IPP INSTITUTE OF PLASMA PHYSICS
OF THE CZECH ACADEMY OF SCIENCES

Collisional sheath model

Report of the project HDsheath

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Aknowledgement to ISFN DivSOL group

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MINISTRY OF EDUCATION,
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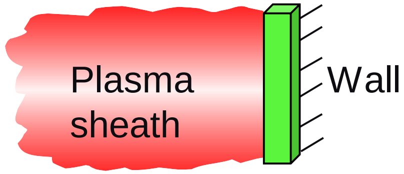
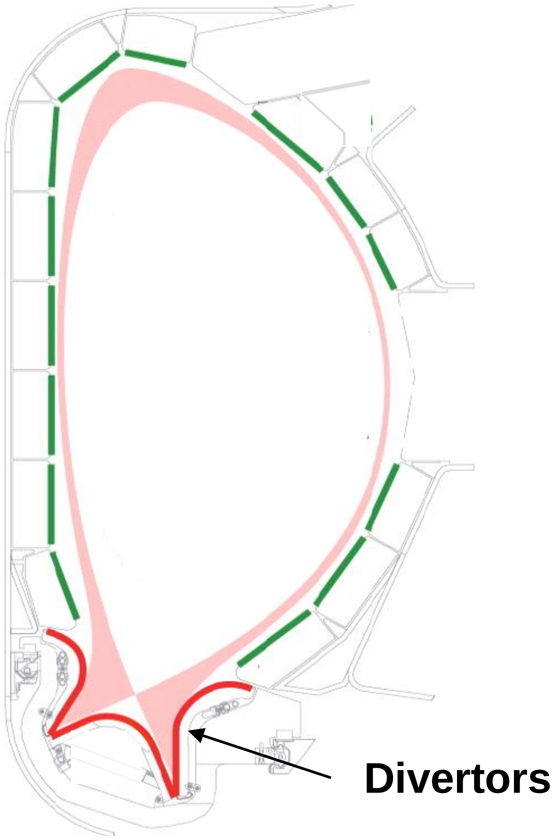
Introduction



- **Plasma sheath** exists in any plasma device and affect **plasma and heat exhaust**
- **plasma probe** theory is based on plasma sheath model



[Chodura Phys. PWI Contr. Fus. 1984]

$$\boxed{M_\parallel = \frac{V_\parallel}{c_s} = 1}$$

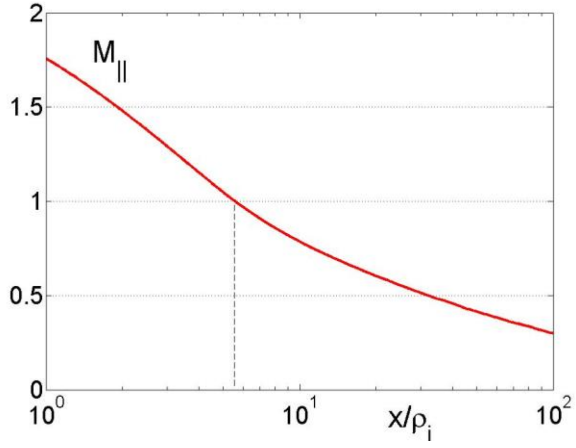
$$c_s = \sqrt{\left(T_i + Z_i T_e\right) / m_i}$$

$$\Gamma_i \sim \underline{M_\parallel} n c_s$$ **- Plasma flux density**

$$q_w \sim \underline{M_\parallel} n c_s T_e$$ **- Heat flux density**

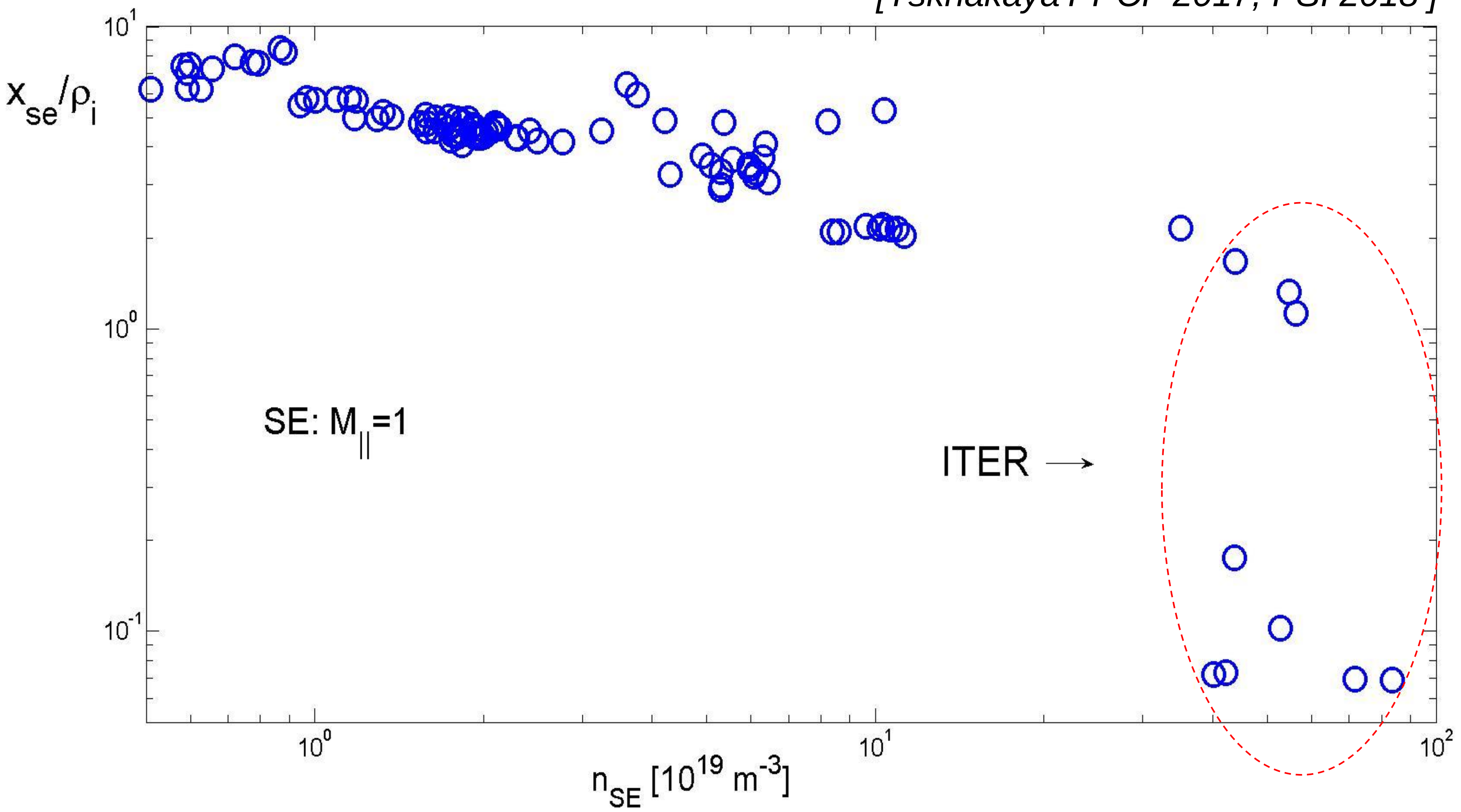
$$\Delta\phi \sim -T_e \ln\left(\underline{M_\parallel} m_e / m_i\right)$$ **- Potential drop**

Motivation



R= 0 – 1, Z_{eff} = 1 – 1.5, θ= 1.5° - 6°

[Tskhakaya PPCF 2017, PSI 2018]

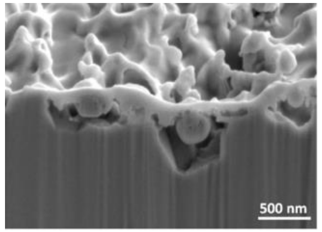


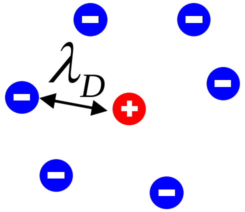
Large set of SOL kinetic modelling indicated subsonic plasma flows for ITER relevant conditions

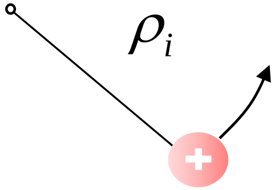
Parameters of the divertor sheath

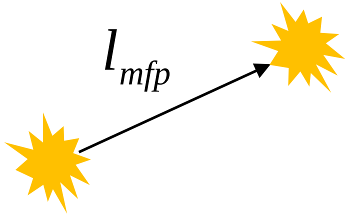
The scaling used for derivation of the classical sheath model

$$h << \lambda_D << \boxed{\rho_i << l_{mfp} \sin(\theta)}$$

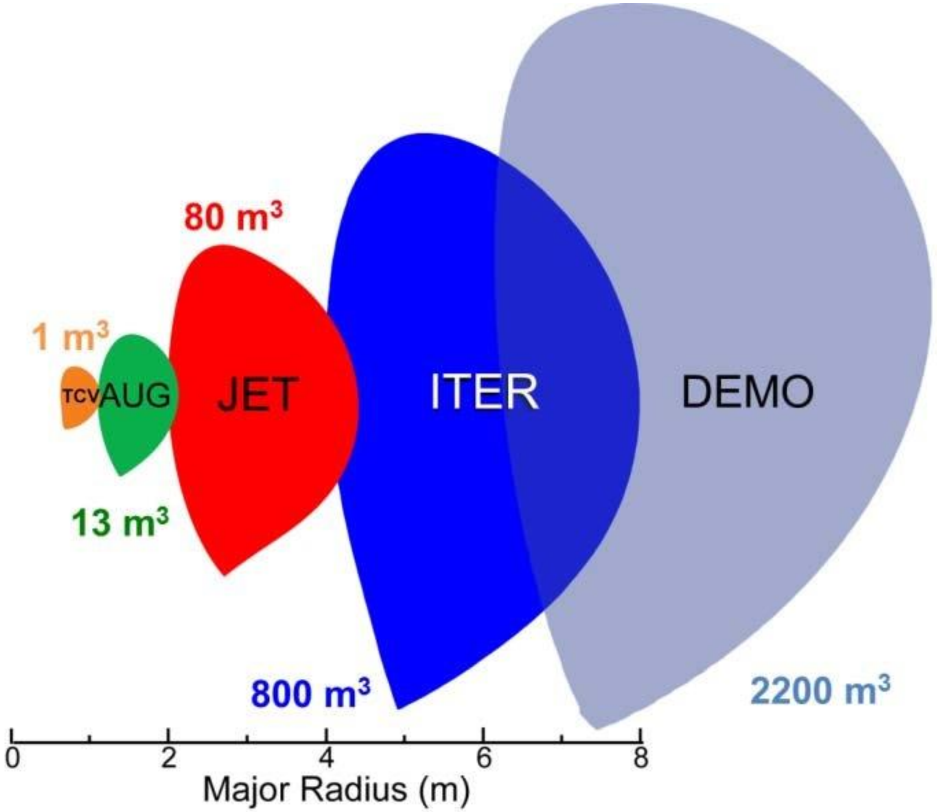








	$B_{0,\,max}$ [T]	Q= P_{fusion}/P_{inj}	Divertor sheath	
			n_{max} [10^{20} m^{-3}]	T_{min} [eV]
COMPASS	2.1	-	0.3	10
ASDEX-U	3.1	-	2	1
JET	3.5	0.67	5	1
ITER	5.2	10	50	0.3
EU DEMO	5.9	25	~100	0.2 (?)



Are sheath conditions satisfied in large fusion devices?

Sheath scaling

$$B = 5\,[T],\quad \theta = 3^\circ$$

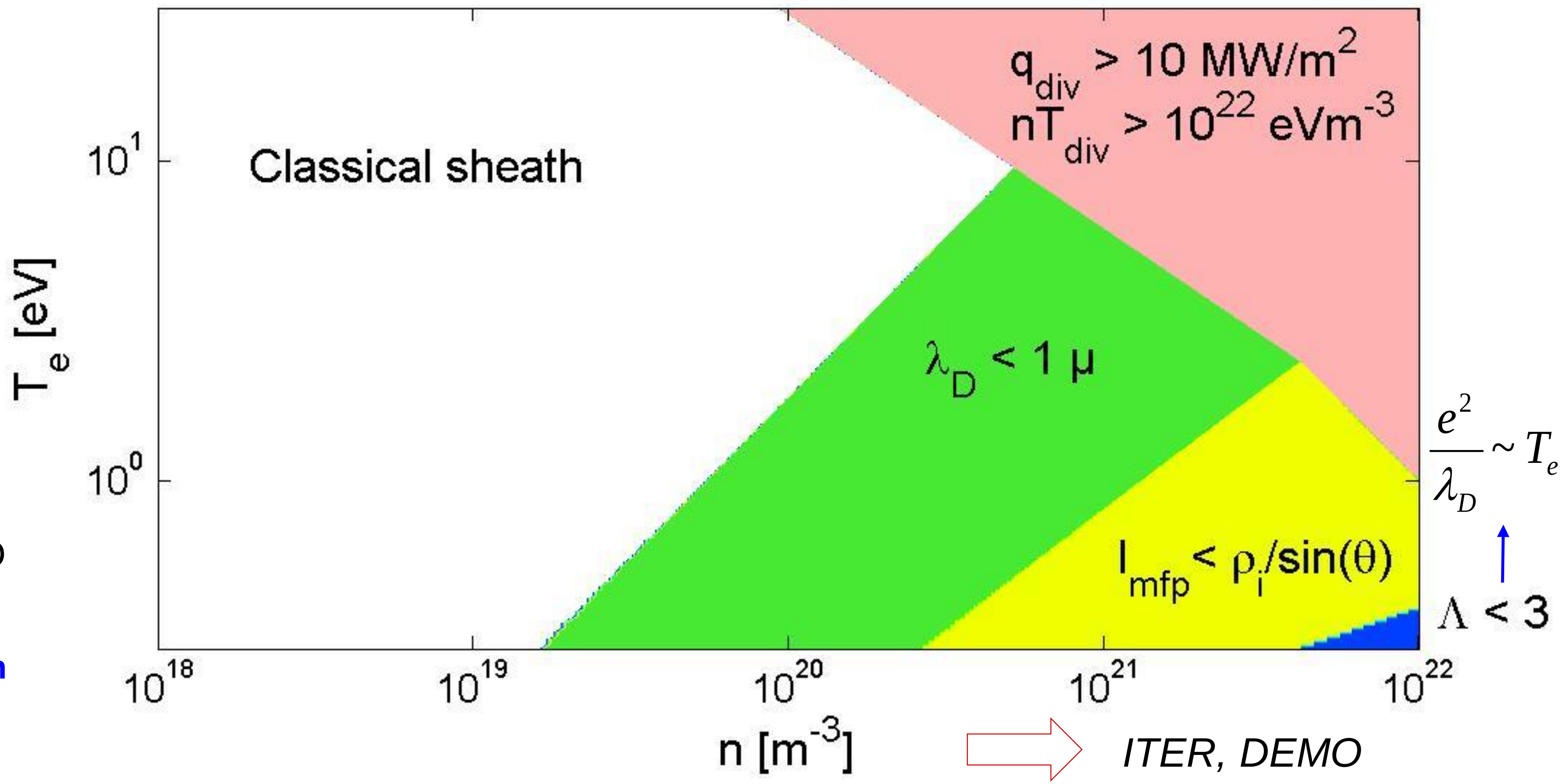
$$\Lambda = 29.9 - 0.5\ln\left(n/T_e^3\right)$$

$$q_{div} = \gamma c_s n T_e$$

$$\lambda_D = 7.4\times10^3\sqrt{\frac{T_e}{n_e}}$$

$$\rho_i = \sqrt{T_i m_i}\,/\,Z_i B$$

$$l_{mfp} = 2.0\times10^{17}\,\frac{T_i^2}{n\Lambda_{ii}}$$

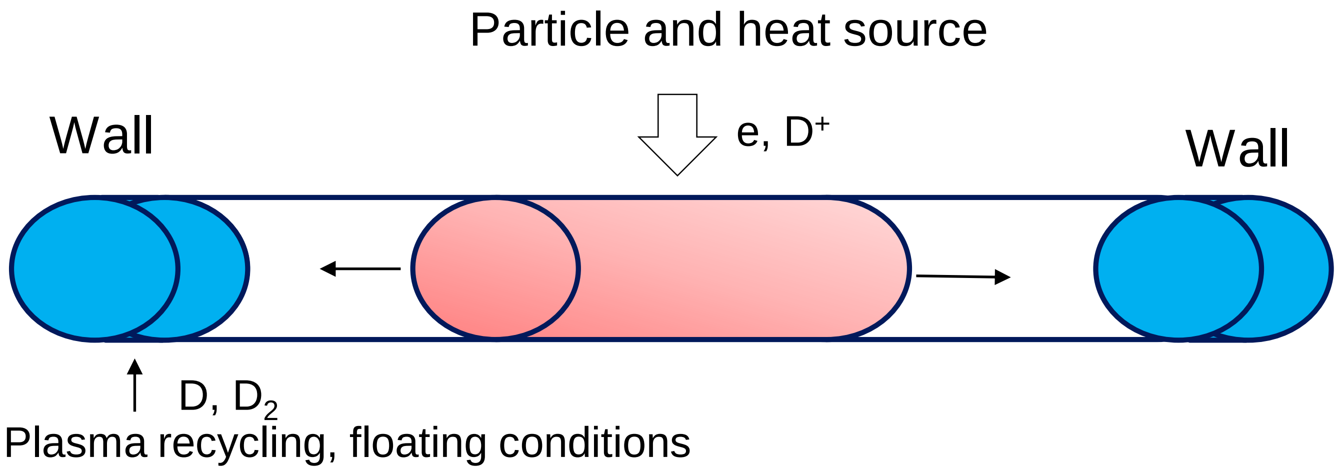


ITER and DEMO divertor sheath is **collisional** with a **rough** divertor surface

Simulation model

BIT1 – electrostatic 1D3V PIC MC code

- Number of simulated particle species is limited by only available atomic and PSI data
- Massively parallel (scaling >4x10^3)
- Nonlinear interaction between plasma, neutral and impurity particles, linear PSI; (~**10^4 processes**)



Particle and heat source

Wall

e, D^+

Wall

D, D_2

Plasma recycling, floating conditions

Atomic and molecular processes used in presented PIC simulations

M – molecule, or atom
A – atom

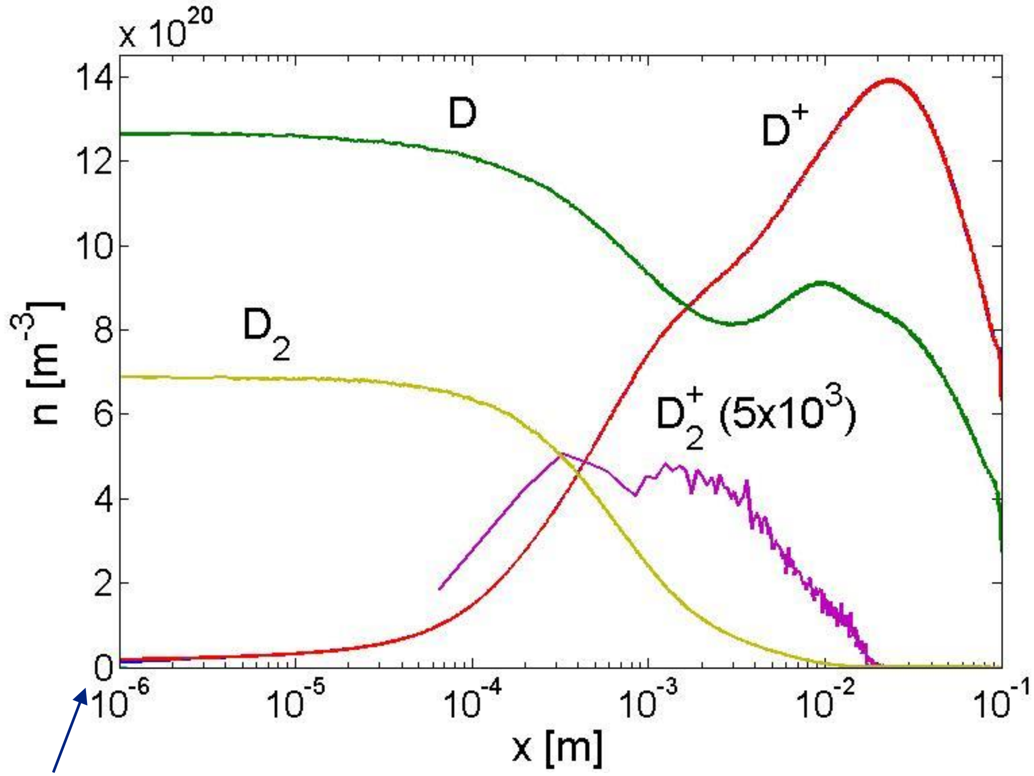
$e + M \rightarrow e + M$	Elastic		
$e + M \rightarrow e + M^*$	Excitation (electronic, vibrational, rotational)		
$e + M \rightarrow 2e + M^+$	Ionization		
$e + M \rightarrow e + A + B$	Dissociation	$A + M \rightarrow A + M$	Elastic
$e + M \rightarrow 2e + A^+ + B$	Dissociative ionization	$A + M \rightarrow A + M^*$	Excitation
$e + M^+ \rightarrow A + B$	Dissociative recombination	$A^+ + M \rightarrow A + M^+$	Charge exchange
$e + M^+ \rightarrow M + vh$	Recombination	$A + M^+ \rightarrow A^+ + M$	Charge exchange
$2e + M^+ \rightarrow e + M$	Three-body recombination	$A + M \rightarrow A + B + G$	Dissociation

PIC simulations

Each simulation took up to 3×10^{6} core hours, 10^{9} particles

Density $[x10^{20}]$	**Plasma recycling**	**Molecules**	**Three-body recombination**	**Current $[J_{sat}]$**
Low 4×10^{-3}	X	X	X	0
Moderate 0.04	X	X	X	0
High 0.17	√	X	X	0
Very high 7.4	√	√	X	0
⬆ Previous	project (KinEdge)		⬇ Present	project
Very high 6.6	√	X	X	0 – 0.24
Very high 7.8	√	√	√	0

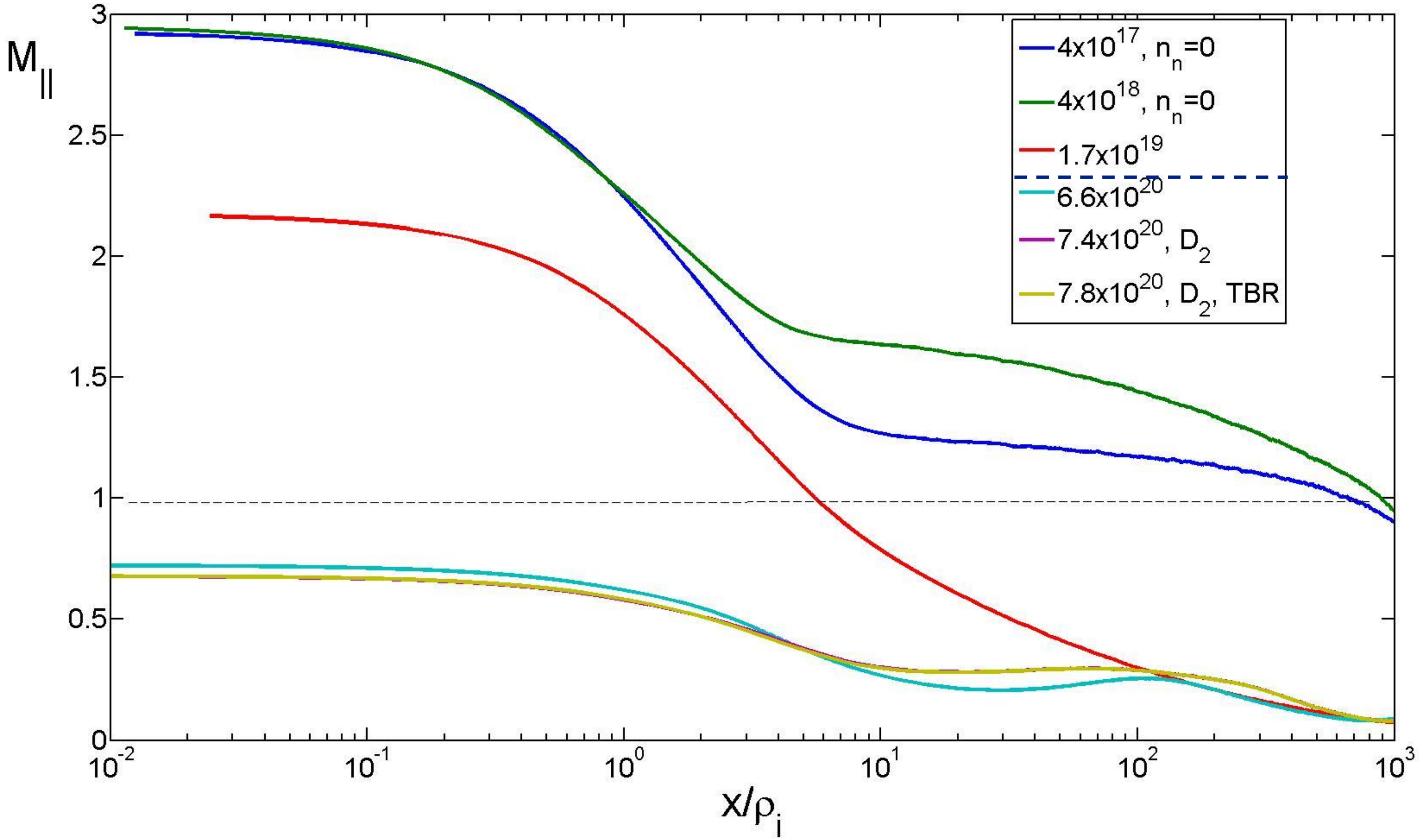
Number of runs were performed on Marconi SC too



Divertor plate

Plasma density profiles

Profiles of Mach number (I=0)



Ion flux in a high density (n_e > $5x10^{20}$ m^{-3}), cold (T_e< 2 eV) floating sheath is sub-sonic

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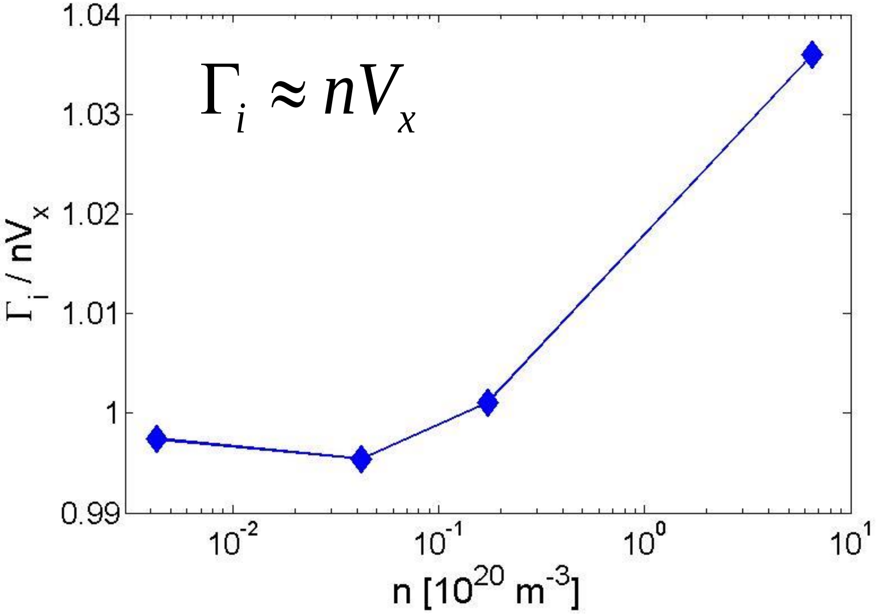
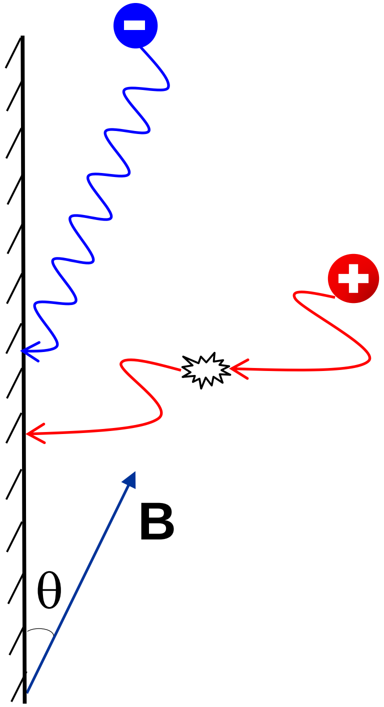
Numerical experiments

Is sheath transport diffusive?

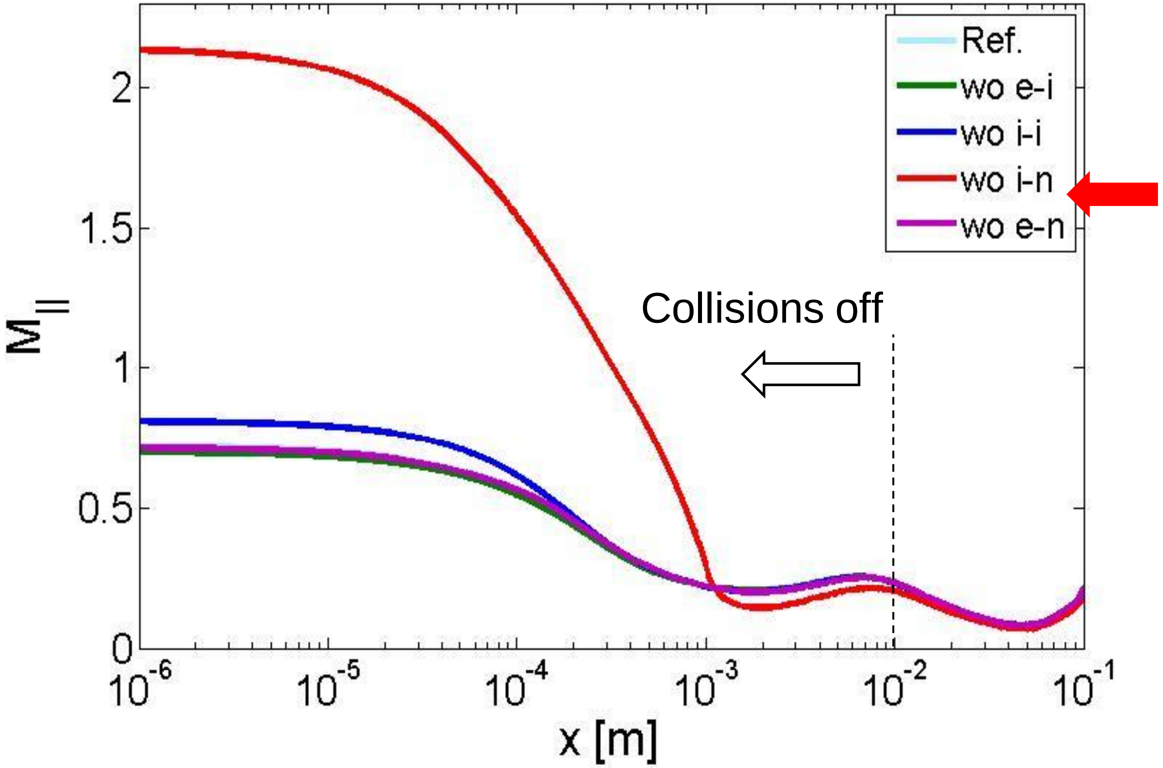
$$\Gamma_i = nV_x + \Gamma_{diff} \ ?$$



Normalized particle flux to the wall

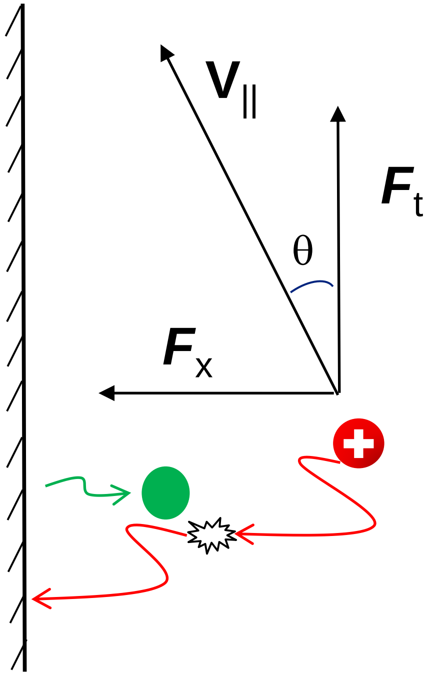


Mach number profiles



Ion-neutral friction is responsible for subsonic plasma flow

Two mechanisms influencing ion flow



Adjusts due to ambipolarity of the fluxes to the wall

$$F_x \approx \overline{eE_x} - m\upsilon_{mt} V_{\|} \sin(\theta) - m\upsilon_{ei} \frac{I_{wall}}{en}$$

$$F_t \approx -m\upsilon_{mt} V_{\|} \cos(\theta) - m\upsilon_{ei} \frac{I_{wall}}{en} \cot(\theta)$$

$$\text{if } I_{wall} \geq 0 \quad \begin{aligned} &V_x \approx c_s \sin(\theta) \\ &V_t < c_s \cos(\theta) \\ &V_{\|} = V_x \sin(\theta) + V_t \cos(\theta) < c_s \end{aligned}$$

$$\text{if } I_{wall} < 0, \quad ?$$

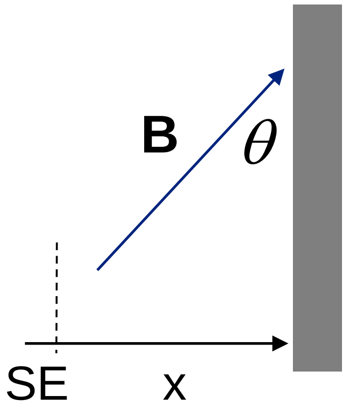
Sub-sonic flow

Analytic model

Particle and momentum conservation equations

$$\vec{\nabla} n \vec{V} = 0, \quad E = \{-T_e \partial_x n / ne,\ 0,\ 0\} \qquad \cancel{\upsilon_i n}, \quad \left|\nabla \cancel{\ln T}\right| << \left|\nabla \ln n\right|, \quad \left|\cancel{\vec{\nabla} \vec{\vec{\pi}}}\right| \sim \tau_{col},$$

$$m_i \vec{V} \vec{\nabla} \vec{V} = e\vec{E} + e\vec{V} \times \vec{B} - T\vec{\nabla} n / n - m_i \upsilon_i \vec{V} - m_i \upsilon_{mt} \left(\vec{V} - \vec{V}^n\right) - m_i \upsilon_{ei} \vec{J} / en$$



1D sheath edge (SE): the point, nearest to the wall, where main ions are magnetized

$$\partial_x \delta\vec{V},\ \delta\vec{V} \to 0, \quad \delta\vec{V} = \vec{V} - \vec{b} V_{\parallel}$$

➡

Boundary condition at the sheath edge

$$\boxed{M_{\parallel} = 1 + \chi - \sqrt{\chi^2 + 2\chi}}$$

➡

$$\upsilon_{mt}, \upsilon_{ei} \to 0, \qquad M_{\parallel}^2 \to 1$$

$$\upsilon_{mt}, \upsilon_{ei} > 0,\ I \sim 0,\ M_{\parallel}^2 < 1$$

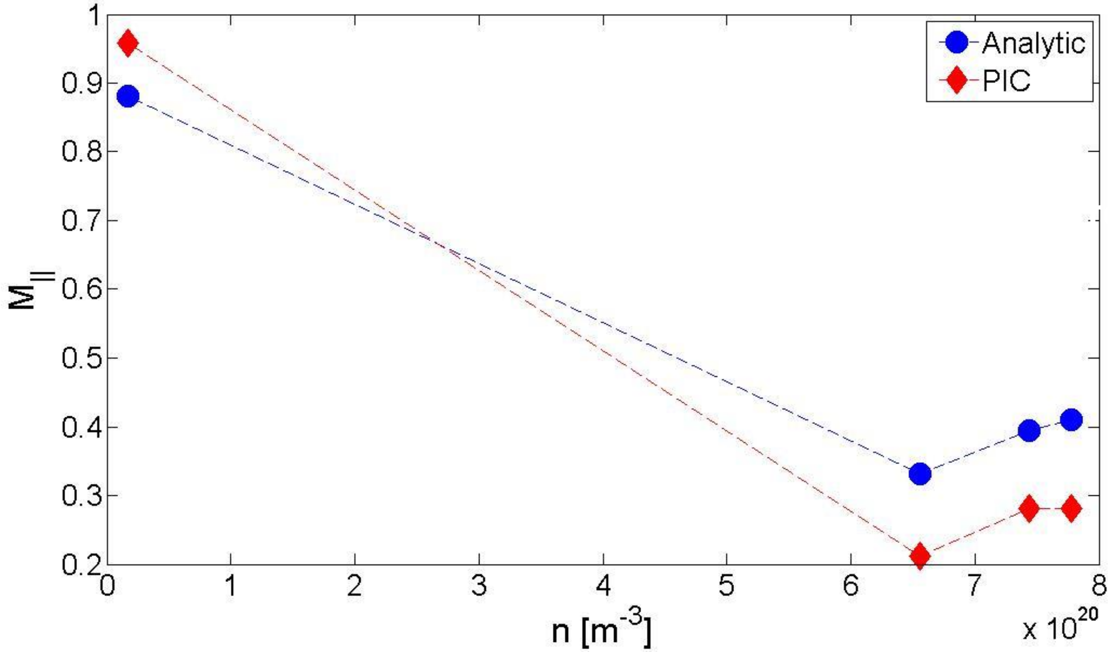
Boundary conditions depend on the **sheath collisionality** as well as on the **current**

$$\chi = \frac{\left(\upsilon_{mt}(1-\alpha) + \upsilon_{ei} I'\right) x_{wall}}{2c_s \sin(\theta)}, \qquad \alpha = V_{\parallel}^n / V_{\parallel}, \quad I' = I / I_{isat}$$

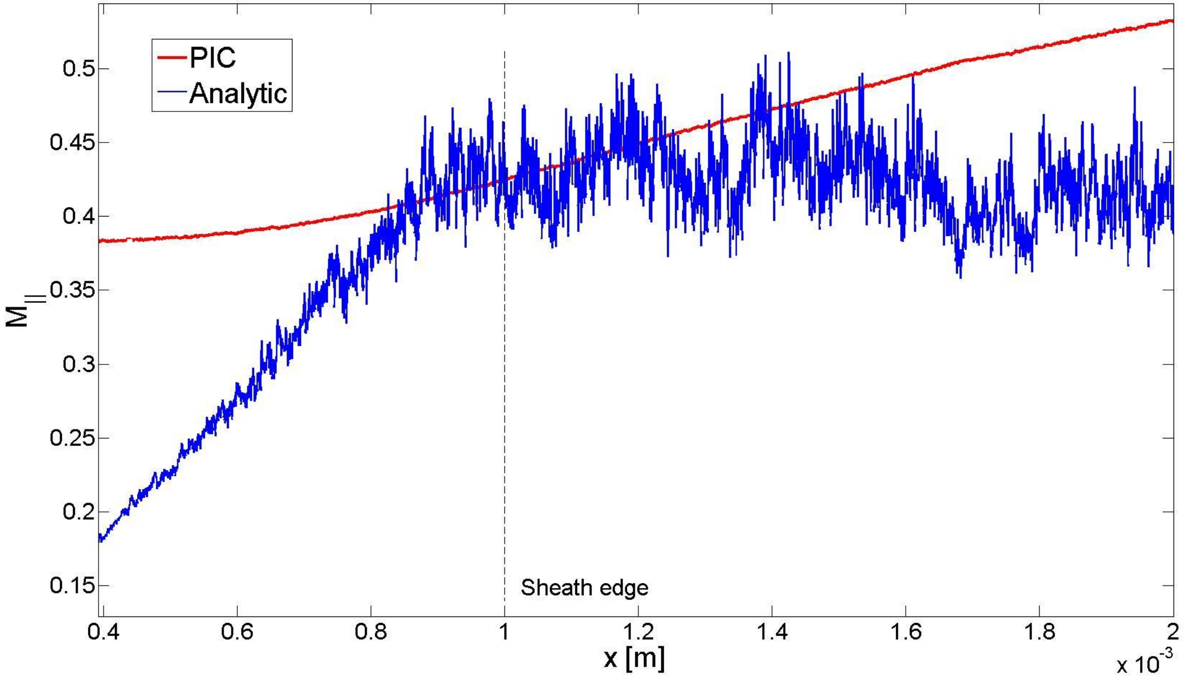
$$M_x(x_0) = \sin(\theta), \quad x_0 \approx x_{wall}$$

comparison with PIC results (I=0)

$$M_{\|} = 1 + \chi - \sqrt{\chi^2 + 2\chi} \qquad \chi = \frac{\upsilon_{mt}\left(1 - V_{\|}^D / V_{\|}\right)}{2c_s \sin(\theta)} x_{wall}$$

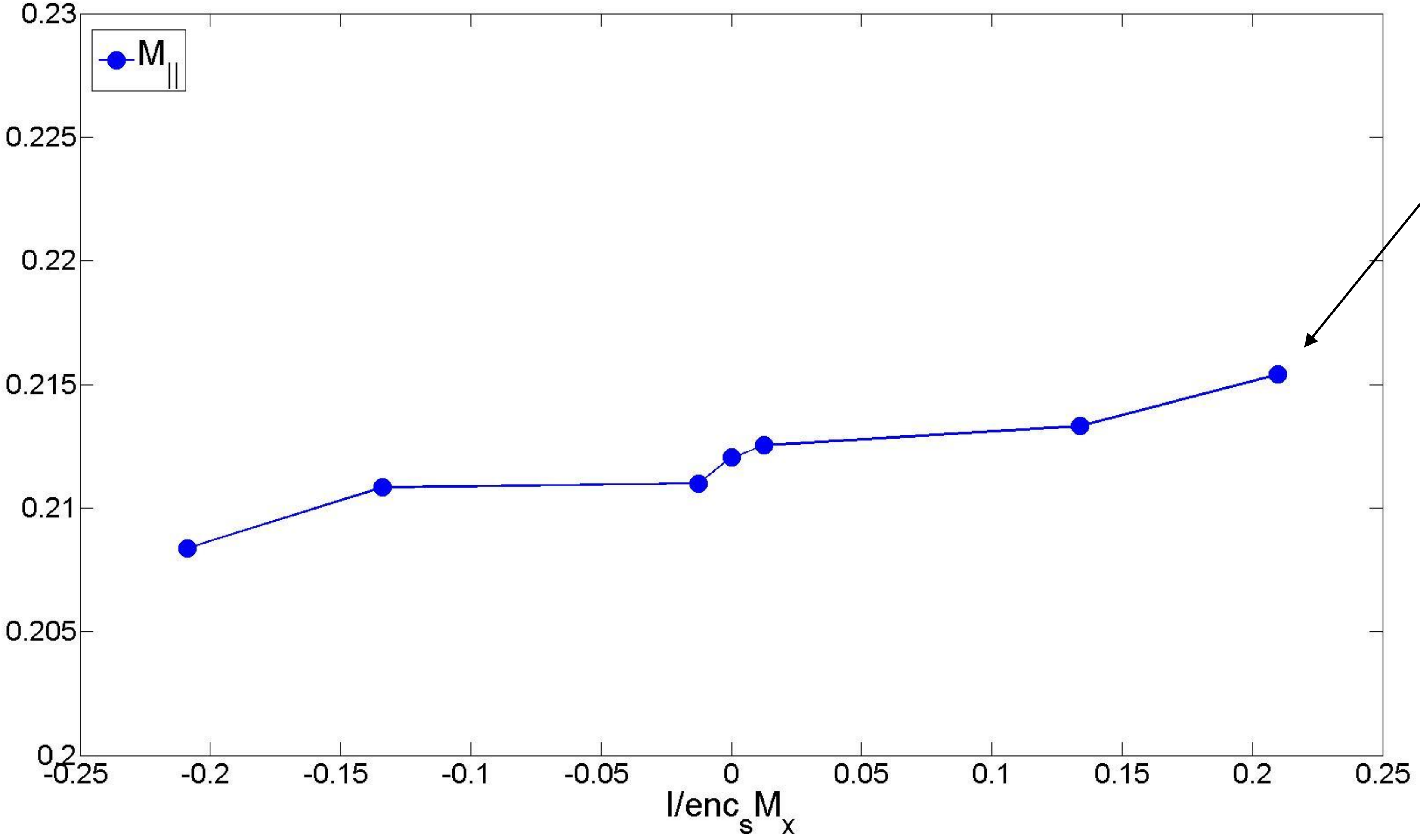


From sheath modelling *[Tskhakaya 47th EPS]*



From SOL modelling *[Tskhakaya PSI 2022]*

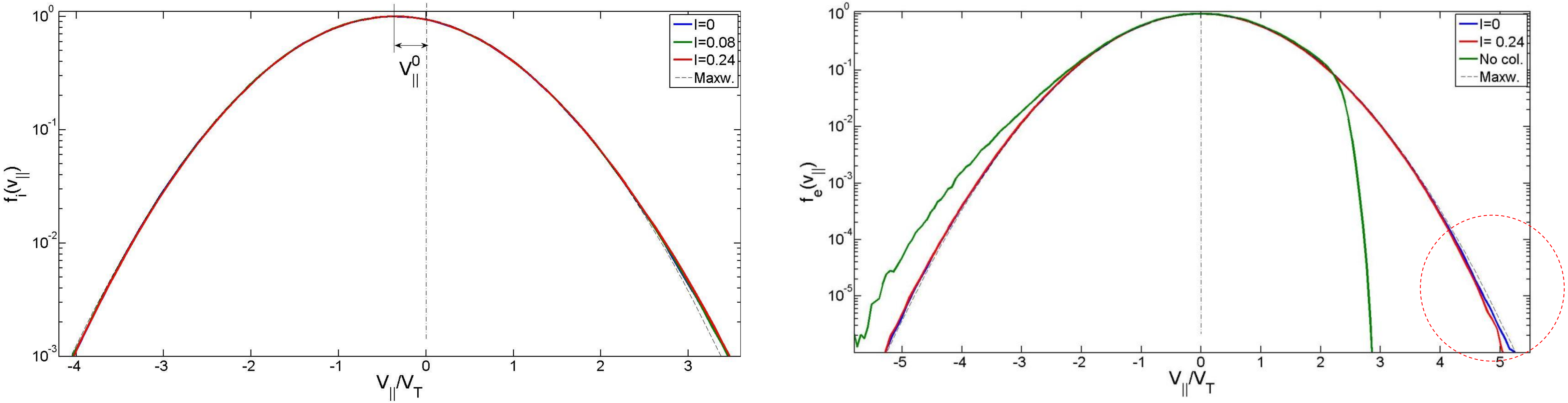
Currents in the sheath



50 V biasing >> $T_{e,div} \sim 1$ eV

On electron-ion friction force at the SE

Electron and ion (D^+) VDFs at the high collisional sheath edge for different current regimes ($I = J/J_{sat}$)



$$R_{\parallel}^{ei} = -mn\upsilon_{ei}\left(V_{\parallel}^{i} - \cancel{V_{\parallel}^{e}}\right) \Rightarrow -mn\upsilon_{ei}V_{\parallel}^{i}$$

Electron-ion friction at the sheath edge is **independent** of the plasma current regime

Consequences

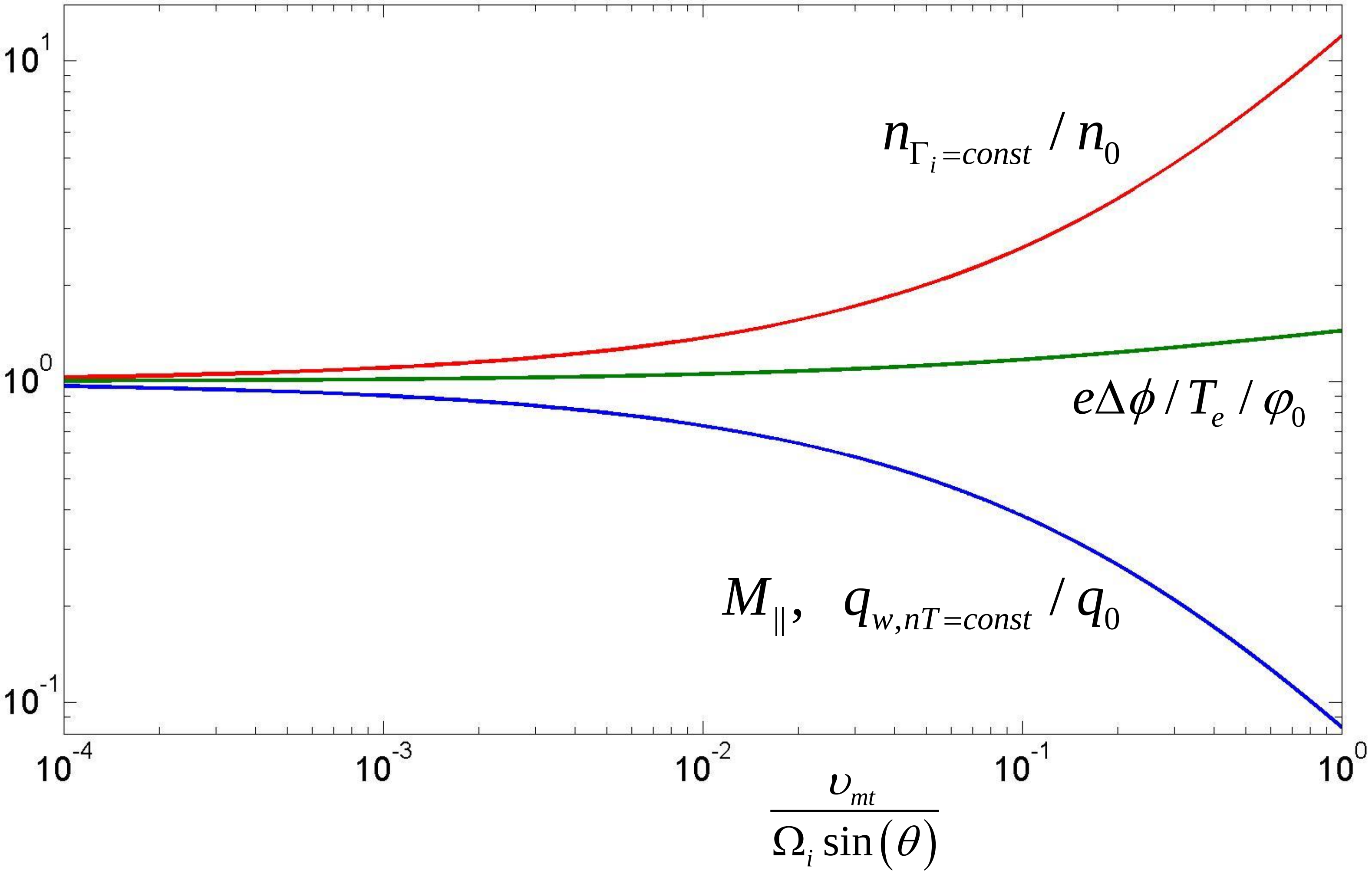
$$\Gamma_i = M_\parallel n c_s$$

$$q_w = \gamma \Gamma_i T_e, \quad \gamma \sim 10$$

$$e\Delta\phi / T_e = \varphi_0 - 0.5 \ln\left(M_\parallel\right)$$

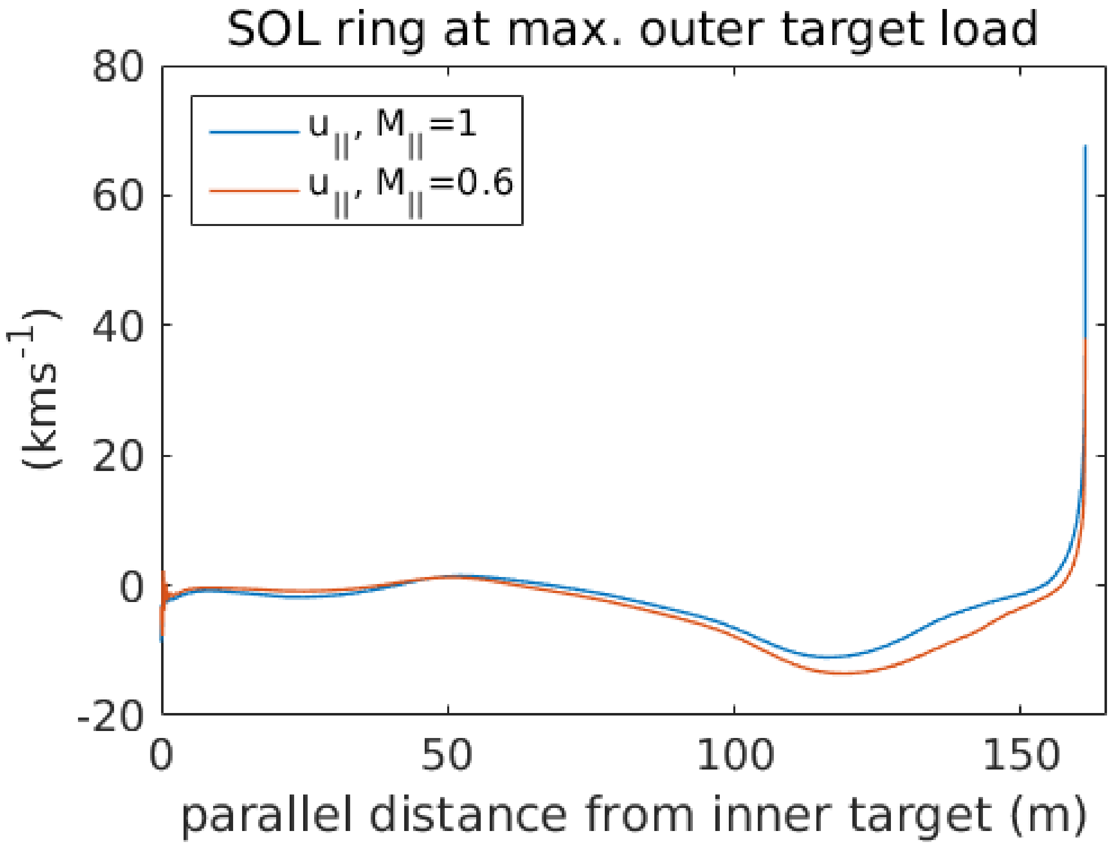
$$M_\parallel = 1 + \chi - \sqrt{\chi^2 + 2\chi}$$

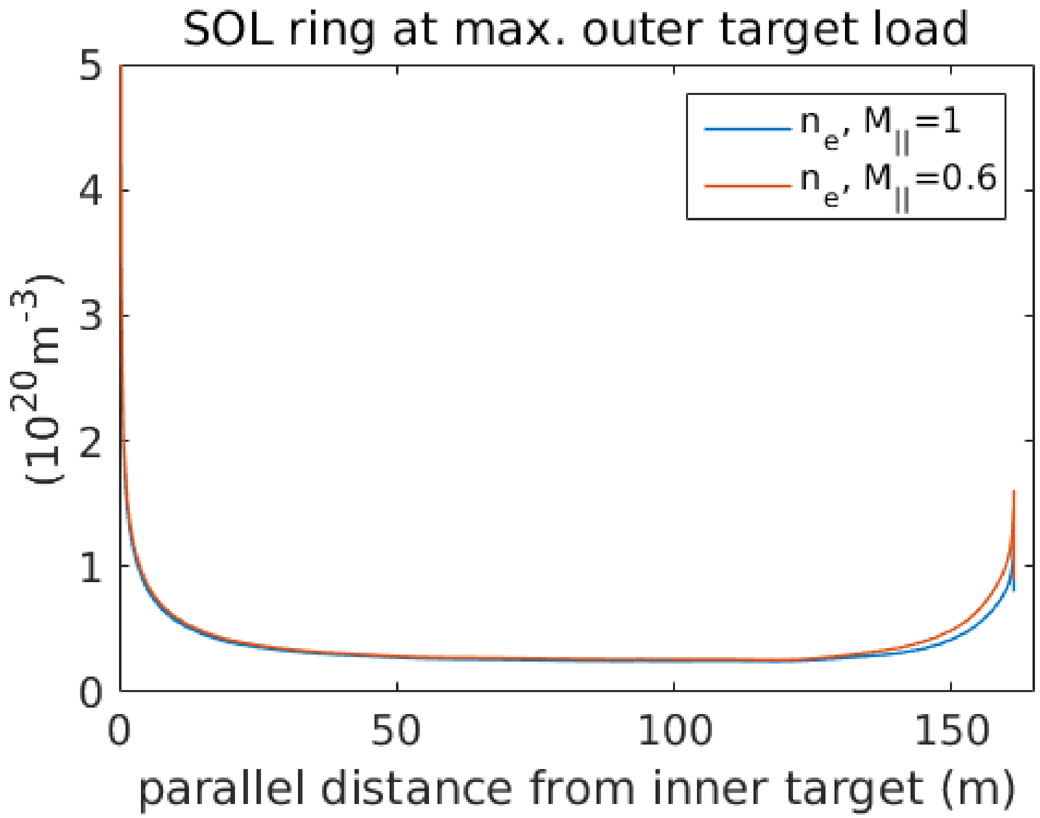
$$\chi = \frac{\upsilon_{mt}\left(1 - V_\parallel^D / V_\parallel\right)}{2 c_s \sin\left(\theta\right)} \; x_{wall}$$



Implementation of collisional sheath in SOLPS-ITER

[D. Moulton, ISFN DivSOL, 2021]





SOLPS-ITER simulation show no changes in particle flux, but increasing of density in divertor and near x-point plasma.

Summary

- A new model of high density collisional plasma sheath (CPS) has been developed. CPS will form mainly in front of divertor tiles in next generation fusion devices (ITER, DEMO,…) and will be characterized by sub-sonic plasma flows.

- A new definition of the magnetic sheath entrance is proposed:

a nearest point to the wall surface, where plasma is still magnetised.

- The Mach number at the sheath entrance depends on the ion-neutral collisionality and reduces to the Bohm–Chodura condition, $M_{||} = 1$, for collisionless case.

- New model of the sheath predicts significant increase of divertor plasma density and/or reduction of plasma and heat fluxes to the divertor plates. As a consequence, the PSI processes can be affected.

- Simulations performed under JFRS project *HDsheath* confirm our analytic findings and helped to explain why electron-ion friction can be neglected in the CPS model.

We thank our colleagues from IFERC-CSC for their support