Coarse-grained gyrokinetics for the critical ion temperature gradient in stellarators: early optimization results

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M. Landreman, A. Zocco, P. Helander for discussions

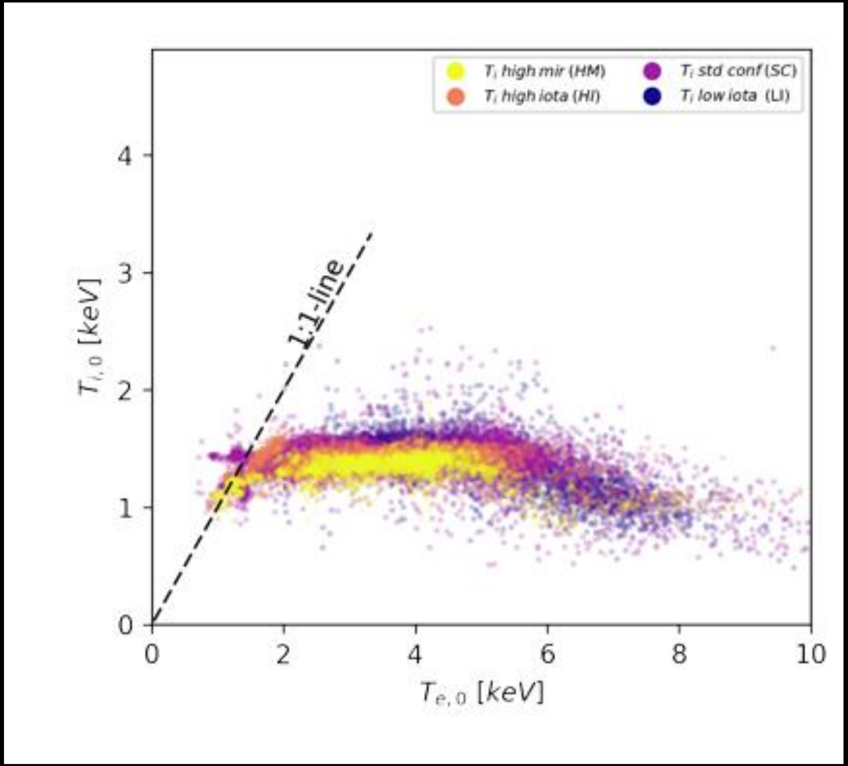
IPP Greifswald

Talk Outline

- Motivation to control the ITG
- Benefits of targeting linear critical gradient
- Physics of marginal stability: simple model
- Results for stellarators
- Modified W7-X-like shape – improved critical gradient
- SIMSOPT applications
- Conclusions

ITG may be culprit for W7-X ion heat losses

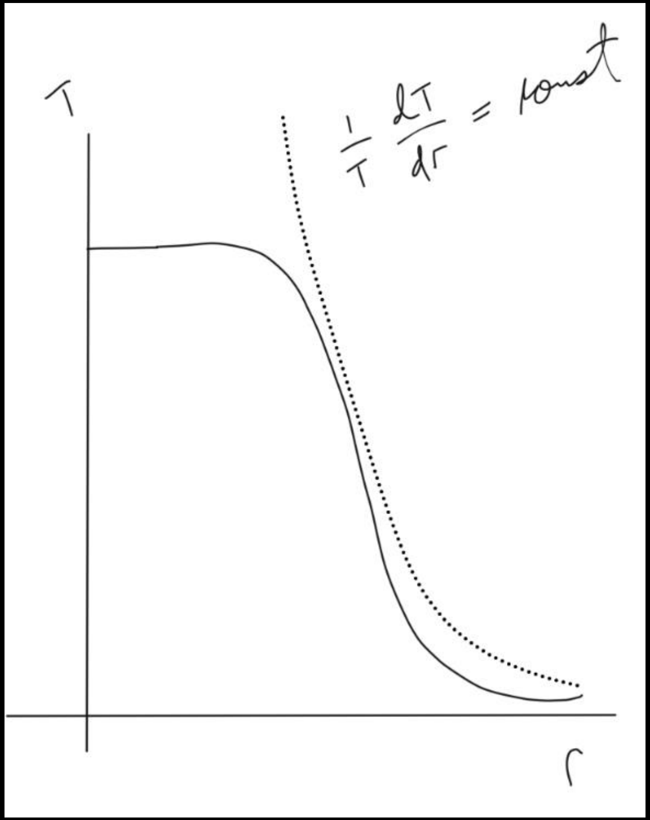
- Plasma can be heated through electron cyclotron resonance heating (ECRH). Electrons transfer heat to ions through collisions
- Heating limit on ions observed in the W7X data – “Ion temperature clamping”
- Thought that small-scale turbulence such as the ITG could be the cause (Beurskens et al. 2021, Bañon-Navarro et al 2022)



Courtesy M. Beurskens

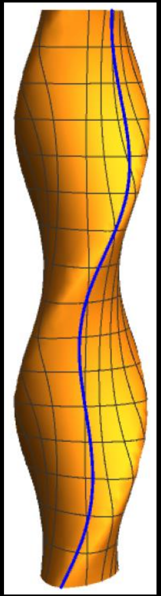
A different approach to ITG reduction

- Traditional (as well as current) stellarator opt. studies have used linear measures (growth rate) or NL estimates / models of transport (Mynick+ 2010, Xanthopoulos+ 2014,Hegna+ 2018, Nunami+ 2013)
- We target the linear critical gradient - the value of the drive parameter where turbulence arises
- Helpful if transport is “stiff” – sharply increasing with drive parameter – above the critical gradient, so turbulence pins gradient to the threshold (Baumgaertel et al. 2013)
- Target a desired radial profile
- Nonlinear threshold likely to be above the linear (Dimits et al. 1998) – linear is a lower bound estimate



ITG Mode

- Electrostatic, driven by plasma gradients in temperature and density
- Studied with gyrokinetic equation (mode has low frequency compared to $\Omega_i = \frac{eB}{m_i c}$, small spatial extent perpendicular to B)
- Thought of in distinct branches:
  - Toroidal (localized along field line, peaks and tapers off within a single drift well, feeds off "bad curvature" – to be defined)
  - Slab (like a sound wave with acoustic speed $C_s = \sqrt{\frac{2T_e}{m_i}}$, spreads along field line)
  - Floquet (Zocco et al. 2018, 2022, Bhattacharjee et al. 1981) – less understood, extended like a slab mode but cares about curvature too



Equations

- Linear electrostatic gyrokinetic equation, coupled with Poisson's equation

- Ignore density gradient, and electron/ion T ratio = 1

- Adiabatic electrons

- Apply ballooning boundary conditions (Connor et al. 1980)

- Vacuum fields

$$iv_{\parallel}\frac{\partial g}{\partial \ell} + (\omega - \widetilde{\omega}_d)g = \varphi J_0(\omega - \widetilde{\omega}_*)f_0 \tag{2.2}$$

with the following definitions: $J_0 = J_0(k_\perp v_\perp/\Omega) = J_0(k_\perp \rho\sqrt{2}v_\perp/v_{\mathrm{T}})$; the thermal velocity is $v_{\mathrm{T}} = \sqrt{2T/m}$ and the thermal ion Larmor radius is $\rho = v_{\mathrm{T}}/(\Omega\sqrt{2})$; n and T are the background ion density and temperature; q is the ion charge; $\varphi = q\phi/T$ is the normalized electrostatic potential. Assuming Boltzmann electrons, the quasineutrality condition is

$$\int d^3\mathbf{v} J_0 g = n(1+\tau)\varphi, \tag{2.3}$$

where $\tau = T_i/(ZT_e)$ with the charge ratio defined as $Z = q_i/q_e$. The equilibrium distribution is the Maxwellian

Coordinates

$$\mathbf{B} = \nabla\psi \times \nabla\alpha$$

$\mathbf{k}_\perp = k_\alpha \nabla\alpha + k_\psi \nabla\psi$, where k_α and k_ψ are constants, so the variation of $\mathbf{k}_\perp(l)$ comes from the geometric quantities $\nabla\alpha$ and $\nabla\psi$. Here ψ is a flux surface label and α is a label for a particular field line on a given surface ψ.

- l is the magnetic field line following coordinate (arc length).
- For simplicity, $k_\psi = 0$

$r = a\sqrt{\psi/\psi_{edge}}$, with a the minor radius corresponding to the flux surface at the edge, and ψ_{edge} the toroidal flux at that location.

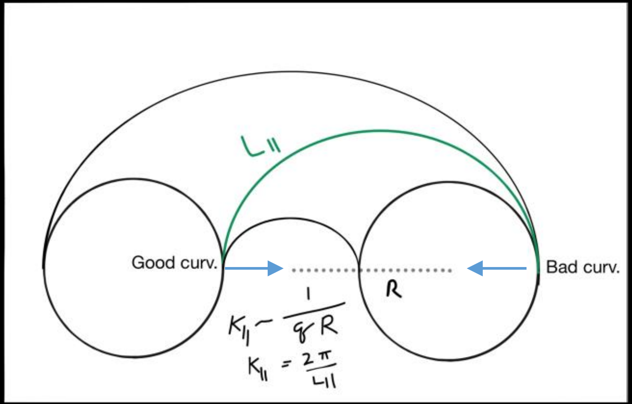
$$a/L_T = -(a/T)dT/dr$$

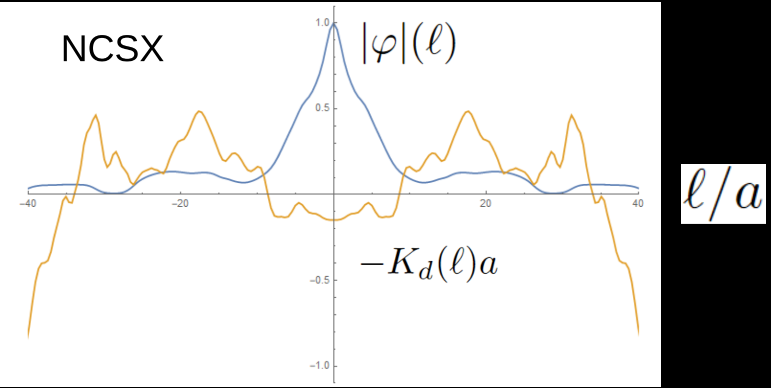
$$\omega_*^T = (Tk_\alpha/q)d\ln T/d\psi\,(v^2/v_T{}^2 - 3/2)$$

$$\omega_d = (1/\Omega)(\mathbf{k}_\perp \cdot \hat{b} \times \boldsymbol{\kappa})(v_\parallel^2 + v_\perp^2/2)$$

Traditional picture: connection length

- Geometry can set the connection (or correlation) length $L_{||}$ of the mode, over which the mode is coherently driven.
- In tokamak literature, $L_{||}$ = inboard to outboard distance along field line
- Normal curvature $\boldsymbol{\kappa}_n = \boldsymbol{\kappa} \cdot \boldsymbol{n} \quad = (\boldsymbol{b} \cdot \nabla \boldsymbol{b}) \cdot \frac{\nabla\psi}{|\nabla\psi|}$
- If curvature and temperature gradient are aligned, “bad curvature” – tends to increase growth rates above marginality. Opposite is “good curvature”
- $\omega_d = (1/\Omega)(\mathbf{k}_\perp \cdot \hat{b} \times \boldsymbol{\kappa})(v_{||}^2 + v_\perp^2/2)$
- “Drift curvature”: $K_d = a^2 \nabla\alpha \cdot \hat{b} \times \boldsymbol{\kappa}$
- Drift resonance (not including parallel variation): $\omega \simeq \omega_d$





Marginal stability – “more bad curvature is good”

- Below and at marginality, the localized mode is stabilized by bad curvature because it suffers a resonant damping, akin to Landau damping (Baumgaertel et al. 2013, Biglari et al. 1988)

$\omega \simeq \omega_d$ Drift resonance

$\omega \sim \omega_*^{\mathrm{T}}$ Marginal stability

→ $\omega_*^T \sim \omega_d$ Matching drive and damping

$$\omega_*^T \sim k_\alpha \rho v_T / L_T$$

$$\omega_d \sim k_\alpha \rho v_T / R_{\mathrm{eff}}$$

→ $$L_{T,\mathrm{crit}}^{-1} \sim R_{\mathrm{eff}}^{-1}$$

- How do we calculate R_{eff}^{-1} ?

Earlier model

- Jenko et al. (2001) looked at large aspect ratio, circular cross section tokamaks (Connor et al 1978). Obtained by fitting results of GK simulations:

$$\frac{R}{L_{T,\mathrm{crit}}} = (1+\tau)\left(1.33 + 1.91\frac{\hat{s}}{q}\right)$$

- Same answer as the estimate from previous slide (first term), assuming R is the radius of curvature
- Additional shear (parallel stabilization) term, Hahm & Tang (1989)
- Did not consider negative or small shear.

$\hat{s} = (r/q)(dq/dr)$, $q(r)$ the safety factor

$$L_s = \hat{s}R/q$$

GK solutions contain a “smoothing effect”

- Integral form of GK eqn. (Connor et al. 1980 -> Romanelli 1989, Plunk 2014):

$$(1+\tau)\varphi(\ell) = \frac{-2i}{v_{\mathrm{T}}\sqrt{\pi}}\int_0^\infty \frac{dx_\parallel}{x_\parallel}\int_0^\infty x_\perp dx_\perp J_0(\omega - \widetilde{\omega}_*)$$
$$\times \int_{-\infty}^{\infty} d\ell' J_0' \exp(-x^2 + i\,\mathrm{sgn}(\ell-\ell')M(\ell',\ell))\varphi(\ell'), \qquad (2.7)$$

where $x_\perp = v_\perp/v_{\mathrm{T}}$ and $x_\parallel = v_\parallel/v_{\mathrm{T}}$, sgn gives the sign of its argument, $J_0 = J_0\left(\sqrt{2b(\ell)}x_\perp\right)$, $J_0' = J_0\left(\sqrt{2b(\ell')}x_\perp\right)$, and $b(\ell) = \rho^2 k_\perp^2(\ell)$. The physics of the drift resonance is contained in the factor

$$M(\ell',\ell) = \int_{\ell'}^{\ell} \frac{\omega - \widetilde{\omega}_d(\ell'')}{v_{\mathrm{T}} x_\parallel} d\ell''. \qquad (2.8)$$

- M exponent averages over small-scale “ripples” in the drift curvature – coarse graining
- To build a theory, we assume scale separation between ripples and the curvature envelope, e.g. “toroidal harmonic” or global shear length
- Should leave a smooth profile of curvature – estimate R_{eff}^{-1} from this

Fitting as an effective coarse graining

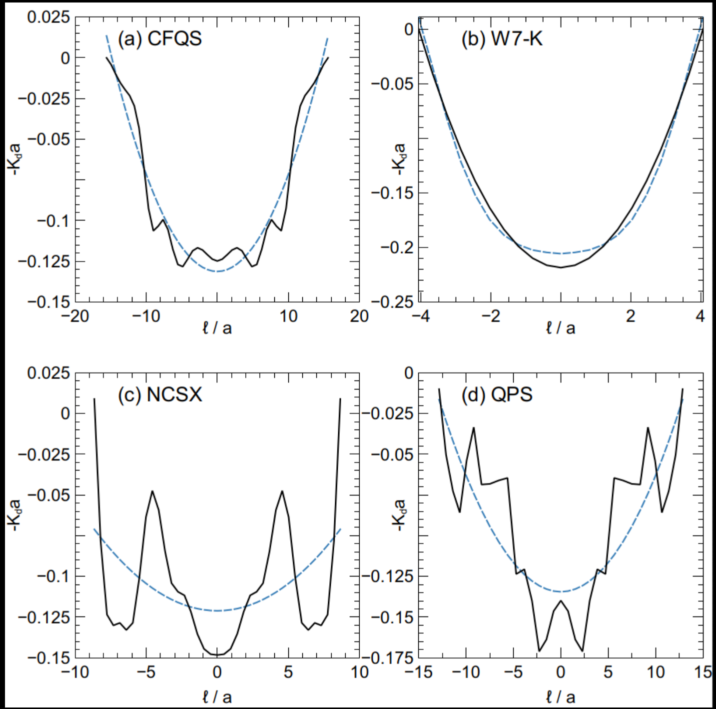
- Quadratic, least squares fit to the drift curvature profile:

$$K_d = a^2 \nabla\alpha \cdot \hat{b} \times \boldsymbol{\kappa}$$

- Use data between the points where the sign of the drift curvature reverses (nominal connection length).
- Scan along flux tube, finding bad curvature wells.
- Amplitude and width of the fit are free.
- Take the peak value of the fitted curve (parabola) to be R_{eff}^{-1}.

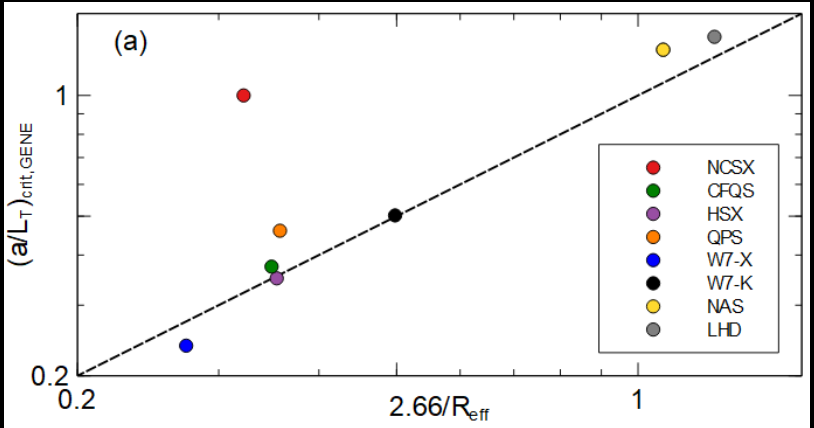
- Estimate critical gradient (minimal value from all wells):

$$\frac{R_{\mathrm{eff}}}{L_{T,\mathrm{crit}}} = (1+\tau)\,(1.33)$$



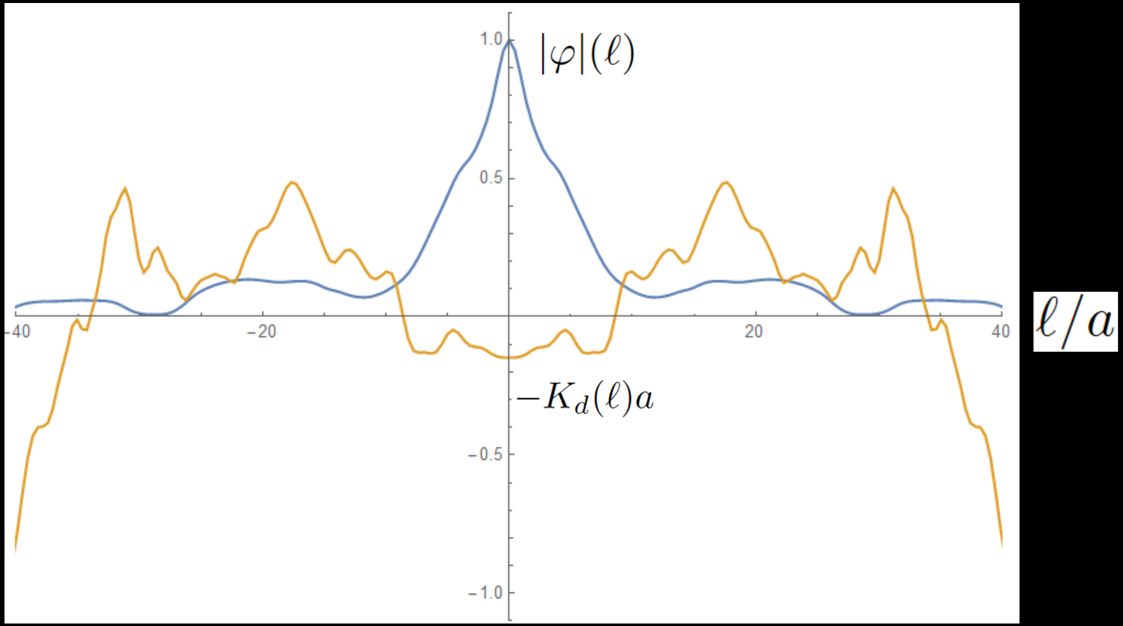
Model Results

- Linear flux-tube simulations using the gyrokinetic code GENE. Outboard midplane, radius of half edge toroidal flux
- Extrapolate growth rate of "last mode standing" to zero → critical gradient
- W7-K is a modified version of 7-X (presented later).
- NAS is a Near-AxiSymm, low aspect ratio, high q config created from the method of Plunk & Helander (2018).
- NCSX needs additional stabilizing term



$$\frac{R_{\text{eff}}}{L_{T,\text{crit}}} = (1+\tau)\,(1.33)$$

Improving estimate for NCSX



- Mode is spreading into the good curvature region – simple connection length estimate will not work
- Must be feeling a stabilizing effect that can nearly triple the predicted critical gradient.
- Related to negative shear?

Parallel Term

$$a/L_{T,crit} = 2.66[(a/R_{\text{eff}}) + 8.00(a/L_{\|})]$$

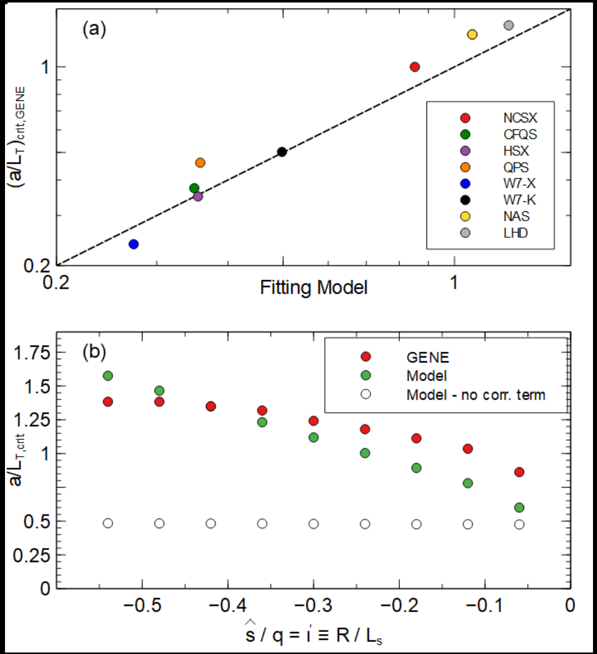
$$\frac{1}{L_{\|}} \sim \frac{1}{R_{\text{eff}}}\Theta(p_1)\Theta\left(\frac{R_{\text{eff}} - R_{\text{good}}}{R_{\text{eff}} + R_{\text{good}}}\right) \times 0.5(1 + \tanh[20\, p_2])$$

$p_1 = R_{\text{eff}}/L_w - 0.20$ $p_2 = R_{\text{eff}}/L_s - 0.15$

$\Theta(x) = xH(x)$ H a Heaviside step function

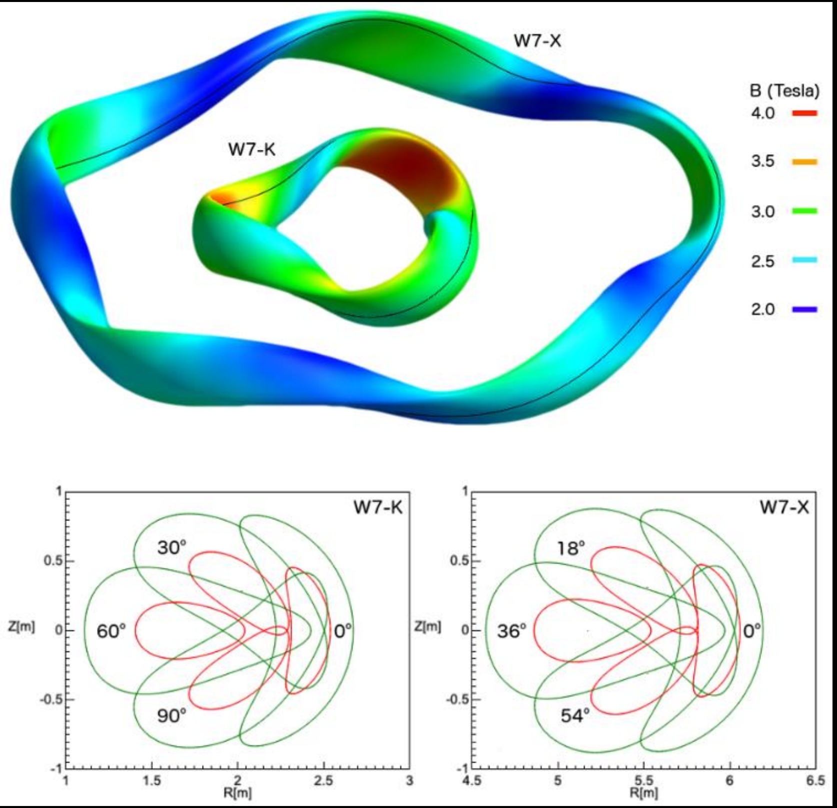
L_w the distance between good and bad curvature

- Add a parallel stabilization term (akin to 2nd term in Jenko formula) to all cases
- Has onset behavior related to neighboring good curvature wells “boxing” the mode in [Plunk et al 2014]
- Shear must cause secular growth of drift curv. along field line (2nd threshold)
- Get rough agreement with neg. shear circular tokamak
- Helps NCSX



W7-K

- Wendelstein 7 “Kompakt”. Started with high mirror W7-X configuration
- Major radius 2.0 meters (aspect ratio shrunk to 2/5.5 of original)
- Reduced field period number to 3
- Critical gradient doubled (as did drift curvature)
- Neoclassical losses reduced



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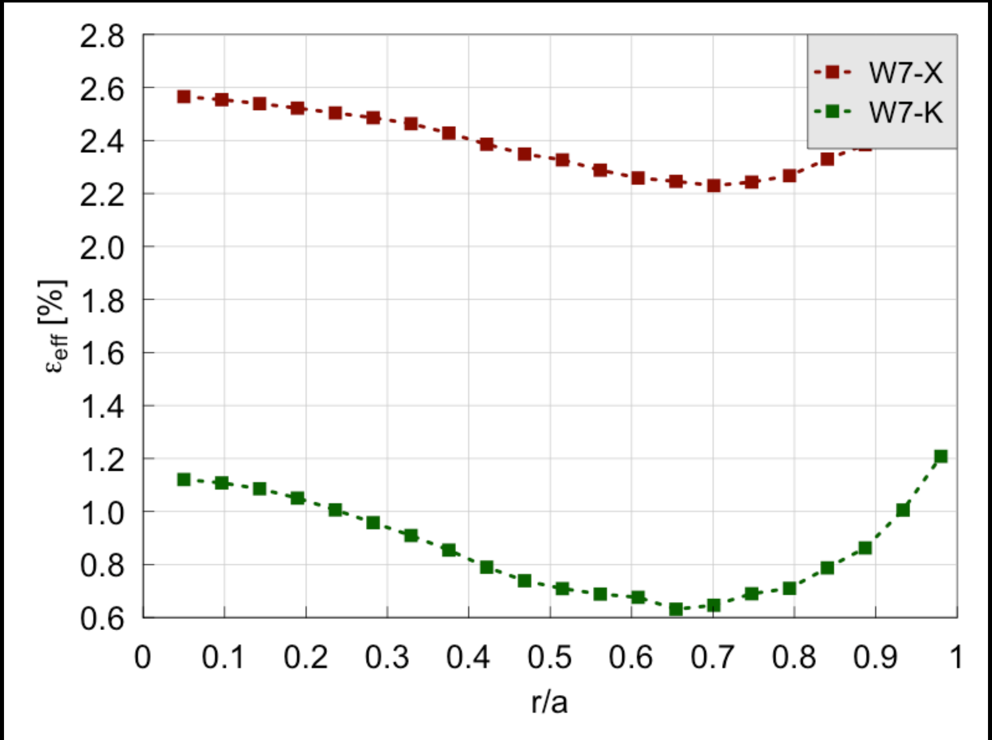

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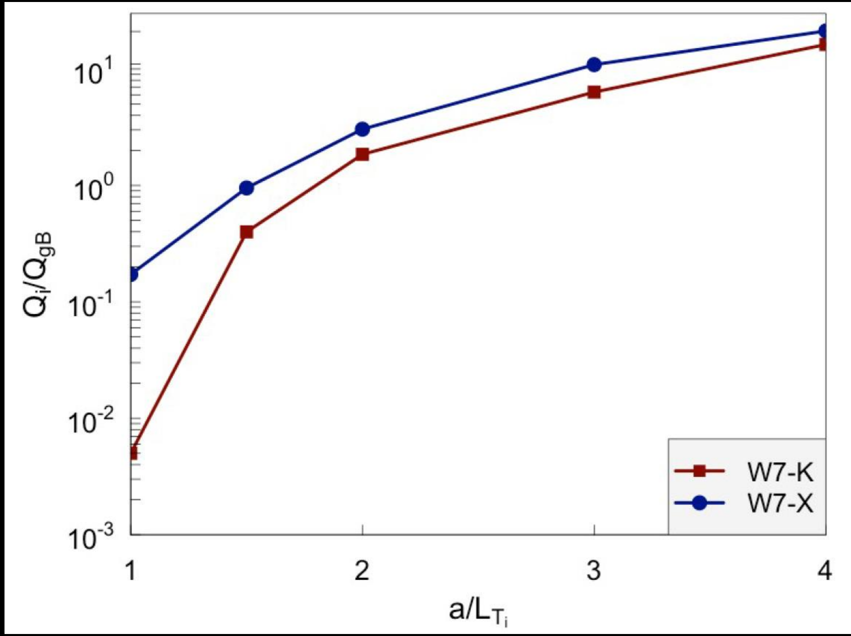


New designs from SIMSOPT

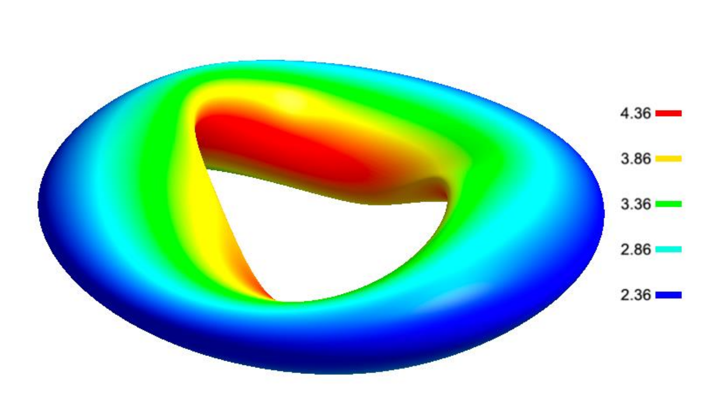
- Objectives: Landreman & Paul QS, aspect ratio, critical gradient.
- Example input files provided in SIMSOPT used to start.

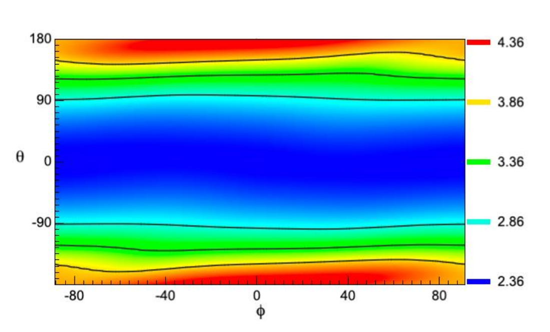
- QA shape, 2 field periods, aspect ratio 3. a/L_T,crit = 1.55 in GENE.

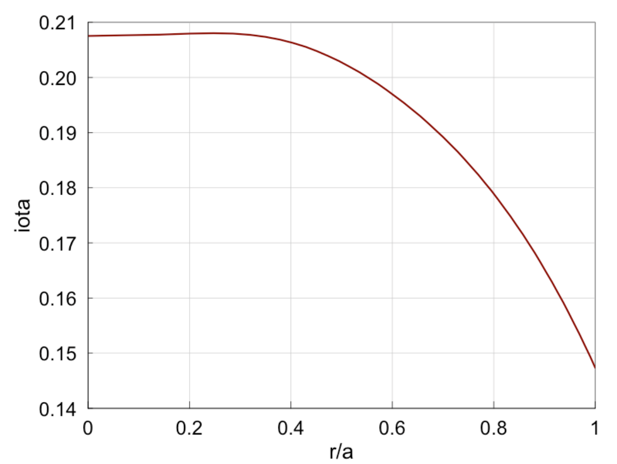
- QH shape, 4 field periods, aspect ratio 4. a/L_T,crit = 1.75 in GENE.

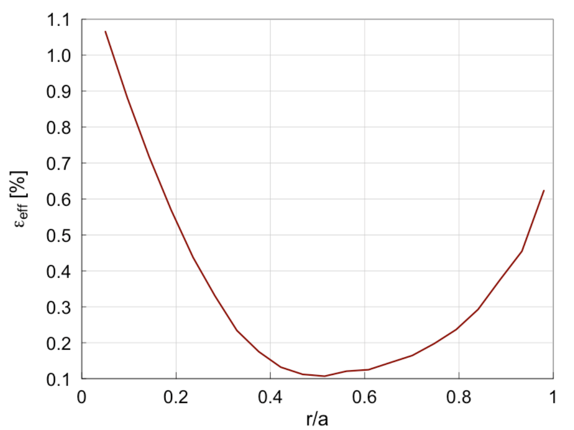
- QH shape, 4 field periods, aspect ratio 6. a/L_T,crit = 0.85 in GENE, has a magnetic well.

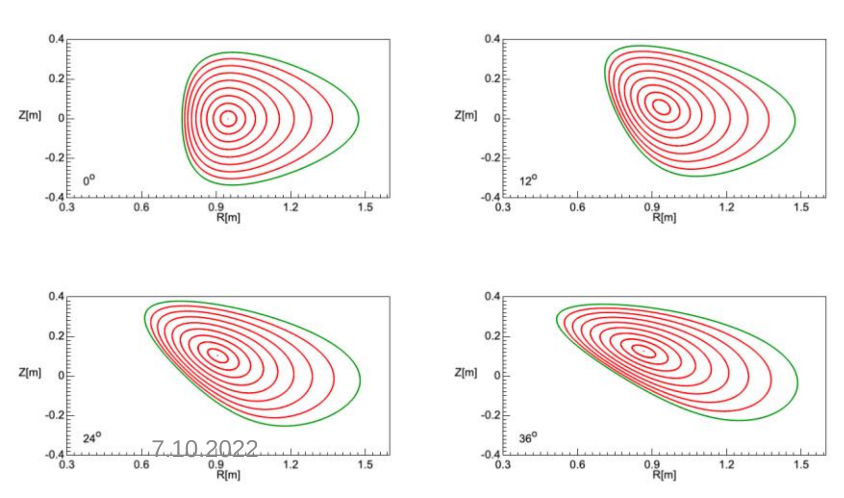
QA, a/L_T,crit = 1.55



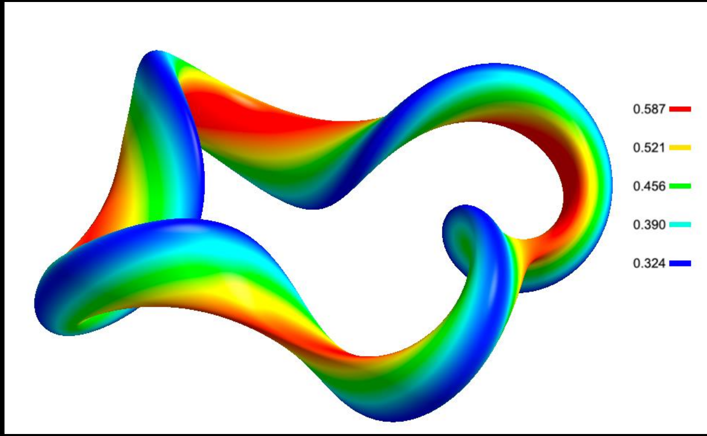


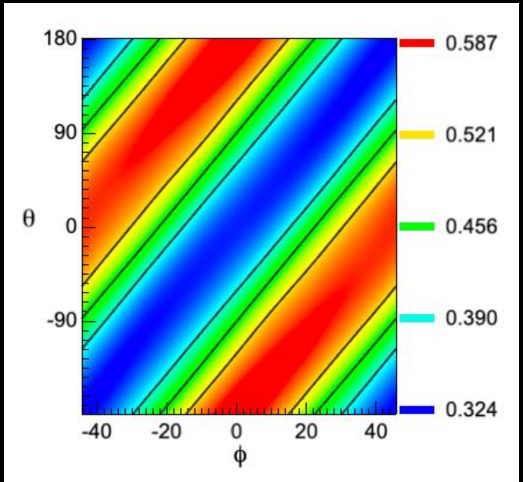


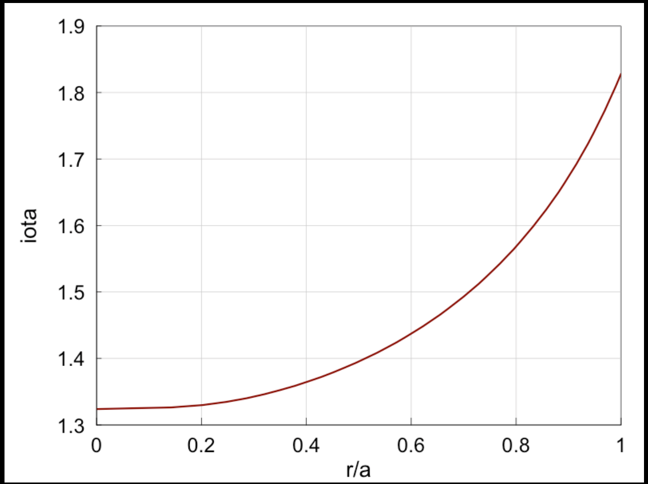


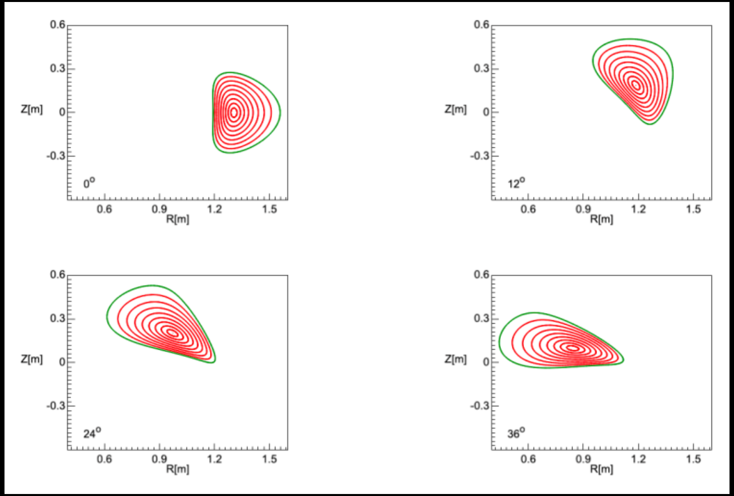


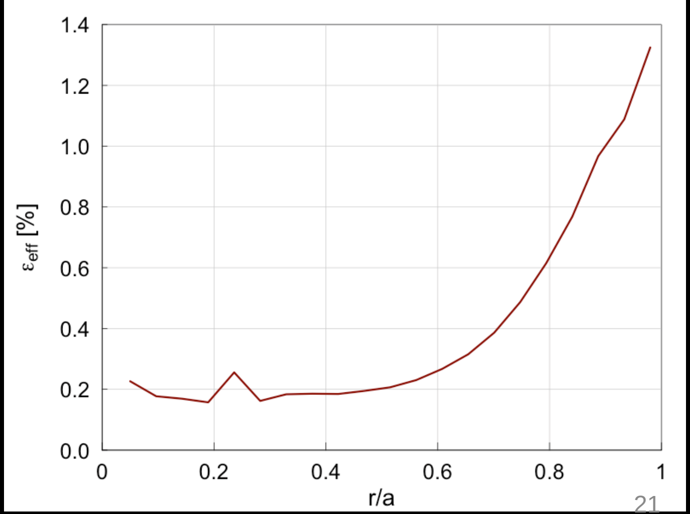
QH, a/L_T,crit = 1.75



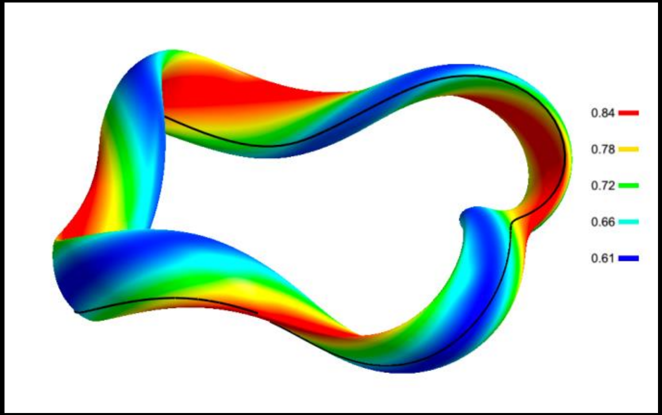


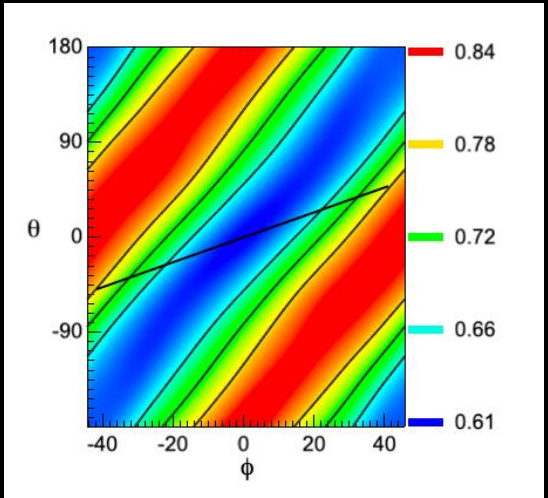


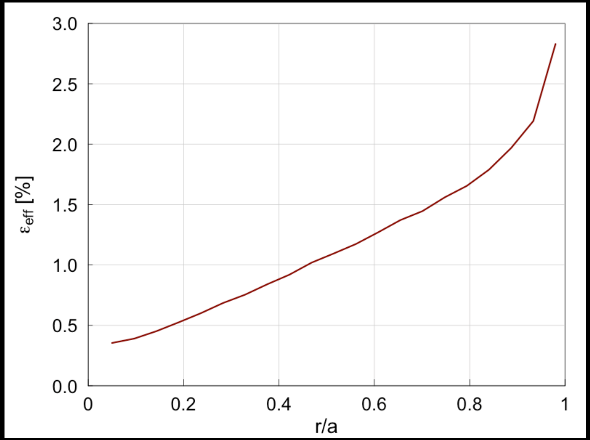


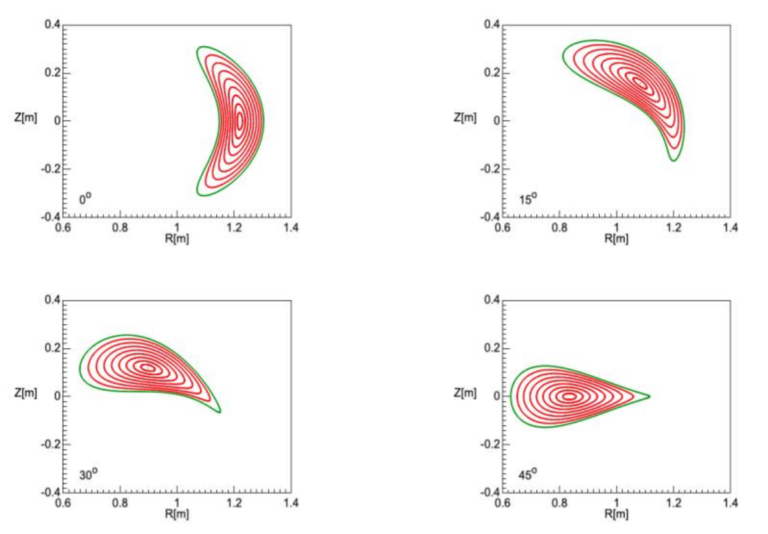


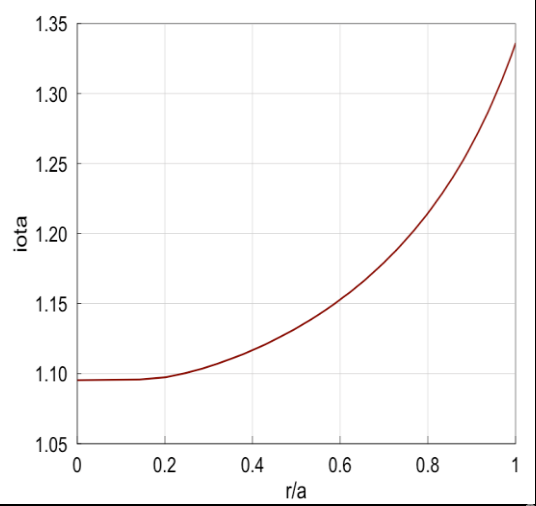
QH, magnetic well, a/L_T,crit = 0.85











Conclusions

- We can use coarse-grained flux tube profiles to quickly find the ITG linear onset of different stellarator geometries.

- Increasing the “drift curvature” along magnetic field lines can be implemented through, e.g., reducing aspect ratio (W7-X → W7-K), thus improving the critical gradient.

- Apparent tradeoff with MHD stability

- Looking at ITG behavior above threshold using heuristics – surface compression (Xanthopoulos et al. 2014), zonal flow efficiency (Plunk et al. 2017)