

Project TOKDGE:

A hybrid Poisson solver for full annular tokamak edge turbulence simulation

Haruki Seto (QST)

- Introduction: BOUT++ code and objectives of TOKEDGE project
- Numerical scheme of a hybrid Poisson solver for full annular tokamak edge simulations
- **[Erratum of CSC WS'21]** impact of flute-ordering assumption in low- n ideal ballooning mode
- Pedestal collapse simulation in full annular torus domain in shifted circular geometry
- Preliminary test of full annular torus turbulence simulation in single-null divertor geometry
- Summary

BOUT++ framework as an edge tokamak simulation code [Dudson CPC2009]

- BOUT++ calculates middle- n ($O(n) > 1$) and high- n ($O(n) \gg 1$) structure with high accuracy in complex boundary region in tokamak plasmas
- BOUT++ employs flute-ordering $k_{\parallel} = 0$ on Poisson solver for $n \neq 0$ modes calculating flow potential from vorticity
 - ✓ Flute-ordering may not be accurate for low- n modes ($O(n) \sim 1$) especially in diverted geometries

TOKEDGE is a two to three years project to extend BOUT++ framework for tokamak edge simulation solving interplay between $n=0$, low- n , middle- n and high- n modes in diverted geometries

➔ **improvement of current-driven ELMs, RMPs, full annular torus edge turbulence simulations, etc...**

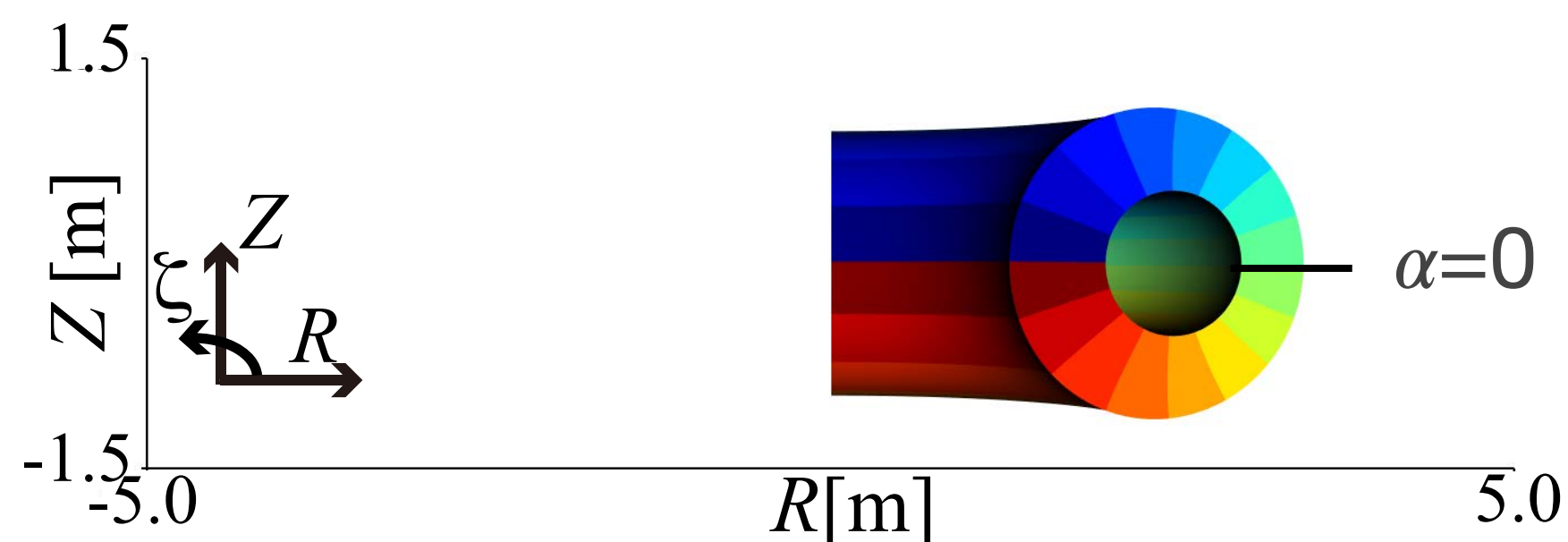
- FY2020: development of flute-ordering-free Poisson solver for low- n modes [CSC WS 2021]
- FY2021: production run of full annular tokamak edge simulation with circular cross section
- FY2021: preliminary test of full annular tokamak edge simulation with single-null divertor geometry

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BOUT++ uses dual coordinate system but has problem in $n=0$ and low- n

BOUT++ calculates middle- n and high- n mode with high accuracy and cheap cost using both **flux surface coords.** (ψ, θ, ζ) and **field-aligned coords.** (x, y, z) with shifted metrics for tokamaks

1/4th annular torus domain by (ψ, θ, ζ)



- Need large poloidal (θ) resolution for parallel differencing for large q and n

❖ Differencing in radial (ψ) direction

➔ 1D Poisson solver for n in radial direction **with flute-ordering** $k_{||}=0$ for calculating flow potential

coord. transform by
FFT: $z = \zeta - \alpha$

field-aligned coords:

$$x = \psi - \psi_0$$

$$y = \theta$$

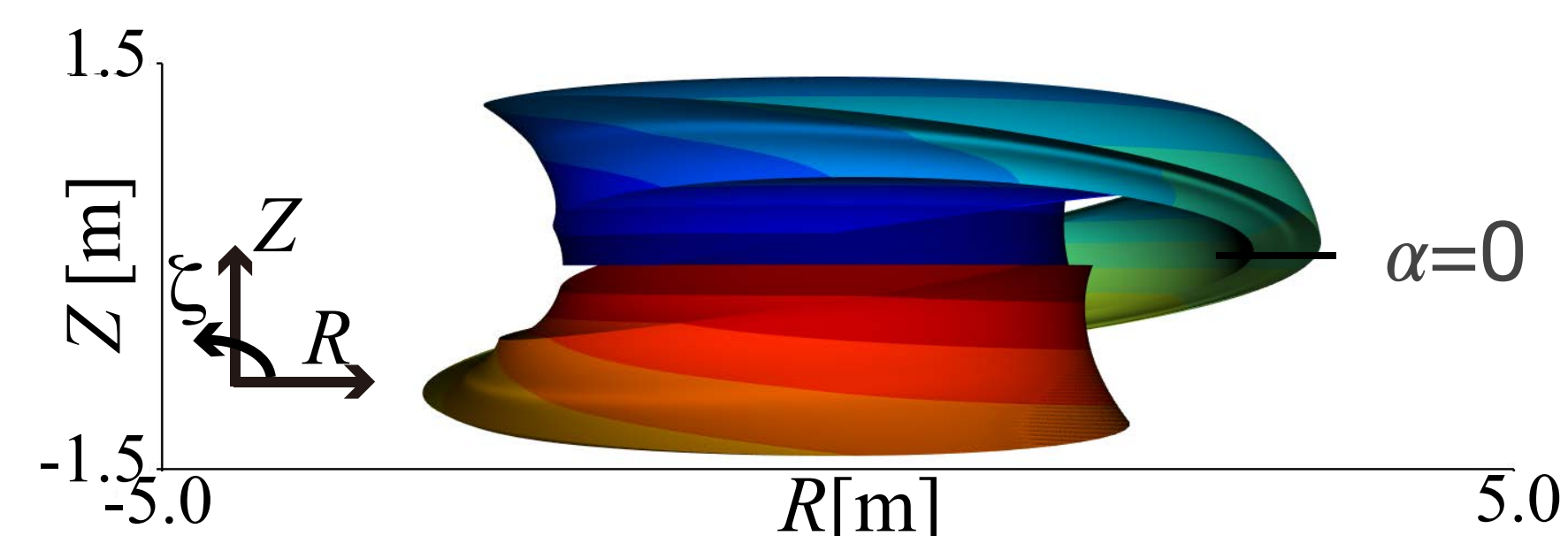
$$z = \zeta - \alpha$$

shift angle for aligning:

$$\alpha = \int_{\theta=\pi}^{\theta} \frac{\mathbf{B} \cdot \nabla \zeta}{\mathbf{B} \cdot \nabla \theta} d\theta$$

coord. transform by
FFT: $\zeta = z + \alpha$

1/4th annular torus defined by (x, y, z)



- Need small parallel (y) resolution for parallel differencing

- Cell deformation by magnetic shear degrades radial (x) differencing

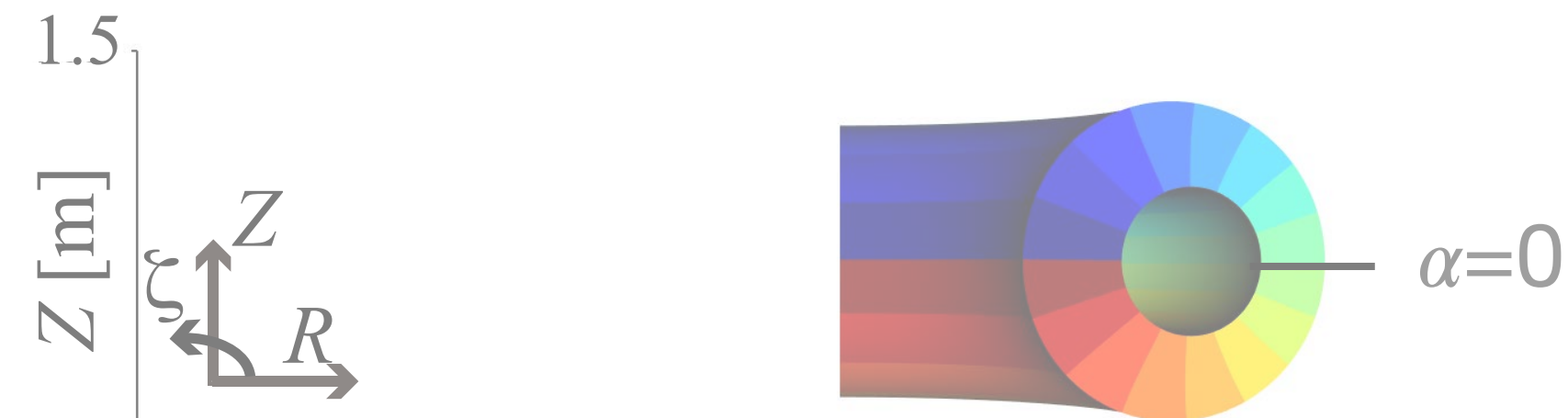
❖ Differencing in parallel (y) dir.

❖ Differencing in binormal (z) dir.

BOUT++ uses dual coordinate system but has problem in n=0 and low-n

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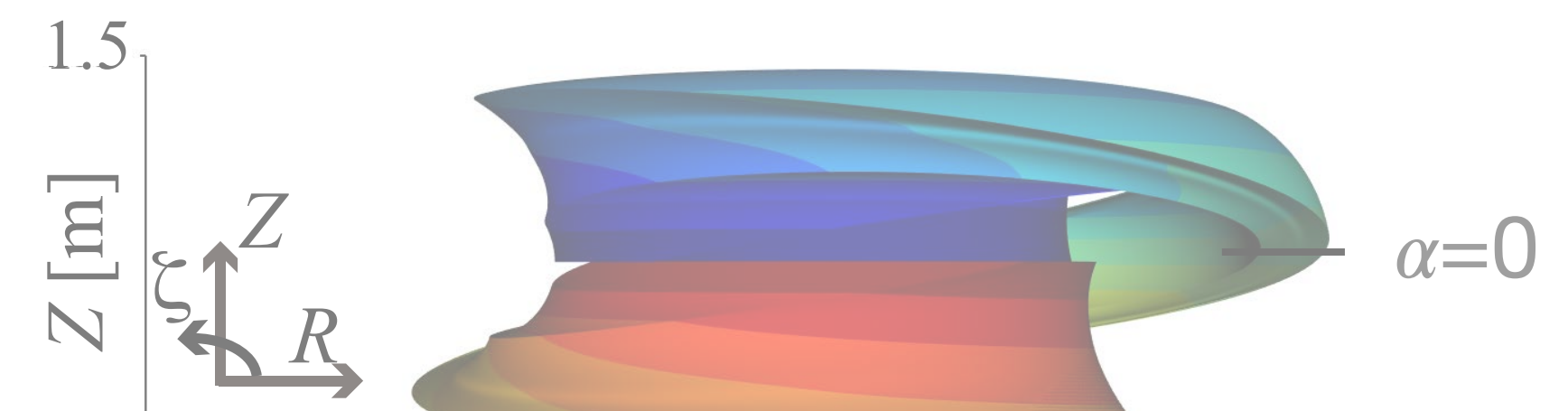
1/4th annular torus domain by (ψ, θ, ζ)



coord. transform by
FFT: $z = \zeta - \alpha$

field-aligned coords:

1/4th annular torus defined by (x, y, z)



Flute-ordering $k_{||}=0$ may not be valid for $n=0$ and low- n modes especially in diverted geometries

Cannot address to nonlinear tokamak edge simulation selfconsistently including interplay between flow ($n=0$), (low- n) MHD, and turbulence

❖ Differencing in radial (ψ) direction

➔ 1D Poisson solver for n in radial direction **with flute-ordering** $k_{||}=0$ for calculating flow potential

$$\alpha = \int_{\theta=\pi}^{\theta=0} \frac{\mathbf{B} \cdot \nabla \zeta}{\mathbf{B} \cdot \nabla \theta} d\theta$$

coord. transform by
FFT: $\zeta = z + \alpha$

degrades radial (x) differencing

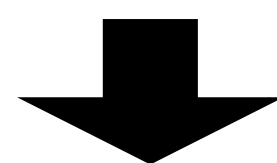
❖ Differencing in parallel (y) dir.

❖ Differencing in binormal (z) dir.

Linearized Poisson solver for $n=n'$ mode vorticity ($n_{i1}/n_{i0} \ll O(1)$) in Fourier space

$$U_1(\cdot, \cdot, n') = \nabla \cdot \left(\frac{n_{i0}}{B_0} \nabla_{\perp} \phi_1 \right) = \mathcal{L}_{\text{shifted}}(\partial_{\psi}, \partial_{\psi}^2, \partial_y, \partial_y^2, n) \phi_1(\cdot, \cdot, n') \longrightarrow \phi_1(\cdot, \cdot, n') = \mathcal{L}_{\text{shifted}}^{-1}(\partial_{\psi}, \partial_{\psi}^2, \partial_y, \partial_y^2, n') U_1(\cdot, \cdot, n') \quad ?$$

Poisson solver however **cannot be defined as a boundary problem straightforwardly**



- **1D Poisson solver in flux-surface coordinates** for $n \neq 0$ modes using flute-ordering approximation ($\partial_y=0$) [Dudson CPC2009]

$$\phi_1(\psi, \theta, n') = \mathcal{L}_{\text{shifted}}^{-1}(\partial_{\psi}, \partial_{\psi}^2, n') U_1(\psi, \theta, n')$$

- **2D Poisson solver in field-aligned coordinates** for $n=0$ mode using toroidal symmetry ($\partial_x = \partial_{\psi} + I \partial_{\zeta} = \partial_{\psi}$) [Dudson PPCF2017]

$$\phi_1(x, y) = \mathcal{L}_{\text{shifted}}^{-1}(\partial_x, \partial_x^2, \partial_y, \partial_y^2) U_1(x, y)$$

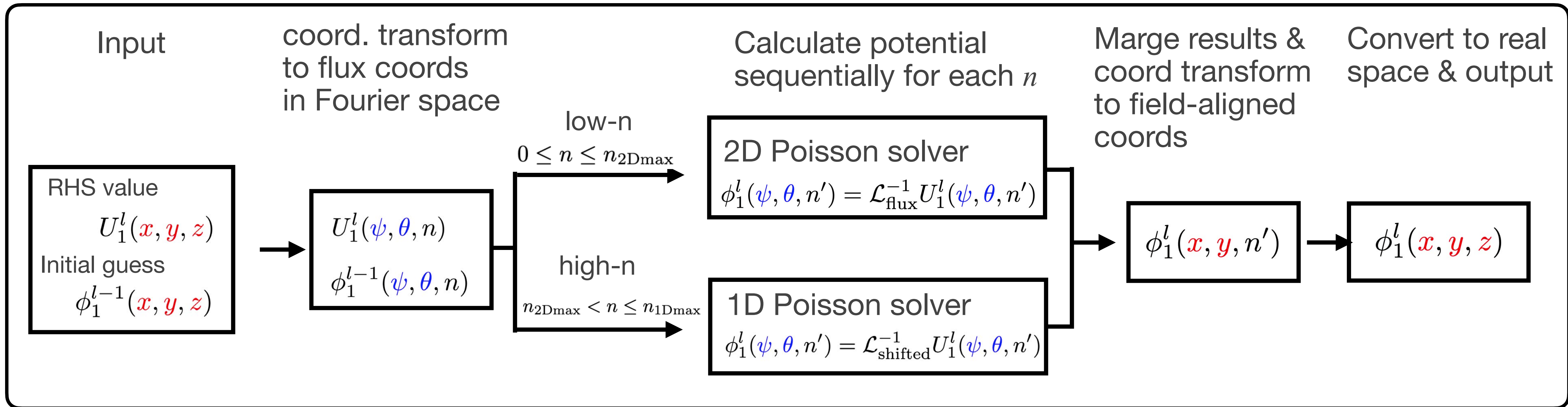
- **2D Poisson solver in flux-surface coordinates** for low-n modes with flux-surface coordinates' metrics [TOKEDGE 2020-cycle]

$$\phi_1(\psi, \theta, n') = \mathcal{L}_{\text{flux surface}}^{-1}(\partial_{\psi}, \partial_{\psi}^2, \partial_{\theta}, \partial_{\theta}^2, n') U_1(\psi, \theta, n')$$

- Poloidal grid resolution must be fine enough to describe poloidal structure of low-n modes
- Iterative solver using GMRES for KSP solver and AMG for preconditioning via PETSc and Hypre

Flute-ordering-free 2D Poisson solver for low- n modes is developed

A hybrid linearized Poisson solver function consisting of flute-ordered 1D Poisson and flute-ordering-free 2D Poisson solver for full annular tokamak edge turbulence simulation



- The highest toroidal mode number solved by 2D Poisson solver n_{2Dmax} and that by 1D Poisson solver n_{1Dmax} are **free parameters to be chosen carefully**
- **A hybrid Laplacian operator consistent with the hybrid Poisson solver** is also important for numerically stable simulation

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Linearized IBM model

$$\frac{\partial U_1}{\partial t} = -B_0 \partial_{\parallel} \left(\frac{J_{\parallel 1}}{B_0} \right) + B_0 \left[A_{\parallel 1}, \frac{J_{\parallel 0}}{B_0} \right] + \mathcal{K}(P_1),$$

$$\frac{\partial A_1}{\partial t} = -\partial_{\parallel} \phi_1,$$

$$\frac{\partial P_1}{\partial t} = -[\phi_1, P_0]$$

$$U_1 = \nabla \cdot \left(\frac{n_{i0}}{B_0^2} \nabla_{\perp} \phi_1 \right),$$

$$J_1 = \nabla_{\perp}^2 A_{\parallel 1},$$

$$[f, g] = \frac{\mathbf{b}_0 \times \nabla_{\perp} f \cdot \nabla_{\perp} g}{B_0}, \quad \mathcal{K}(f) = \frac{\mathbf{b}_0 \times \boldsymbol{\kappa}_0 \cdot \nabla f}{B_0},$$

- constant ion number density $n_{i0} = 10^{19} \text{ [m}^{-3}\text{]}$
- normalized with poloidal Alfvén unit
- original BOUT++ employs **flute-ordering** in

❖ Poisson solver for electrostatic potential

$$(\nabla_{\perp}^2 + B_0^2 \nabla B_0^{-2} \cdot \nabla_{\perp}) \phi_{1,n} = B_0^2 U_{1,n}$$

❖ Laplacian operator for parallel current
(for consistency with Poisson solver)

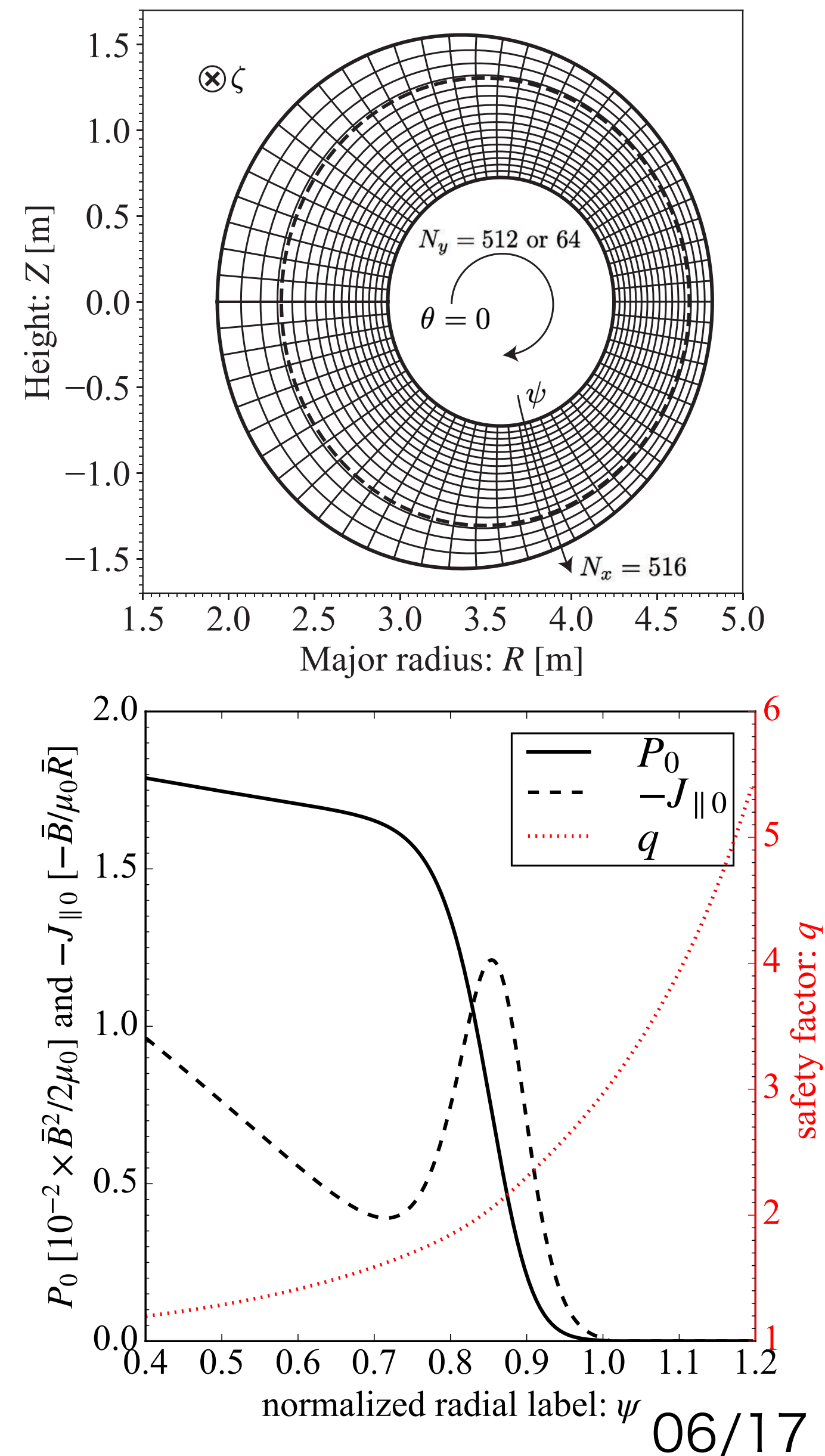
$$J_1 = \nabla_{\perp}^2 A_{\parallel 1}$$

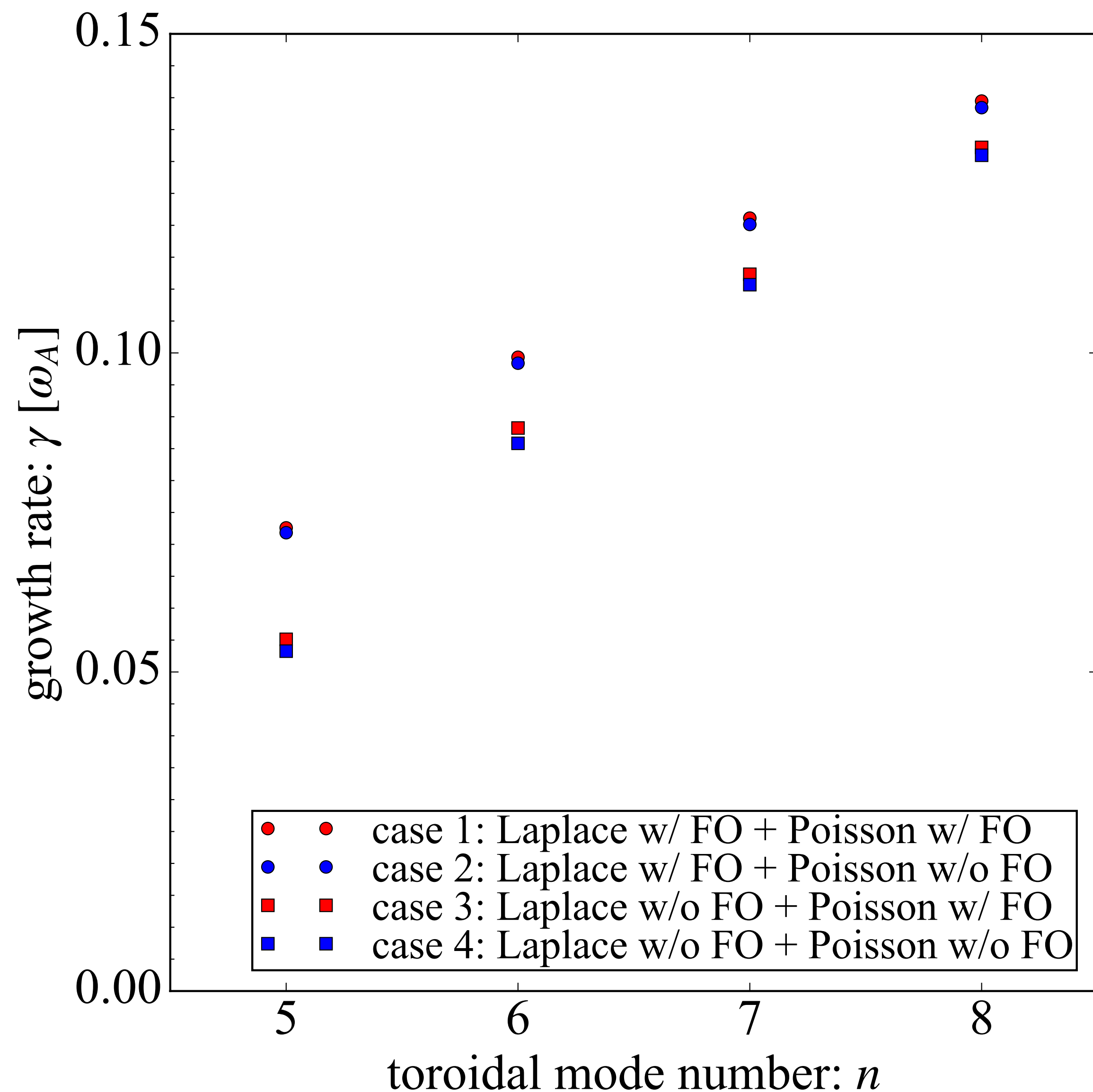
cbm18_dens8: a well-benchmarked equilibrium with circular cross section strongly unstable for ideal ballooning mode [Dudson CPC2009 PPCF2011]

- $1/n$ -th annular wedge domain in z for n mode with following resolutions

Grid Resolution	Nx	Ny	Nz
Case 1: Laplace w/ FO + Poisson w/ FO (same flute-ordering rule used in original BOUT++)	512	64	32
Case 2: Laplace w/ FO + Poisson w/o FO	512	512	32
Case 3: Laplace w/o FO + Poisson w/FO (reported in CSC workshop 2021 as case 1 by mistake)	512	64*	16
Case 4: Laplace w/ FO + Poisson w/ FO	512	512	32

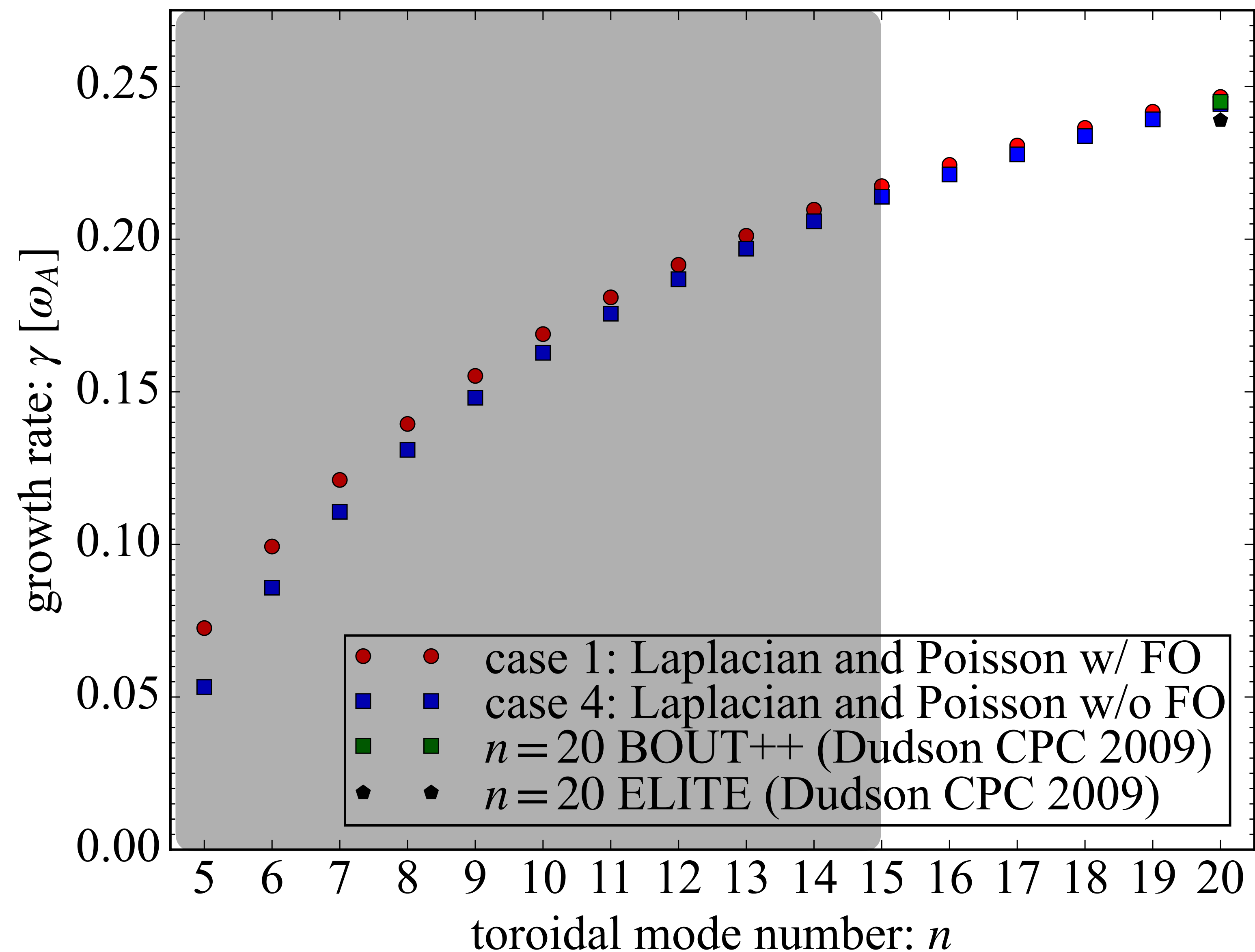
- In CSC WS2021, case 3 was reported as case 1 by mistake and it was concluded that flute-ordering has little impact in this equilibrium





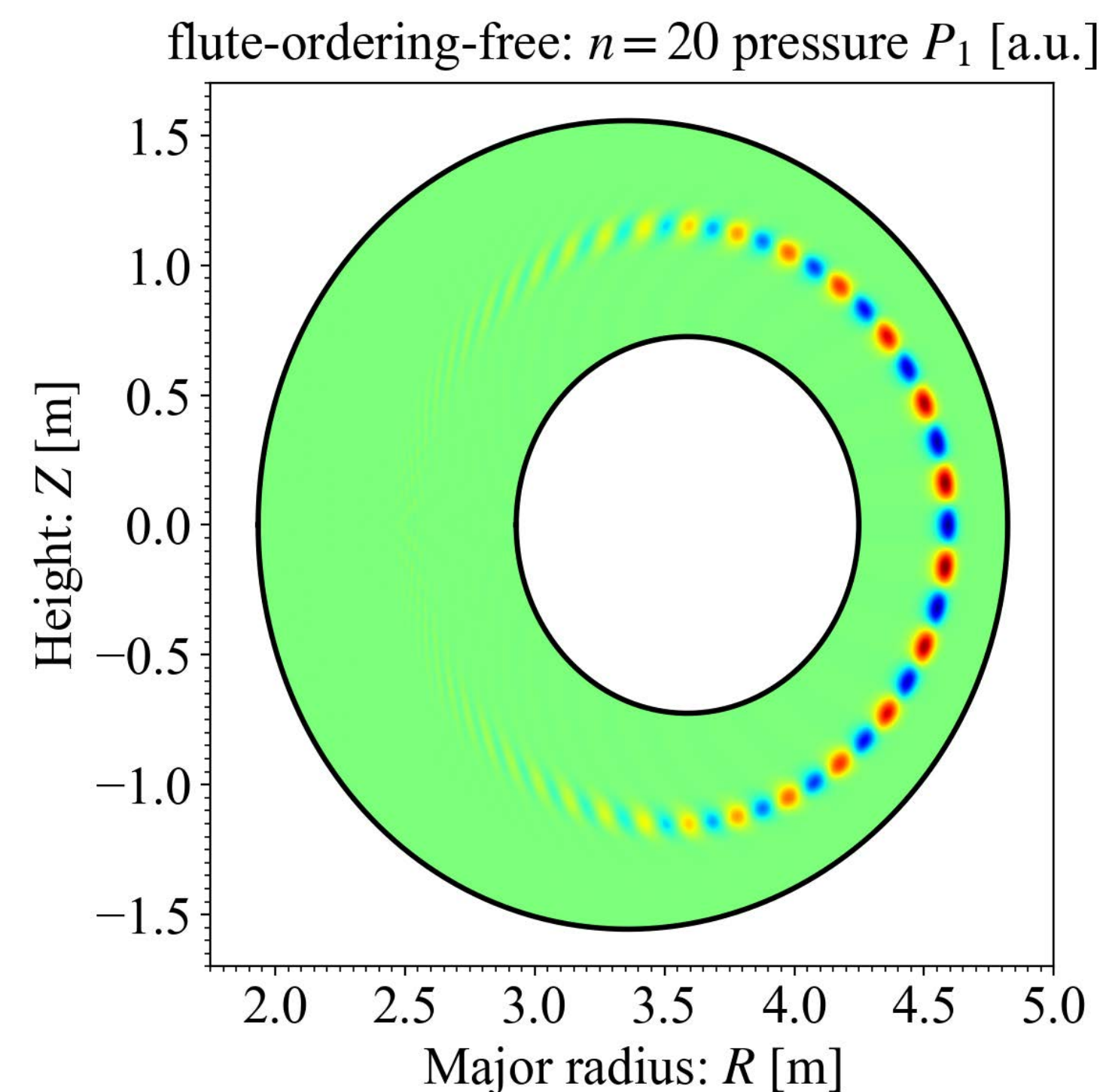
- Flute-ordering in Laplacian for $J_{//}$ has large impact in low- n regime (difference between circles and squares)
- Flute-ordering in Poisson solver for ϕ has little impact in even low- n regime (difference between red symbols and blue ones)

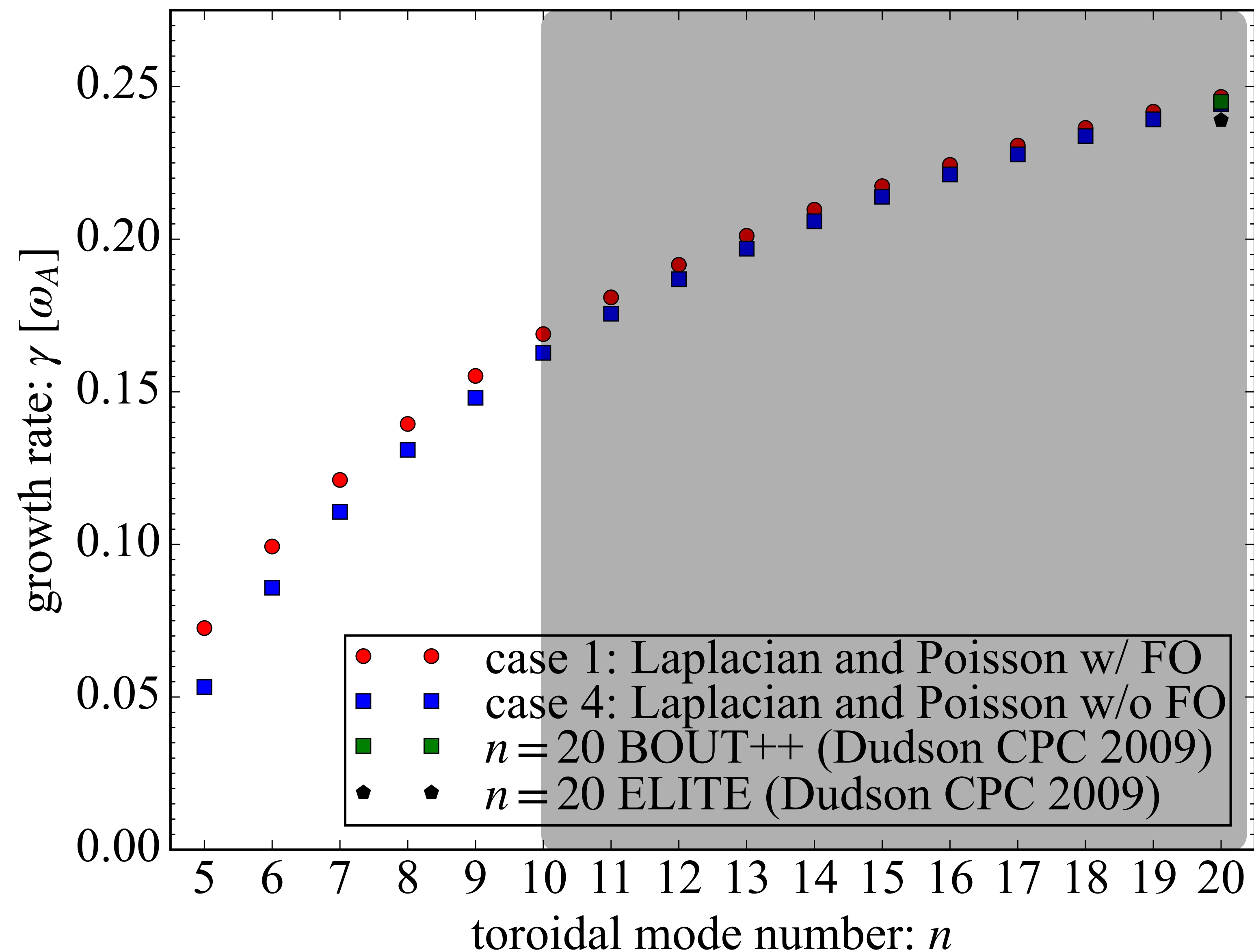
Asymmetric usage of flute-ordering (case 2 and 3) can give disruptive numerical instabilities



- Flute-ordering seems good for high- n regimes ($\sim 1\%$ difference at $n=20$)

➔ FO-free Laplacian and Poisson solver (case 4) can capture IBM eigenfunction for high- n ($n=20$)





- Flute-ordering seems good for high- n regimes ($\sim 1\%$ difference at $n=20$)
 - ➔ FO-free Laplacian and Poisson solver (case 4) can capture IBM eigenfunction for high- n ($n=20$)
- Flute-ordering gives large difference for low- n regimes ($\sim 36\%$ at $n=5$)
 - ➔ BOUT++ overestimates growth rate in low- n regime [Dudson PPCF'11] and FO-free scheme can suppress this overestimation to some level

Test of FO-free Poisson solver and Laplacian against low- n linear RBM in a single-null diverted geometry was also finished [reported in CSC WS'21]

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• Scale separated four-field RBM+DW model [Seto+ CPP'20]

$$\frac{\partial}{\partial t} \varpi_1 = - [F_1, \varpi] - [F_0, \varpi_1] + \mathcal{C}(p_1, F) + \mathcal{C}(p_0, F_1) - B_0 \partial_{\parallel} \left(\frac{J_{\parallel 1}}{B_0} \right) + B_0 \left[A_{\parallel 1}, \frac{J_{\parallel}}{B_0} \right] + \mathcal{K}(p_1) + \mu_{\parallel} \partial_{\parallel}^2 \varpi_1 + \mu_{\perp} \nabla_{\perp}^2 \varpi_1,$$

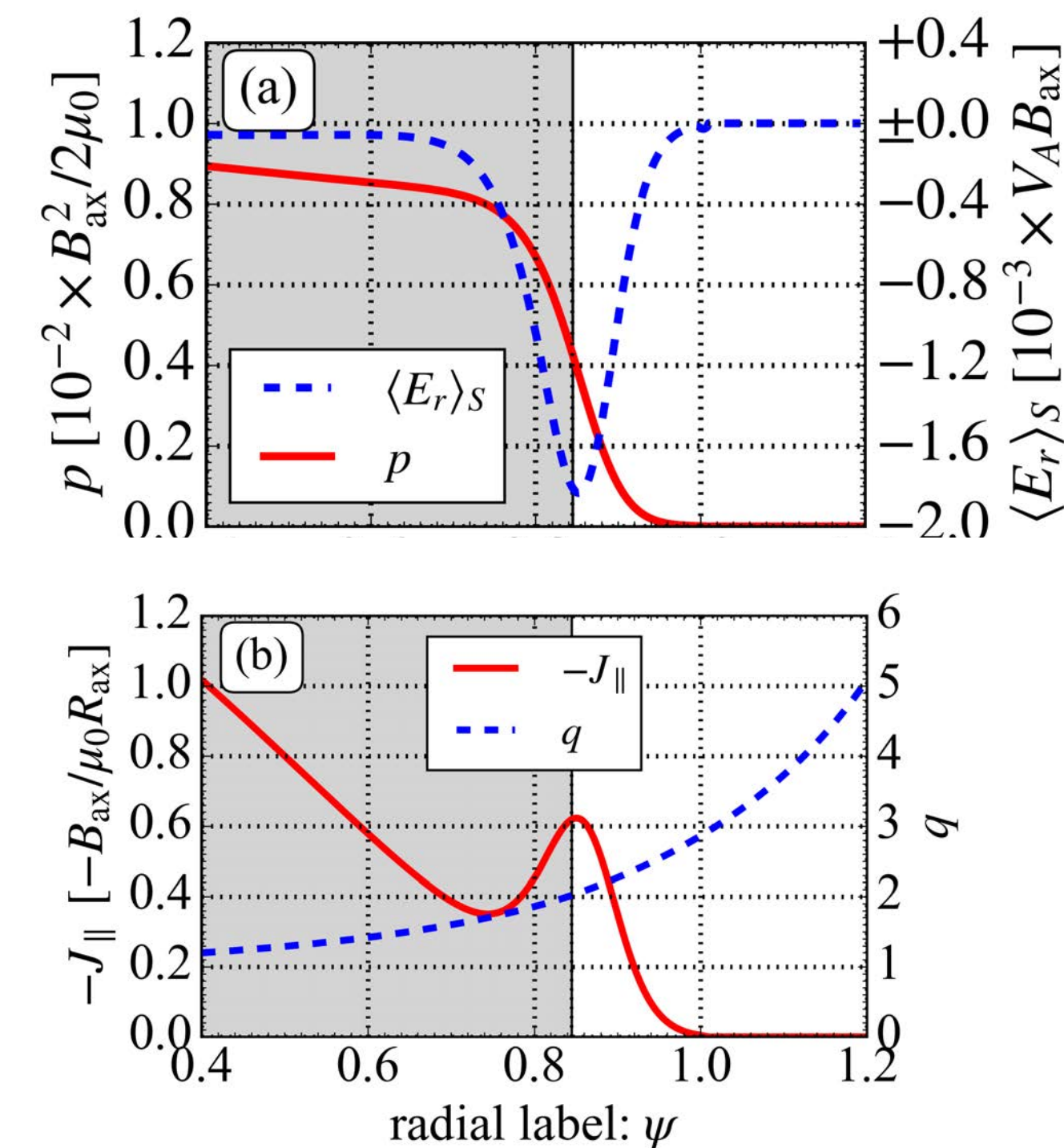
$$\frac{\partial}{\partial t} p_1 = - [\phi_1, p] - [\phi_0, p_1] - 2\beta_* \left(\mathcal{K}(p_1) - B_0 \partial_{\parallel} \left(\frac{v_{\parallel 1} + d_i J_{\parallel 1}}{2B_0} \right) + B_0 \left[A_{\parallel 1}, \frac{v_{\parallel 1} + d_i J_{\parallel 1}}{2B_0} \right] \right) + \chi_{\parallel} \partial_{\parallel}^2 p_1 + \chi_{\perp} \nabla_{\perp}^2 p_1,$$

$$\frac{\partial}{\partial t} A_{\parallel 1} = - [\phi, A_{\parallel 1}] - \partial_{\parallel} \phi_1 + \delta_e (\partial_{\parallel} p_1 - [A_{\parallel 1}, p]) + \eta J_{\parallel 1} - \lambda \nabla_{\perp}^2 J_{\parallel 1},$$

$$\frac{\partial}{\partial t} v_{\parallel 1} = - [\phi, v_{\parallel 1}] - \frac{1}{2} (\partial_{\parallel} p_1 - [A_{\parallel 1}, p]) + \nu_{\perp} \nabla_{\perp}^2 v_{\parallel 1},$$

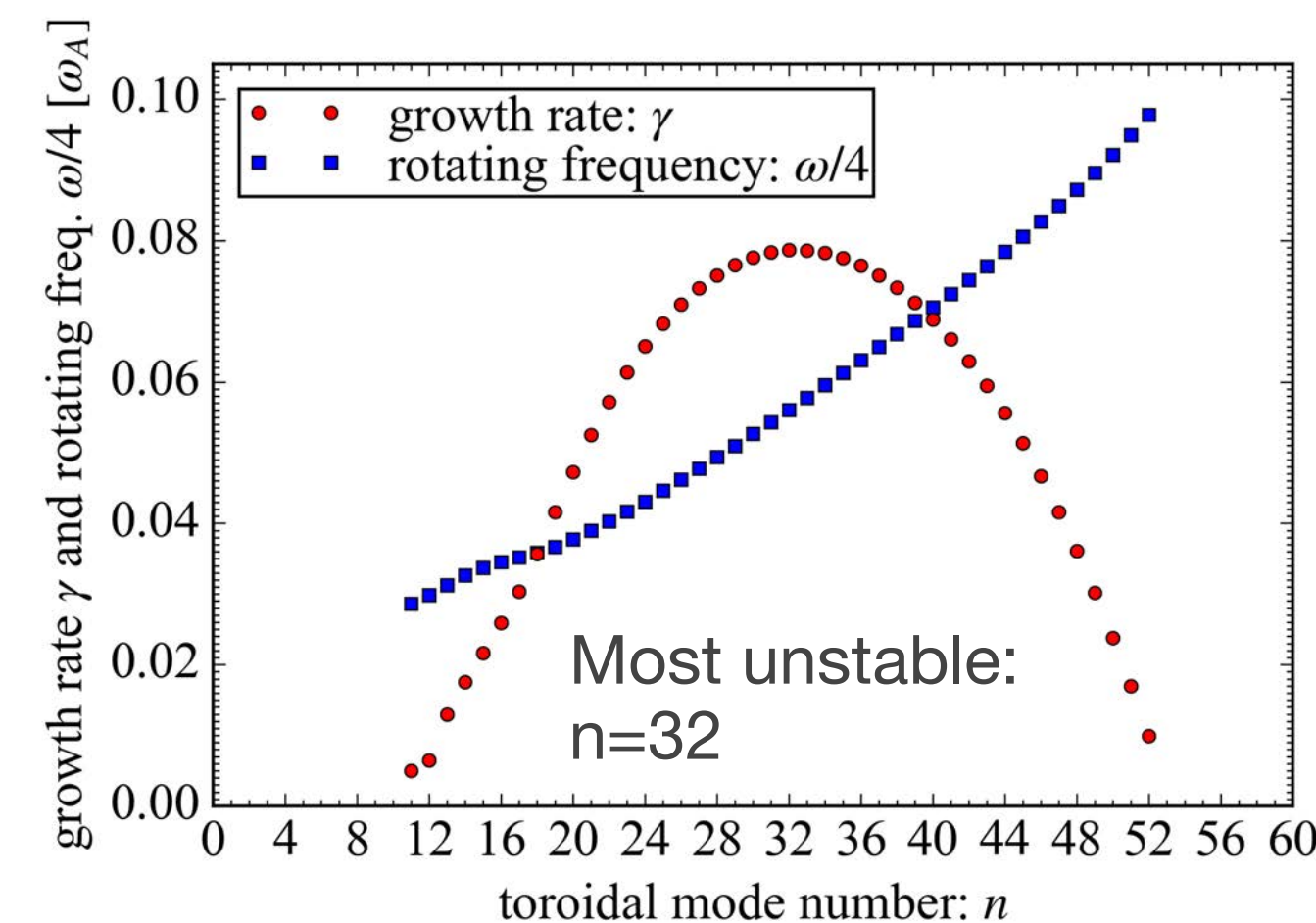
$$\varpi = \nabla_{\perp}^{*2} F, \quad J_{\perp 1} = \nabla_{\perp}^2 A_{\parallel}, \quad F = \phi + \delta_i p, \quad \phi = \phi_0 + \phi_1, \quad p = p_0 + p_1, \quad \mathbf{B} = \mathbf{B}_0 + \nabla A_{\parallel 1} \times \mathbf{b}_0, \quad J_{\parallel} = J_{\parallel 0} + J_{\parallel 1},$$

$$n_{i0} = 10^{19} [m^{-3}], \quad \eta = 10^{-8}, \quad \lambda = 10^{-12}, \quad \mu_{\perp} = \chi_{\perp} = \nu_{\perp} = 10^{-7}, \quad \mu_{\parallel} = \chi_{\parallel} = 10^{-1}$$

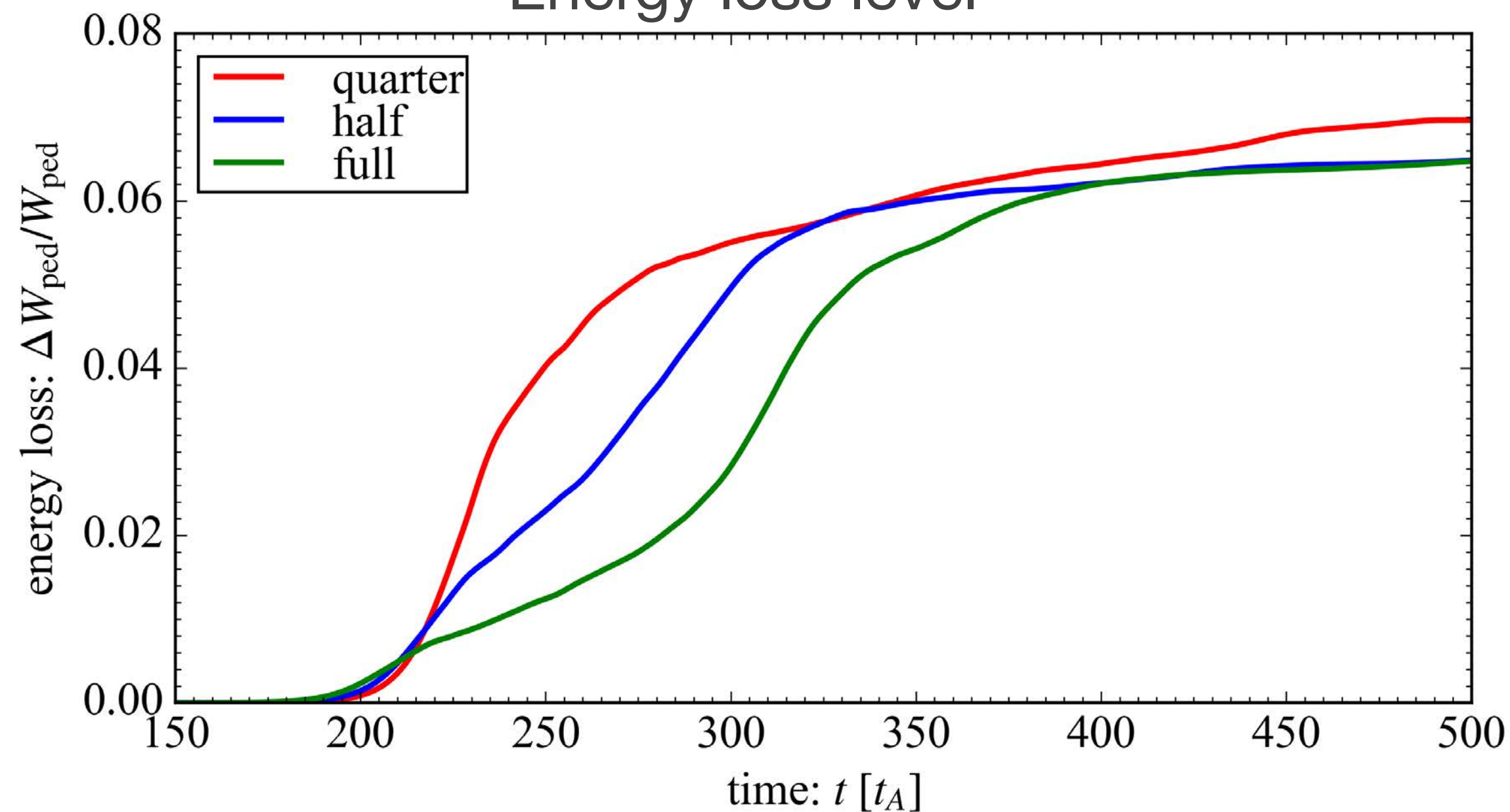


• Grid resolution and toroidal mode numbers to be solved

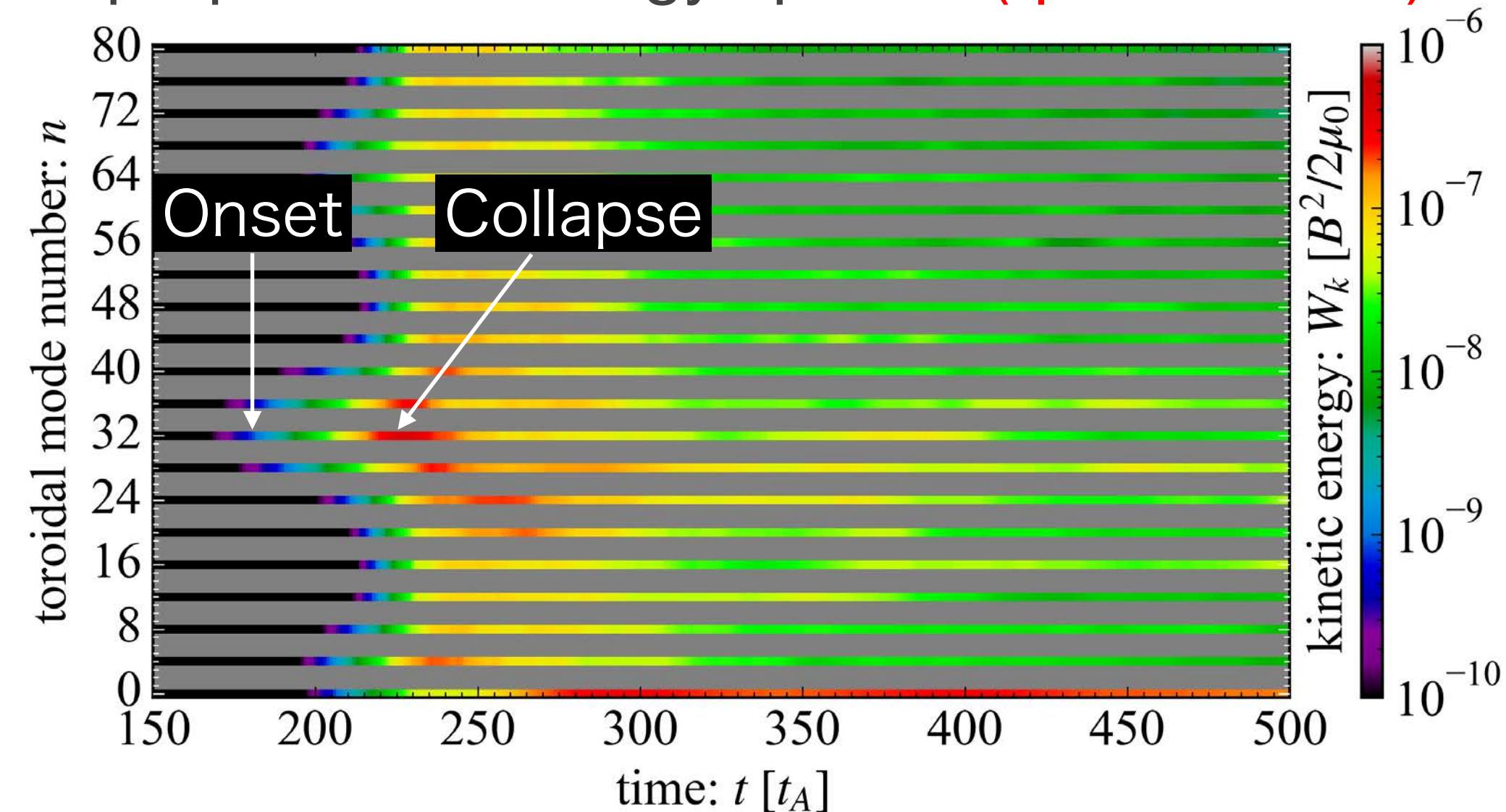
binormal domain length	Nx	Ny	Nz	tor. mode # to be solved	2D Poisson +2D Laplace	1D Poisson +1D Laplace
full torus ($0 \leq z < 2\pi/1$)	1028	128	256	$n=0, 1, \dots, 80$	$n=0, 1, 2, 3, 4$	$n=5, 6, \dots, 80$
half torus ($0 \leq z < 2\pi/2$)	1028	128	128	$n=0, 2, \dots, 80$	$n=0, 2, 4$	$n=6, 8, \dots, 80$
quarter torus ($0 \leq z < 2\pi/4$)	1028	128	64	$n=0, 4, \dots, 80$	$n=0, 4$	$n=8, 12, \dots, 80$



Energy loss level

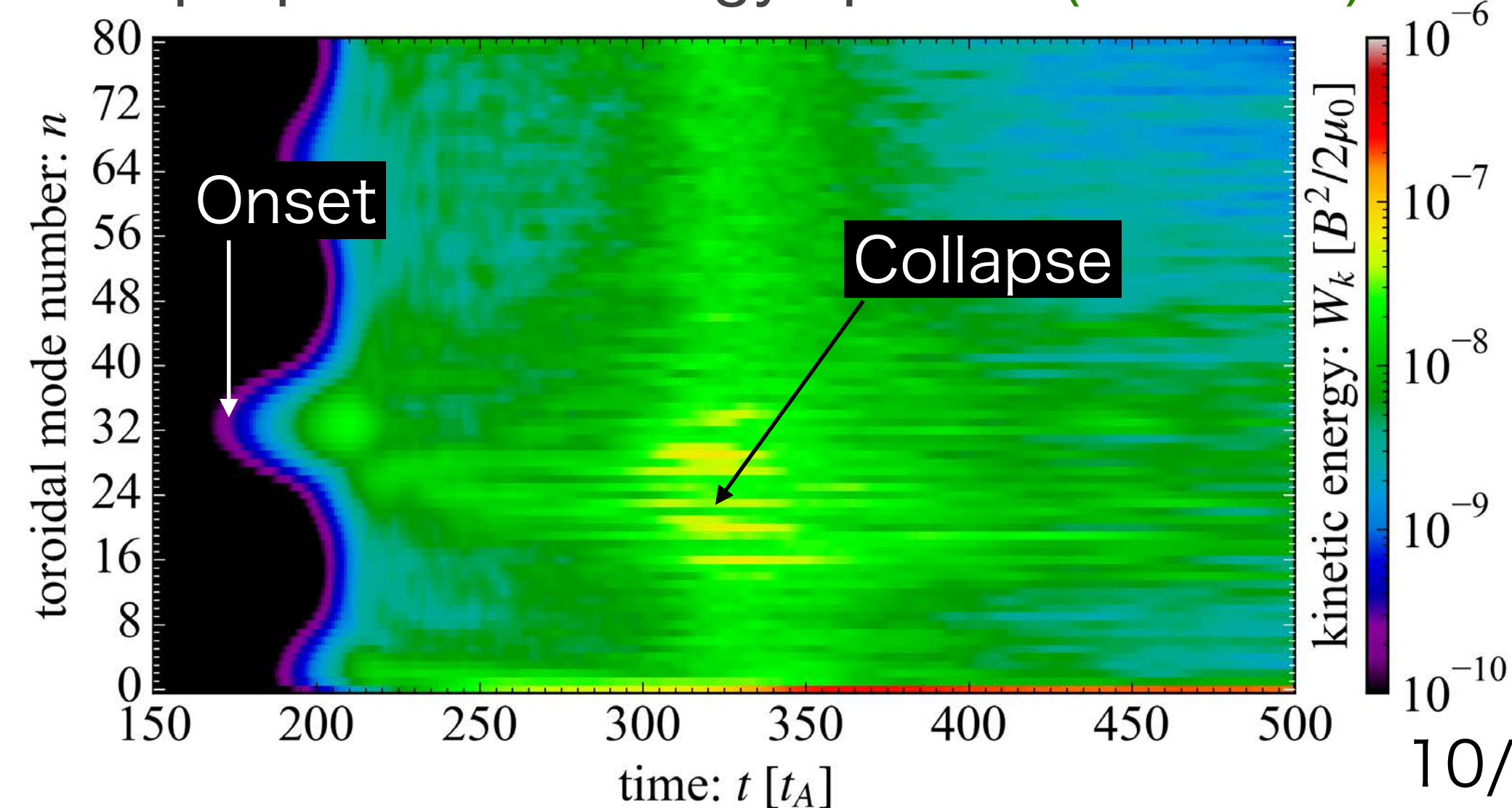


perp. kinetic energy spectra (quarter torus)

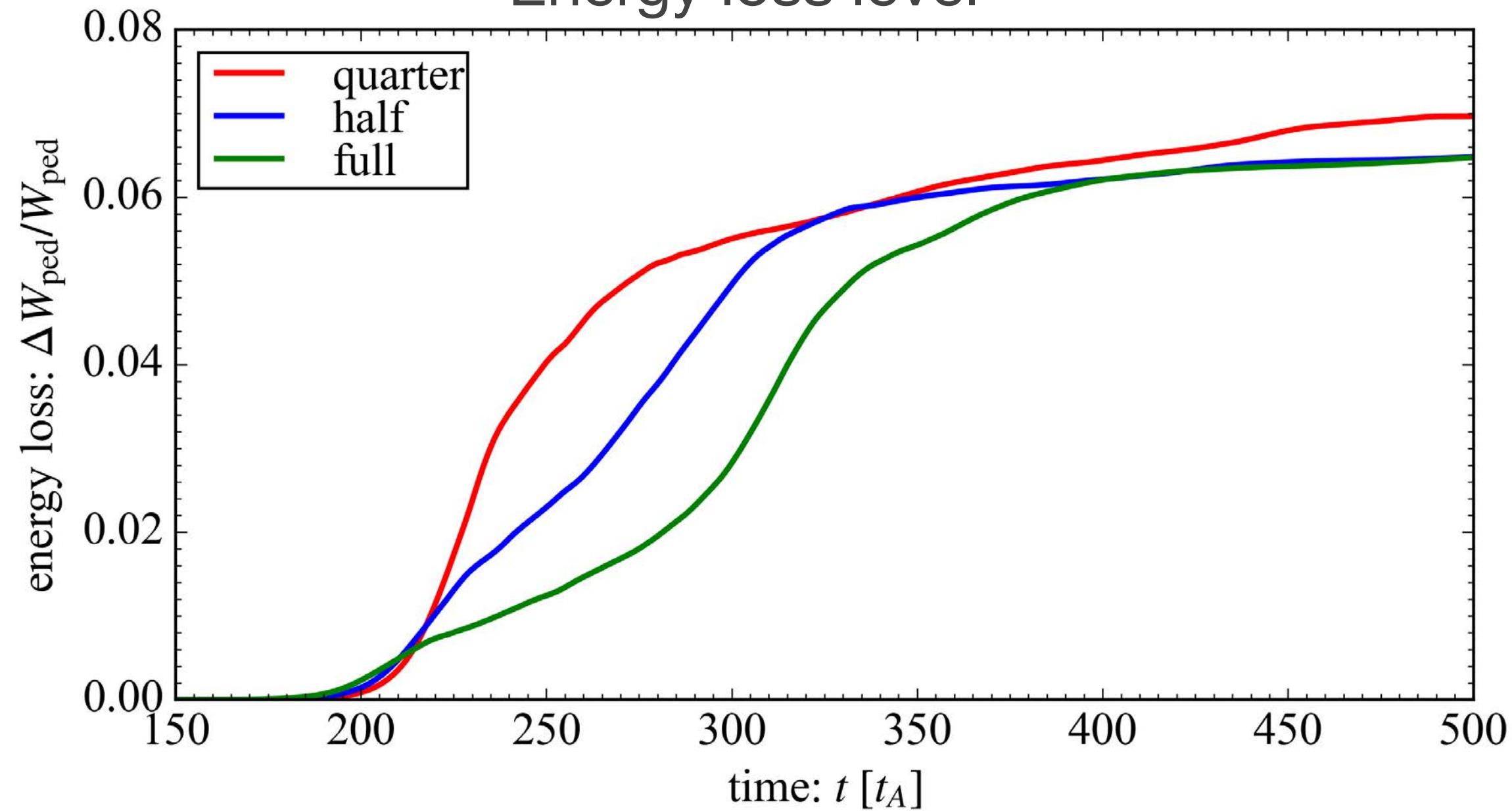


- **Initially unstable modes (IUMs) ($n \sim 32$)** sets the pedestal collapse onset at $t \sim 200 t_A$
- IUMs are strongly excited and directly drive the collapse for **quarter** case
- Low- n ($n \sim 1$) modes are generated via nonlinear coupling between IUMs and collapse is triggered by down-shifted modes ($n = 20 \sim 30$) for **full** torus case (and **half** torus case)

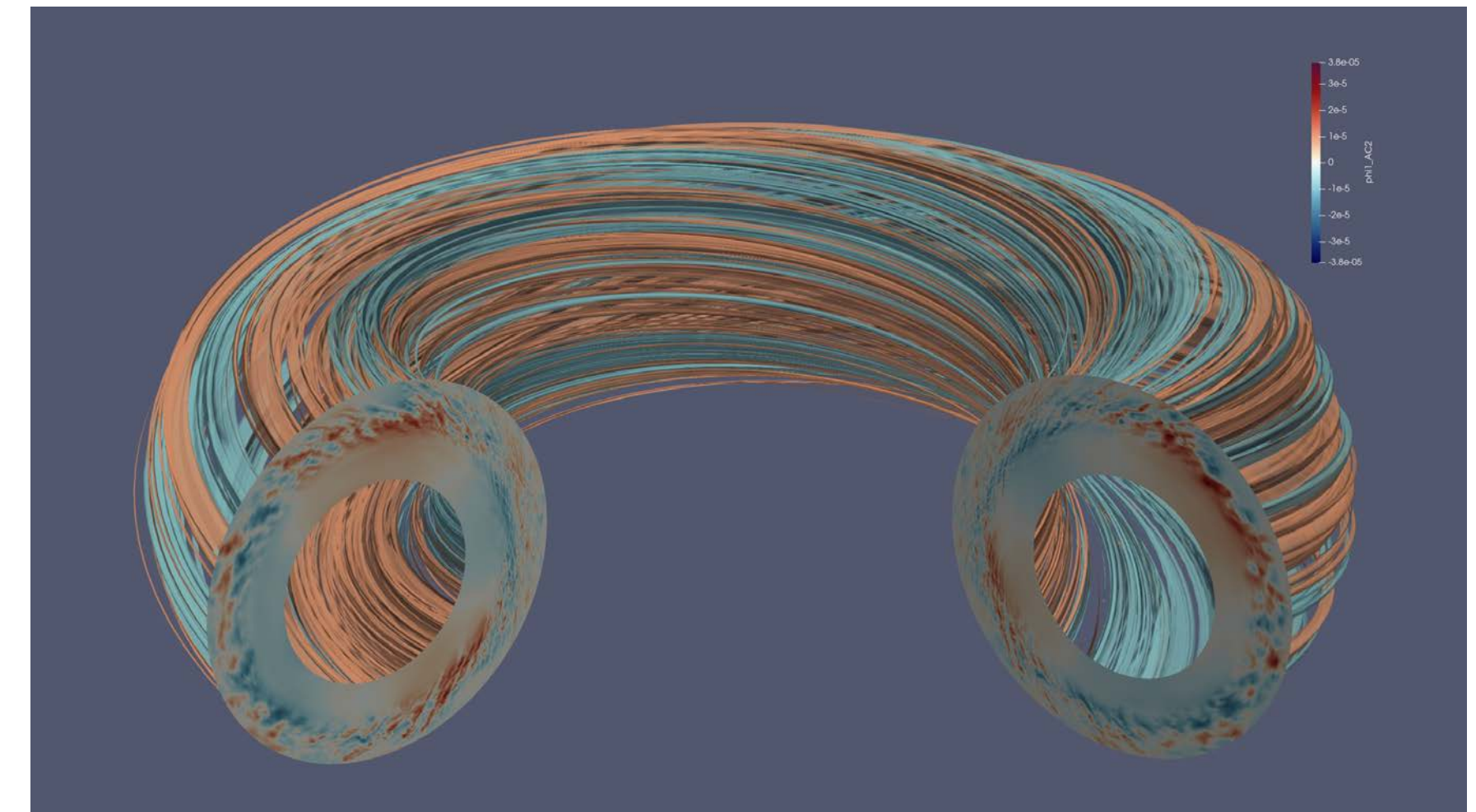
perp. kinetic energy spectra (full torus)



Energy loss level

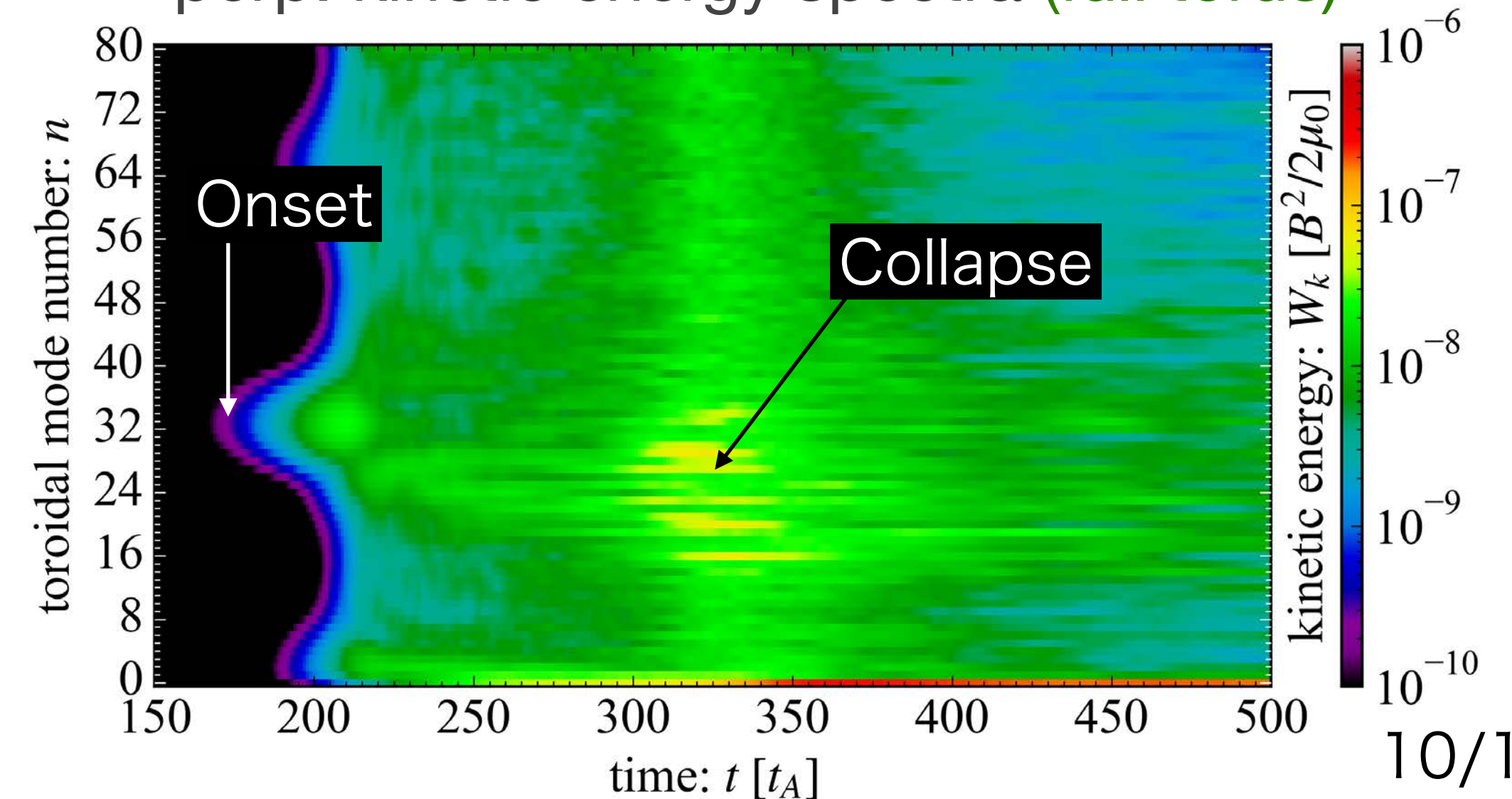


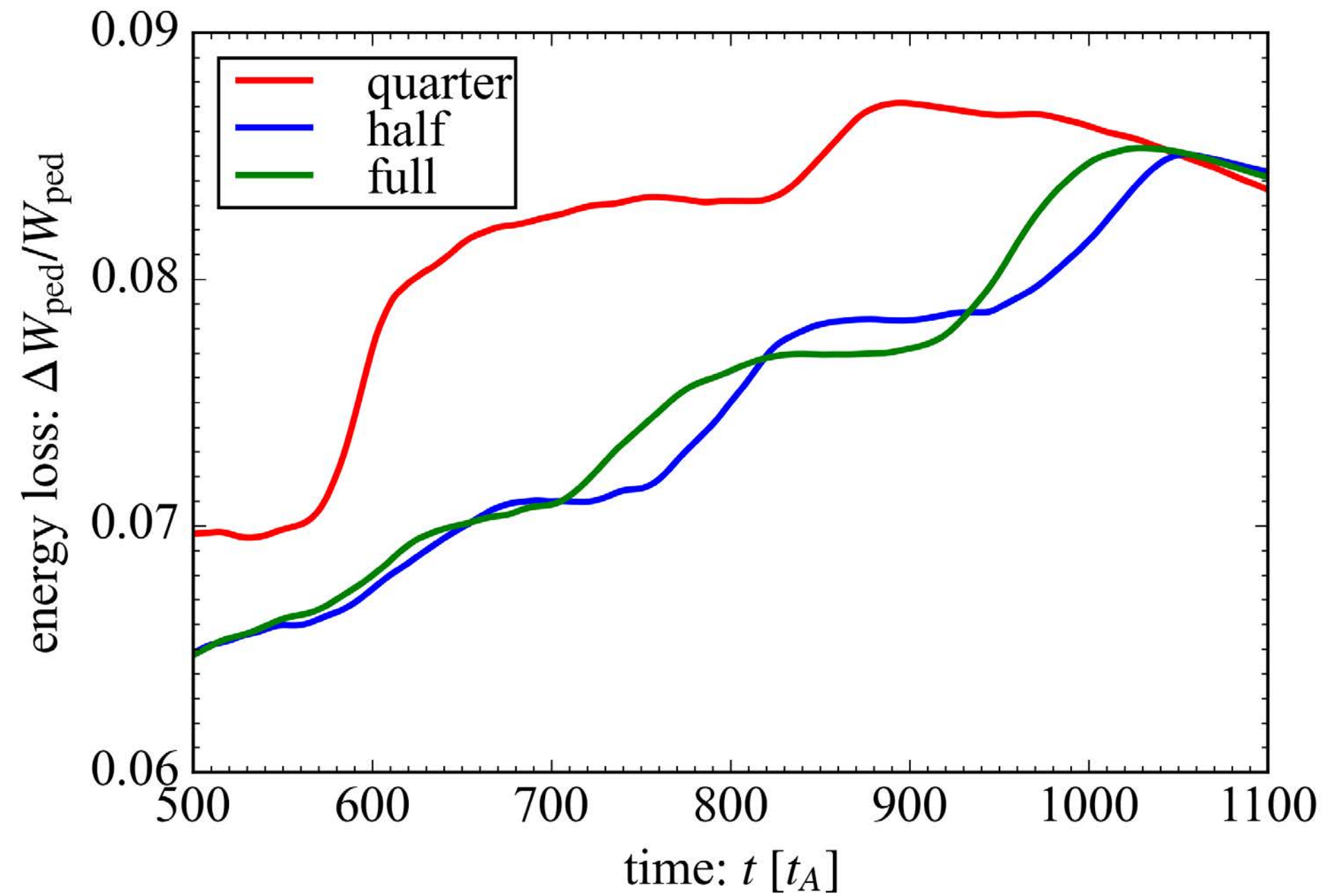
ϕ w/o $n=0$ in full torus during collapse $t=330$



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- IUMs are strongly excited and directly drive the collapse for **quarter** case
- Low- n ($n \sim 1$) modes are generated via nonlinear coupling between IUMs and collapse is triggered by down-shifted modes ($n=20 \sim 30$) for **half** and **full** case

perp. kinetic energy spectra (full torus)

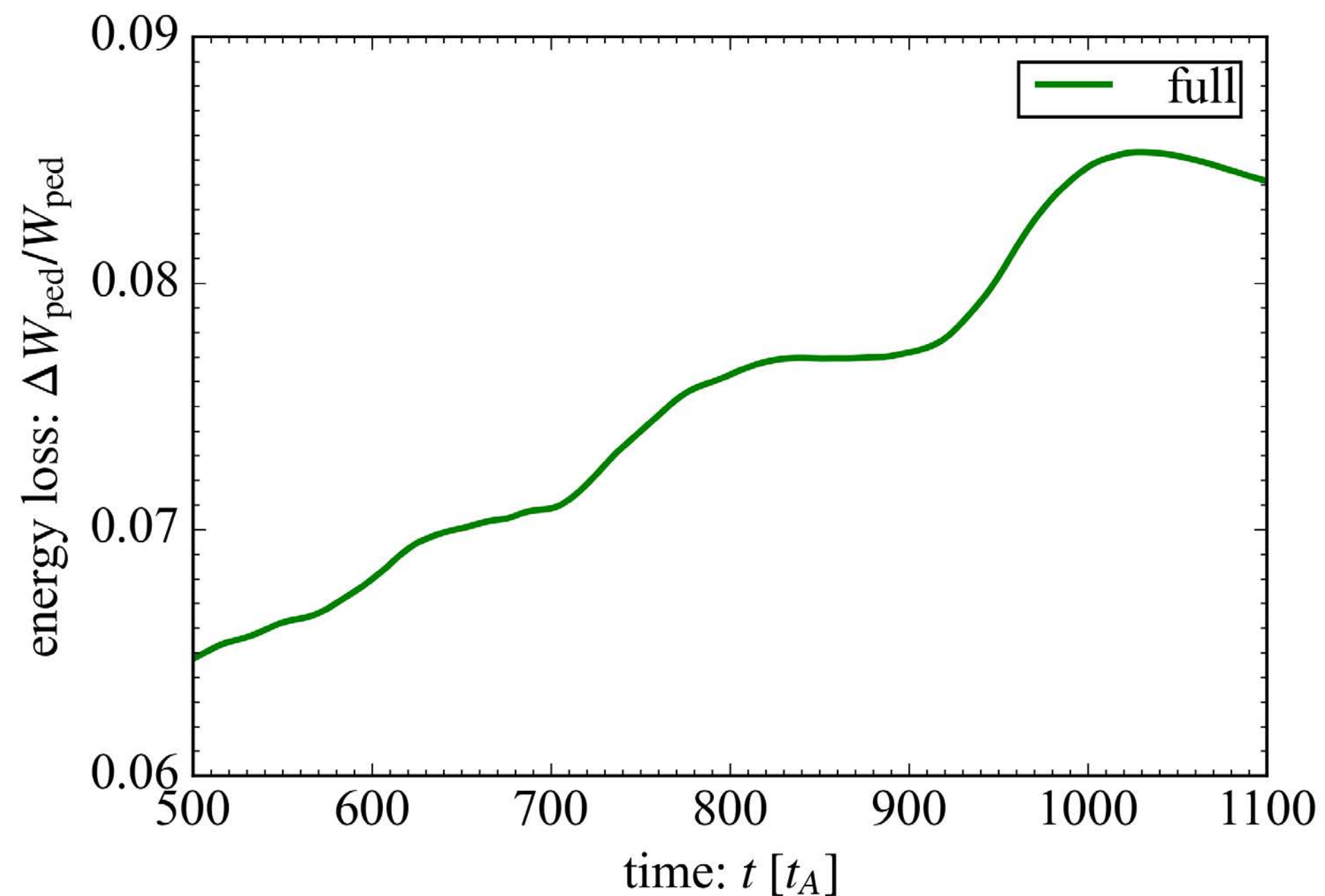




Energy loss enhancement by interplay between zonal flow (ZF) and turbulence [Seto PoP'19] is also observed in **full** torus case as well as **half** and **quarter** torus cases

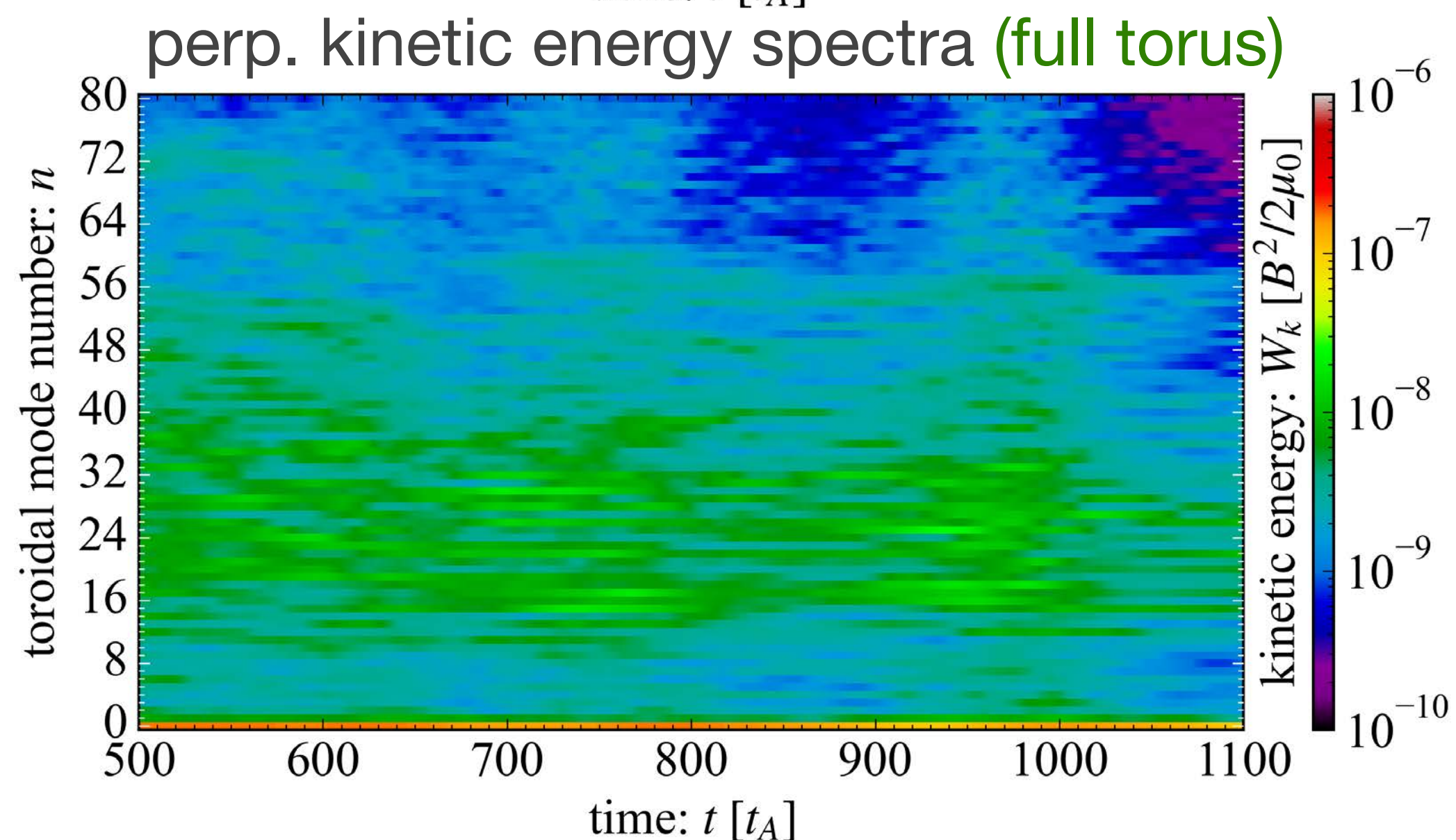
- Similar energy loss level
- cf.) generation mechanism of ZF [Yagi PET2021]
 - ❖ Reynolds stress cancels with Maxwell stress
 - ❖ **Pressure gradient drives ZF via geodesic curvature**

Impact of binormal domain length on energy loss after pedestal collapse



Energy loss enhancement by interplay between zonal flow (ZF) and turbulence [Seto PoP'19] is also observed in **full** torus case as well as **half** and **quarter** torus cases

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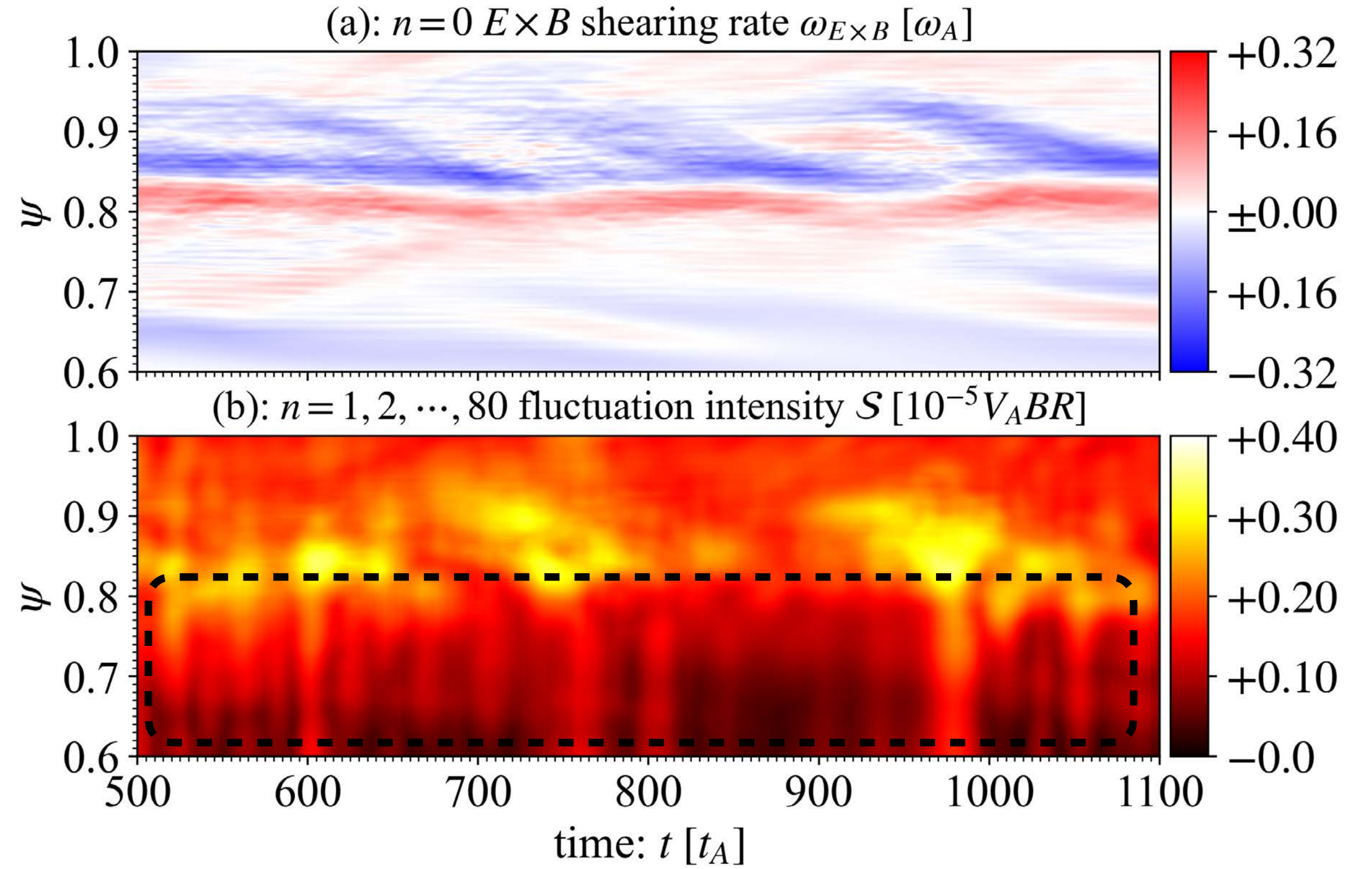
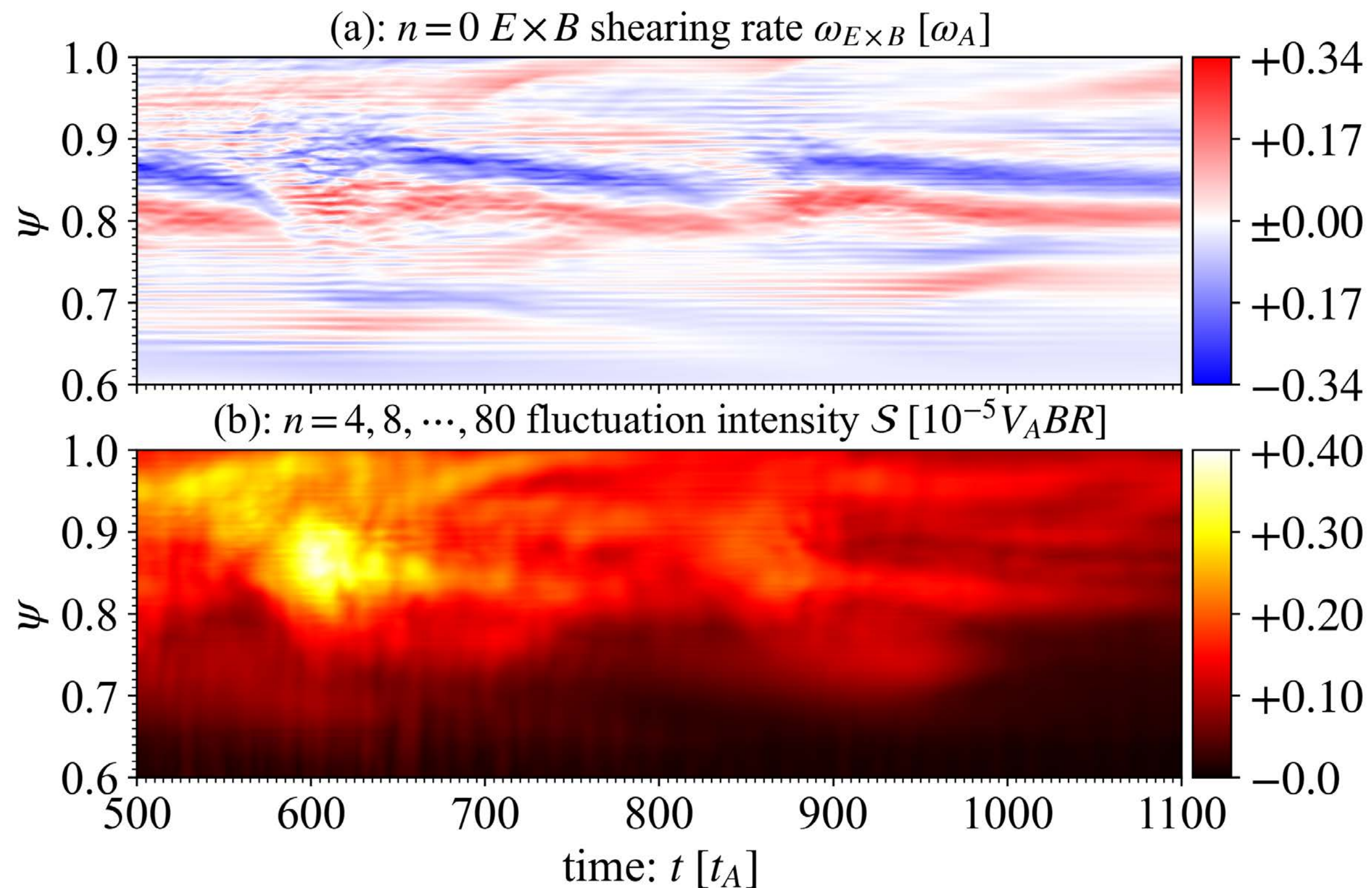


Turbulence burst occurs periodically after the pedestal collapse

Spatio-temporal structure of flow and turbulence after pedestal collapse

flow & turbulence structure: **quarter torus**

flow & turbulence structure: **full torus**

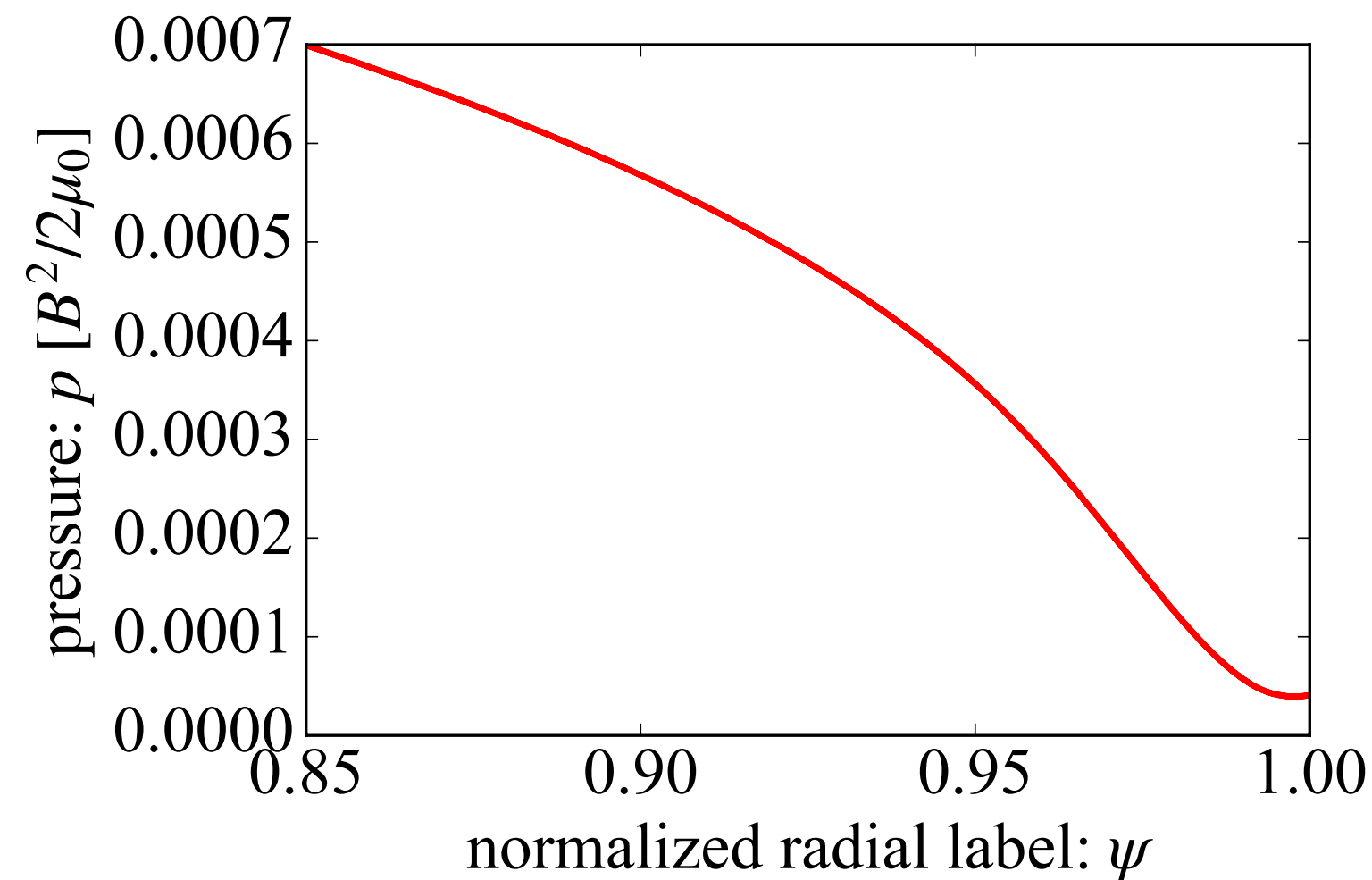
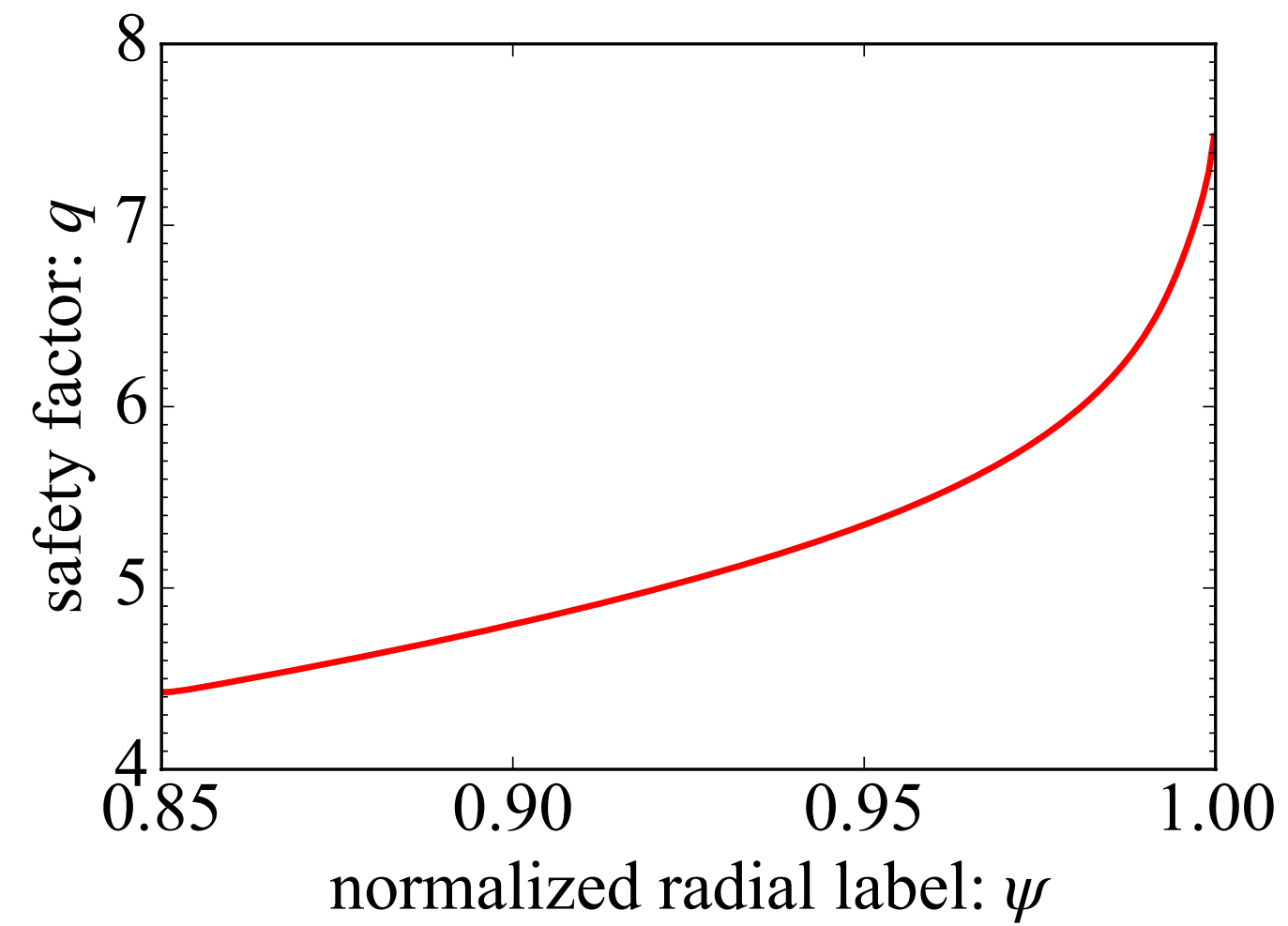
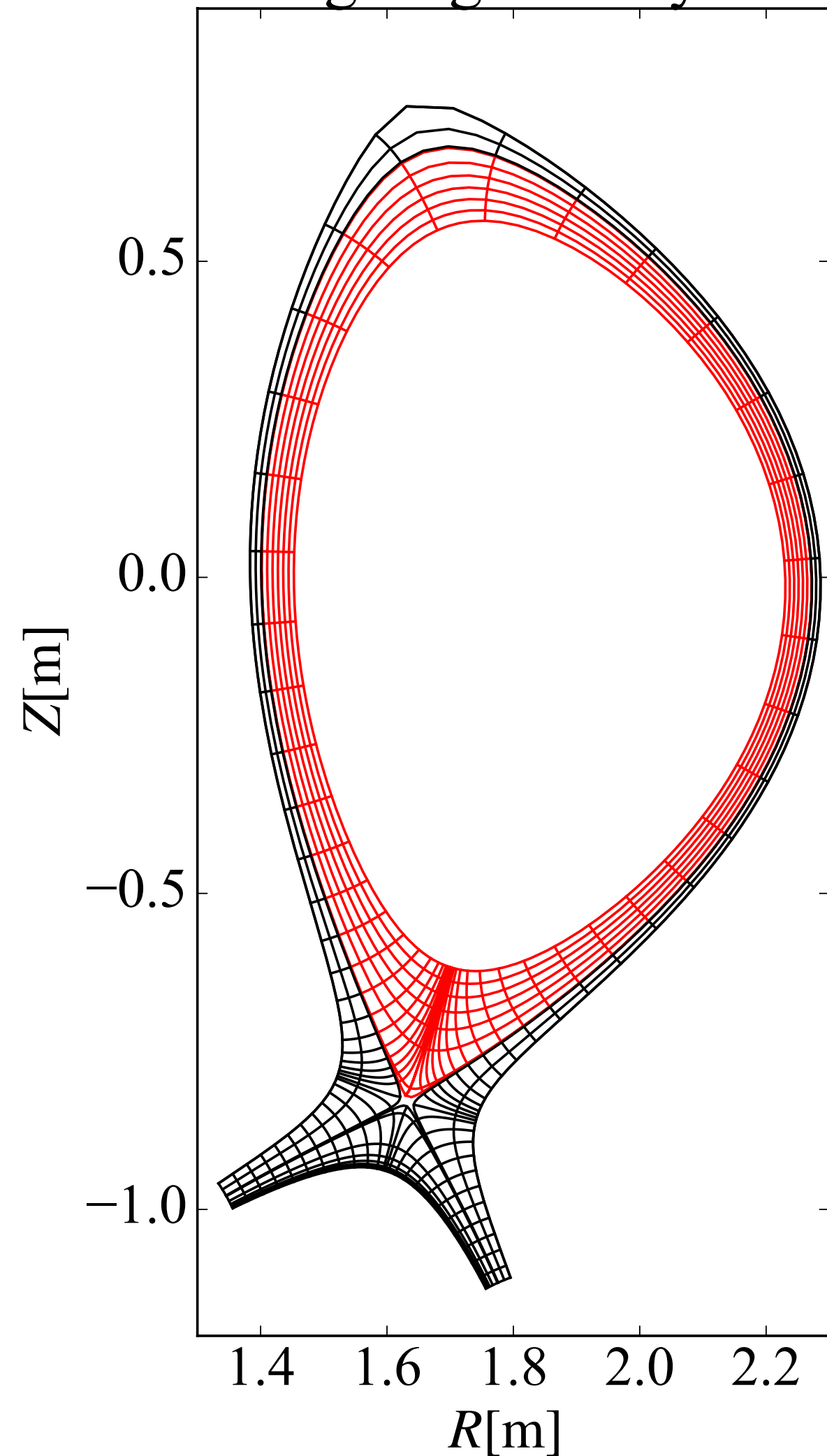


Fluctuation intensity:
$$\mathcal{S}(\psi) = \sqrt{\sum_{n \neq 0} \sum_m \phi(\psi, m, n) \phi(\psi, -m, -n)}$$

- Turbulence bursts with cyclic oscillation between ZF and turbulence are observed in all cases
- $n=1$ fluctuation has global structure whose peak is located at turbulence bursts in **full** torus case

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grid geometry



Grid information

- $N_x=260$ ($N_{y_core}=195$) for $[0.85, 1.05]$
- $N_y=288$ ($N_{y_core}=192$)
- $N_z=256$ for $zperiod=1$, $[0, 2\pi]$
- $lowpass=80$, $n=0, 1, 2, \dots, 80$ are taken
 - ✓ $n=0, 1, \dots, 4$: 2D Poisson solver
 - ✓ $n=5, 6, \dots, 80$: 1D Poisson solver

Scale separated 3-field DW+RBM model (simplified ion diamagnetism + electron Hall effect)

$$\frac{\partial}{\partial t} \varpi_1 = - [\phi_1, \varpi] - B_0 \partial_{\parallel} \left(\frac{J_{\parallel 1}}{B_0} \right) + B_0 \left[A_{\parallel 1}, \frac{J_{\parallel}}{B_0} \right] + \mathcal{K}(p_1) + \mu_{\parallel} \partial_{\parallel}^2 \varpi_1 + \mu_{\perp} \nabla_{\perp}^2 \varpi_1,$$

$$\frac{\partial}{\partial t} p_1 = - [\phi_1, p] + \chi_{\parallel} \partial_{\parallel}^2 p_1 + \chi_{\perp} \nabla_{\perp}^2 p_1,$$

$$\frac{\partial}{\partial t} A_{\parallel 1} = - [\phi, A_{\parallel 1}] - \partial_{\parallel} \phi_1 + \delta_e (\partial_{\parallel} p_1 - [A_{\parallel 1}, p]) + \eta J_{\parallel 1}$$

$$\varpi = \nabla_{\perp}^{*2} F, \quad J_1 = \nabla_{\perp}^2 A_{\parallel}, \quad F = \phi + \delta_i p, \quad \phi = \phi_0 + \phi_1, \quad p = p_0 + p_1,$$

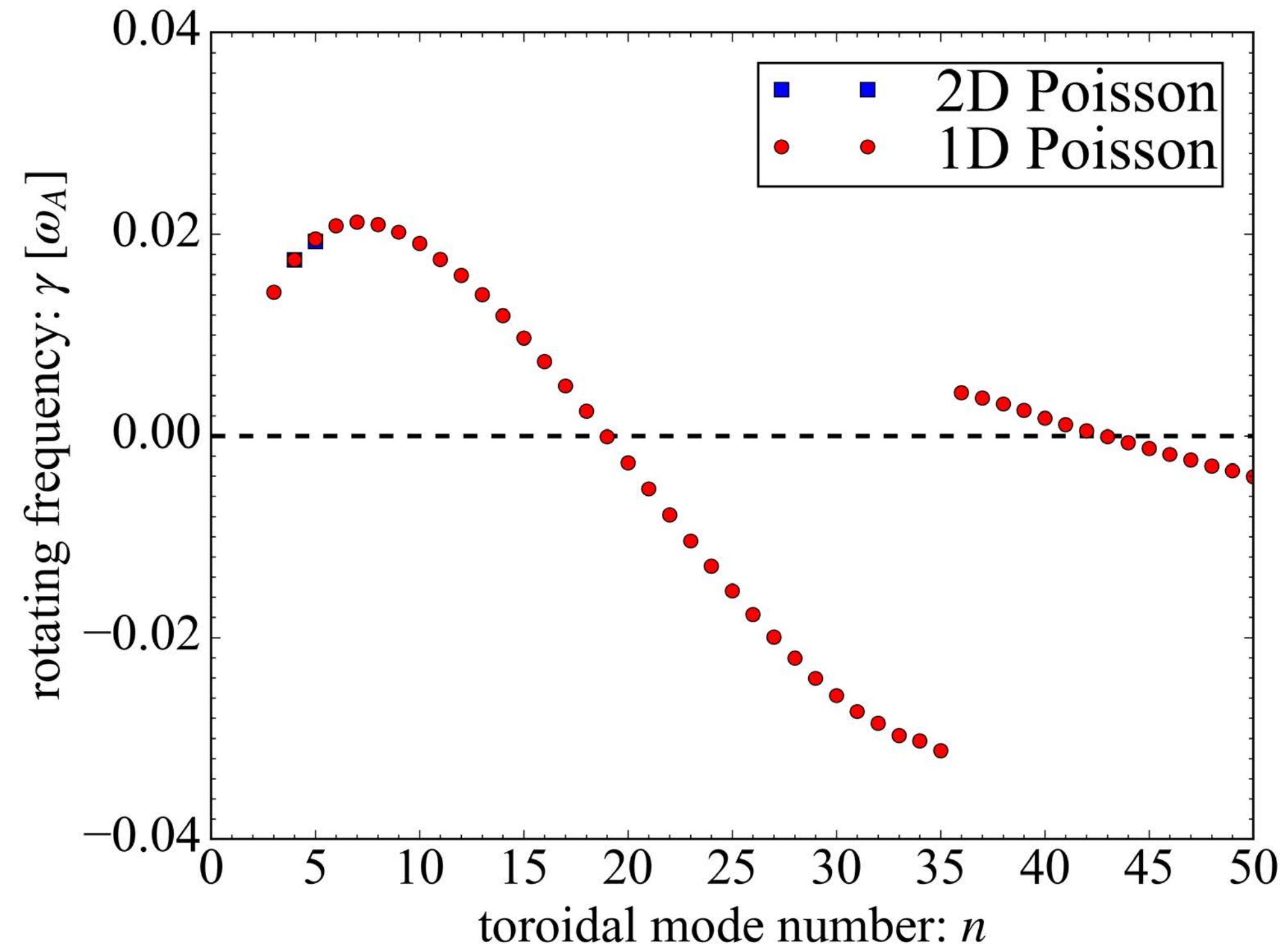
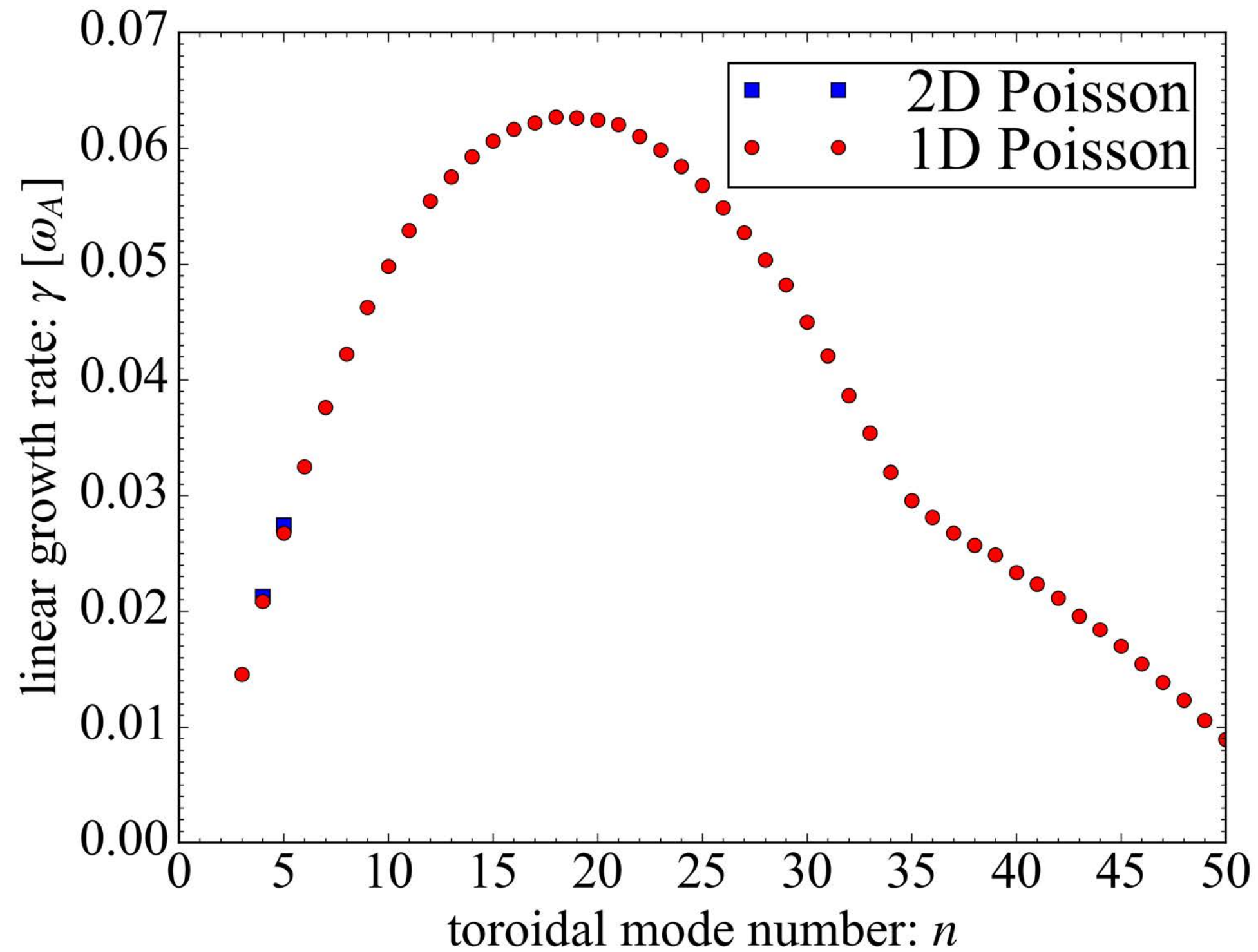
$$\mathbf{B} = \mathbf{B}_0 + \nabla A_{\parallel 1} \times \mathbf{b}_0, \quad J_{\parallel} = J_{\parallel 0} + J_{\parallel 1},$$

Bmag	3.129918 [T]
Rmag	2.286076 [m]
n ₀	10 ¹⁹ [m ⁻³]
μ_{\parallel}	1.0×10 ⁻¹
μ_{\perp}	1.0×10 ⁻⁷
χ_{\parallel}	1.0×10 ⁻¹
χ_{\perp}	1.0×10 ⁻⁷
η	1.0×10 ⁻⁶

- Normalized with poloidal Alfvén unit
- Use equilibrium pressure and parallel current profiles
- Use modeled constant ion number density and dissipations (not based on equilibrium)
- No equilibrium radial electric field, SOL/DIV physics

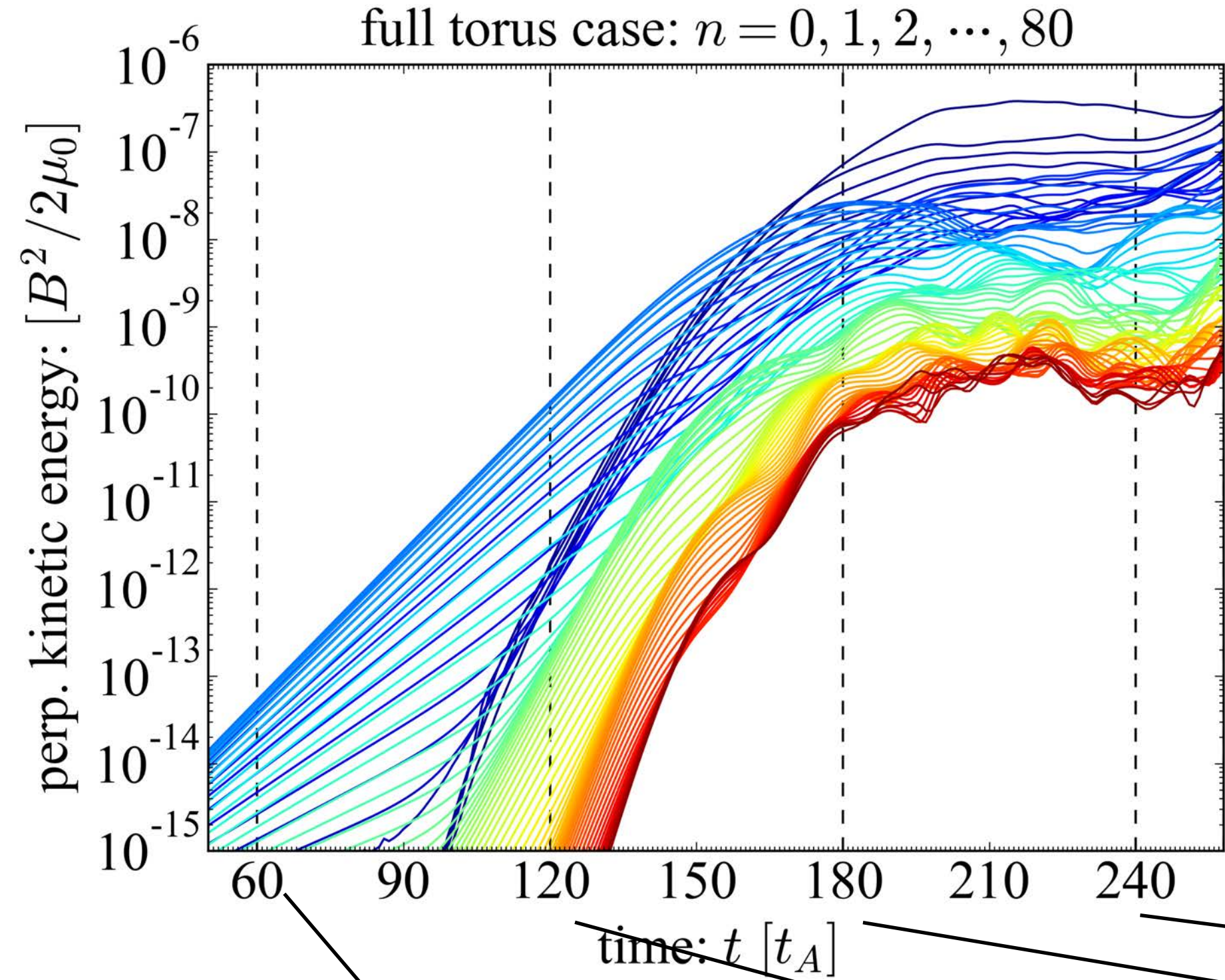
Preliminary test only for checking whether the hybrid Poisson solver makes full annular torus simulation possible

Dispersion relation of preliminary pedestal collapse simulation

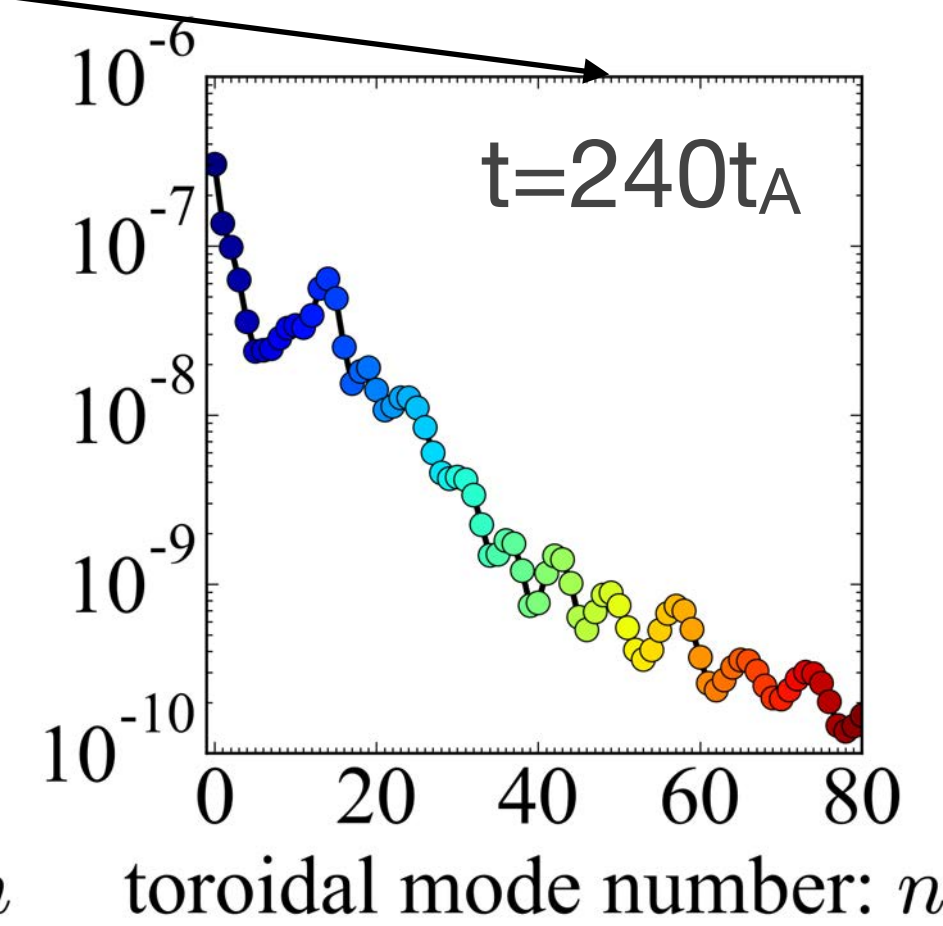
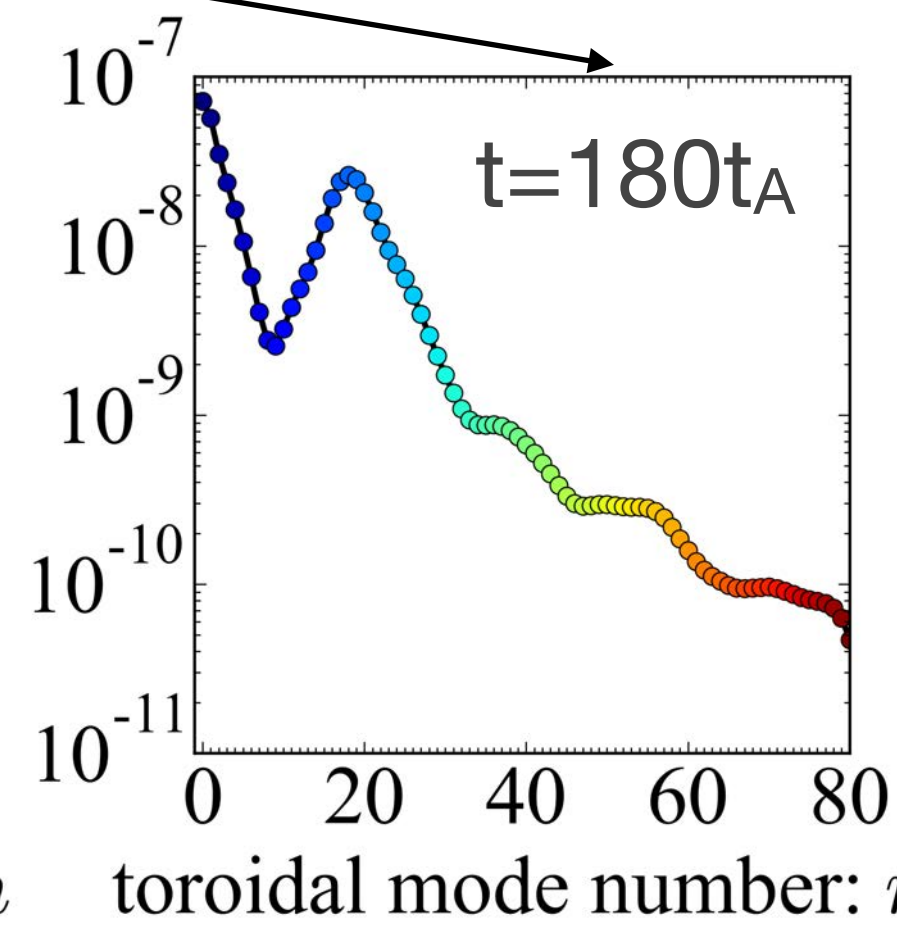
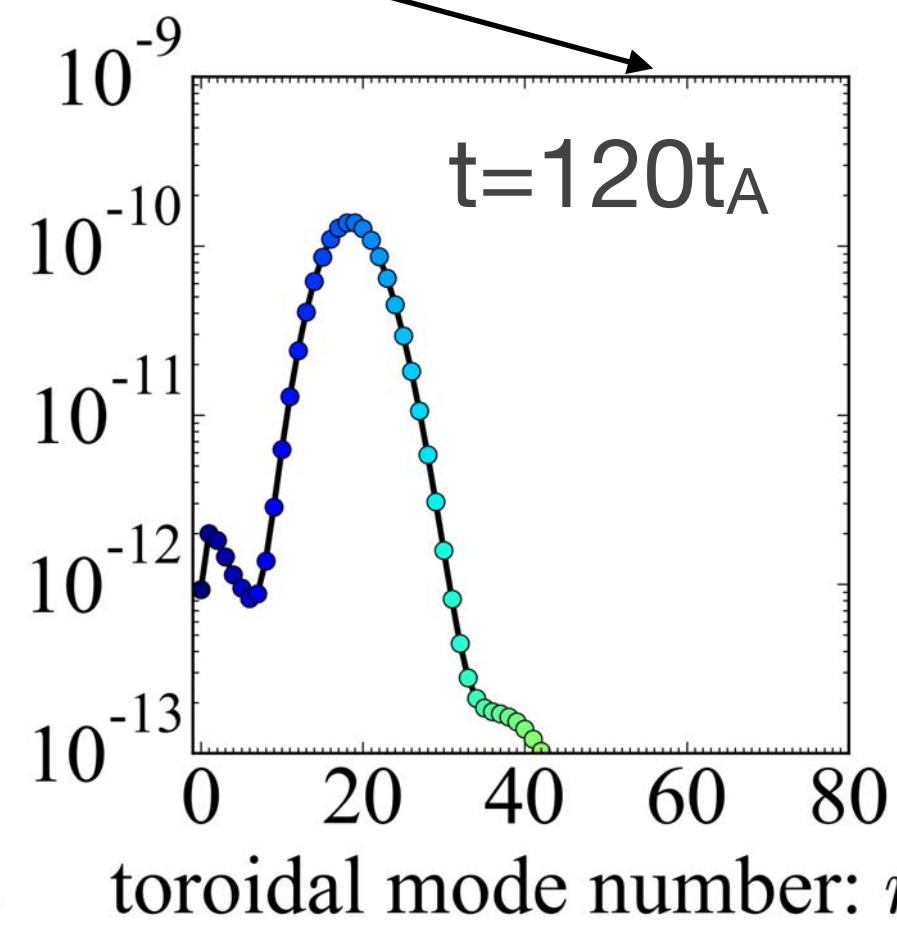
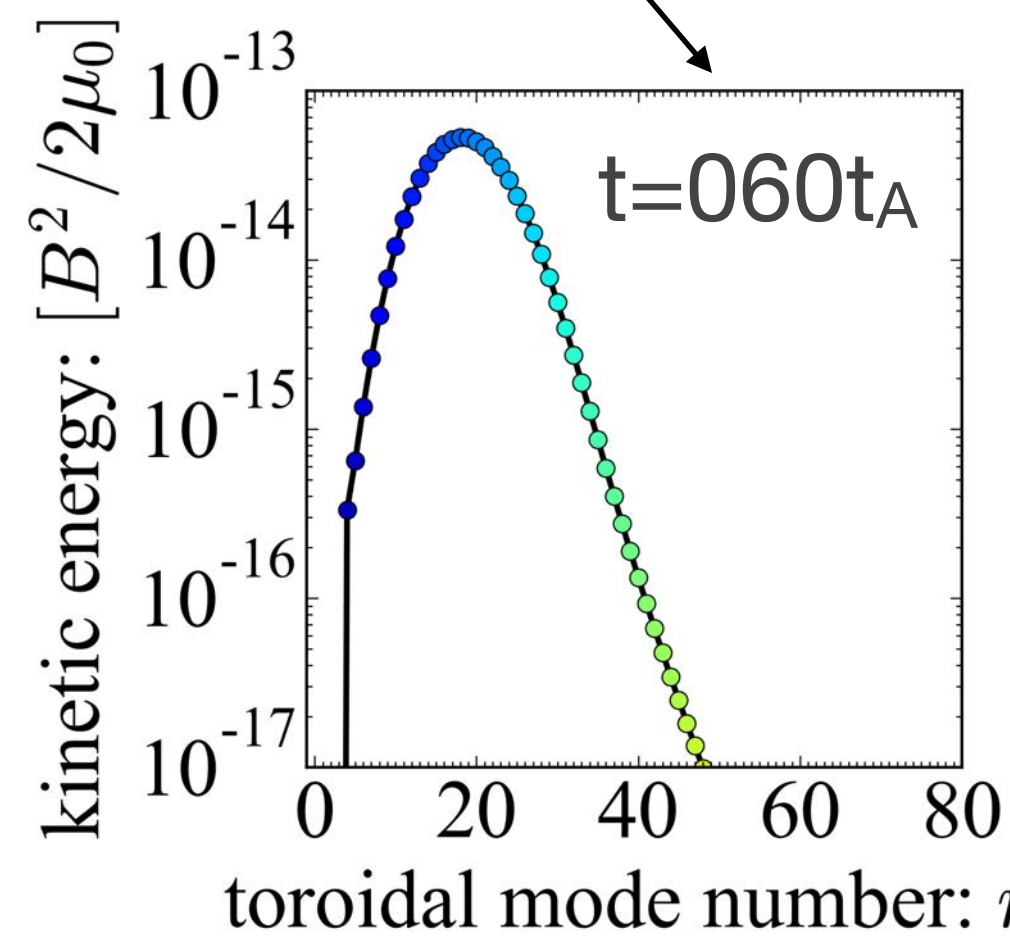


- $n \geq 36$ modes are no longer in ballooning branch, they have broad structures in radial direction
 - ➔ Pedestal collapse is triggered mainly by DWRBMs spectra whose peak is $n=18$
- $n=3$ is stable for DWRBM with 2D Poisson solver while $n=3$ is unstable for DWRBM with 1D solver
- Small ($\sim 3\%$) differences in $n=4$ and $n=5$ growth rate between 2D and 1D Poisson solver
 - ➔ Using 2D Poisson solver for $n \leq 4$ seems to be appropriate

Kinetic energy grows in reasonable manner and low- n modes get excited



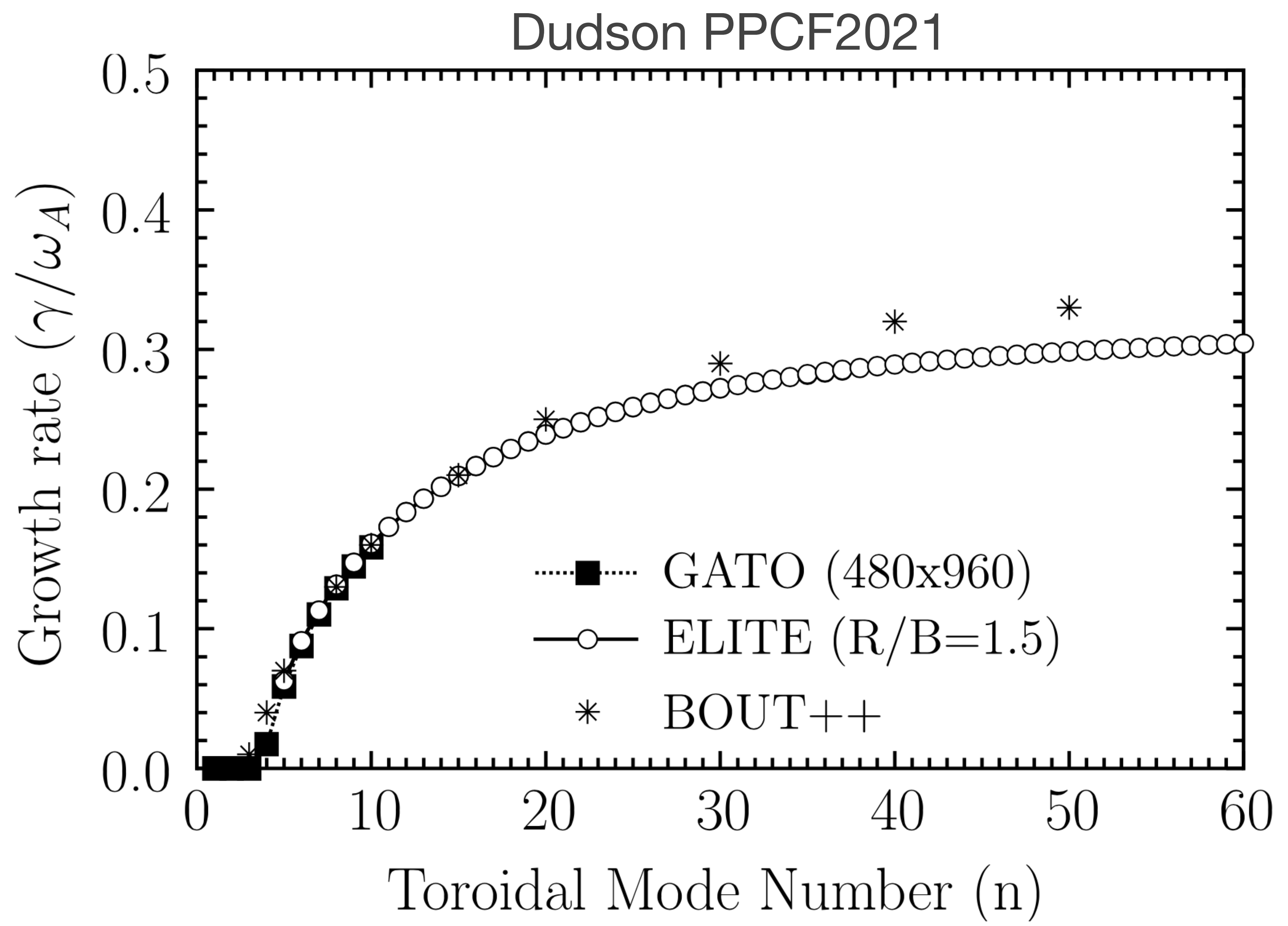
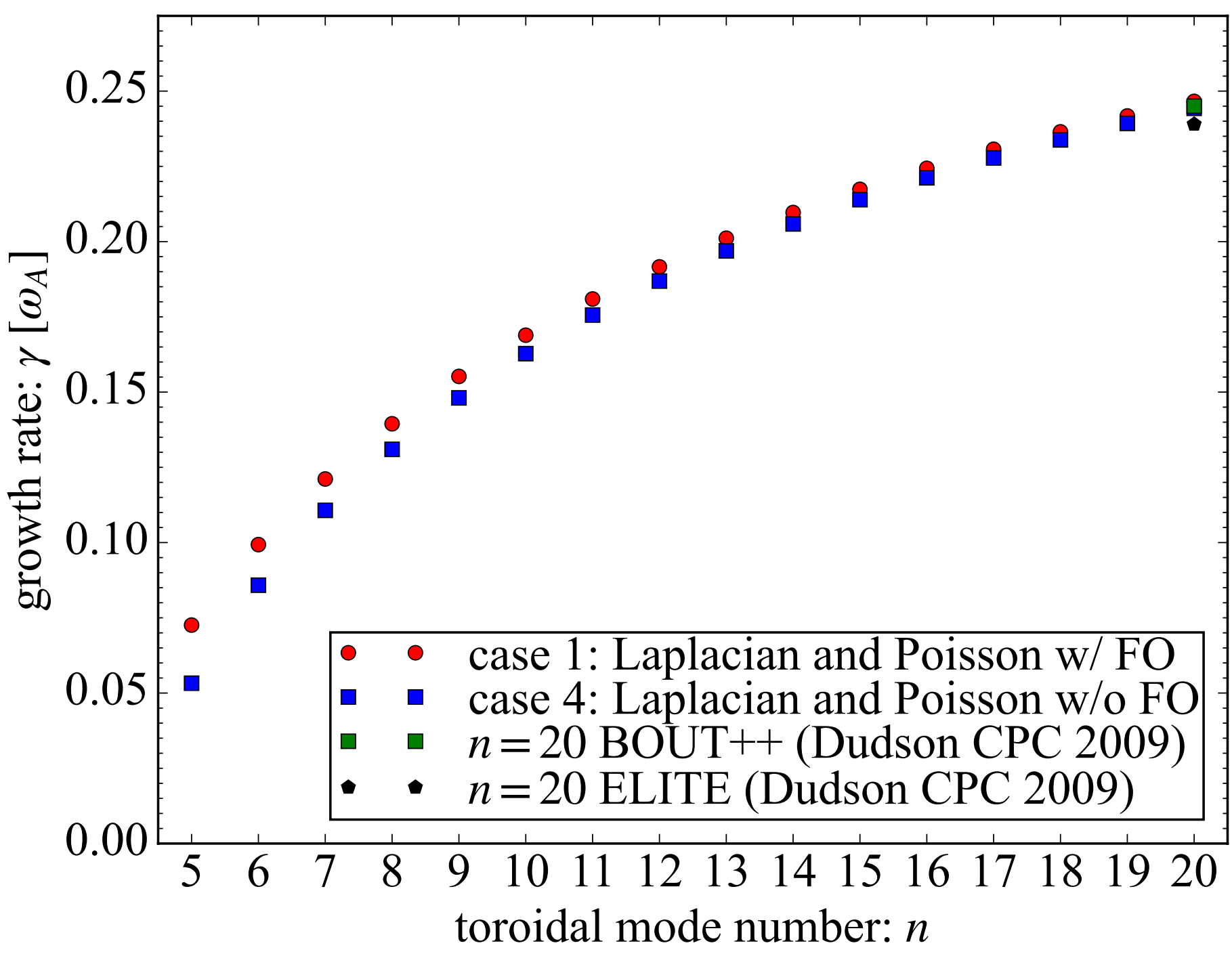
Full annular edge turbulence simulations in ITER baseline case is in preparation partially under JIFT collaboration between QST and LLNL



- Introduction: BOUT++ code and objectives of TOKEDGE project
- Numerical scheme of a hybrid Poisson solver for full annular tokamak edge simulations
- **[Erratum of CSC WS'21]** impact of flute-ordering assumption in low- n ideal ballooning mode
- Pedestal collapse simulation in full annular torus domain in shifted circular geometry
- Preliminary test of full annular torus turbulence simulation in single-null divertor geometry
- **Summary**

A hybrid Poisson solver for full annular tokamak edge turbulence simulation has been developed

- [Erratum of CSC WS'21] impact of flute-ordering assumption in low- n ideal ballooning mode
 - ❖ Flute-ordering (in Laplacian for $J//$) has large impact on IBM growth rate in the presented case
- Pedestal collapse simulation in full annular torus domain in shifted circular geometry
 - ❖ Low- n modes generated via nonlinear couplings among initially unstable modes before pedestal collapse for half and full tori
 - ❖ $n=1$ fluctuation has global radial structure during / after collapse in full torus case
 - ❖ Energy loss enhancement by interplay between flow and turbulence is observed after the pedestal collapse and energy loss levels are saturated to a similar level in all cases
- Preliminary test of full annular torus turbulence simulation in single-null divertor geometry
 - ❖ Hybrid Poisson solver enables nonlinear full annular torus simulation in diverted geometry
 - ❖ Production run with ITER baseline equilibrium is in preparation



2D Poisson solver is tested in single-null geometry by comparing linear RBM growth rates by 2D Poisson solver and those by 1D flute-ordered Poisson solver

Linearized RBM model with dissipation

$$\frac{\partial \varpi_1}{\partial t} = B_0 \partial_{\parallel} \left(\frac{J_{\parallel}}{B_0} \right) - B_0 \left[A_{1\parallel}, \frac{J_{\parallel 0}}{B_0} \right] + \frac{\mathbf{b}_0 \times \boldsymbol{\kappa}_0 \cdot \nabla P_1}{B_0} + \mu_{\perp} \nabla_{\perp}^2 \varpi_1 + \mu_{\parallel} \partial_{\parallel}^2 \varpi_1$$

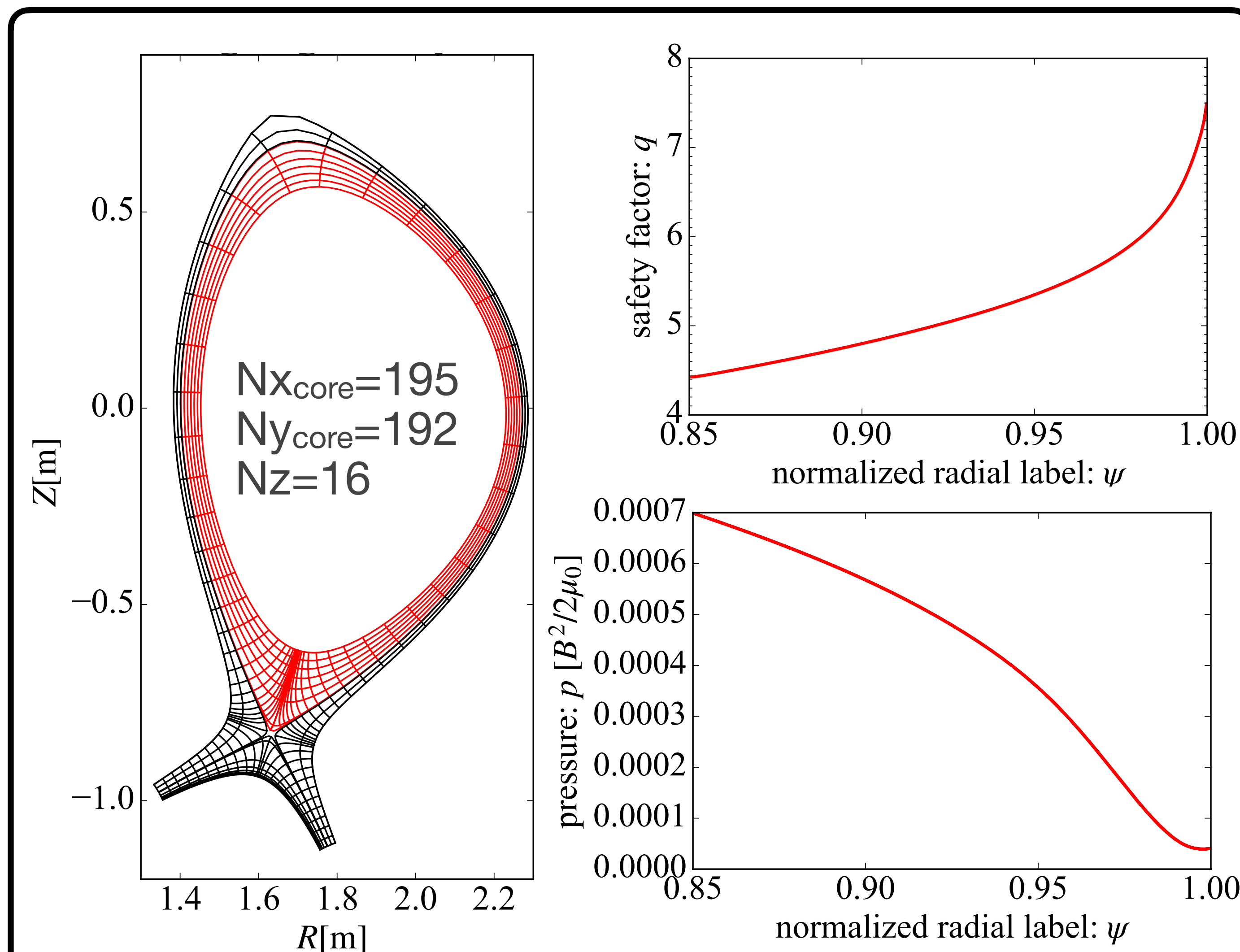
$$\frac{\partial P_1}{\partial t} = -[\phi_1, P_0]$$

$$\frac{\partial A_{\parallel 1}}{\partial t} = -\partial_{\parallel} \phi_1 + \eta J_{\parallel 1}$$

$$\varpi = \nabla \cdot \left(\frac{1}{B_0^2} \nabla_{\perp} \phi \right), \quad J_{\parallel 1} = \nabla_{\perp}^2 A_{\parallel 1}$$

$$\eta = 10^{-6}, \quad \mu_{\perp} = 10^{-7}, \quad \mu_{\parallel} = 10^{-1}$$

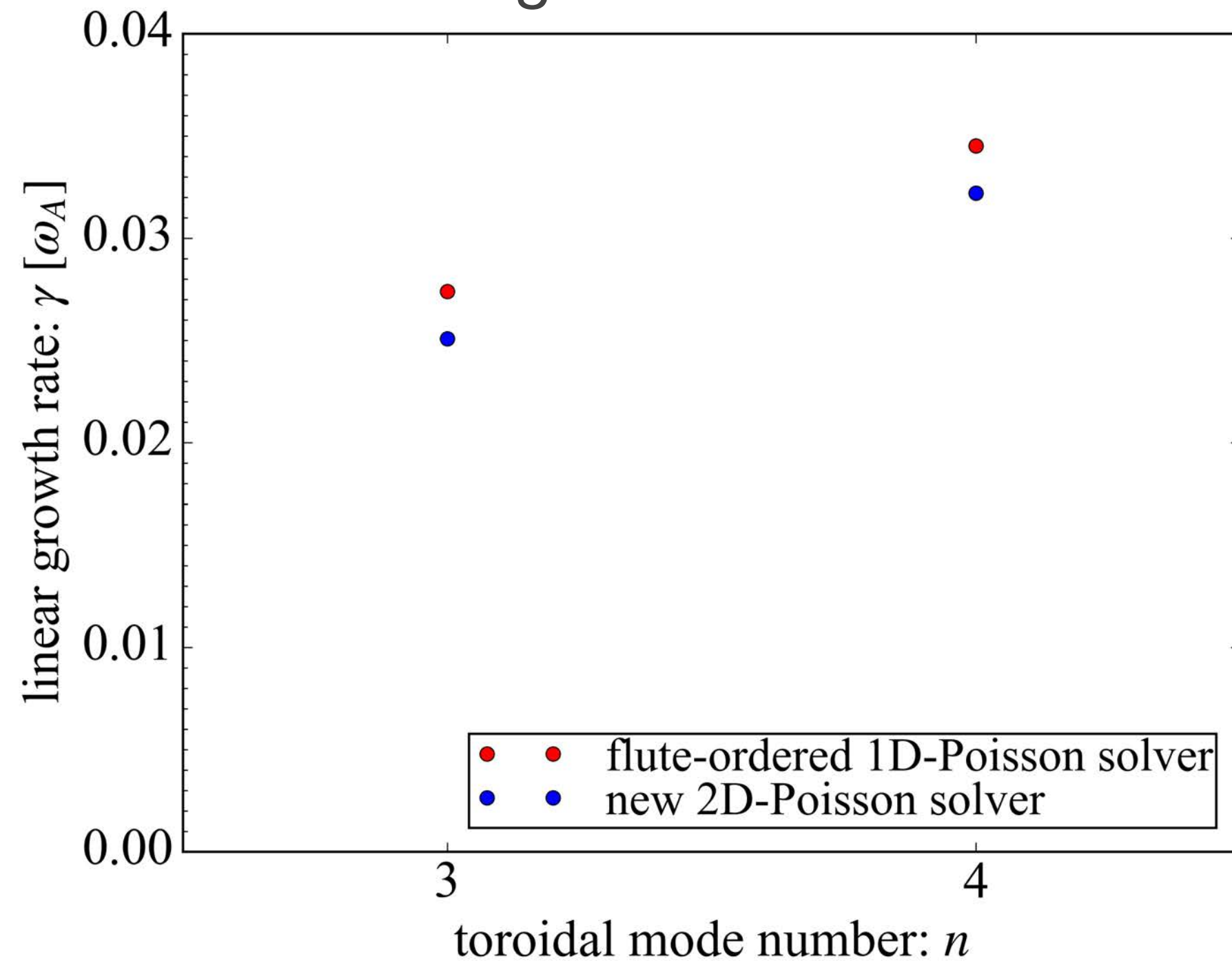
- constant ion density $n_{i0} = 10^{19} \text{ [m}^{-3}\text{]}$
- Dissipations in vorticity equation are required for numerical stability in both 2D and 1D Poisson solver.



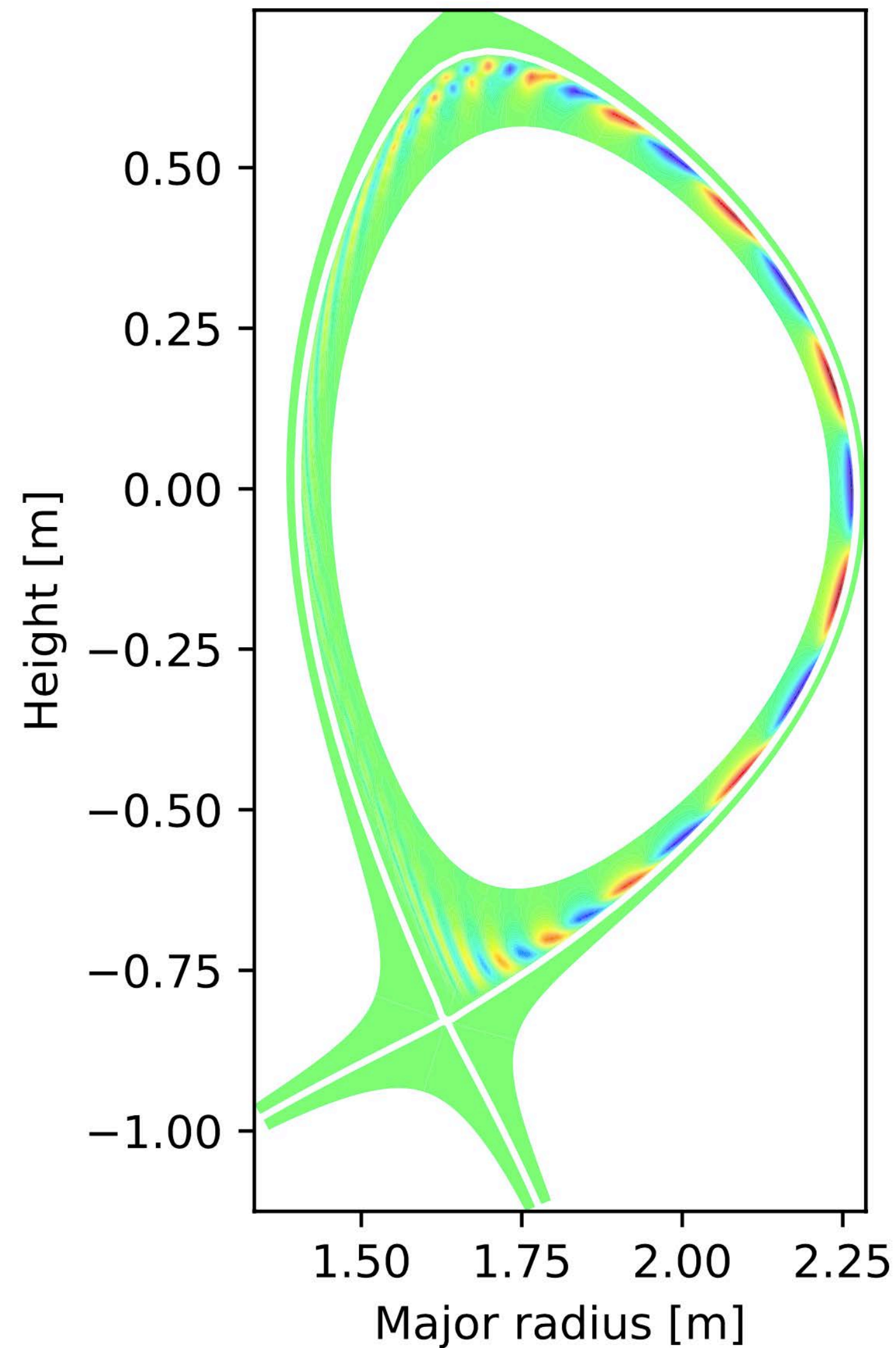
- $1/n$ -th annular wedge domain for $n=3,4$
- z -derivatives are evaluated with FFT

2D Poisson solver captures RBM instability but growth rates are different

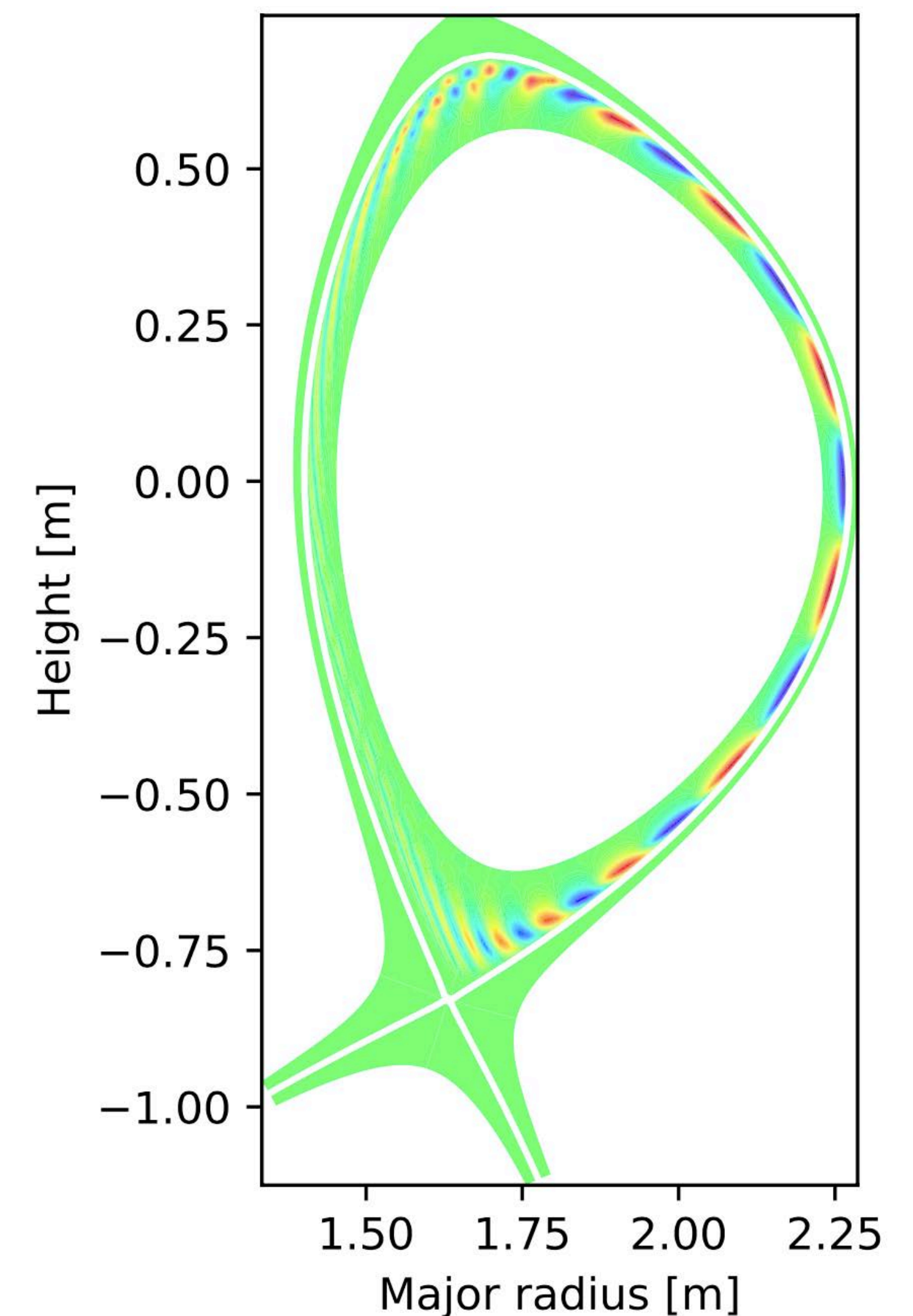
Linear growth rate of RBM



$n=4$ perturbed pressure by 1D Poisson solver



$n=4$ perturbed pressure by 2D Poisson solver

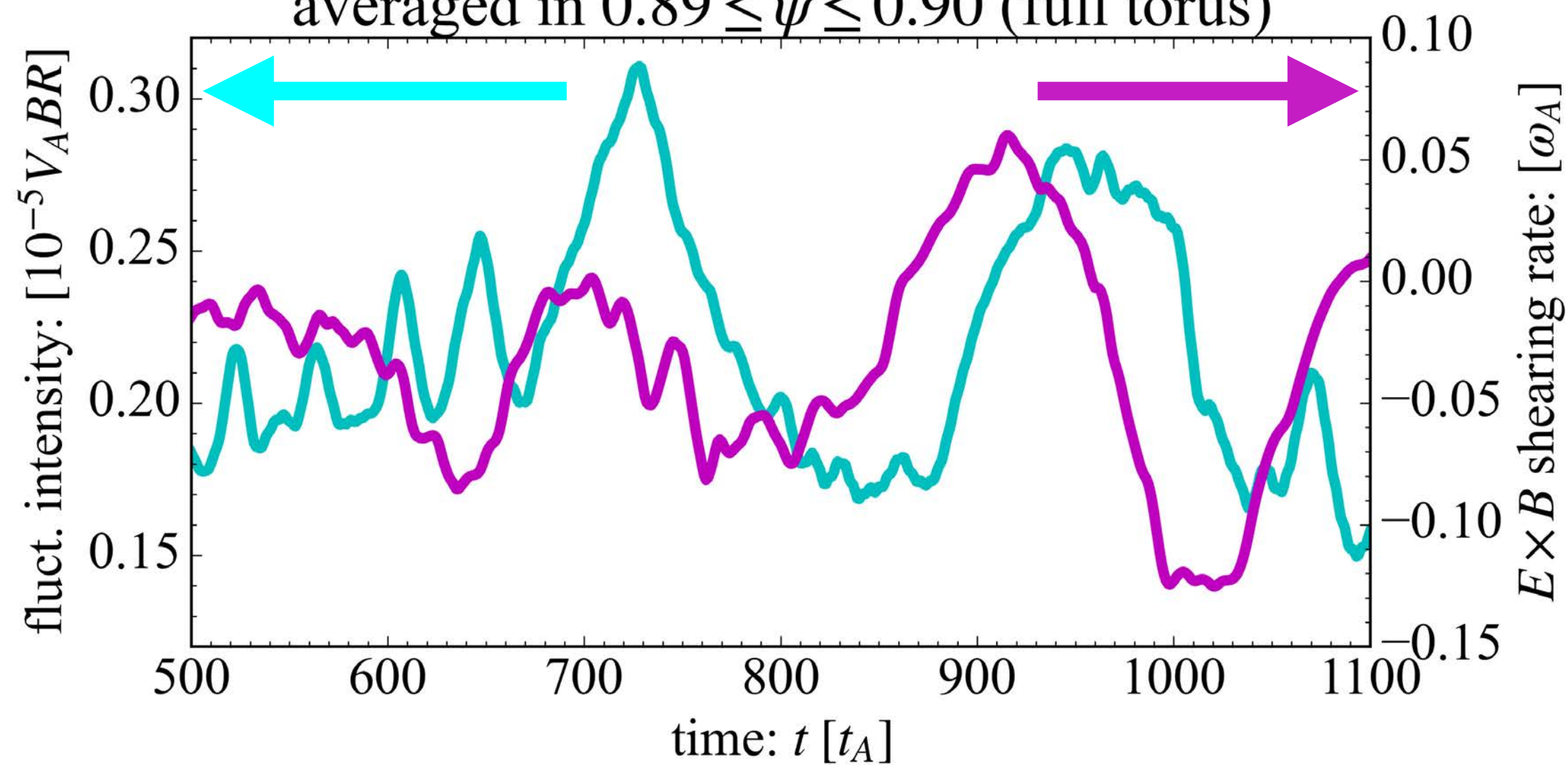


RBM eigen-functions are clearly obtained by both Poisson solvers but their growth rates are different by 6~8%

➔ Further tests (mesh convergence etc...) are required to clarify impact of flute-ordering in complex geometries

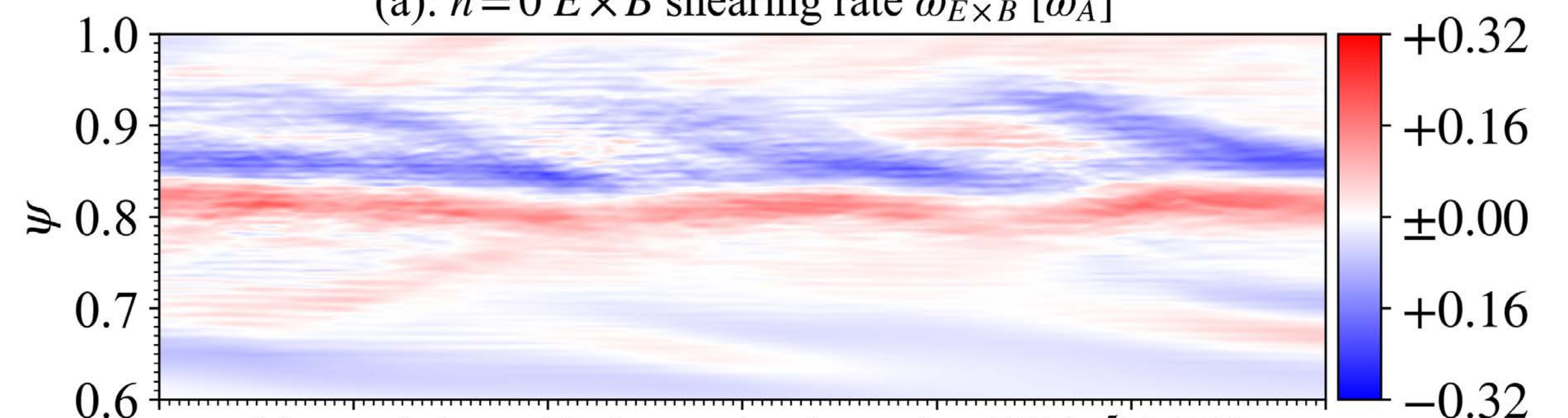
flow & turbulence phase: **full torus**

averaged in $0.89 \leq \psi \leq 0.90$ (full torus)

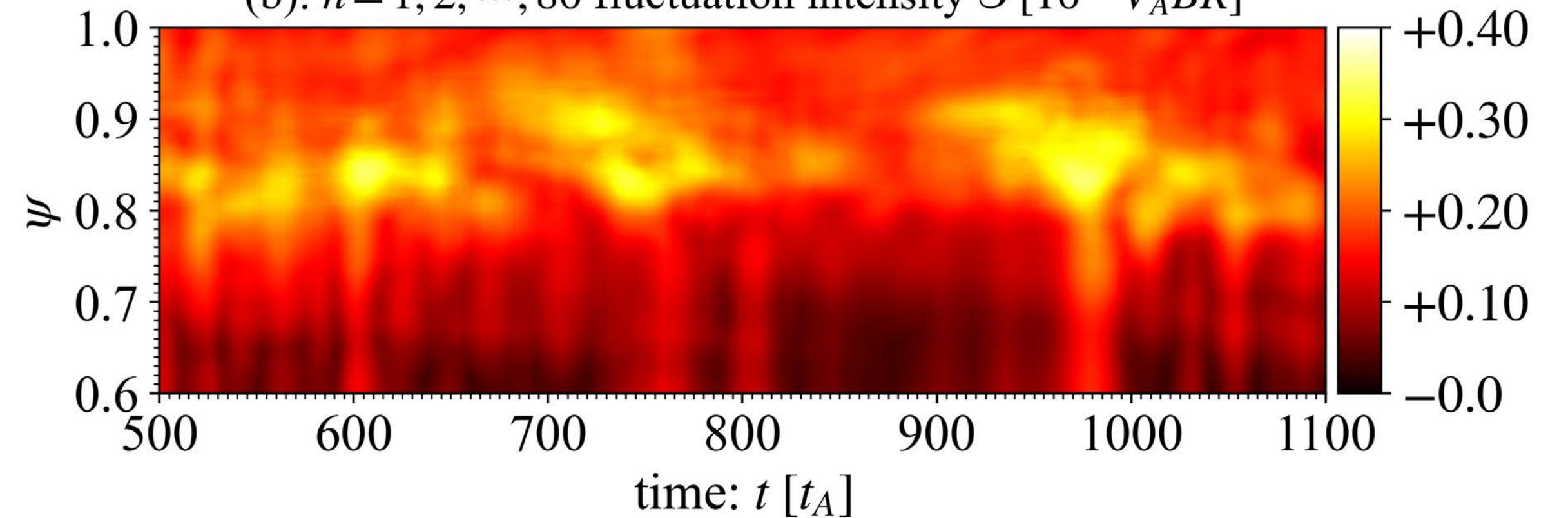


flow & turbulence structure: **full torus**

(a): $n=0$ $E \times B$ shearing rate $\omega_{E \times B} [\omega_A]$



(b): $n=1, 2, \dots, 80$ fluctuation intensity $\mathcal{S} [10^{-5} V_A BR]$

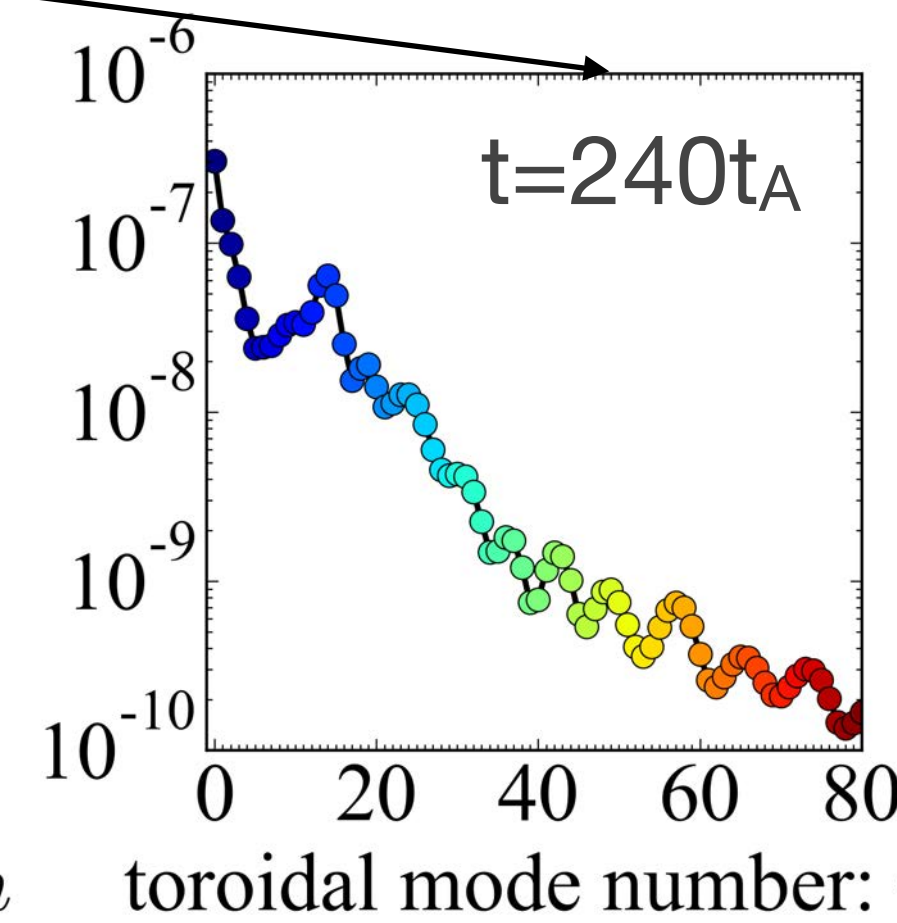
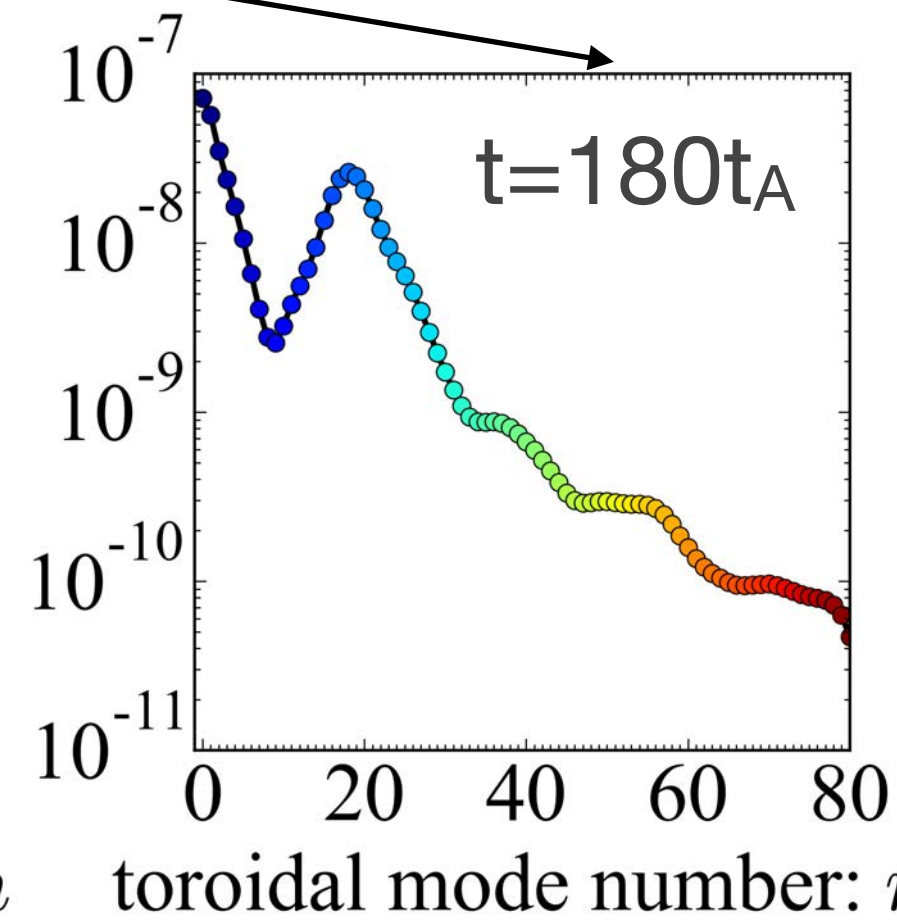
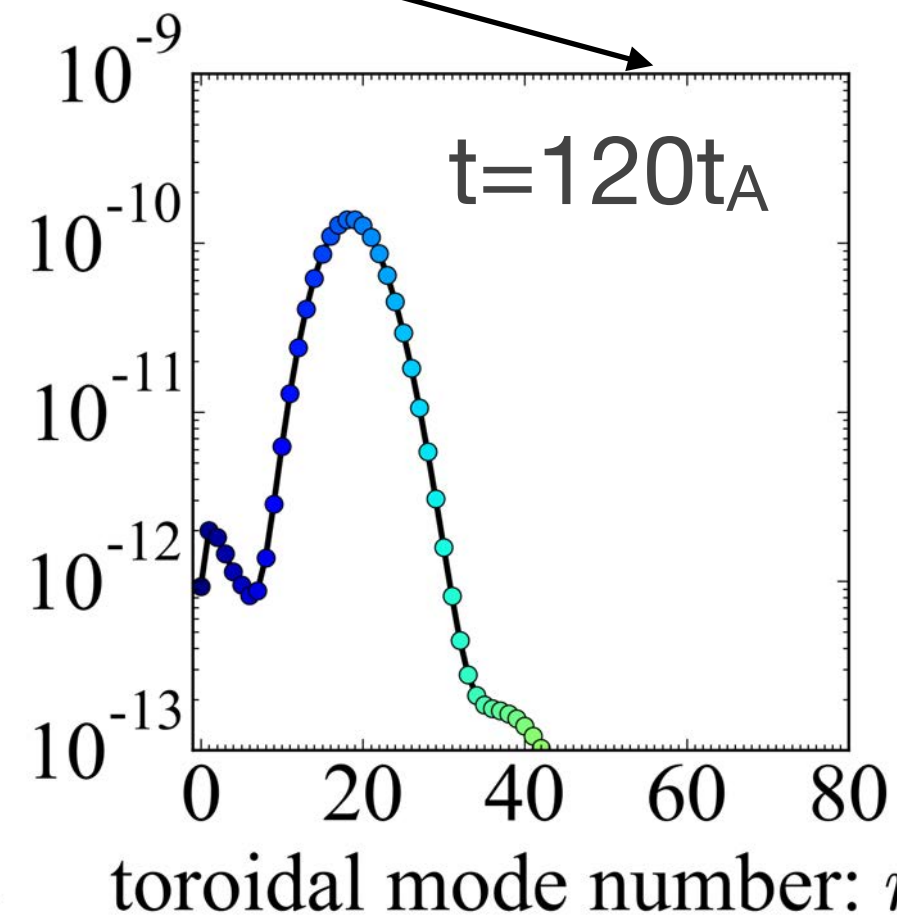
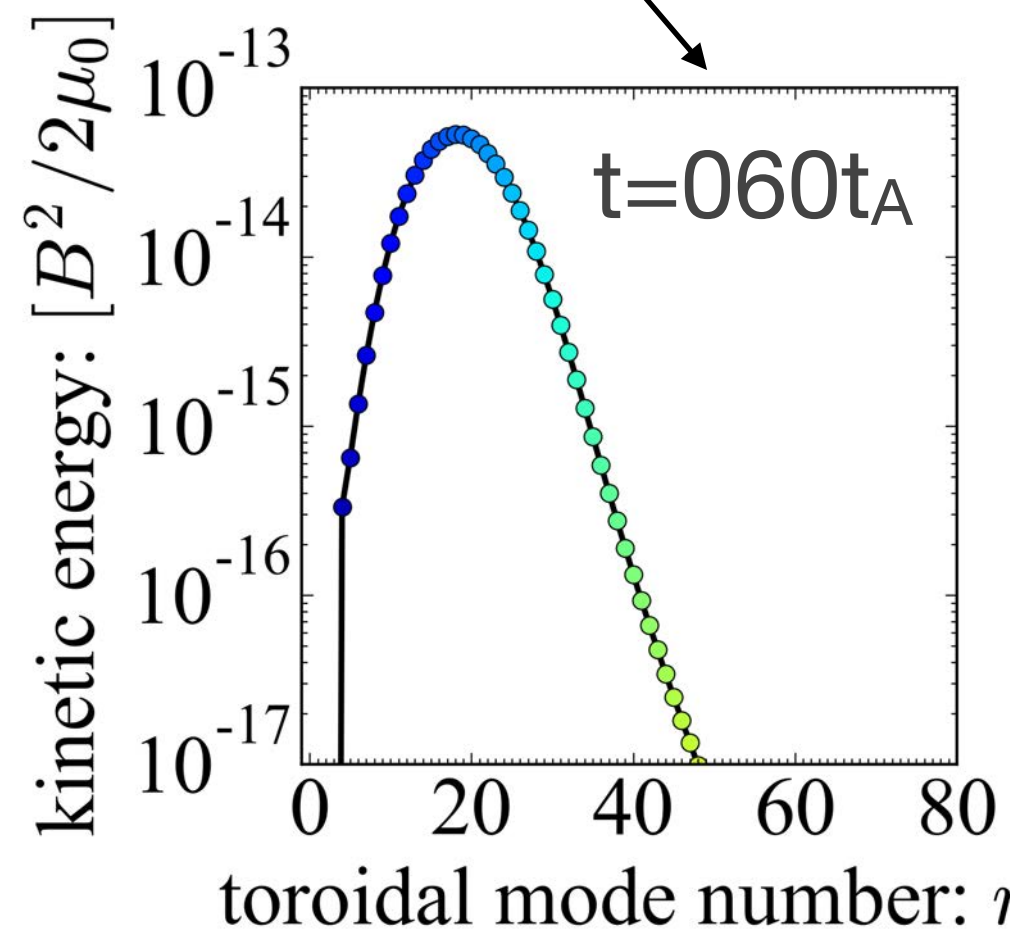
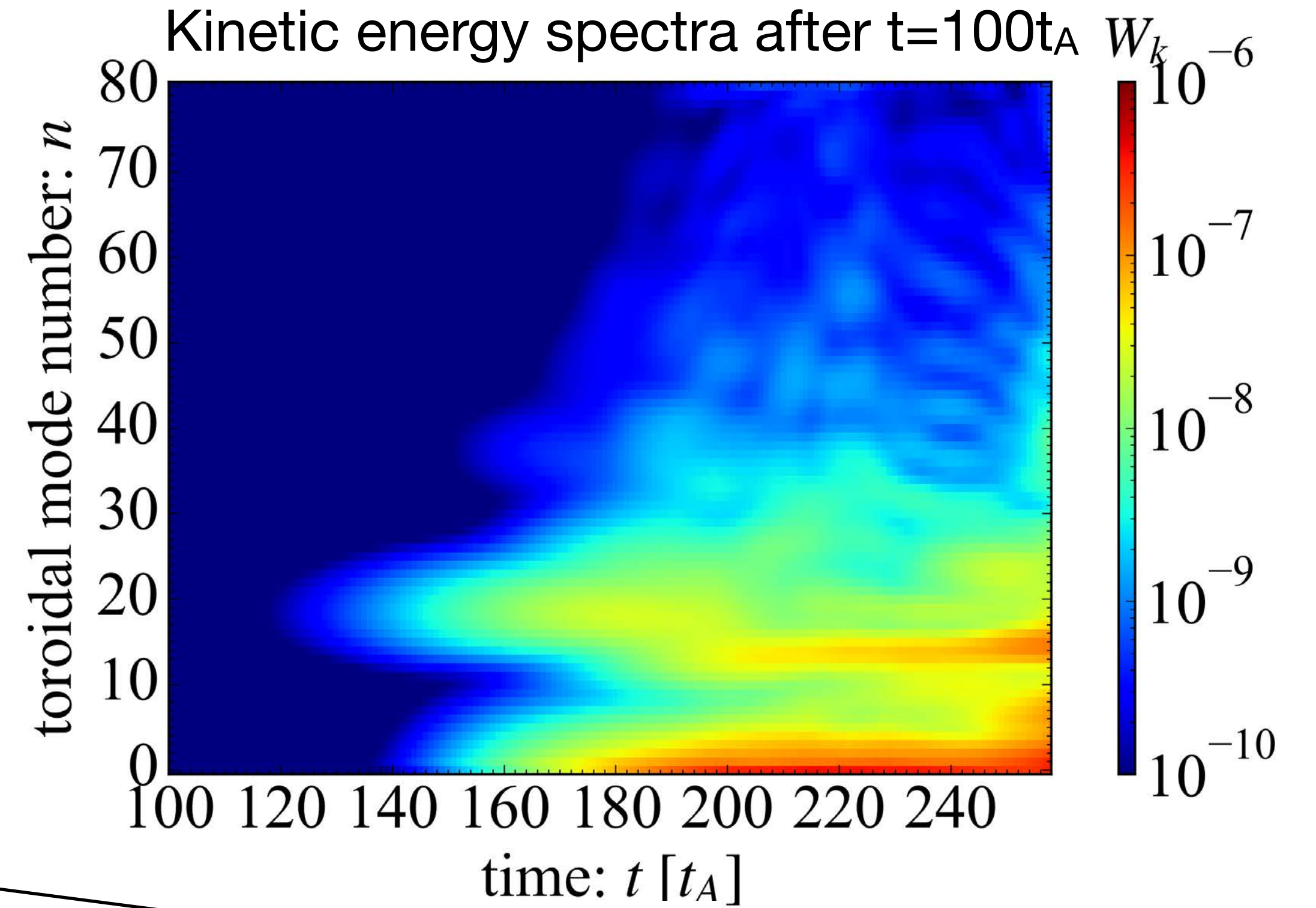
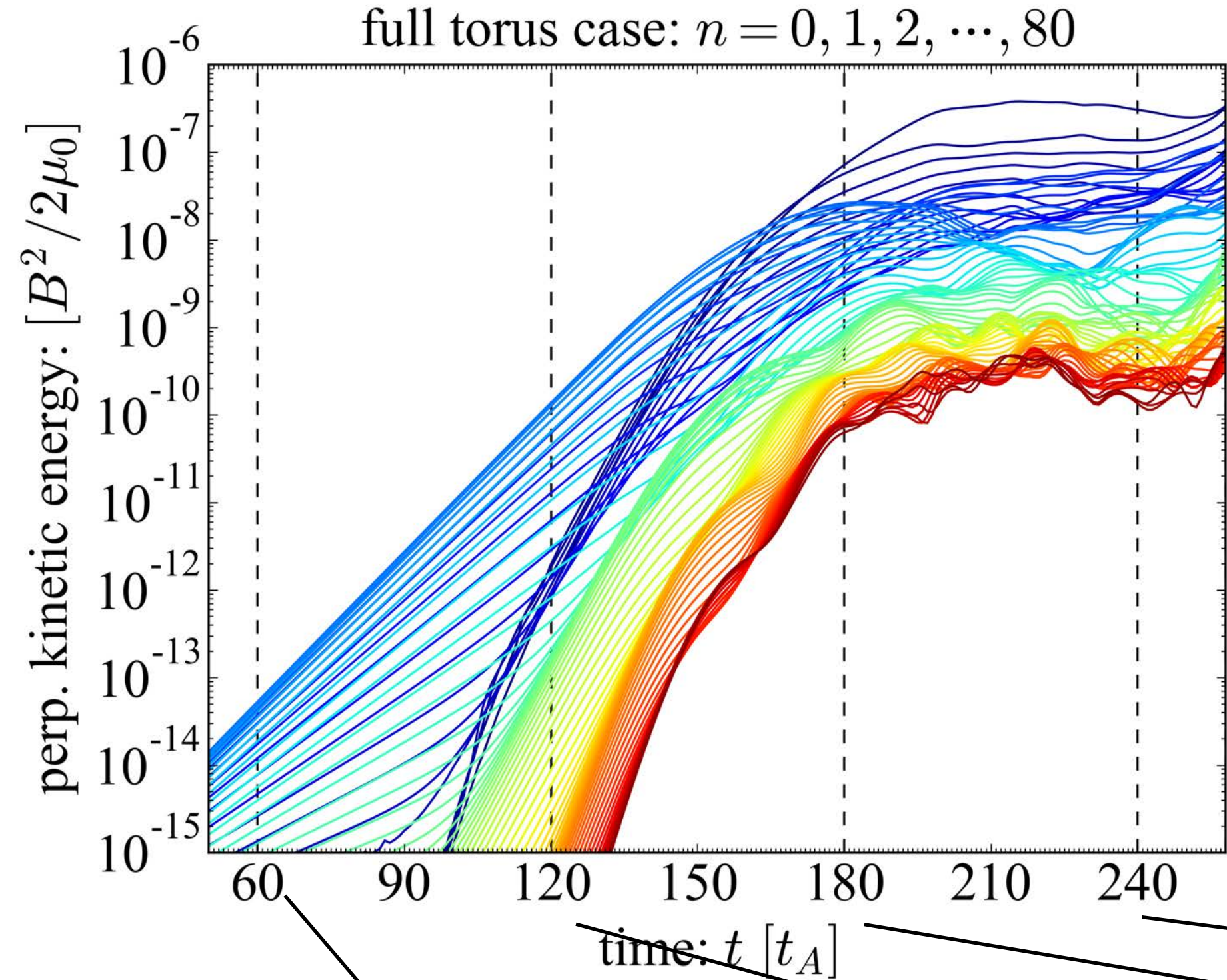


Electromagnetic oscillations (ZF \rightarrow turb.) rather than electrostatic oscillations (turb. \rightarrow ZF)

$$\text{Fluctuation intensity: } \mathcal{S}(\psi) = \sqrt{\sum_{n \neq 0} \sum_m \phi(\psi, m, n) \phi(\psi, -m, -n)}$$

- Turbulence bursts with cyclic oscillation between ZF and turbulence are observed in all cases
- $n=1$ fluctuation has global structure whose peak is located at turbulence bursts in **full** torus case

Kinetic energy grows in reasonable manner and low- n modes get excited



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