- Operating mode $TE_{34,19}$
- Frequency 170 GHz
- Output power 2 MW
- Magnetic field **6.9 T** ➡ **3.5 T**
- No second harmonic operation possible, because of the fundamental competitors

➡ **New cavity design for second harmonic interaction necessary**

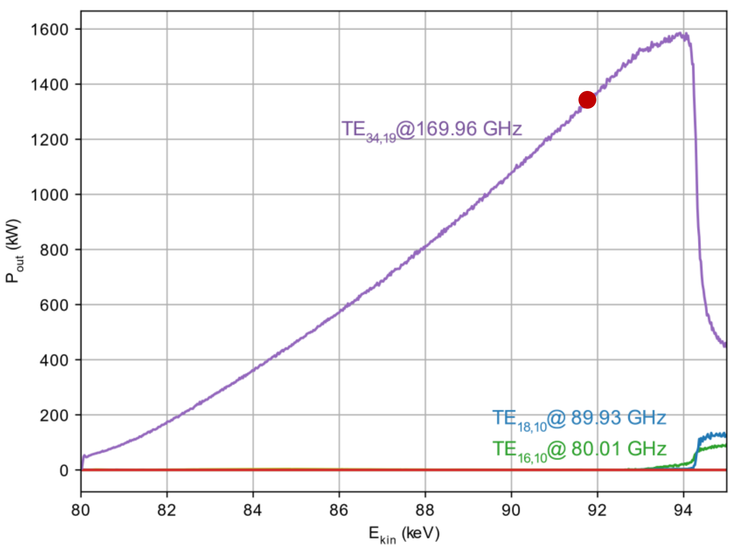
New Second Harmonic Cavity

- Coaxial inner conductor has to be thicker compared to fundamental
- Improvement of *inner* corrugations
- Longer effective interaction length
- Operation with similar electron velocity ratio $\frac{v_\perp}{v_z}$ as fundamental

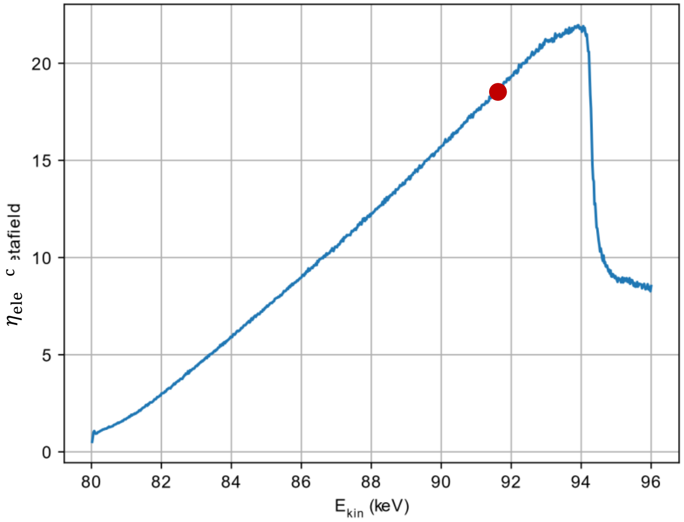


Efficiency and Output Power

P_{out}



η_{elec}



Limitation: thermal loading on inner conductor (230 W/cm² / red dot)

Inner Corrugation Results

- **Output**
  - Output Power : $P_{out} = 1.33\ \mathrm{MW}$
  - Frequency: $f = 170.01\ \mathrm{GHz}$
  - Electronic Efficiency : $\eta_{elec} \approx 18\ \%$ (without Collector)
  - Total Efficiency: $\eta_{tot} \gtrsim 50\ \%$ (with Multi-stage depressed collector)
- Design limited by thermal loading on inner conductor (~ 230 W/cm²)

➡ **Additional suppression of fundamental modes by outer corrugations?**

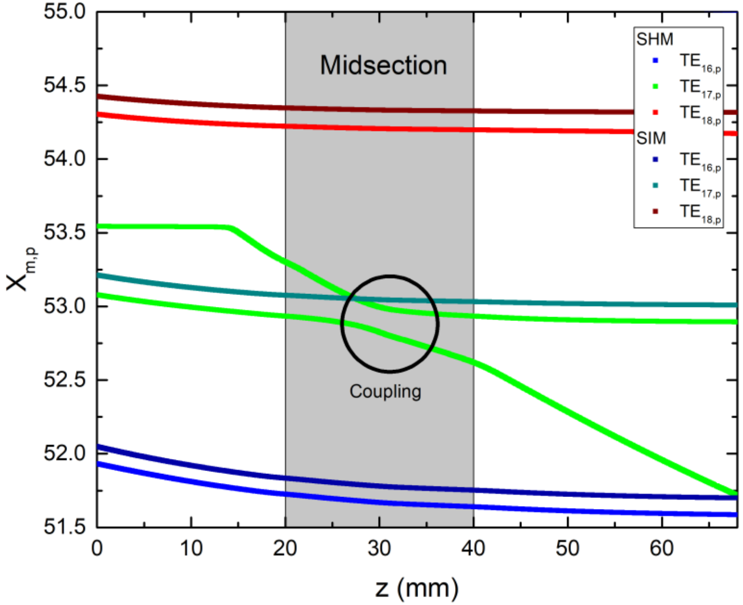
Mode Converting Outer Corrugations

Investigation of the concept

- By introducing corrugations on the outer wall, the critical fundamental modes are coupled with lower order ones and degenerate.
- The degenerated modes present low quality factor *Q* and cannot be excited.
- The 2nd harmonic modes should be left unaffected.
- As the baseline, the 2 MW-170 GHz coaxial cavity ($TE_{34,19}$ –mode, eigenvalue $\chi \approx 105$) is used, with $B \approx 3.43$ T.
- The first-harmonic competing modes ($\chi \approx 52$) are calculated.
- This selection is performed by employing the “Surface Harmonic Model” (SHM), which relates the number of the corrugations with the azimuthal indices of the coupled modes.

Derivation of the appropriate coupling scheme

- Multiple types of outer corrugations have been investigated.
- The appropriate coupling is achieved by introducing M=27 outer corrugations and enforcing $C=R_{out}/R_{in}$ ratio in the range 3.64-3.84 in the midsection.
- It has been observed that $TE_{17,10}$ can be coupled with $TE_{-10,p}$ in the midsection. According to SHM criterion this can be achieved by introducing M=27 corrugations.



Comparison of eigenvalue spectrums versus z, without (SIM) and with (SHM) N=27 outer corrugations

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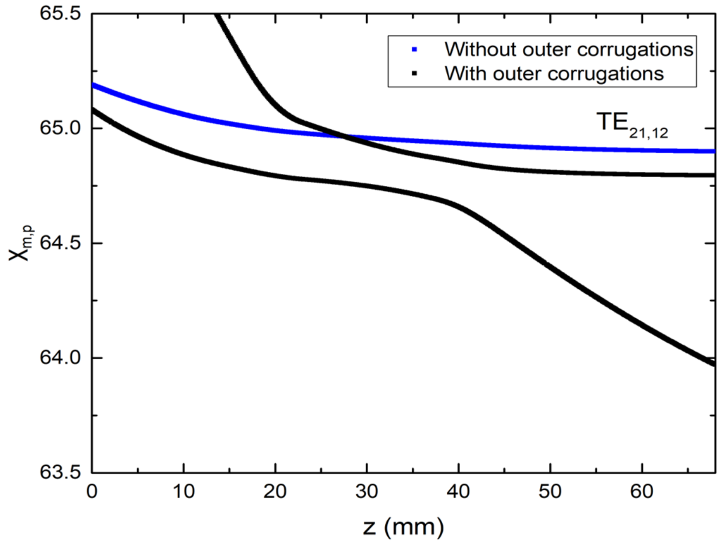
Quality factors

- The quality factor ("Q-factor") quantifies the stored energy of the resonator at a given power.
- The *Q*-factors of the smooth and corrugated cavity design have been calculated.
- The main first-harmonic competitor ($TE_{17,10}$) shows significantly larger relative decrease of the *Q* factor compared to the operating mode.

Mode	Q_{smooth}	$Q_{corrugated}$	Relative decrease (%)
$TE_{17,10}$	719	321	55.4
$TE_{34,19}$	3431	2570	25.1

Studies for operation at 204 GHz

- 204 GHz / 2 MW $TE_{40,23}$ coaxial cavity design used as baseline.
- More challenging case: eigenvalue spectrum quite dense ($\chi_{40,23} \approx 126$).
- Several coupling schemes are under investigation; a promising choice is M=36 outer corrugations. With this, the main first-harmonic competitor $TE_{21,12}$ is strongly coupled with $TE_{-15,p}$ modes in the midsection.



Task T1.2.2 Outlook

- Optimize 170 GHz second harmonic interaction
- Further investigations for 204 GHz operation
- Design of a $>$ 204 GHz cavity → New challenges
  - Higher mode order → More competing modes
  - Smaller cavity size → Higher ohmic loading
  - Smaller corrugation depth → More tolerance sensitivity
- Initiation of studies with regard to the application of the double-corrugation concept targeting multi-frequency operation.

T1.1.1:
Studies on MW-class 2nd Harmonic Operation with Injection Locking

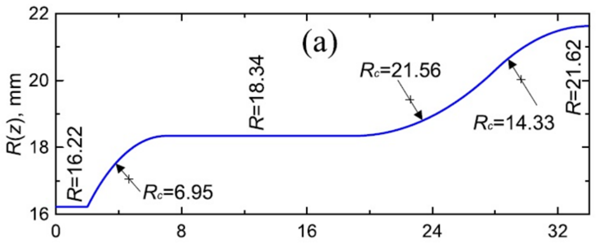
T1.1.1 Fundamental theoretical studies on injection locking

- “Generic studies on the possibilities for driving a gyrotron by injection of signals at fundamental and second harmonic, taking the existing DEMO-relevant gyrotron cavities (e.g. 170 GHz and/or 204 GHz operation) as starting point.”

2nd harmonic operation with injection locking
Current state-of-the-art designs

Institute	Time of publication	Harmonic	Mode	Frequency	Power	Efficiency	Ohmic loading	Injected Power
IAP-RAS [1]	Sept. 2021	Second-harmonic	$TE_{31,8}$	230 GHz	0.65 MW	~20%	2.0 kW/cm^2	30 kW
*IAP-RAS [2]	Feb. 2022	Second-harmonic	$TE_{34,14}$	230 GHz	0.95 MW	~20%	2.5 kW/cm^2	50 kW

- A 0.95 MW 2nd-harmonic cavity design was published this year.
- Try to reproduce numerically the performance in [2].
  - Benchmark our code with respect to injection locking.
  - Acquire experience with the state of the art.



*Cavity contour is given in the paper

[1] G.G. Denisov, et. al., "Possibility of MW-Level Second-Harmonic Generation in a Gyrotron Locked by an External Signal," 46th IRMMW-THz, 29 Aug.-3 Sept. 2021.

[2] G. G. Denisov, et.al., "Phase-Locking of Second-Harmonic Gyrotrons for Providing MW-Level Output Power," in IEEE Transactions on Electron Devices, vol. 69, no. 2, pp. 754-758, Feb. 2022.

2^{nd} harmonic operation with injection locking Simulation of the IAP-RAS design

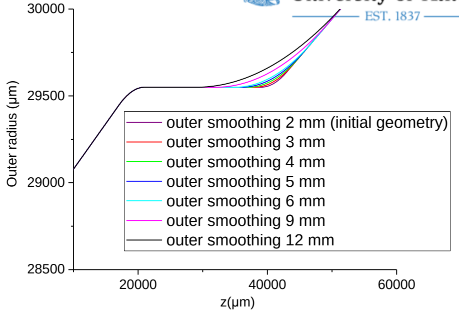
- Large discrepancy in the calculated starting currents.
  - EURIDICE-ISTART calculates starting currents with the fixed-field approximation.
  - To resolve this discrepancy, **the development of a new self-consistent code has been initiated**.
- The ideal copper conductivity value has been assumed in the original design.
  - From simulations it was found that the authors have used the ideal copper conductivity value and not the usual pessimistic one that takes into account the possible surface roughness.
  - **This implies lower power than 0.95 MW**, if the ohmic loading limit (~2.5 kW/cm^2) is respected.
- Simulations with EURIDICE code gave slightly reduced performance:
  - Mode loss at slightly lower voltage (~1.5 kV lower voltage).
  - Maximum power 0.9 MW (published 0.95 MW), maximum efficiency 18% (published ~20%).
  - Additional simulations with code SimpleRick are planned for the near future to further compare.

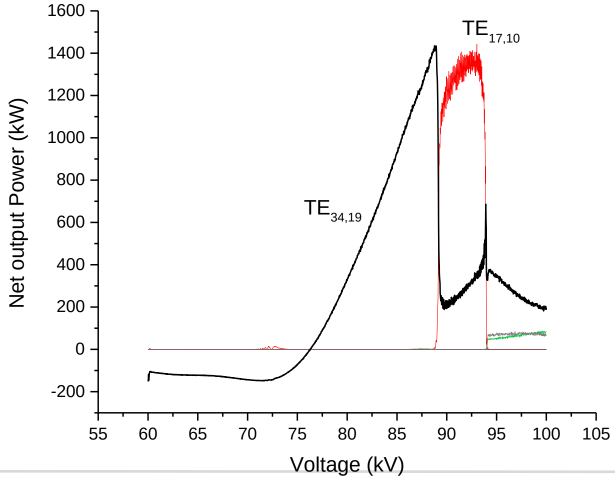
2^{nd} harmonic operation with injection locking
Existing 170 GHz coaxial cavity

- First approach: Keep the same cavity and same operating mode $TE_{34,19}$ and try to achieve 2^{nd} harmonic operation at 170 GHz with injection locking.
- It was found necessary to elongate the existing cavity by 6 mm.
- Multi-mode simulations show that mode competition is very strong.
  - Very high injected powers >300 kW are required to excite the 2^{nd} harmonic mode.
  - Injected powers ~500 kW are required to get ~1.2 MW of net output power.
  - Unacceptable ohmic loading >4.0 kW/cm^2 with such high injected powers.
- Solution: Introduce large cavity smoothings to reduce the quality factor (keeping the larger cavity length) and mitigate the strong mode competition.

2^{nd} harmonic operation with injection locking Modified 170 GHz coaxial cavity

- Seven different values of the outer smoothing (cavity-uptaper interface) were checked.
  - ➢ Cavity with smaller smoothing exhibit strong mode competition.
  - ➢ Cavity with larger smoothing give low power/efficiency.
- Best performance with a smoothing of 9 mm. By optimizing slightly the beam parameters (radius, current) we could achieve the following performance:
  - ➢ Necessary injected power **150 kW**.
  - ➢ **1.6 MW** of total output power, **1.45 MW** net output power,
  - ➢ **20%** of electronic efficiency,
  - ➢ Ohmic loading **2.2 kW/cm²**.





Task T1.1.1 Outlook

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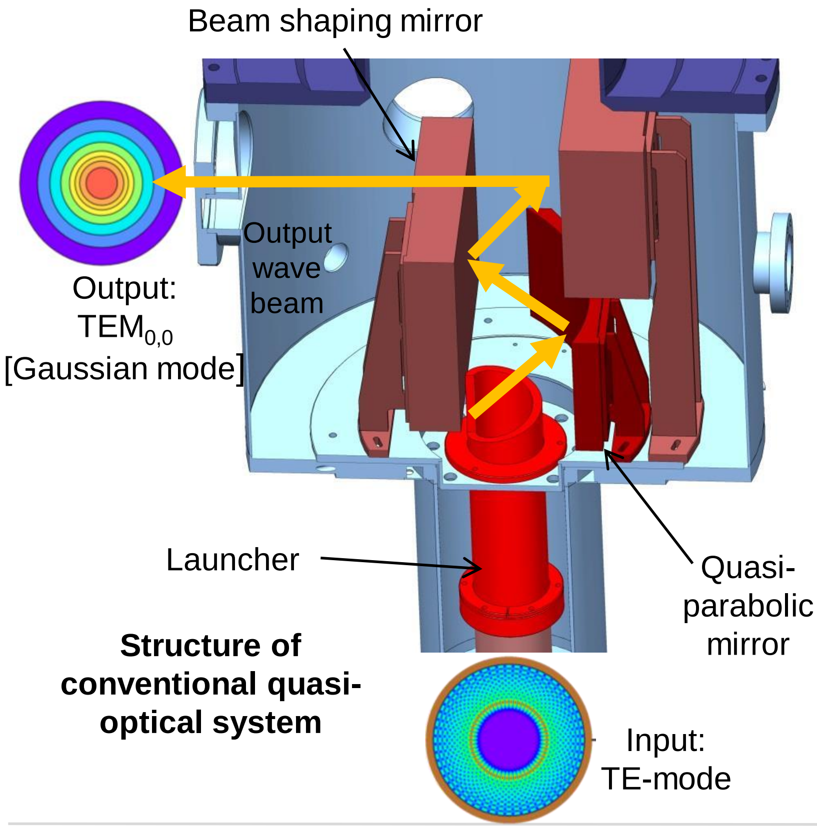
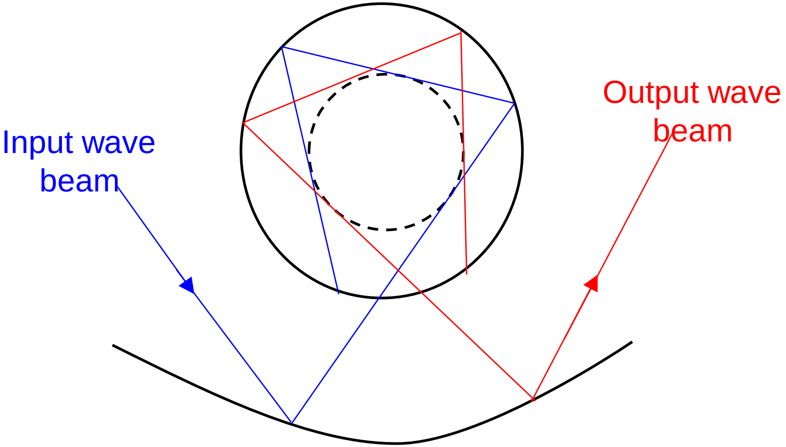
- Develop a generic strategy for optimum mode selection including injection locking.
- Investigate the concept of a variable injected frequency to suppress both neighboring competing modes during start-up. EURIDICE upgrade to support a variable injected frequency has been completed.
- Investigate the influence of the injected signal mode purity on the performance.
- Combine injection locking with double-corrugation concept.

T1.1.2:
Research on a Coupling System for Injection Locking

Research on Coupling System for Injection Locking



Structure of conventional quasi-optical system



Launcher for co- and counter-rotation operating modes used for phase locking operation.

Research on Coupling System for Injection Locking

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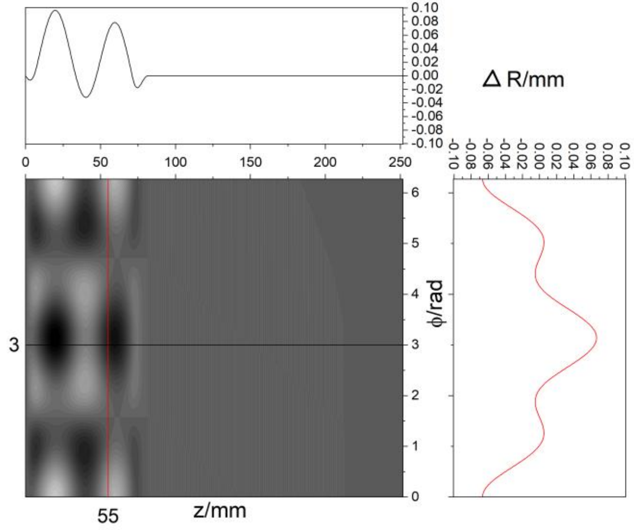
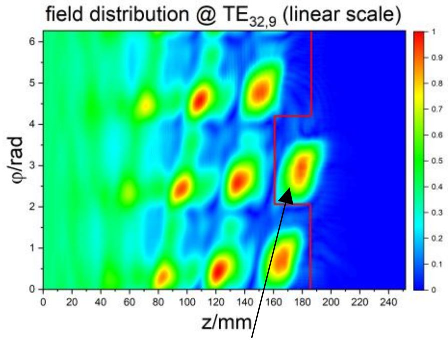
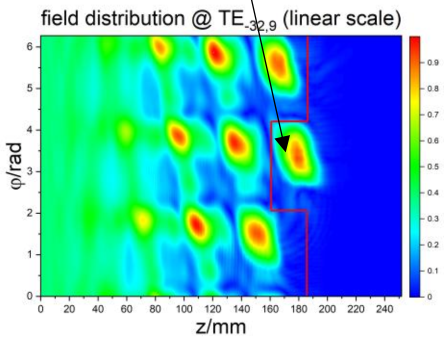
Design of a *Denisov-type* launcher
for co- ($TE_{32,9}$ mode) and counter- ($TE_{-32,9}$ mode), 170 GHz

Profile on unrolled launcher wall:





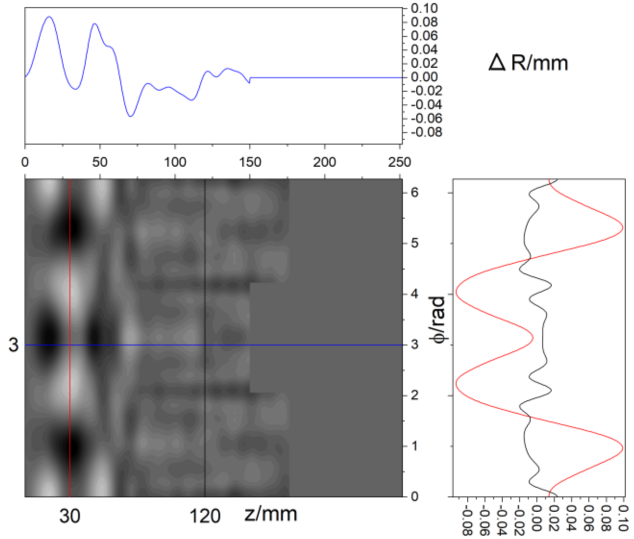
Gaussian mode content: **96.65 %**

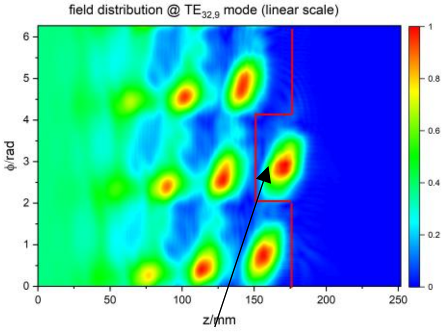


Research on Coupling System for Injection Locking

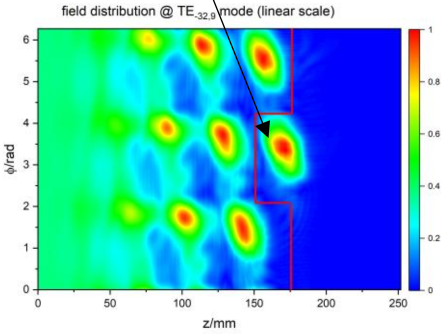
Design of a *Hybrid-type* launcher
for co- ($TE_{32,9}$ mode) and counter- ($TE_{-32,9}$ mode), 170 GHz

Profile on unrolled launcher wall:





Gaussian mode content: **98.24 %**



Research on Coupling System for Injection Locking

Next Steps:

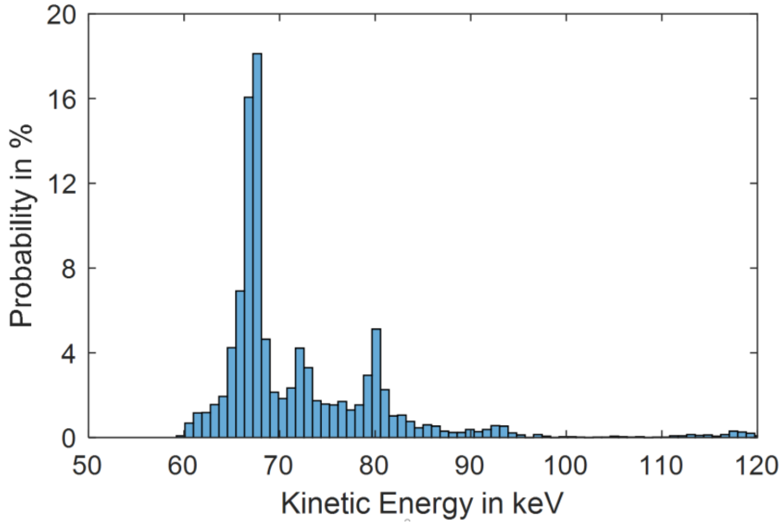
- Design of a Mirror-line type launcher for 204 GHz / coaxial cavity (Denisov- and Hybrid-type launchers are not possible).
- Design of two-way mirror systems for 170 GHz / 204 GHz systems.

T1.3: Multistage-Depressed Collector

Collector Theory

- Energy of the spent electron beam is recovered in depressed collectors
- Electrical efficiency of the overall tube is increased
- Single-stage collector efficiency is limited by:
  - Slowest electron
  - Widely spread energy spectrum

➡ Multi-stage depressed collector
  - Higher deceleration potential possible
  - Reflected electrons are collected at first electrode
  - Reduced thermal losses
  - Increased collector & gyrotron efficiency



Collector Geometry

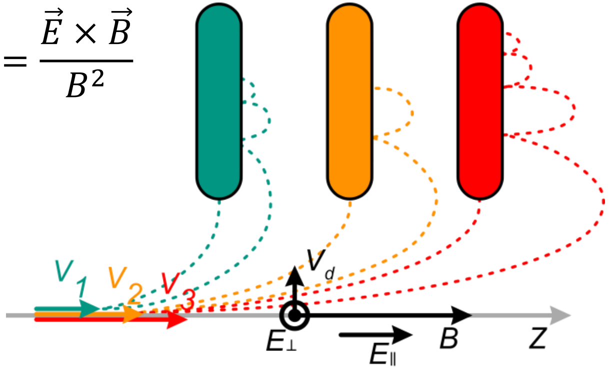
Solution for MDC in gyrotron

- Sorting electrons based on $E \times B$ drift
- Spatial separation according to the kinetic energy

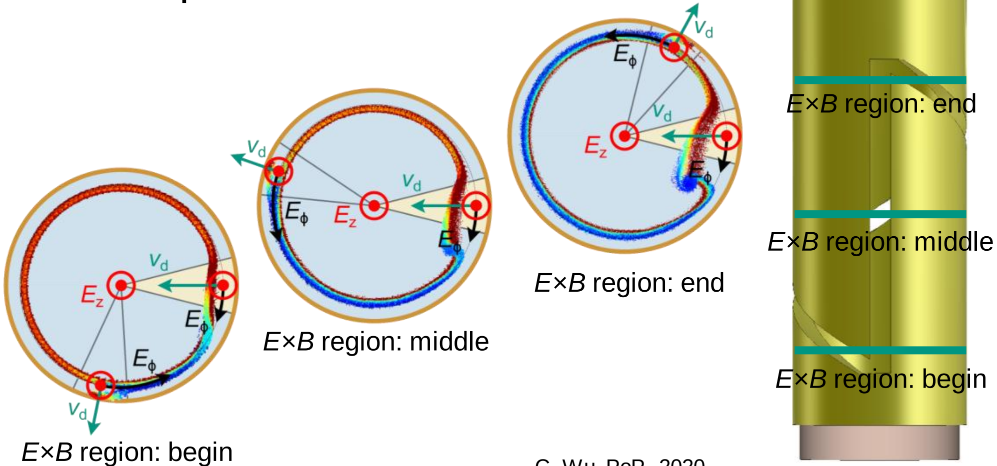
Basic Geometry Definition

- Non-axisymmetric E-field
- Faraday's law: E_ϕ must flip
- Electrons drifts radially outwards except at a small section

$$\vec{v_d} = \frac{\vec{E} \times \vec{B}}{B^2}$$



I. Gr. Pagonakis, IEEE TPS, 2008

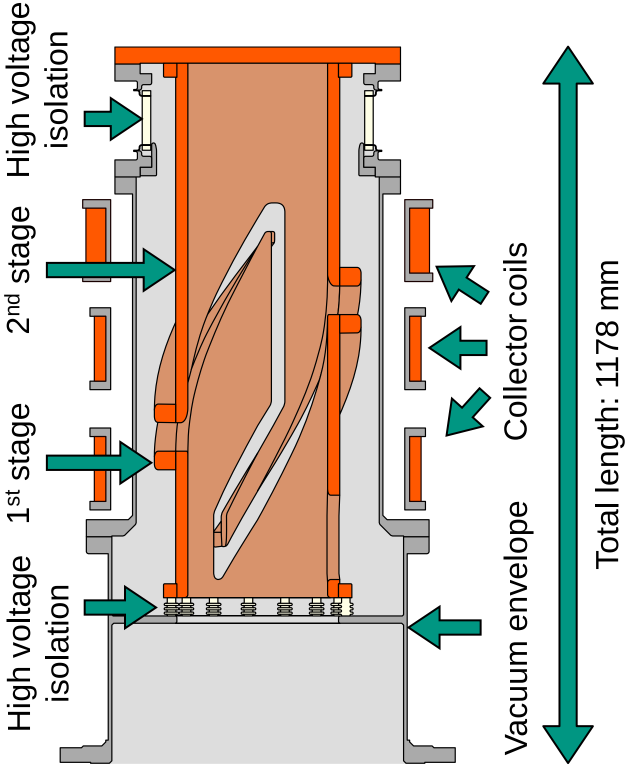


C. Wu, PoP, 2020

Collector Prototype

- Prototype two-stage collector is built
  - Validation of *E*×*B* drift concept
  - Short pulse operation
  - Optimized for fundamental operation

- Operation possible at 2^{nd} harmonic
  - 1^{st} depression potential: 44 kV
  - 2^{nd} depression potential: 63 kV
  - Gyrotron efficiency enhancement: 24 % → 67 % (without RF losses)
  - Low reflected current: 0.027 %
  - Power loading 1^{st} stage: 94 kW



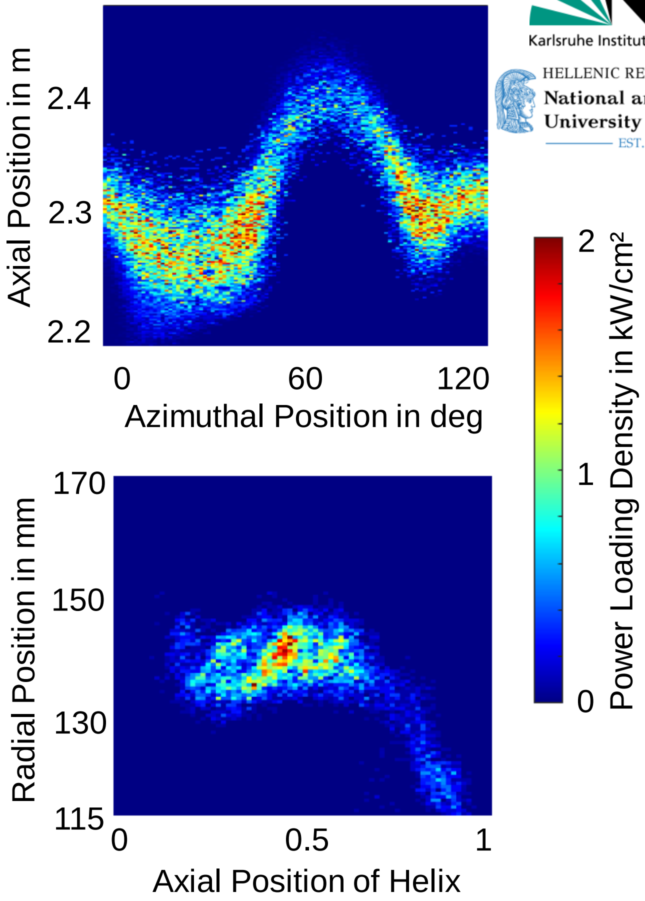
Collector Prototype Limitation

- Power loading density of short pulse prototype up to 2 kW/cm²
- Limit for CW at 500 W/cm²

Outlook:

➡ New design concept for Continuous Wave (CW)

- Increased collector size
- Implementation of sweeping system for 2nd stage
- Operation point with reduced power on 1st stage
- Optimized design for harmonic operation



Conclusion and Outlook

- Significant progress has been made for all defined tasks, including the development of tools and methods.

- Outlook: continuation according to task specification.
  - Check feasibility of experiments with coaxial cavity (170 GHz).
  - Focus on higher frequencies.
  - Investigate possibility of multi-frequency operation.
  - Further enhancement of numerical tools.

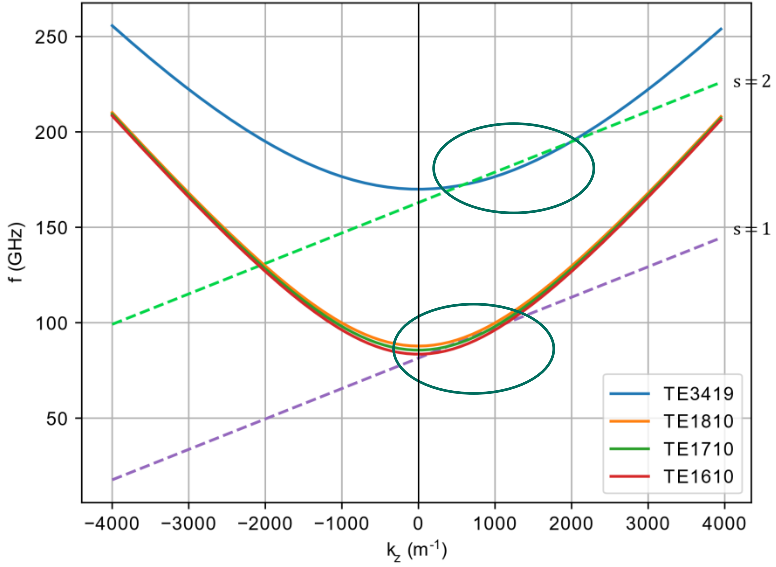
Thank you for your attention!

Second Harmonic Interaction

- Cyclotron resonance condition

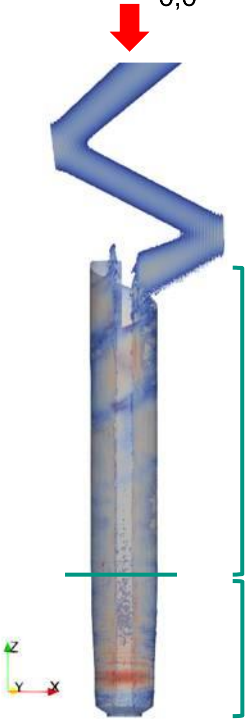
$$\omega \approx s \underbrace{\frac{eB}{m_e \gamma}}_{\Omega_c} - k_z v_z$$

  - Half of the magnetic field needed for second harmonic operation
- Problem first harmonic competitors
- **So far only fundamental fusion gyrotron**



T1.1.2: Research on coupling system

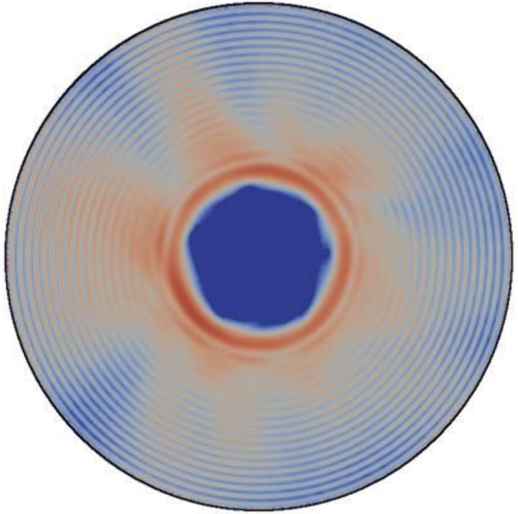
$TEM_{0,0}$



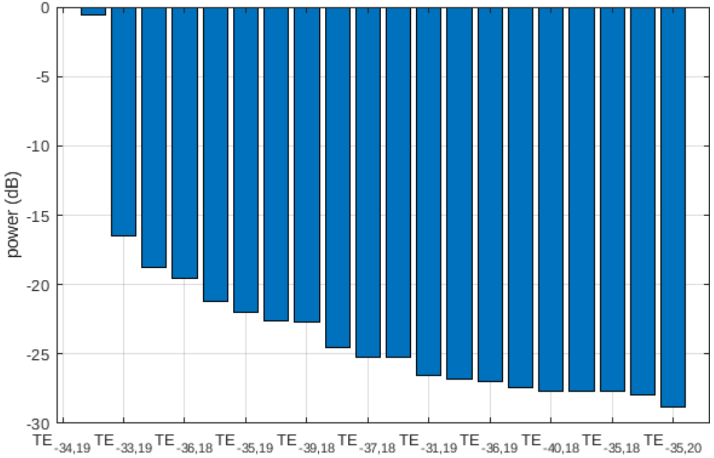
Launcher

Cavity

Mode pattern at the launcher entrance

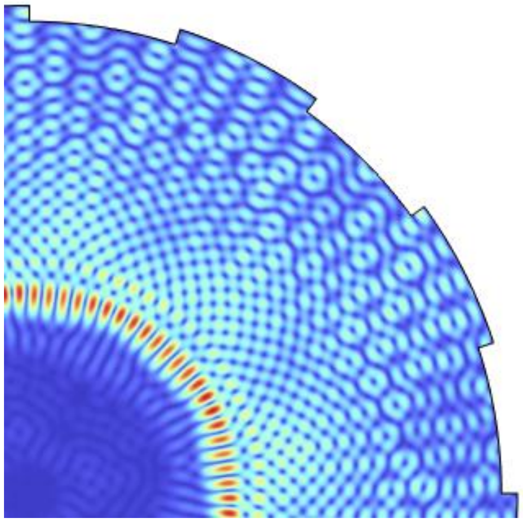


Mode content

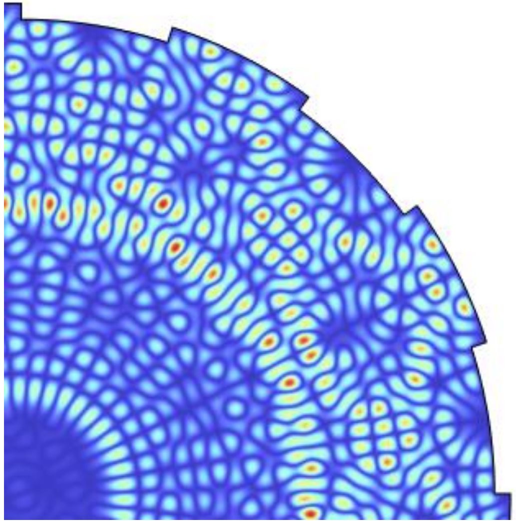


- **Mode patterns in a resonator with different outer corrugations (exemplary)**

Corrugation depth: 1.0 mm

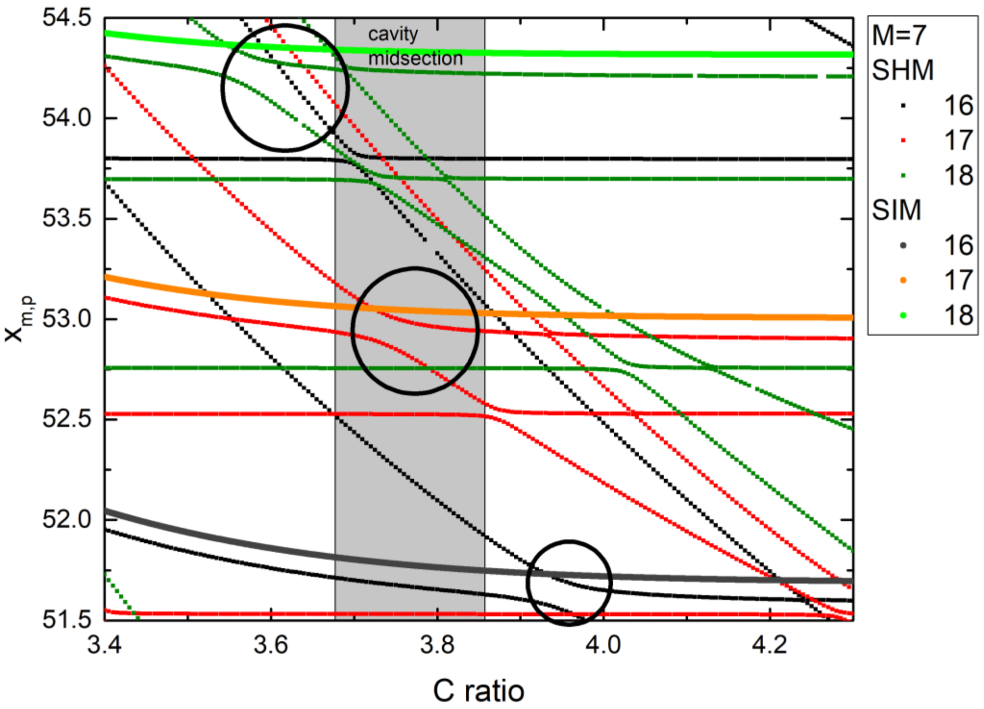


Corrugation depth: 1.5 mm



T1.2.1: Mode converting azimuthal corrugations

Eigenvalues versus C ratio (outer to inner diameter) of the case with inner corrugations only (SIM) and when M=7 outer corrugations are introduced. The coupling regions are also visible (encircled areas).



Work already performed

- Identification of the competing modes of the first and second harmonic.
- Investigation of various coupling schemes.
- It was found that only the first harmonic competitors should be coupled and the coaxial insert geometrical properties should be appropriately changed.

Project Roadmap

Karlsruhe Institute of Technology

HELLENIC REPUBLIC
National and Kapodistrian University of Athens
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1. The three elements (injection locking, advanced corrugated cavity, MDC) will be comprehensively studied aiming at the development of the theoretical background and the required tools.
2. A high-frequency, high-power and high-efficiency gyrotron operating at second-harmonic will be designed based on the theory and tools developed in the first part.
   The possibility to design a gyrotron operating at very high frequency (above 280 GHz) for future Electron Cyclotron Heating (ECH) systems and Collective Thomson Scattering (CTS) diagnostics will be investigated.
3. Preparation of the first worldwide experiments of a high-power gyrotron operating in second harmonic with the support of injection locking technique. The presence of two operational high-power gyrotron test stands at KIT is an important advantage for that purpose.

ENR-TEC.01.KIT-T001 2021-06-01 – 2021-12-31

Task 1 - Fundamental theoretical research on individual methods

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- *T1.1 Injection locking*
  - *T1.1.1 Fundamental theoretical studies on injection locking (K. Avramidis)*
    - Generic studies on the possibilities for driving a gyrotron by injection of signals at fundamental and second harmonic, taking the existing DEMO-relevant gyrotron cavities (e.g. 170 GHz and/or 204 GHz operation) as starting point.
  - *T1.1.2 Enabling research on coupling system (J. Jin)*
    - Development of the TWLDO code for the synthesis of Denisov-type launchers for the conversion of the co- and counter-rotating mode in the same design.
    - Development of the TWLDO code for the synthesis of hybrid-type launchers for the conversion of the co- and counter-rotating mode in the same design.
    - Improvement of KARLESSS code for the simulation of the quasi-optical (q.o.) system with ideal Gaussian mode injection from the launcher, forward and backward propagation of the wave beam in the Q.O. system.
- *T1.2 Corrugated cavity*
  - *T1.2.1 Enabling research on corrugations for second harmonic operation (D. Peponis, K. Avramidis, A. Marek)*
    - Investigation of the second harmonic operation of the existing 2 MW coaxial gyrotron at KIT, operating at 170 GHz with the $TE_{34,19}$ mode, with the same mode at 170 GHz. The updated cavity design will incorporate mode converting corrugations on the outer wall and impedance corrugations on the coaxial insert.
    - Initiation of the studies of the eigenvalues and azimuthal indices of operating modes appropriate for efficient MW-class second-harmonic operation at 170 GHz, 204 GHz, and >280 GHz.

ENR-TEC.01.KIT-T001 2021-06-01 – 2021-12-31
Deliverables

The first-year deliverable will include the results of the tasks performed in 2021, such as the fundamental theoretical studies on injection locking and on advanced corrugated cavities for second harmonic operation.

ENR-TEC.01.KIT-T002 2022

Task 1 - Fundamental theoretical research on individual methods

Karlsruhe Institute of Technology

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- *T1.1 Injection locking*
  - *T1.1.1 Fundamental theoretical studies on injection locking (K. Avramidis)*
    - Systematic research on the influence of major design parameters, concerning the cavity geometry and the electron beam properties, on the performance of an injection-locked gyrotron at fundamental and second harmonic operation.
    - Studies on the influence of mode purity of the external signal on gyrotron operation.
  - *T1.1.2 Enabling research on coupling system (J. Jin)*
    - Design a Denisov-type launcher for the co- and counter-rotating mode, investigate the change of the field distribution due to the influence of the perturbation corresponding to the anti-rotating mode.
    - Design a hybrid-type launcher for the co- and counter-rotating mode to achieve a higher conversion efficiency in comparison to the Denisov-type launcher as much as possible.
    - Investigation of the possibility for mirror-line launchers to be used for the conversion of the co- and counter-rotating mode, the possibility for a mirror-line launcher to provide a higher conversion efficiency in comparison to the Denisov-type and hybrid-type launchers.
    - Design of mirror systems, and analysis of the q.o. system.

ENR-TEC.01.KIT-T002 2022

Task 1 - Fundamental theoretical research on individual methods

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- *T1.2 Corrugated cavity*
  - *T1.2.1 Enabling research on corrugations for second harmonic operation (D. Peponis, K. Avramidis, A. Marek)*
    - Investigation of the second harmonic operation of the existing coaxial gyrotron design by KIT, which allows for 2 MW operation at 204 GHz with the $TE_{40,23}$ mode, with the same mode at 204 GHz. The updated cavity design will incorporate mode converting corrugations on the outer wall and impedance corrugations on the coaxial insert.
    - Continuation of the studies of the eigenvalues and azimuthal indices of operating modes appropriate for efficient MW-class second-harmonic operation at 170 GHz, 204 GHz, and >280 GHz.
  - *T1.2.2 Enabling research on multi-frequency operation (I. Chelis, A. Marek)*
    - Search for suitable mode pairs/triplets for multi-frequency operation, based on the sets of suitable modes found in T1.2.1.
    - Investigation of possible compromises on the design strategies to achieve the best possible performance of the same design in two/three frequencies.

ENR-TEC.01.KIT-T002 2022

Task 1 - Fundamental theoretical research on individual methods

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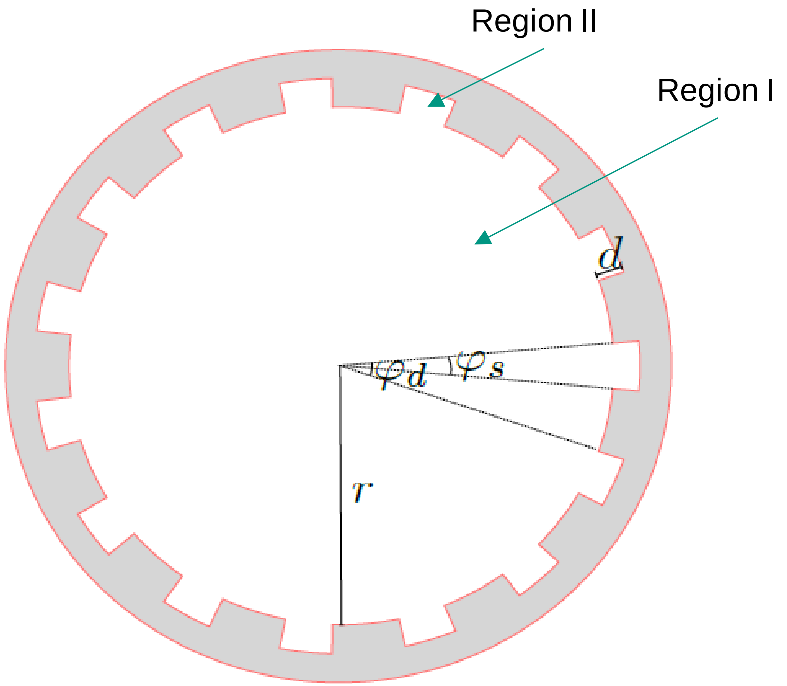
- *T1.3 Multistage depressed collector (B. Ell, G. Latsas)*
  - Fundamental research on different MDC design approaches based on E×B concept for gyrotron operation at higher harmonics. Such design approaches are the coaxial and cylindrical helical MDCs, MDC approaches based on the transformation of the annular beam to sheet beam, MDC approach based on helicoid magnetic field, and novel/alternative design approaches.
  - The most appropriate design approach will be chosen considering the manufacturing complexity and the operational efficiency. A MDC will be optimized for a typical spent beam of the second-harmonic gyrotron operation.
  - Fundamental research on the influence of secondary electrons on MDC operation and comparison with the fundamental harmonic results will be performed.
  - The influence of stray magnetic field and misalignments on the MDC operation and comparison with the corresponding results obtained in older WP for the fundamental harmonic will take place.

ENR-TEC.01.KIT-T002 2022
Deliverables

The annual deliverable will include the results of the tasks performed in 2022: the continuation of the fundamental theoretical studies on injection locking and advanced corrugated cavities, as well as the theoretical investigation on the multistage depressed collector for second harmonic operation.

Surface Harmonic Model

- The eigenmodes of a corrugated waveguide depend on the number of corrugations M and their depth d
- The "corrugated" eigenmodes can be described by superimposing eigenmodes of
  - Region I $r \in [0, R)$
  - Region II $r \in [R, R+d]$
- Due to periodicity, Region I can be solved by Floquet theorem
  - Can be described as coupled eigenmodes of a cylindrical resonator
  - Azimuthal dependence of the modes change with M
- Region II is expressed in terms of Fourier harmonics



About coupling (1)

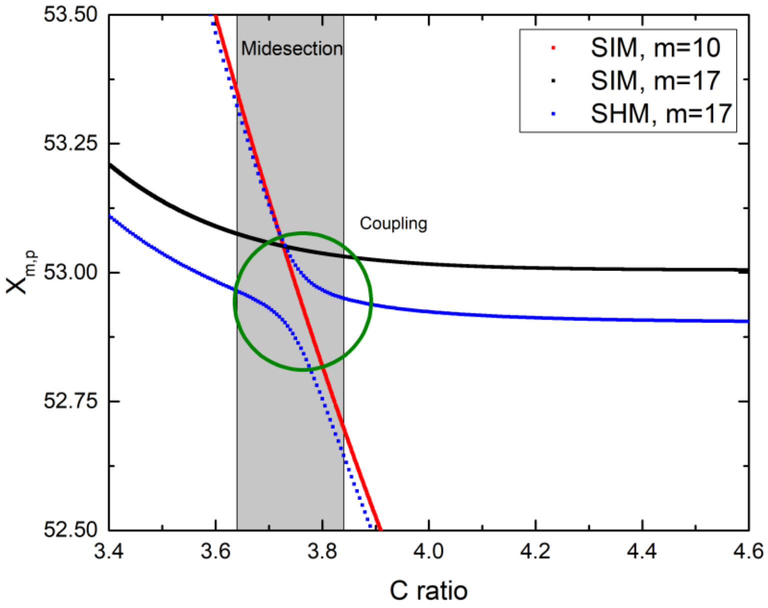
- The coaxial cavity presents variable inner (R_{in}) and outer (R_{out}) radius with respect to z, and the cutoff frequency of each mode depends on the ratio of these radii C.
- It is preferable to use the eigenvalue χ instead of the cutoff frequency $f_c = \frac{\chi}{2\,\pi\,R_{out}}\,c$, since it depends only on C making the analysis global. Therefore, a specific eigenvalue curve $\chi(C)$ corresponds to each mode.
- In a coaxial cavity without azimuthal corrugations on the outer wall, the field pattern of each mode has an azimuthal dependence of exp($jm\varphi$), where the integer m is the azimuthal index of the mode.
- In a coaxial cavity with M outer corrugations, the eigenvalues depend also on M and azimuthal dependence of the eigenmode becomes $\Sigma_q A_q \times \exp\{j(m+qM)\varphi\}$.

About coupling (2)

- In other words, the outer corrugations introduce additional modes with azimuthal index $m+qM$. These modes are related (coupled) with the mode under consideration (m).
- The coupling results in modified eigenvalue curves $\chi(C)$. By choice of M, we can couple the unwanted competing modes with lower order ones resulting in eigenvalue curves with increased negative slopes.
- The negative slopes correspond to lower Quality factors, because of the increasing group velocity of the wave towards the cavity output. The lower the Q-factor, the more difficult for the competing mode to be excited.

Derivation of the appropriate coupling scheme (1)

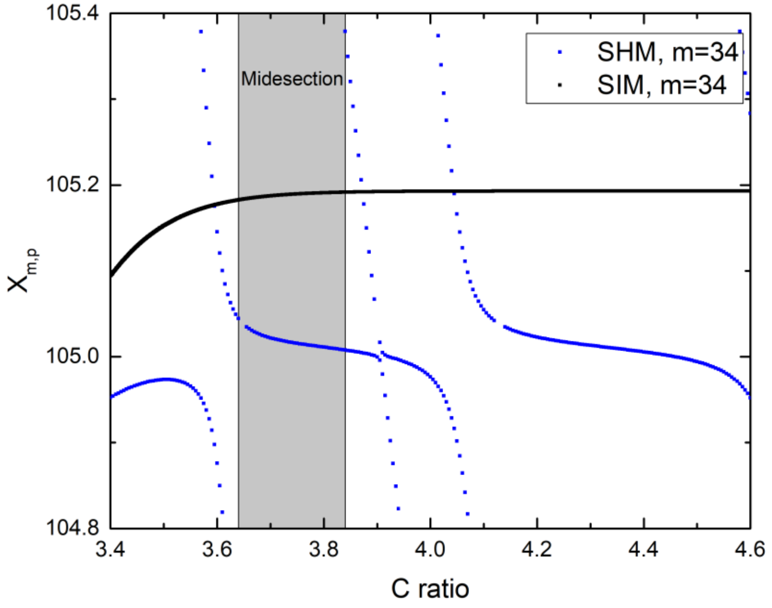
- The eigenvalue curves versus *C* for the case without outer corrugations (SIM) are calculated.
- We find the eigenvalue curve with m_c smaller than that of the competitor (m) which intersects the curve of the latter. Here, $TE_{17,p}$ curve is intersected by $TE_{10,p}$ curve at $C \approx 3.76$.
- According to the SHM criterion, $m_c=|m \pm qM|$, we calculate the number of corrugations needed for such coupling. For q=1, M=27 or M=7.



Eigenvalues versus C ratio of the $TE_{17,p}$, $TE_{10,p}$ modes (without corrugations, SIM) and $TE_{17,p}$ with corrugations (SHM)

Derivation of the appropriate coupling scheme (2)

- Next, we check whether this number of corrugations introduces unwanted lower order modes that couple with the operating mode $TE_{34,19}$.
- We adjust the *C* interval of the midsection in order to avoid possible unwanted couplings of the operating mode.
- As it can be seen, by introducing *M*=27 outer corrugations, $TE_{17,10}$ is coupled and degenerated whereas $TE_{34,19}$ remains almost unaffected.



Comparison of eigenvalue spectra versus z, without (SIM) and with (SHM) N=27 outer corrugations for $TE_{34,19}$.