MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Turbulence optimisation

Per Helander

This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.

Importance of turbulence in W7-X

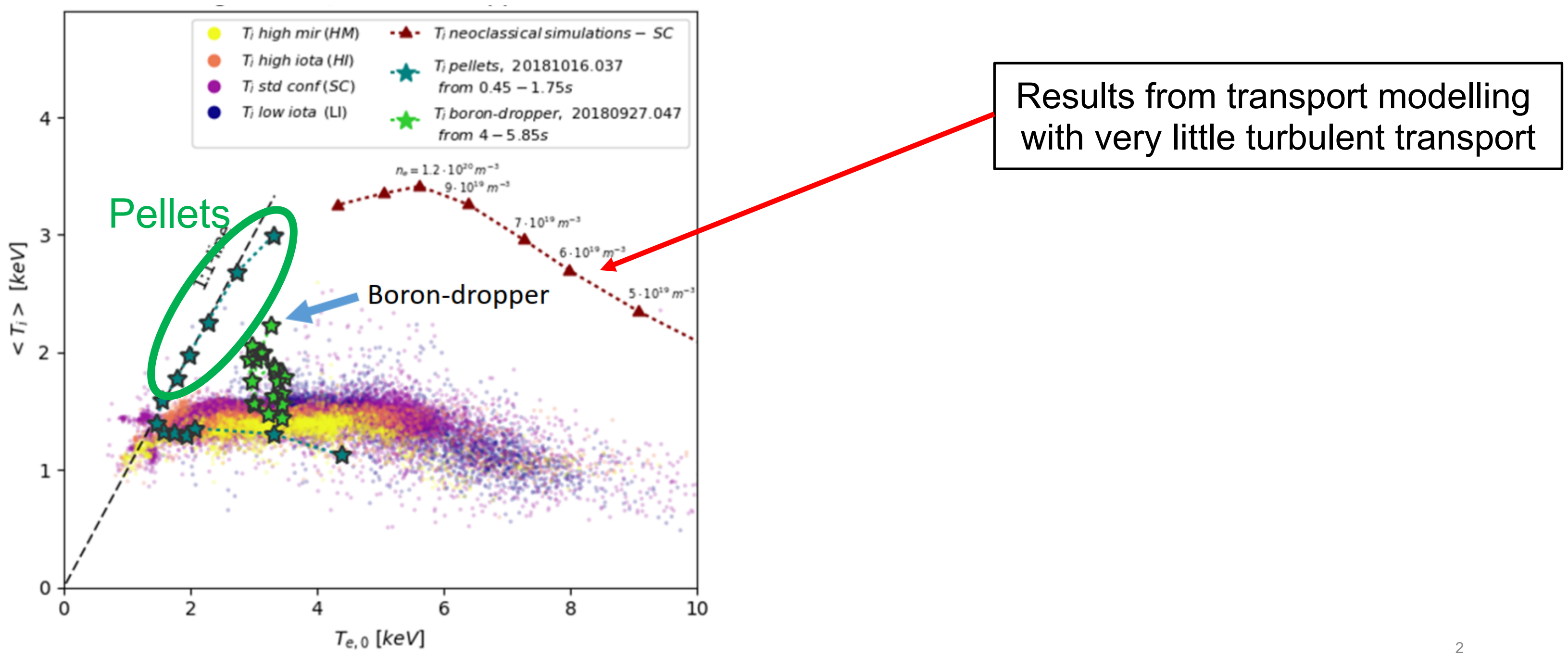
Turbulence causes most of the transport in W7-X.

- Transport modelling of plasmas with P = 5 MW predict, in the absence of turbulence

$$\langle\beta\rangle = 2 - 3\%$$

whereas in most plasmas $\langle\beta\rangle < 1\%$.

Best confinement is achieved in plasmas with pellet injection, which reduces ITG turbulence by steepening the density gradient.



Turbulence and stellarator optimisation

- Most of the transport in W7-X is due to turbulence.

- If we could make a stellarator less prone to turbulence, the energy confinement time could increase substantially.

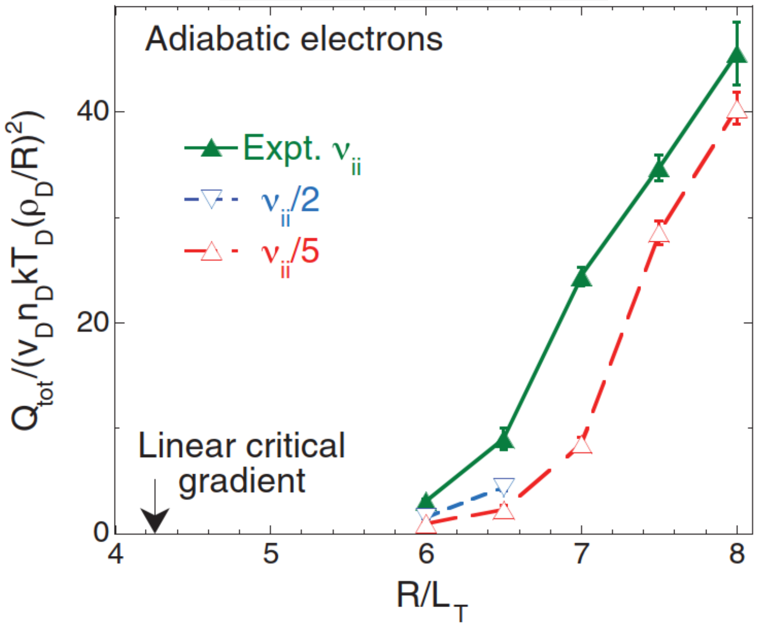
- Two obvious possibilities:
  - Increase linear stability threshold.
  - Reduce turbulence above threshold

- Gyrokinetic simulations are too slow to include inside a stellarator optimisation loop.
  - Instead, try to understand basic trends from fundamental considerations.
  - Implement simple criteria in stellarator optimisation codes.

Reduce turbulence?

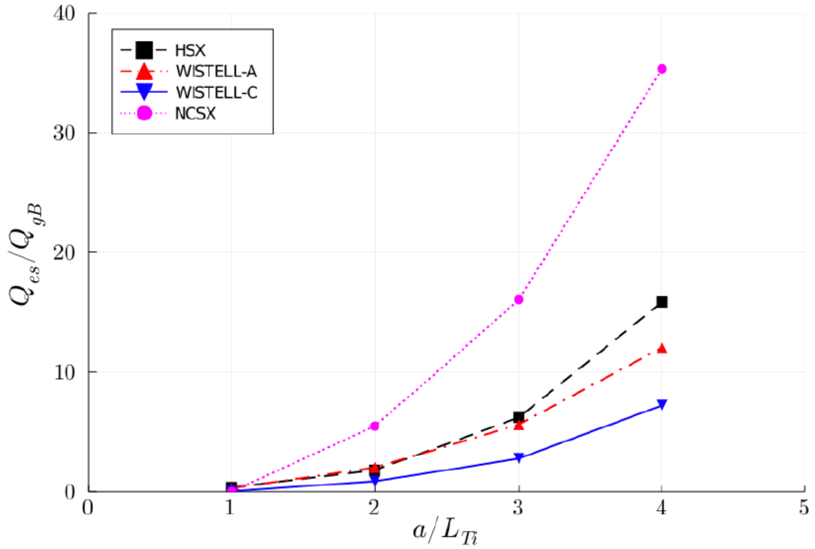
Onset of turbulent transport is usually softer in stellarators than in tokamaks.

Tokamak



Mikkelsen and Dorland, PRL (2008)

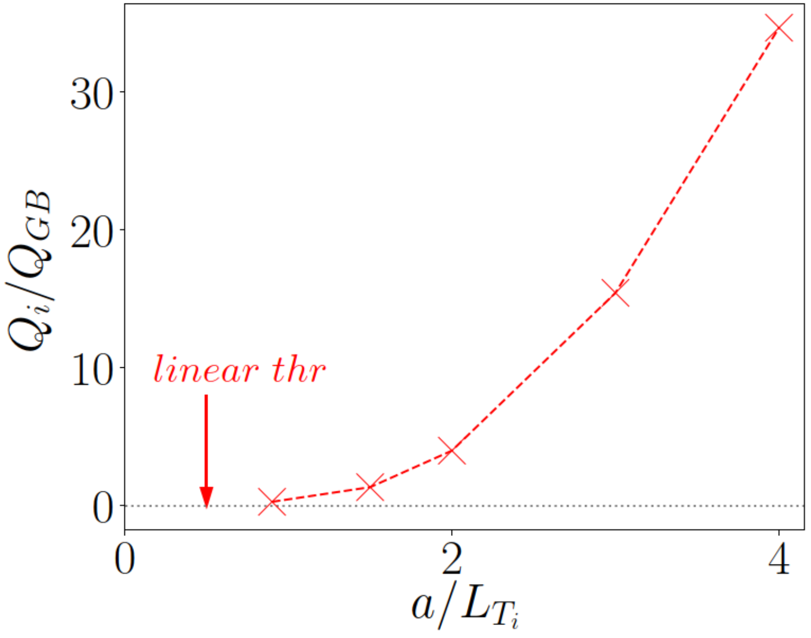
Stellarators

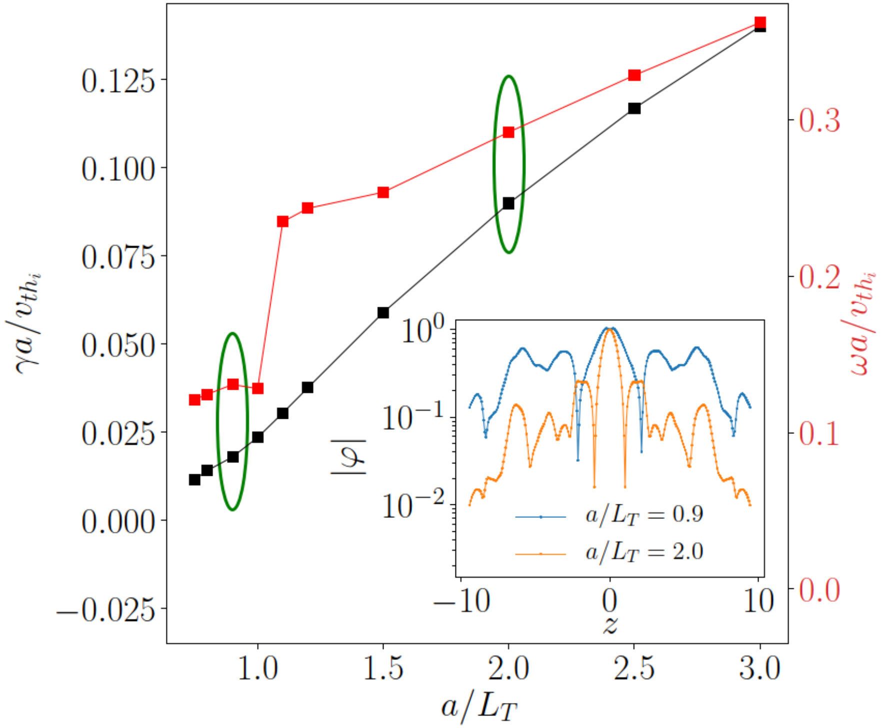


C. C. Hegna et al, Nucl. Fusion (2022)

Near-threshold behaviour of ITG instability & turbulence

- Close to the linear ITG stability threshold, eigenmodes are particularly extended along the field and narrow across it.
- Linear growth rates are weak.
- Turbulent fluxes are small.
    - "Soft" onset of turbulent transport





Zocco et al. PRE (2022).

A simple way to reduce ITG instability and turbulence

- General upper bounds on all gyrokinetic instabilities have recently been derived.

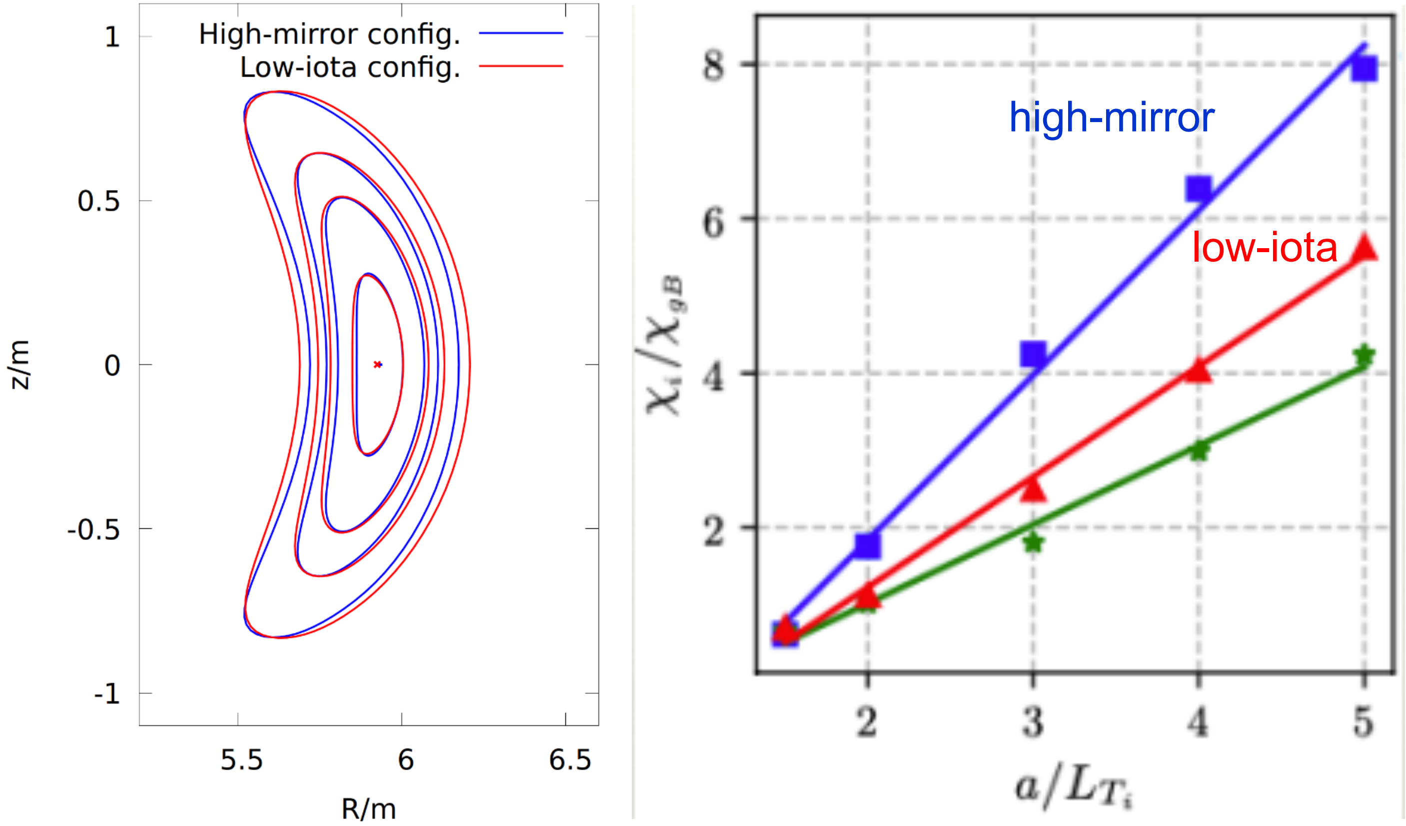
  Helander and Plunk, PRL 2021, JPP 2022;

  Plunk and Helander, JPP 2022

- These bounds are insensitive to magnetic-field geometry
  - …except for depence on

$$\eta_i \omega_{*i} \sim \frac{k_\perp |\nabla T_i|}{eB}$$

- Discriminates between configurations with different flux-surface compression.
  - Example: low-iota and high-mirror configurations in W7-X.
  - Also: difference in surface-to-volume ratio



Stroteich, Xanthopoulos, Plunk and Schneider JPP (2022)

Instability threshold

Basic frequencies:

- Diamagnetic frequency $\omega_*^T \sim k_\perp \rho_i v_{Ti}/L_T,$ $L_T = T/|\nabla T|$
- Drift frequency $\omega_d \sim k_\perp \rho_i v_{Ti}/R,$ $R = \text{curvature radius of field lines}$

For fluid-type instabilities $(k_{||} v_{Ti} \ll \omega)$, the growth rate is of order

$$\gamma \sim \sqrt{\omega_d(\omega_*^T - \omega_d)}$$

Drift frequency occurs twice:

- Destabilising if $\omega_*^T \omega_d > 0$ (“bad“ curvature, convex field lines)
- Sets stability limit

$$\omega_*^T \sim \omega_d \quad \Rightarrow \quad L_T \sim R$$

Coarse-grained model for predicting ITG stability threshold

Somehat ad hoc, but seems to works in practice.

$$\frac{1}{L_T^{\mathrm{crit}}} = \frac{2.66}{R_{\mathrm{eff}}} + \frac{1}{L_{\|}}$$

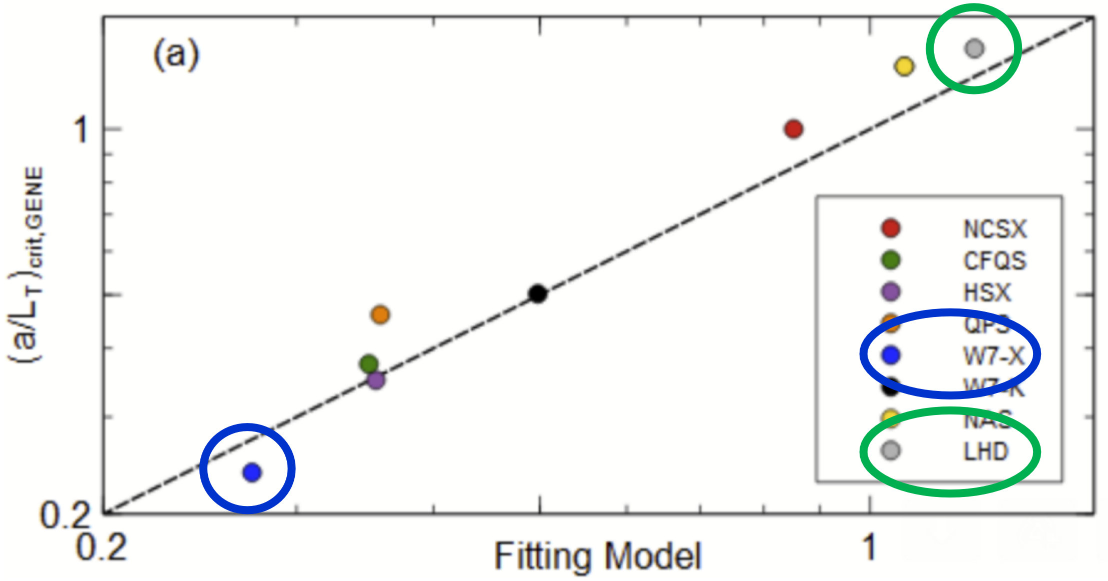
where

R_{eff} = appropriate average of curvature radius

$L_{\|}$ = appropriate average of distance along B between regions of good and bad curvature

Reproduces the instability threshold computed by gyrokinetic codes well in all stellarators considered. There is a wide range!

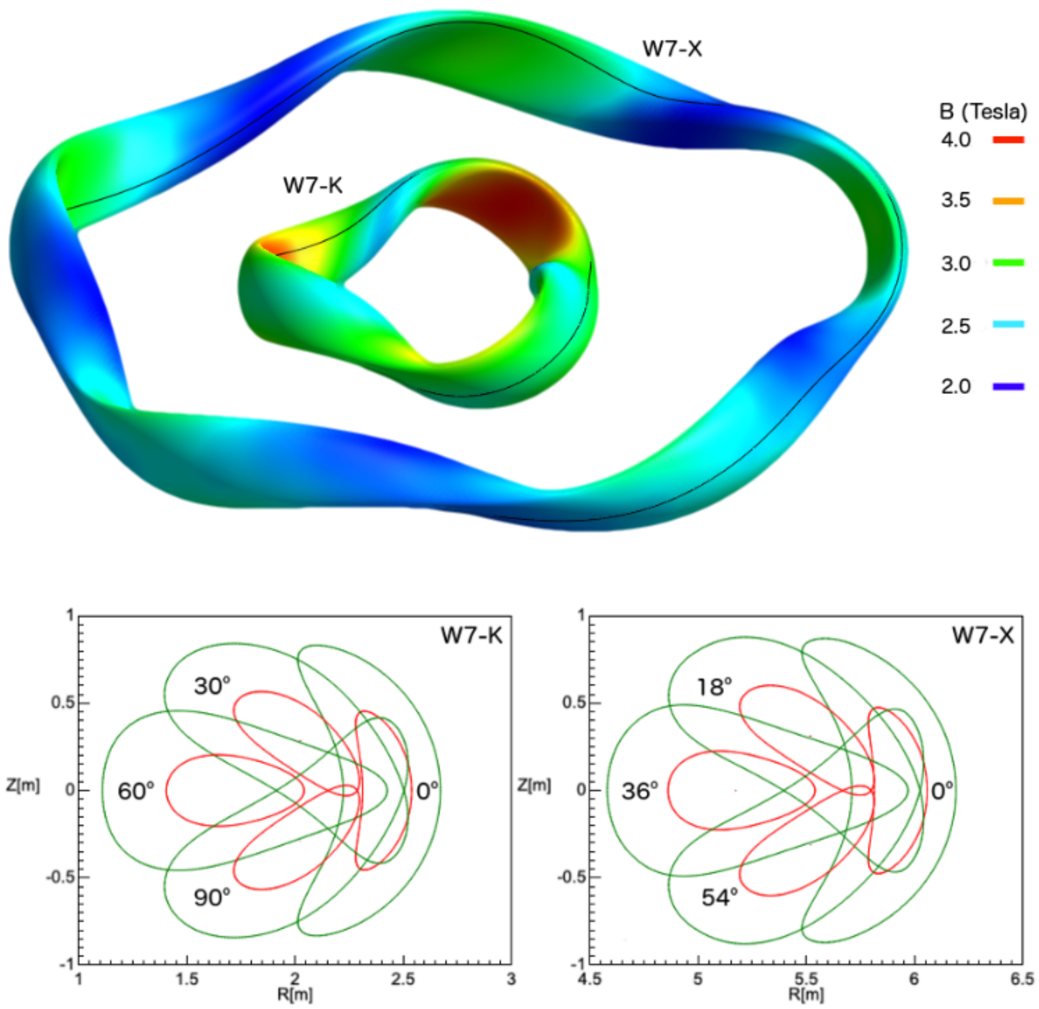
- Room for improving W7-X!
- Criterion implemented in SIMSOPT.



Roberg-Clark et al, PRR (2022)

Bad curvature can be good

- Large curvature raises the instability threshold.
- Suggests decreasing the aspect ratio.
- Tried reducing the number of field periods in W7-X from 5 to 3.
    - Doubles the curvature and the instability threshold.
    - Aspect ratio 3.6 instead of 10.
    - Neoclassical transport is still small.



Roberg-Clark et al, PRR (2022)

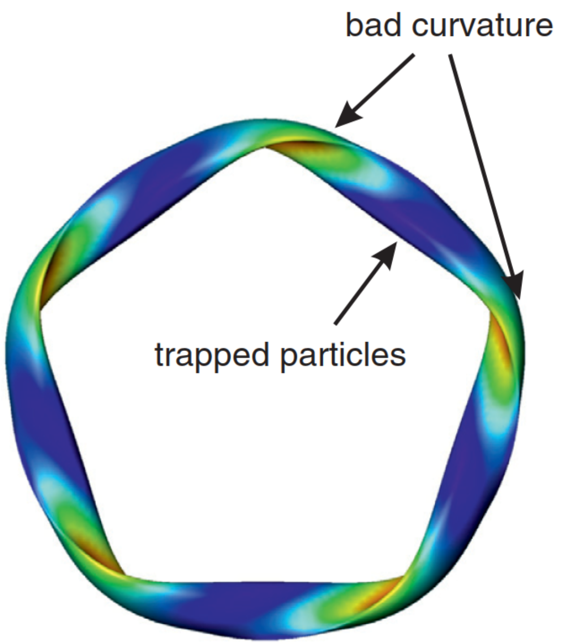
Trapped-electron modes

Trapped-electron modes

Caused by electron trapping in bad-curvature regions

Overlap minimised in W7-X

Can be understood in terms of energy extraction from the trapped electron population.



Ordinary density-gradient-driven TEMs are stable if

$$\overline{\omega}_{de}\omega_{*e} < 0$$

implying that trapped particles experience average good curvature.

This is rarely satisfied for all orbits.

- How do we assess a configurations were it is not?

Rosenbluth, PoF 1968
Proll et al, PRL 2012
Helander et al, PoP 2013

Available energy of trapped electrons

How much of the thermal electron energy can, in principle, be converted into instabilities and turbulence?

Constraints:

- Liouville's theorem (phase-space volume conservation) in the absence of collisions
- Conservation of magnetic moment and *J*.

Result expressible as an integral over trapped electron orbits, which is easily implemented numerically

$$\widehat{A} = \iint \mathrm{d}z\mathrm{d}\lambda \sum_{\mathrm{wells}(\lambda)} \exp(-z) z^{5/2} \times \left[\hat{\omega}_\alpha^2 \left(\frac{\hat{\omega}_*^T}{\hat{\omega}_\alpha} - 1 + \hat{F}\right) + \hat{\omega}_\psi^2 \left(-1 + \hat{F}\right)\right]$$

$$\hat{F} \equiv \frac{\sqrt{(\hat{\omega}_*^T - \hat{\omega}_\alpha)^2 + \hat{\omega}_\psi^2}}{\sqrt{\hat{\omega}_\alpha^2 + \hat{\omega}_\psi^2}}$$

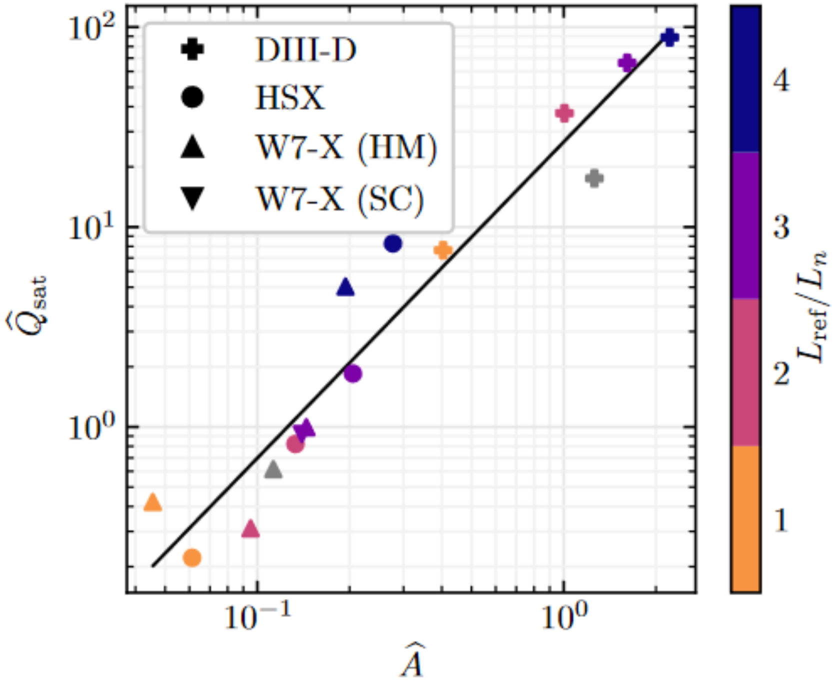
$$z = \frac{m_e v^2}{2T_e}, \qquad \lambda = \frac{v_\perp^2}{v^2 B}$$

$\hat{\omega}_\psi$ = average radial drift

$\hat{\omega}_\alpha$ = average poloidal drift

$\hat{\omega}_*^T$ = diamagnetic frequency

Available energy vs energy flux from simulations of density-gradient-driven turbulence



Mackenbach et al, PRL (2022)

Summary

Several approaches:

- **Basic theory**
  - Rigorours upper bounds on gyrokinetic instabilities
  - Understanding ITG instability and turbulence close to marginality
- **Raising the ITG instabilty threshold**
  - optimisation studies underway, show great promise
  - often in conflict with MHD stability
    - will likely need to choose between ITG and KBMs
- **Reduce TEMs**
  - Already accomplished in W7-X
  - Figure of merit developed and implemented, can now be used in optimisation studies