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Improved collision operator for full-f HAGIS simulations

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This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.

Milestones and deliverables

- **TSVV1 sub project D2.3, D2.7:**

  Use HAGIS as a full-f code with non-linear collision model

  for calculating the bootstrap current in the pedestal

- **D2.3: First version of full-f HAGIS code available, due: 12/2022**

  Full-f code with linearized collision operator - ready in 2021

  Full-f code with improved (non-linear) collision operator - this talk

- **M2.11: Extend delta-f neoclassical studies with HAGIS to full-f, due: 12/2022**

- D2.7: Neoclassical bootstrap current studies with full-f HAGIS code

  in support of GK studies, due: 6/2024

Full-f code with linearized collisions (last year)

- HAGIS: integration of equations of motion in Boozer coordinates, variable time steps
- **Full-f: all particle have the same fixed weight**

  Number of particles in real space grid cell is porportional to the volume
- **Full-f: particles are created with a Maxwellian velocity distribution**

  (with a finite flow velocity for shortening the transient phase)
- Separate simulations for ions and electrons as before
- Linearized Coulomb collision operator

  Each particle collides with a Maxwellian distribution with the same flow velocity and temperature as the actual distribution function (in moving frame)
- Full-f code requires for low noise much more num. particles than delta-f code, particularly for electrons (flow velocity $\ll$ ion thermal velocity)

Fokker-Planck collision operator

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Collisions of particles of species a by particles of species b:

$$C_{ab}(f_a, f_b) = -\nu_{ab0}\,\frac{v_{T_a}^3}{n_b}\nabla_{\boldsymbol{v}}\cdot\left[\left(1+\frac{m_a}{m_b}\right)f_a(\boldsymbol{v})\nabla_{\boldsymbol{v}}H_b(\boldsymbol{v}) - \frac{1}{2}\nabla_{\boldsymbol{v}}\cdot\left(f_a(\boldsymbol{v})\nabla_{\boldsymbol{v}}\nabla_{\boldsymbol{v}}\,G_b(\boldsymbol{v})\right)\right]$$

Rosenbluth potentials:

$$G_b(\boldsymbol{v}) = \int f_b(\boldsymbol{w})|\boldsymbol{v}-\boldsymbol{w}|d^3w \qquad H_b(\boldsymbol{v}) = \int \frac{f_b(\boldsymbol{w})}{|\boldsymbol{v}-\boldsymbol{w}|}d^3w \qquad \nu_{ab0} = \frac{Z_a^2 Z_b^2 e^4 n_b \ln\Lambda_{ab}}{4\pi\varepsilon_0^2 m_a^2 v_{T_a}^3}$$

$$\Delta H_b(\boldsymbol{v}) = -4\pi f_b(\boldsymbol{v}) \qquad \Delta G_b(\boldsymbol{v}) = 2H_b(\boldsymbol{v})$$

For collisions within a species, C_{aa} is a nonlinear operator for f_a

Calculating the Rosenbluth potentials numerically is time-consuming

For avoiding this, $f_b(v)$ is approximated by an analytic expression [Donnel et al. PPCF63]

Approximation of the distribution function

For calculation of the Rosenbluth potentials, $\boldsymbol{f}(\boldsymbol{v})$ is approximated by

$$\tilde{f} = f_M(v^2)\left(1 + \frac{m\boldsymbol{v}\cdot\boldsymbol{V}}{T} - \frac{m\boldsymbol{v}\cdot\boldsymbol{q}}{nT^2}\left(1 - \frac{mv^2}{5T}\right) - \frac{mV^2}{2T}\left(1 - \frac{mv^2}{3T}\right)\right) \tag{5.1}$$

(first 4 terms of polynomial expansion of f/f_M)

- Due to strong anisotropy in the plasma, we can assume $\boldsymbol{V} = V_\parallel \boldsymbol{b}$, $\boldsymbol{q} = q_\parallel \boldsymbol{b}$:

$$\tilde{f} = \frac{n\exp\left(-\frac{mv^2}{2T}\right)}{(2\pi T/m)^{3/2}}\left(1 + \frac{mv_\parallel V_\parallel}{T} - \frac{mv_\parallel}{T}\frac{q_\parallel}{nT}\left(1 - \frac{mv^2}{5T}\right) - \frac{mV_\parallel^2}{2T}\left(1 - \frac{mv^2}{3T}\right)\right) \tag{5.2}$$

- This approximative distribution function has the properties:

$$\int \tilde{f} d^3v = n \qquad \int v_\parallel \tilde{f} d^3v = nV_\parallel \qquad \int v_\parallel\left(\frac{mv^2}{2} - \frac{5}{2}T\right)\tilde{f}\, d^3v = q_\parallel \tag{5.3}$$

$$\int mv^2 \tilde{f} d^3v = n(3T + mV_\parallel^2) \qquad \text{but:} \quad \int mv_\parallel^2 \tilde{f} d^3v = n(T + \frac{1}{3}mV_\parallel^2) \tag{5.4}$$

Approximation of the distribution function (2)

- The **scattering is done in the frame moving with the field particles**
- In the frame with $V_{\|b} = 0$ the distribution function (5.2) becomes

$$f_b \approx f_{bt} = f_M(v^2)\left(1 - \frac{m_b v_\|}{T_b}\frac{q_{\|b}}{n_b T_b}\left(1 - \frac{m_b v^2}{5T_b}\right)\right) \tag{6.1}$$

- This corresponds to a modified shifted Maxwellian in the lab frame

$$f_{bt} = \hat{f}_M \exp\left(-\frac{m_b(\boldsymbol{v} - \boldsymbol{b}V_{\|b})^2}{2T_b}\right)\left[1 - \frac{m_b(v_\| - V_{\|b})}{T_b}\frac{q_{\|b}}{n_b T_b}\left(1 - \frac{m_b(\boldsymbol{v} - \boldsymbol{b}V_{\|b})^2}{5T_b}\right)\right] \tag{6.2}$$

- **Advantage of calculating the collisions in the moving frame:**
  Expressions for friction and diffusion coefficients are reduced
  $V_{\|b}$ does not have to be small

Scattering in moving frame: friction

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- Coordinate system with $\boldsymbol{v} = v\boldsymbol{e}_z$ (before collisions)
- $\boldsymbol{b} = \boldsymbol{B}/B = (v_\parallel/v)\boldsymbol{e}_z - (v_\perp/v)\boldsymbol{e}_y, \quad \boldsymbol{e}_x = \boldsymbol{e}_y \times \boldsymbol{e}_z,\ v_\parallel = \boldsymbol{v}\cdot\boldsymbol{b},\ v_\perp = |\boldsymbol{v}\times\boldsymbol{b}|$

$$\boldsymbol{\Gamma}_{ab} = \nu_{ab0}\left(1+\frac{m_a}{m_b}\right)\left(\Gamma^y_{ab}\boldsymbol{e}_y + \Gamma^z_{ab}\boldsymbol{e}_z\right) \qquad \nu_{ab0} = \frac{Z_a^2 Z_b^2 e^4 n_b \ln\Lambda_{ab}}{4\pi\varepsilon_0^2 m_a^2 v_{T_a}^3} \tag{7.1}$$

$$\Gamma^z_{ab} = -v\frac{F(s_b)}{s_a^3} + u_{\parallel b}\frac{v_\parallel}{v}(2s_b^2-1)\frac{v_{T_a}^3}{v_{T_b}^3}\frac{2}{\sqrt{\pi}}e^{-s_b^2} \qquad u_{\parallel b} = \frac{2q_{\parallel b}}{5n_b T_b} \tag{7.2}$$

$$\Gamma^y_{ab} = u_{\parallel b}\frac{v_\perp}{v}\frac{v_{T_a}^3}{v_{T_b}^3}\frac{2}{\sqrt{\pi}}e^{-s_b^2} \qquad s_b = \frac{v}{v_{T_b}} \qquad s_a = \frac{v}{v_{T_a}} \tag{7.3}$$

[Donnel et al., NF63]

$$F(s_b) = \mathrm{erf}(s_b) - s_b\,\mathrm{erf}'(s_b) = \mathrm{erf}(s_b) - \frac{2s_b}{\sqrt{\pi}}e^{-s_b^2} \tag{7.4}$$

Scattering in moving frame: diffusion

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$$2\mathbb{D}_{ab} = \nu_{ab0} \begin{pmatrix} D_{ab}^{xx} & 0 & 0 \\ 0 & D_{ab}^{yy} & D_{ab}^{yz} \\ 0 & D_{ab}^{yz} & D_{ab}^{zz} \end{pmatrix} \qquad \nu_{ab0} = \frac{Z_a^2 Z_b^2 e^4 n_b \ln\Lambda_{ab}}{4\pi\varepsilon_0^2 m_a^2 v_{T_a}^3} \tag{8.1}$$

$$D_{ab}^{yy} = v^2 \frac{G(s_b)}{s_a^3} - u_{\parallel b} v_\parallel \frac{H(s_b) - G(s_b) + F(s_b)}{s_a^3} = D_{ab}^{xx} \qquad u_{\parallel b} = \frac{2q_{\parallel b}}{5n_b T_b} \tag{8.2}$$

$$D_{ab}^{yz} = u_{\parallel b} v_\perp \frac{H(s_b) - G(s_b) + F(s_b)}{s_a^3} \qquad s_a = \frac{v}{v_{T_a}} \tag{8.3}$$

$$D_{ab}^{zz} = v^2 \frac{H(s_b)}{s_a^3} + 2u_{\parallel b} v_\parallel \left(\frac{H(s_b) - G(s_b) + F(s_b)}{s_a^3} - \frac{v_{T_a}^3}{v_{T_b}^3} \frac{2}{\sqrt{\pi}} e^{-s_b^2} \right) \qquad s_b = \frac{v}{v_{T_b}} \tag{8.4}$$

[Donnel et al., NF63]

$$G(s_b) = \mathrm{erf}(s_b)\left(1 - \frac{1}{2s_b^2}\right) + \frac{1}{s_b\sqrt{\pi}} e^{-s_b^2} \qquad H(s_b) = \frac{\mathrm{erf}(s_b)}{s_b^2} - \frac{2}{s_b\sqrt{\pi}} e^{-s_b^2} \tag{8.5}$$

$$H(s_b) - G(s_b) + F(s_b) = \frac{3\,\mathrm{erf}(s_b)}{2s_b^2} - \frac{3 + 2s_b^2}{s_b\sqrt{\pi}} e^{-s_b^2} \qquad F(s_b) = \mathrm{erf}(s_b) - \frac{2s_b}{\sqrt{\pi}} e^{-s_b^2} \tag{8.6}$$

Equation for $(\Delta \boldsymbol{v})_{\mathrm{coll}}$

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$$\left.\frac{\partial f_a}{\partial t}\right|_{\mathrm{coll}} = \sum_b C_{ab}(f_a, f_b) = -\sum_b \nabla_{\boldsymbol{v}} \cdot \Big(\boldsymbol{\Gamma}_{ab} f_a - \nabla_{\boldsymbol{v}} \cdot (\mathbb{D}_{ab} f_a)\Big) \tag{9.1}$$

$$\boldsymbol{\Gamma}_{ab} = \nu_{ab0}\,\frac{v_{T_a}^3}{n_b}\left(1 + \frac{m_a}{m_b}\right)\nabla_{\boldsymbol{v}} H_b(\boldsymbol{v}) \qquad 2\mathbb{D}_{ab} = \nu_{ab0}\,\frac{v_{T_a}^3}{n_b}\,\nabla_{\boldsymbol{v}}\nabla_{\boldsymbol{v}}\, G_b(\boldsymbol{v}) \tag{9.2}$$

In a PIC code: particle velocities $v_{\|i}$ and $v^2_{\perp i}$ have to be changed

Use corresponding stochastic differential equation:

$$\left.\frac{d\boldsymbol{v}}{dt}\right|_{ab} = \boldsymbol{\Gamma}_{ab} + \mathbb{K}_{ab}\, d\boldsymbol{W} \qquad \boldsymbol{W}: \text{ stochastic Wiener process,} \qquad 2\mathbb{D}_{ab} = \mathbb{K}_{ab}\,\mathbb{K}_{ab}^T$$

$$\Delta\boldsymbol{v} = \boldsymbol{\Gamma}_{ab}\Delta t + \mathbb{K}_{ab}\sqrt{\Delta t}\,\boldsymbol{R} \qquad \boldsymbol{R}: \text{ vector of random numbers,} \quad \langle R_i \rangle = 0, \ \langle R_i^2 \rangle = 1$$

$$\Delta v_i = (\boldsymbol{\Gamma}_{ab})_i \Delta t + \sqrt{(2\mathbb{D}_{ab})_{ii}\Delta t}\, R_i \qquad \text{if } \mathbb{D}_{ab} \text{ is diagonal}$$

Procedure for the scattering

1. Friction: $v_z = v + \Gamma_z \Delta t, \;\; v_y = \Gamma_y \Delta t, \;\; v_x = 0$

2. Diffusion:

   $x: \;\; v_x = \sqrt{D_{ab}^{yy} \Delta t} R_1 \quad R_1:$ random number

   $y, z:$

   1. transform (v_y, v_z) to (v_2, v_3) so that $2\mathbb{D}$ becomes diagonal, $2\mathbb{D} = \begin{pmatrix} \lambda_2 & 0 \\ 0 & \lambda_3 \end{pmatrix}$

   2. $v_2 \to v_2 + \sqrt{\lambda_2 \Delta t} R_2, \;\; v_3 \to v_3 + \sqrt{\lambda_3 \Delta t} R_3$

   3. transfrom back to(v_y, v_z)

   4. retrieve $v_\parallel, v_\perp^2: \;\; v^2 = (v_x^2 + v_y^2 + v_z^2), \;\; v_\parallel = \boldsymbol{v} \cdot \boldsymbol{b} = b_z v_z + b_y v_y, \;\; v_\perp^2 = v^2 - v_\parallel^2$

Collisions: scattering part and correction part

In a delta-f code with linearized collision operator we have

$$C(f_a, f_b) = \underbrace{C(\delta f_a, f_{Mb})}_{\text{scattering part}} + \underbrace{C(f_{Ma}, \delta f_b) + C(\delta f_a, \delta f_b) + C(f_{Ma}, f_{Mb})}_{\text{replaced by correction part}} \tag{11.1}$$

In the full-f code with the approximation f_{bt} for f_b we have

$$C(f_a, f_b) = \underbrace{C(f_a, f_{bt})}_{\text{scattering part}} + \underbrace{C(f_a, (f_b - f_{bt}))}_{\text{replaced by correction part}} \tag{11.2}$$

The correction part is needed for momentum/energy conservation and for correct momentum/energy loss/gain

Full-f: $C(f_a, f_{bt})$ is closer to $C(f_a, f_b)$ than $C(\delta f_a, f_{Mb})$, because f_{bt} is closer to f_b than f_{Mb} and the particles scattered represent the full distribution function f_a

Momentum and energy conservation

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Velocity dependent correction for momentum and energy conservation:

Before and after scattering: $(V_{\|1}, E_1)$ and $(V_{\|2}, E_2)$, $\quad V_\| = \frac{1}{N}\sum_j v_{\|j}, \quad E = \frac{1}{N}\sum_j v_j^2$

Corrections for $v_\|$ and $v_\perp^2$:

$$\Delta v_{\|j} = -\frac{v_{\|j}\delta v_{\|j}\Delta V_\|}{\frac{1}{N}\sum_j v_{\|j}\delta v_{\|j}}, \quad \frac{1}{N}\sum_j \Delta v_{\|j} = -\Delta V_\|, \quad \Delta V_\| = V_{\|2} - V_{\|1} \tag{12.1}$$

$$\Delta v_j^2 = -\frac{v_j^2\delta v_j^2\Delta E}{\frac{1}{N}\sum_j v_j^2\delta v_j^2}, \quad \frac{1}{N}\sum_j \Delta v_j^2 = -\Delta E, \quad \Delta E = \frac{3T_2}{m} + V_{\|2}^2 - \left(\frac{3T_1}{m} + V_{\|1}^2\right) \tag{12.2}$$

$$\Delta v_{\perp j}^2 = \Delta v_j^2 - 2v_{\|j}\Delta v_{\|j} - (\Delta v_{\|j})^2 \tag{12.3}$$

Theoretical changes of momentum and energy (next slides)

$$\delta v_{\|j} = (\Delta v_\|)_j^{\mathrm{th}}, \quad \delta v_j^2 = (\Delta v^2)_j^{\mathrm{th}} \tag{12.4}$$

Average momentum/energy loss per particle

In the frame with $V_{\|b} = 0$ (**obtained with $\langle \boldsymbol{R_i} \rangle = 0$, $\langle \boldsymbol{R_i^2} \rangle = 1$**):

$$(\Delta v_\|)^{\rm th}_{ab} = \nu_{ab0}\Delta t \left(1 + \frac{m_a}{m_b}\right) \left[-v_\| \frac{F(s_b)}{s_a^3} + \frac{2q_{\|b}}{5n_b T_b} \left(2\frac{v_\|^2}{v_{T_b}^2} - 1 \right) \frac{2}{\sqrt{\pi}} \frac{v_{T_a}^3}{v_{T_b}^3} e^{-s_b^2} \right] \tag{13.1}$$

$$(\Delta v^2)^{\rm th}_{ab} = -\nu_{ab0}\Delta t \left[v^2 \left(2\left(1 + \frac{m_a}{m_b}\right) \frac{F(s_b)}{s_a^3} - \frac{2E_b}{s_a^3} \right) - \frac{2q_{\|b} v_\|}{5n_b T_b} \left(\left(1 + \frac{m_a}{m_b}\right)(2s_b^2 - 1) - 1 \right) \frac{v_{T_a}^3}{v_{T_b}^3} \frac{4}{\sqrt{\pi}} e^{-s_b^2} \right] \tag{13.2}$$

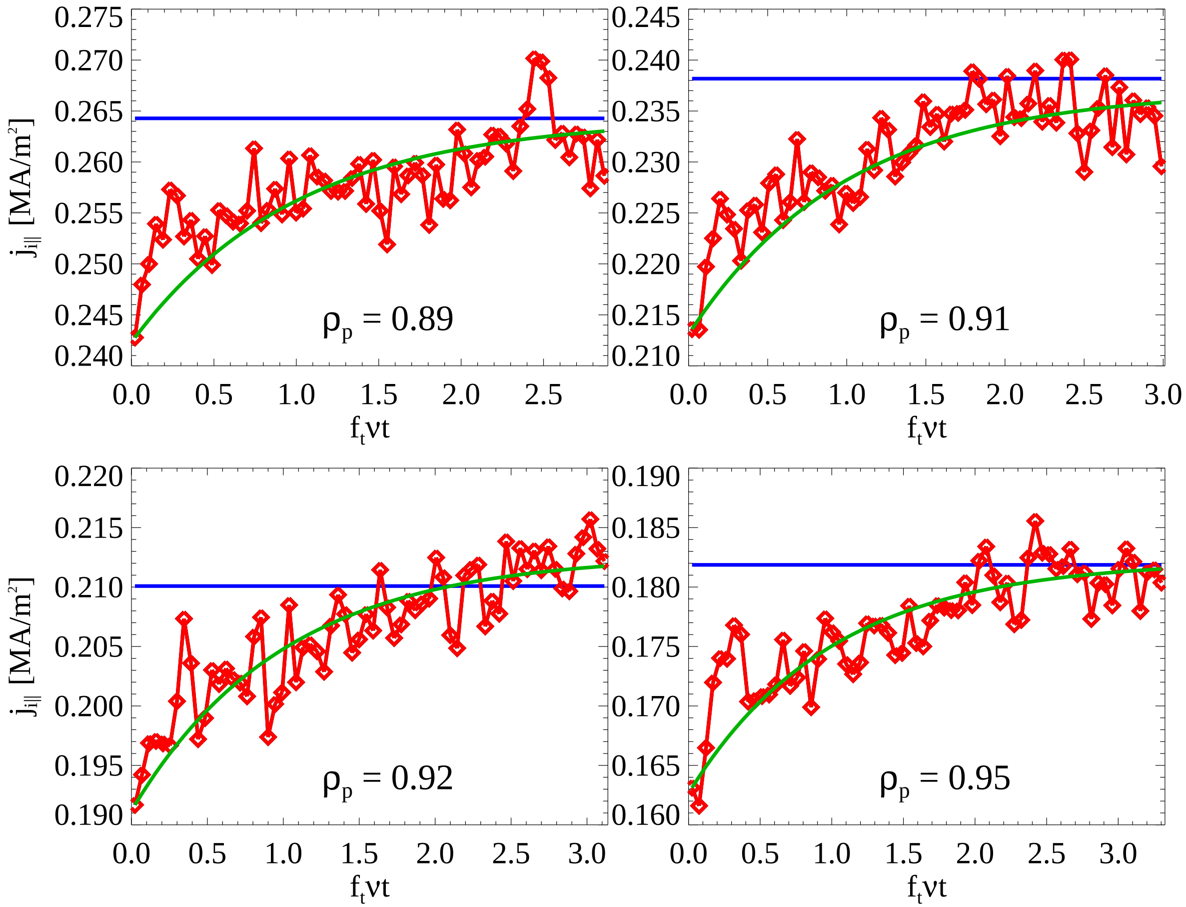
Neglecting q_b:

$$(\Delta v_\|)^{\rm th}_{ab} = -v_\| \nu_{ab0}\Delta t \left(1 + \frac{m_a}{m_b}\right) \frac{F(s_b)}{s_a^3} \tag{13.3}$$

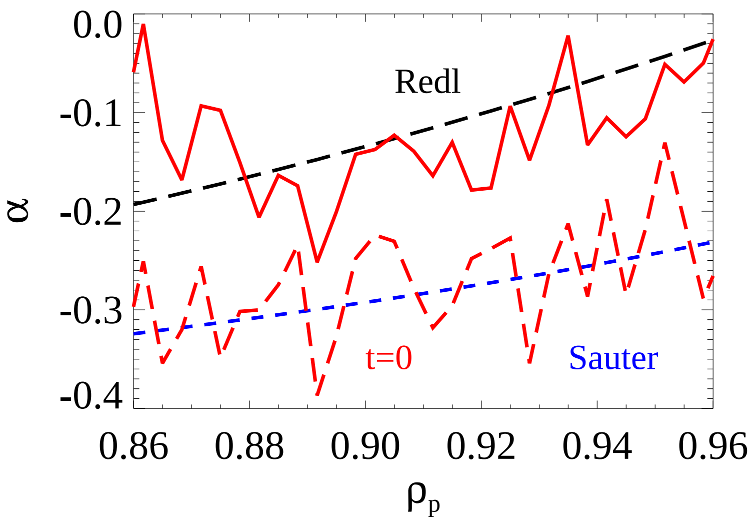
$$(\Delta v^2)^{\rm th}_{ab} = -v^2 \nu_{ab0}\Delta t \left(2\left(1 + \frac{m_a}{m_b}\right) \frac{F(s_b)}{s_a^3} - \frac{2E_b}{s_a^3} \right) \tag{13.4}$$

Ion simulation with weak gradients

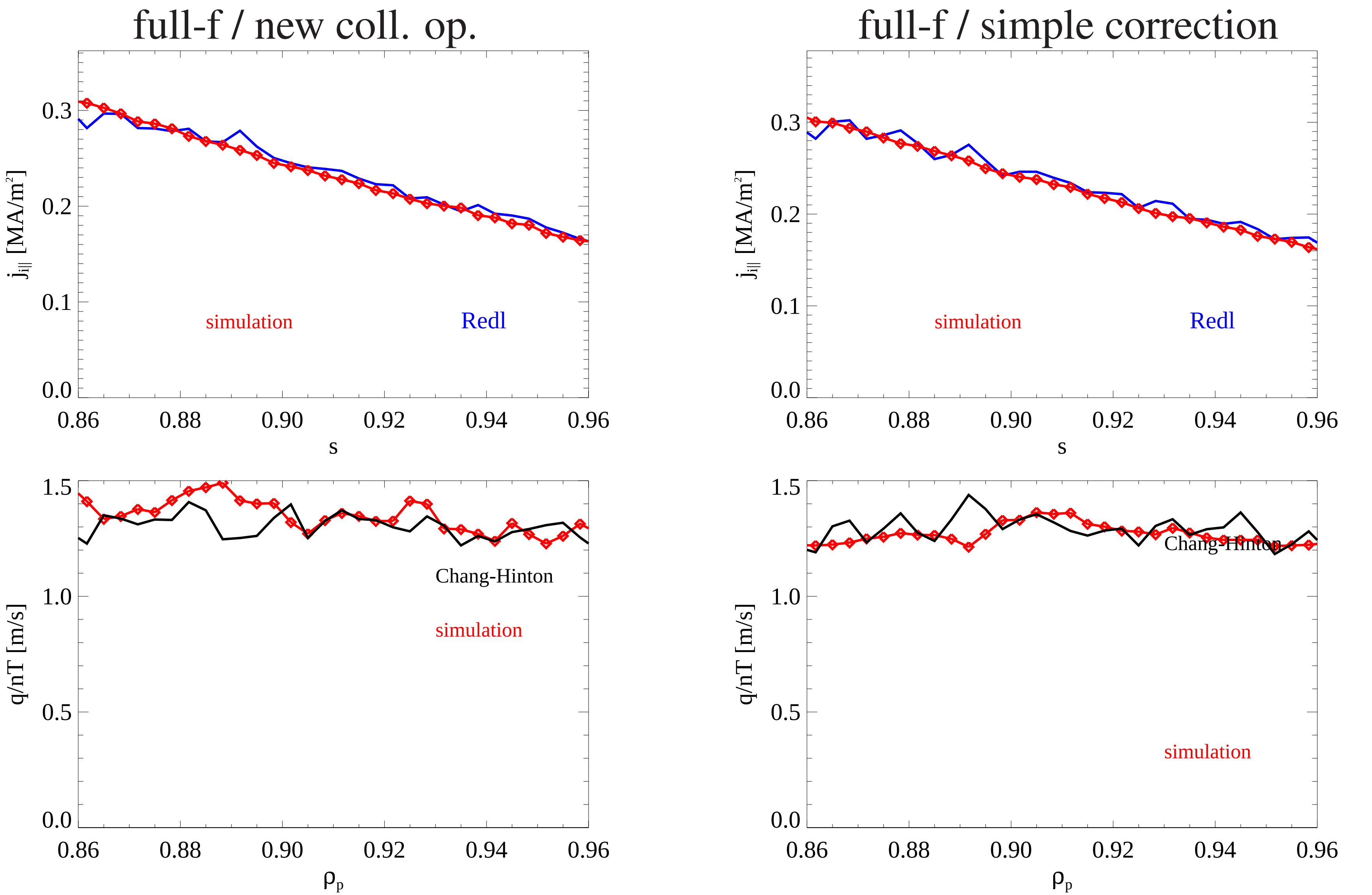
Characteristic time for ion current: $\tau_{\mathrm{coll}}/f_t \; [dj_i/dt = (\nu_{ii}/f_t^2) j_{i\mathrm{tr}} - f_t\, \nu_{ii}\, j_i]$



Coefficient for poloidal flow



Ion simulation with weak gradients



For comparison: simple scheme

Simple version: correction for momentum/energy conservation/transfer:

- Intra-species collisions: momentum and energy conservation

$$v_\parallel \to V_{\parallel 1} + (v_\parallel - V_{\parallel 2})\sqrt{\frac{T_1}{T_2}} \qquad v_\perp^2 \to v_\perp^2 \frac{T_1}{T_2} \qquad [1,2: \text{before/after scatt.}] \tag{16.1}$$

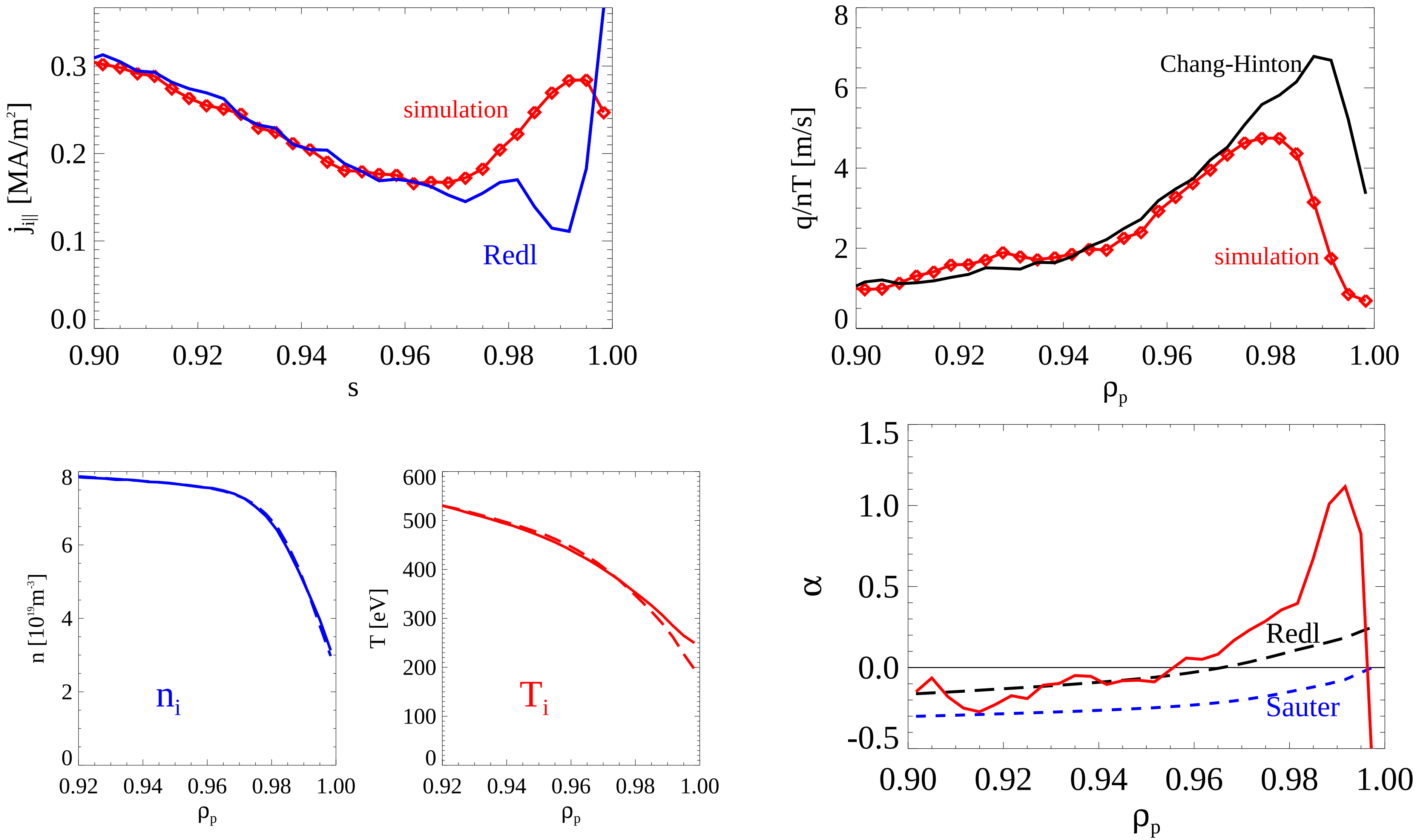
- electron - ion collisions: momentum and energy loss/gain

$$T_{e1n} = T_{e1} + 2(T_i - T_{e1})\frac{m_e}{m_i}0.752\,\nu_{ei0}\Delta t \qquad v_{\perp e}^2 \to v_{\perp e}^2 \frac{T_{e1n}}{T_{e2}} \tag{16.2}$$

$$v_{\parallel e} \to V_{\parallel e1} + (v_{\parallel e} - V_{\parallel e2})\sqrt{\frac{T_{e1n}}{T_{e2}}} + (V_{\parallel i} - V_{\parallel e1} + \frac{3q_{\parallel e}}{5n_eT_e})0.752\,\nu_{ei0}\Delta t \tag{16.3}$$

- This procedure works, but was shown for the linearized operator to be inaccurate

Ion simulation for pedestal



Energy dependent correction for momentum and energy:

Before and after scattering: $(V_{\|1}, T_1)$ and $(V_{\|2}, T_2)$

After correction: $V_{\|1} + (\Delta V_\|)_{ab}, T_1 + (\Delta T)_{ab}$

Theoretical losses of momentum and energy

$$(\Delta V_\|)_{ab}, \quad (\Delta T)_{ab} \tag{18.1}$$

Corrections for $v_\|$ and $v_\perp^2$:

$$\Delta v_{\|j} = -\Delta V_\| \frac{v_{\|j}\delta v_{\|j}}{\frac{1}{N}\sum_j v_{\|j}\delta v_{\|j}}, \quad \Delta V_\| = V_{\|2} - (V_{\|1} + (\Delta V_\|)_{ab}) \tag{18.2}$$

$$\Delta v_j^2 = -\frac{v_j^2 \delta v_j^2 \Delta E}{\frac{1}{N}\sum_j v_j^2 \delta v_j^2}, \quad \Delta E = \frac{3T_2}{m} + V_{\|2}^2 - \left(\frac{3(T_1 + (\Delta T)_{ab})}{m} + (V_{\|1} + (\Delta V_\|)_{ab})^2\right) \tag{18.3}$$

$$\Delta v_{\perp j}^2 = \Delta v_j^2 - 2v_{\|j}\Delta v_{\|j} - \Delta v_{\|j}^2 \tag{18.4}$$

Different species: change of average velocity

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In the frame with $V_{\|b} = 0,\ V_{\|a} = \Delta V_\|$ (4-term approximation (6.1) also used for f_a):

$$(\Delta V_\|)_{ab} = \nu_{ab}\Delta t \left[-\Delta V_\| + \frac{3q_{\|a}}{5n_a T_a} \frac{T_a m_b}{T_a m_b + T_b m_a} - \frac{3q_{\|b}}{5n_b T_b} \frac{T_b m_a}{T_a m_b + T_b m_a} \right] + \ldots \tag{19.1}$$

In the frame with $V_{\|a} = 0,\ V_{\|b} = -\Delta V_\|$

$$(\Delta V_\|)_{ba} = \nu_{ba}\Delta t \left[\Delta V_\| - \frac{3q_{\|a}}{5n_a T_a} \frac{T_a m_b}{T_a m_b + T_b m_a} + \frac{3q_{\|b}}{5n_b T_b} \frac{T_b m_a}{T_a m_b + T_b m_a} \right] + \ldots \tag{19.2}$$

Two terms in the expressions from [Donnel et al., NF63] are omitted for obtaining

$$(\Delta V_\|)_{ba}\Big|_{V_{\|a}=0} = -\,(\Delta V_\|)_{ab}\Big|_{V_{\|b}=0}$$

For $m_a \ll m_b$ terms with $q_{\|b}$ are very small:

$$(\Delta V_\|)_{ei} = \nu_{ei}\Delta t \left[V_{\|i} - V_{\|e} + \frac{3q_{\|e}}{5n_e T_e} \right] = \nu_{ei}\Delta t \left[\frac{J_\|}{en_e} + \frac{3q_{\|e}}{5n_e T_e} \right] \tag{19.3}$$

Different species: change of average energy

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In the frame with $V_{\|b} = 0,\ V_{\|a} = \Delta V_{\|}$ (f_a approx. by f_{at}):

$$(\Delta E)_{ab} = \frac{m_a}{m_a + m_b} \nu_{ab} \Delta t \left\{ 3(T_b - T_a) - \frac{(\Delta V_{\|})^2}{v_{T_a}^2 + v_{T_b}^2} \left[\left(2\frac{m_a}{m_b} + 3 \right) T_b - T_a \right] \right.$$
$$- 3 \frac{2 q_{\|b} \Delta V_{\|}}{5 n_b T_b} \frac{X_{ba}^2}{(v_{T_a}^2 + v_{T_b}^2)} \left[\left(\frac{m_a}{m_b} + 2 \right) T_b - \left(3\frac{m_b}{m_a} + 4 \right) T_a \right]$$
$$\left. + \frac{15}{2} \frac{2 q_{\|a}}{5 n_a T_a} \frac{2 q_{\|b}}{5 n_b T_b} \frac{X_{ab}^2 X_{ba}^2}{(v_{T_a}^2 + v_{T_b}^2)} \left[3\left(\frac{m_a}{m_b} + 4 \right) T_b - \left(3\frac{m_b}{m_a} + 4 \right) T_a \right] \right\} \tag{20.1}$$

$$X_{ab}^2 = \frac{T_a m_b}{T_a m_b + T_b m_a}, \quad X_{ba}^2 = \frac{T_b m_a}{T_a m_b + T_b m_a} \tag{20.2}$$

Electron - ion collisions:

$$X_{ei}^2 \approx 1, \quad X_{ie}^2 \approx m_e/m_i, \quad (\Delta V_{\|})^2 \ll v_{T_e}^2 \tag{20.3}$$

$$(\Delta E)_{ei} = \frac{m_e}{m_i} \nu_{ei} \Delta t \, 3(T_i - T_e), \quad (\Delta T)_{ei} = \frac{m_e}{m_i} \nu_{ei} \Delta t \, 2(T_i - T_e) \tag{20.4}$$

Bootstrap current

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From neoclassical theory:

$$j_\parallel = -\frac{RB_t}{B}\left\{\left(\frac{dp_e}{d\psi}+\frac{dp_i}{d\psi}\right)+\frac{B^2}{\langle B^2\rangle}\left[(L_{31}-1)\left(\frac{dp_e}{d\psi}+\frac{dp_i}{d\psi}\right)\right.\right.$$

$$\left.\left.+L_{32}\,n_e\frac{dT_e}{d\psi}+\alpha\,L_{34}\,n_i\frac{dT_i}{d\psi}\right]\right\}$$

$$\frac{\langle j_\parallel B\rangle}{B_0} = -R_0\left\{L_{31}\left(\frac{dp_e}{d\psi}+\frac{dp_i}{d\psi}\right)+L_{32}\,n_e\frac{dT_e}{d\psi}+\alpha\,L_{34}\,n_i\frac{dT_i}{d\psi}\right\}$$

$$L_{3x} = L_{3x}(\nu_*, f_t)$$

- No explicit dependence on the mass ratio m_i/m_e.
- Possible effect of finite electron orbit width, if electron mass is large.
- We can use smaller mass ratio if $\rho_{pe} \ll L$ still holds.

Bootstrap current - simulation with weak gradients

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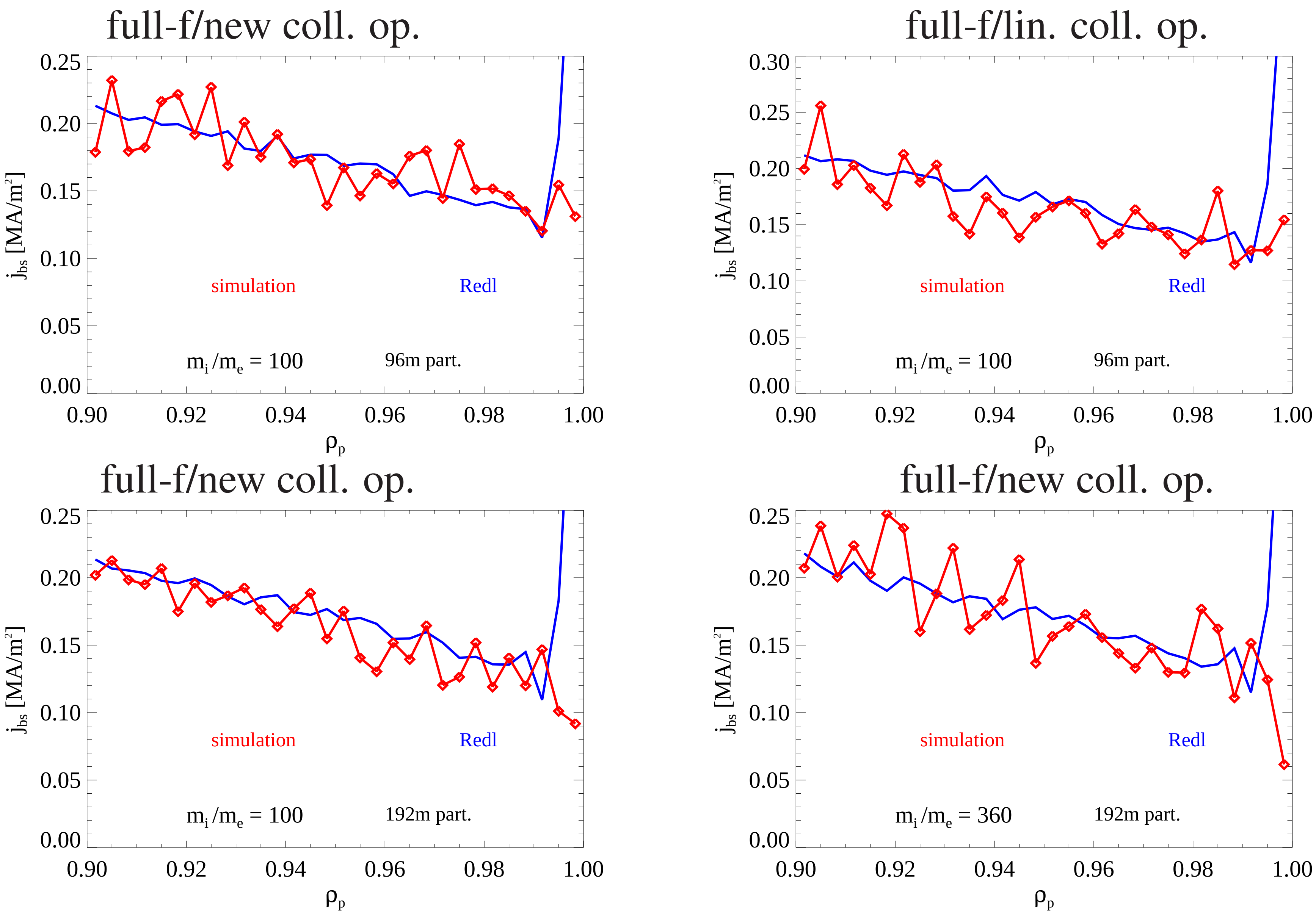

Bootstrap current in pedestal

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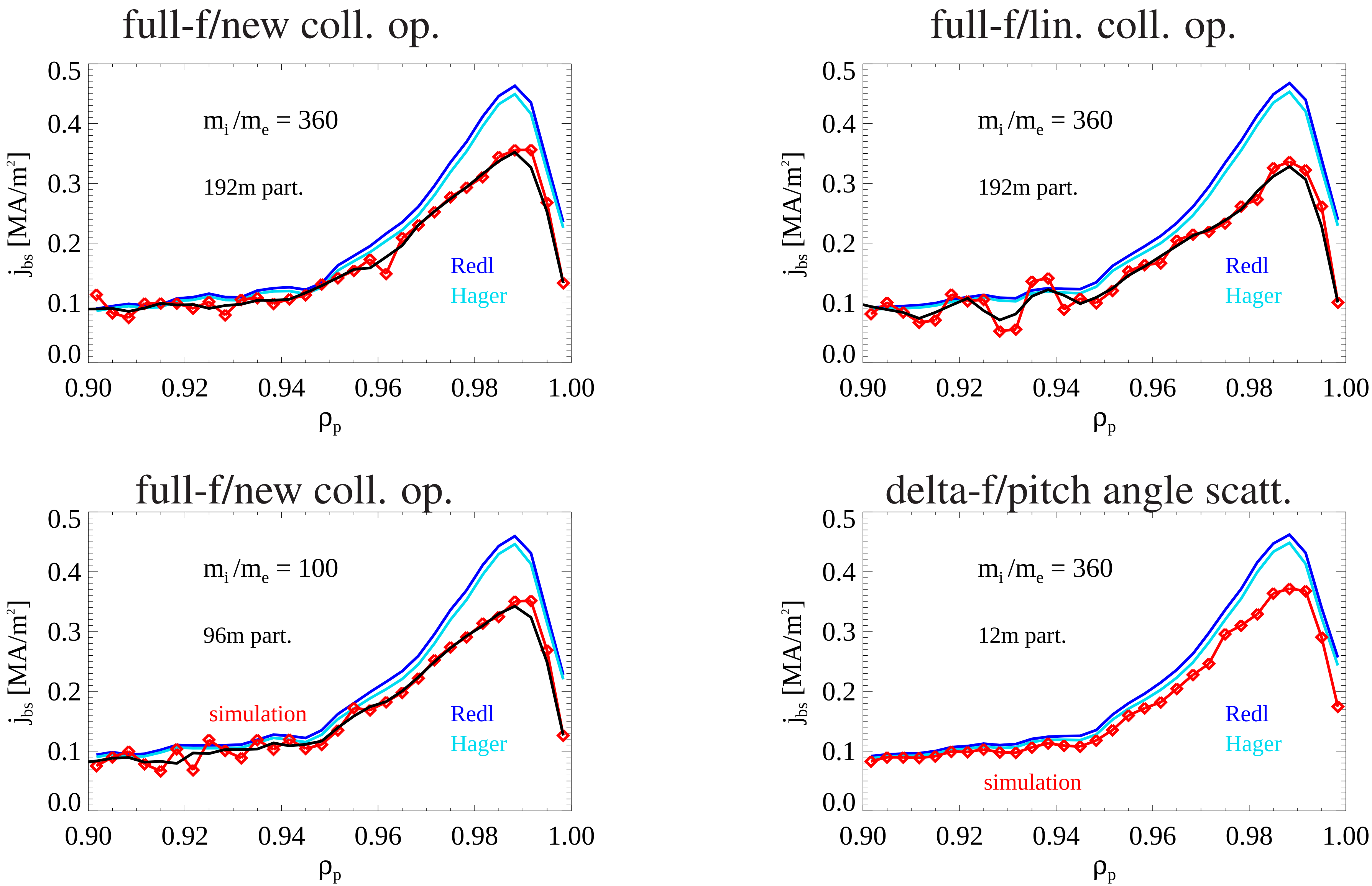

Summary

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- Improved collision operator for full-f HAGIS simulations was implemented
- Collisions are calculated in the frame moving with the field particles
- New versus linear collision operator: additional terms due to the heat flux
- Corrections for momentum/energy conservation/exchange proportional to theoretical rates are smaller than for linearized operator
- For bootstrap current a large number of marker particles is necessary
- Currently without noise reduction procedure, but
- smaller mass ratio (360, 100) can be used in bootstrap current calculation for reducing the numerical noise