

CRM and photon tracing module in EIRENE

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H colrad or the CRM in Eirene

- Based on the collisional radiative model by Sawada (1995)
- Resolve population coefficients for each Eirene cell
- Derive effective rates such as effective ionization and recombination rates to be used in EIRENE
- Provide population densities of excited species as a bulk ion species for the Photon tracing module



CRM for atomic hydrogen

$$\frac{dn_H(p)}{dt} = - \left[\sum_{p>q} (A_{(p,q)} + F_{(p,q)} n_e) + S_{(p)} n_e + \sum_{p<q} C_{(p,q)} n_e \right] n_{H(p)} + \sum_{q<p} C_{(q,p)} n_e n_{H(q<p)} \\ + \sum_{q>p} [A_{(q,p)} + F_{(q,p)} n_e] n_{H(q>p)} + [\alpha(p) n_e + \beta(p)] n_e n_{H^+} + \Gamma_{H(p)}$$

A = Spontaneous emission rate

F = De-excitation rate coefficient

S = ionization rate coefficient

C = Excitation rate coefficient

α = 3-body recombination rate coefficient

β = radiative recombination rate



CRM in steady-state, matrix form

$$\begin{bmatrix} -(S_{(1)} + \sum C_{(1)}) n_e & A_{(1,p)} + F_{(1,p)} n_e & A_{(1,q)} + F_{(1,q)} n_e & (\alpha_{(1)} n_e + \beta_{(1)}) n_e \\ C_{(1,p)} n_e & -A_{(1,p)} - (S_{(p)} + F_{(1,p)} + C_{(p,q)}) n_e & A_{(p,q)} + F_{(p,q)} n_e & (\alpha_{(p)} n_e + \beta_{(p)}) n_e \\ C_{(1,q)} n_e & C_{(p,q)} n_e & -\sum A_{(q)} - (S_{(q)} + \sum F_{(q)}) n_e & (\alpha_{(q)} n_e + \beta_{(q)}) n_e \\ S_{(1)} n_e & S_{(p)} n_e & S_{(q)} n_e & -(\sum \alpha n_e + \sum \beta) n_e \end{bmatrix} \begin{bmatrix} n_H \\ n_{H(p)} \\ n_{H(q)} \\ n_{H^+} \end{bmatrix} = \begin{bmatrix} -\Gamma_H \\ -\Gamma_{H(p)} \\ -\Gamma_{H(q)} \\ -\Gamma_{H^+} \end{bmatrix}$$

$$\begin{bmatrix} -A_{(1,p)} - (S_{(p)} + F_{(1,p)} + C_{(p,q)}) n_e & A_{(p,q)} + F_{(p,q)} n_e \\ C_{(p,q)} n_e & -\sum A_{(q)} - (S_{(q)} + \sum F_{(q)}) n_e \end{bmatrix} \begin{bmatrix} n_{H(p)} \\ n_{H(q)} \end{bmatrix} = \begin{bmatrix} C_{(1,p)} n_e \\ C_{(1,q)} n_e \end{bmatrix} n_H + \begin{bmatrix} (\alpha_{(p)} n_e + \beta_{(p)}) n_e \\ (\alpha_{(p)} n_e + \beta_{(p)}) n_e \end{bmatrix} n_{H^+} + \begin{bmatrix} -\Gamma_{H(p)} \\ -\Gamma_{H(q)} \end{bmatrix}$$

$$\begin{bmatrix} n_{H(p)} \\ n_{H(q)} \end{bmatrix} = R_1 n_H + R_0 n_{H^+} + R_{ext}$$

R_1 , R_0 and R_{ext} are population coefficients

General EIRENE test case for testing rate coefficients

- EIRENE 2D grid with n_e and T_e varied along x and y dimensions
- n_e , 20 points, $\sim 1e8 - 1e16 \text{ cm}^{-3}$
- T_e , 200 points, $\sim 0.5 - 1e4 \text{ eV}$

The effective ionization rate of H colrad line perfectly with AMJUEL

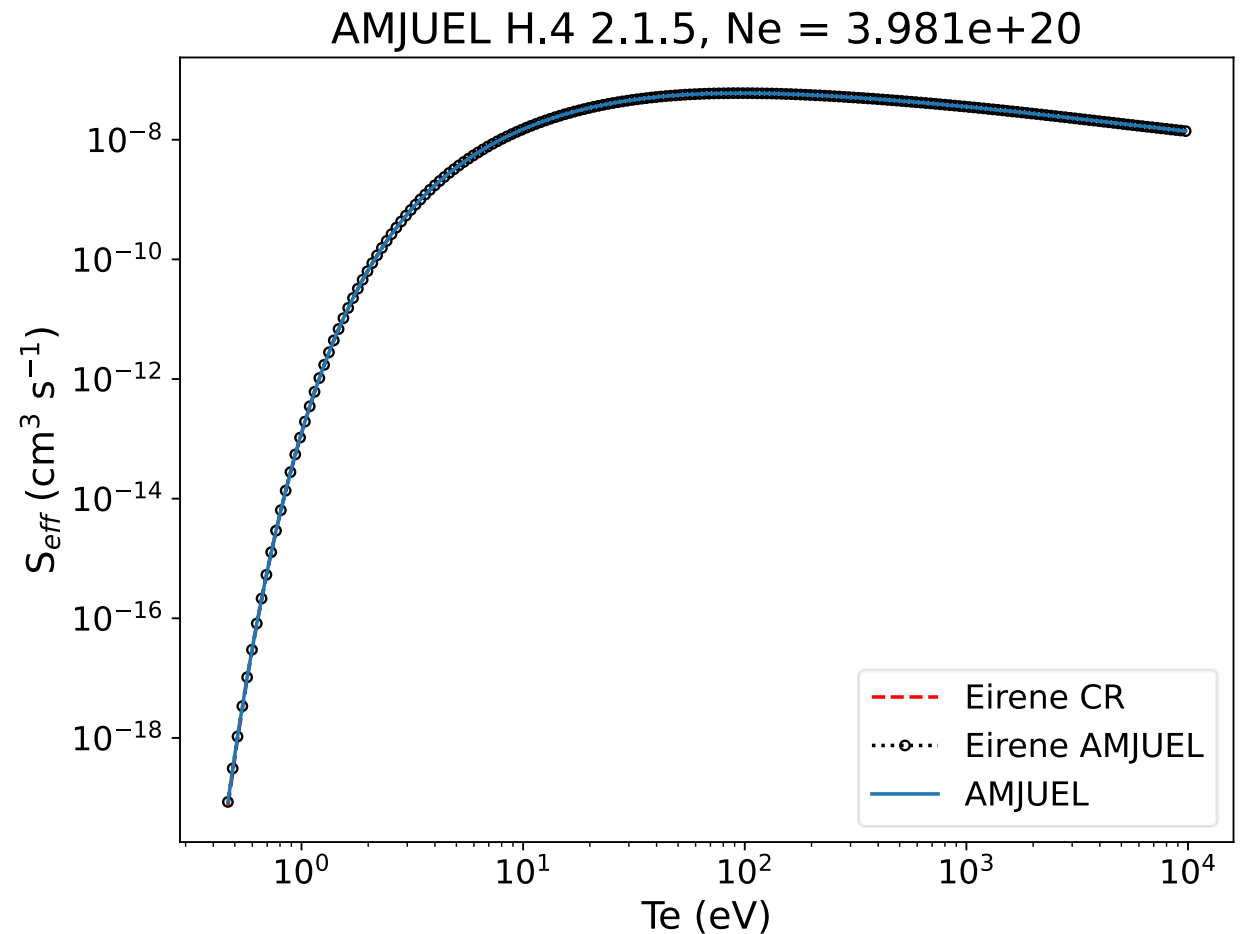
Effective ionization rate S_{eff} :

$$S_{\text{eff}} = S_{(1)} + \sum_p \left(C_{(1,p)} - R_{1(p)} \left(F_{(p,1)} + \frac{A_1}{n_e} \right) \right)$$

Eirene CR -> H colrad

Eirene AMJUEL -> using AMJUEL rates within Eirene

AMJUEL – last update entry: May 18

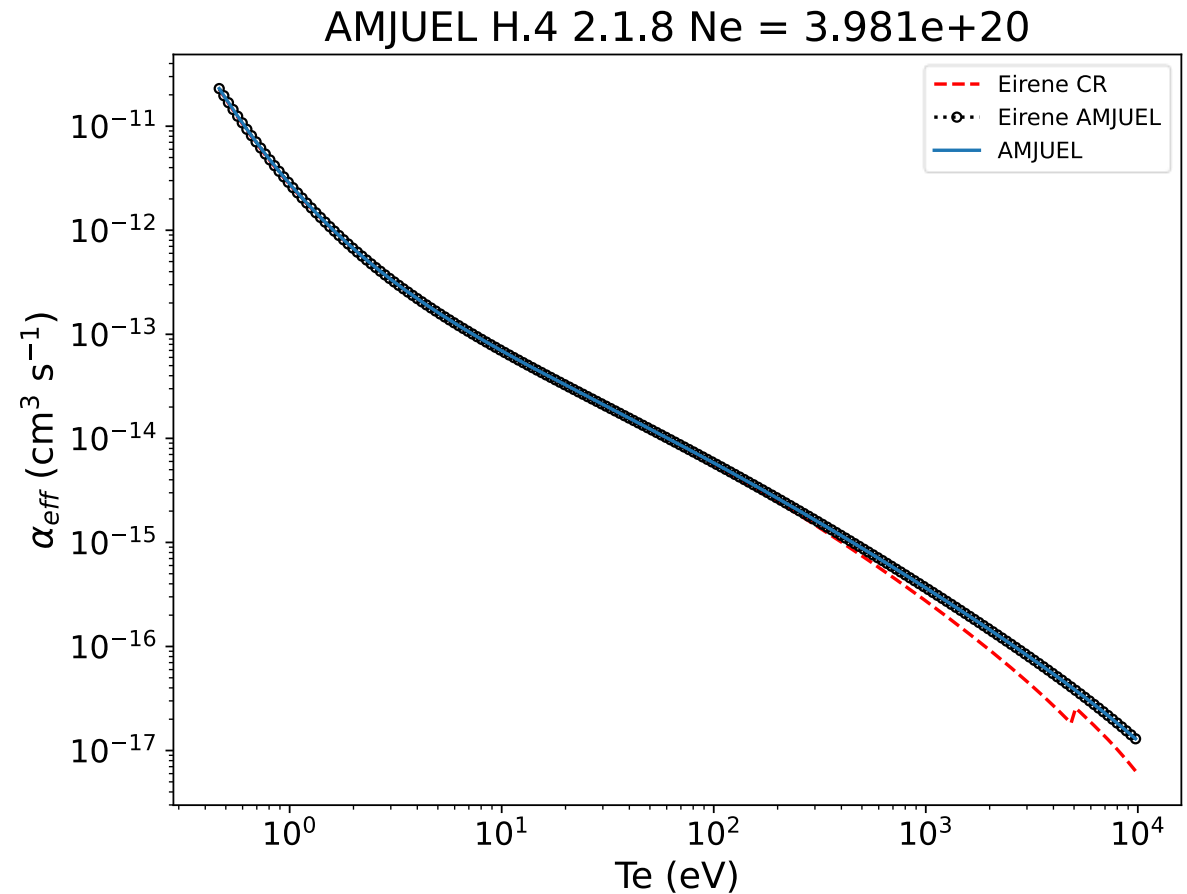


The H colrad recombination rate starts to diverge at $T_e > 400$ eV

Effective recombination rate α_{eff} :

$$\alpha_{\text{eff}} = \alpha_1 n_e + \beta_1 + \sum_p R_{0(p)} \left(F_{(p,1)} + \frac{A_1}{n_e} \right)$$

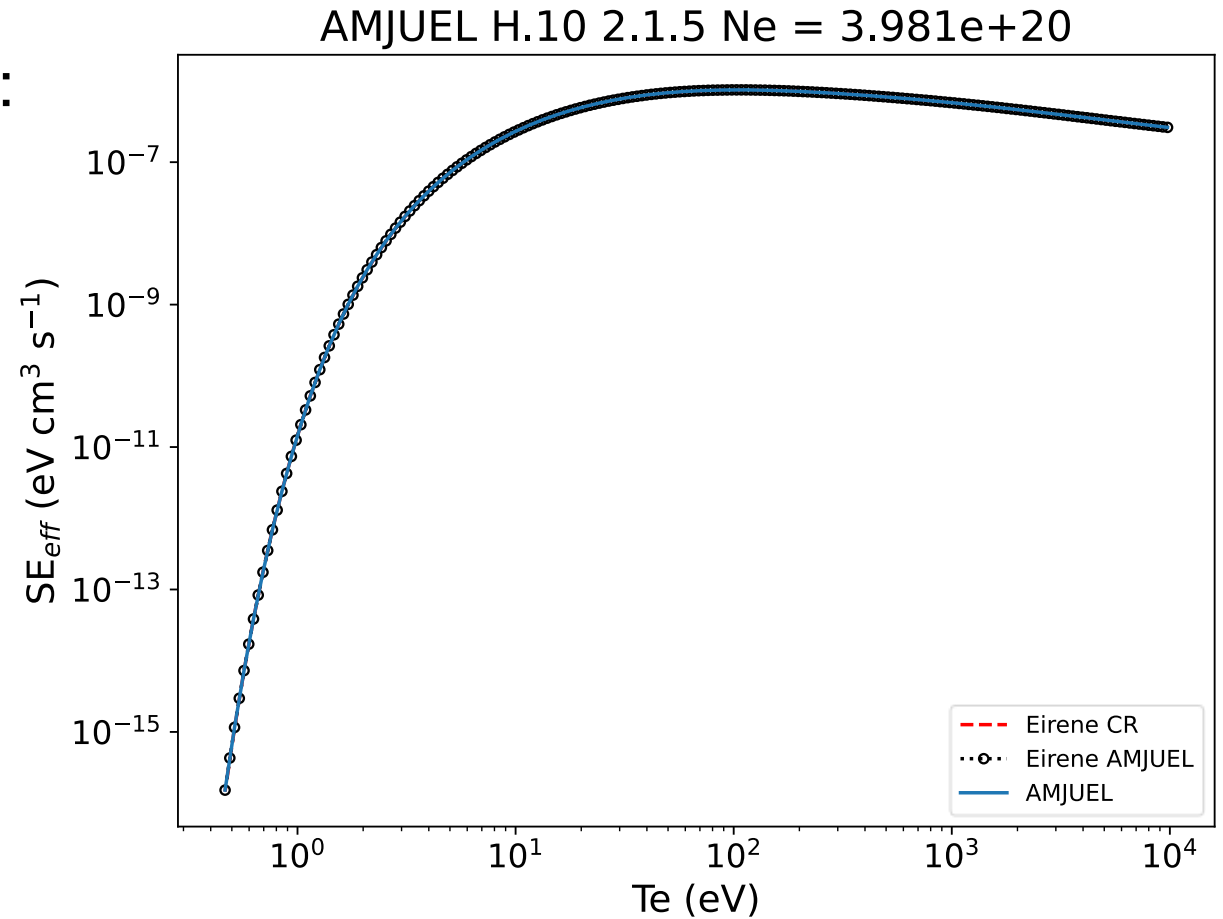
- Discrepancy can be due to the different expressions of the recombination rate coefficient



The effective ionization cooling rate of H colrad line perfectly with AMJUEL

Effective ionization cooling rate SE_{eff} :

$$SE_{\text{eff}} = S_{(1)}E_{\alpha} + \sum_p \left(R_{1(p)} (S_p E_{p-\alpha} + C_{(1,p)} E_p) \right) + \sum_{q>p} \left(R_{1(p)} C_{p,q} E_{p-q} - R_{1(q)} F_{(p,q)} E_{p-q} \right)$$

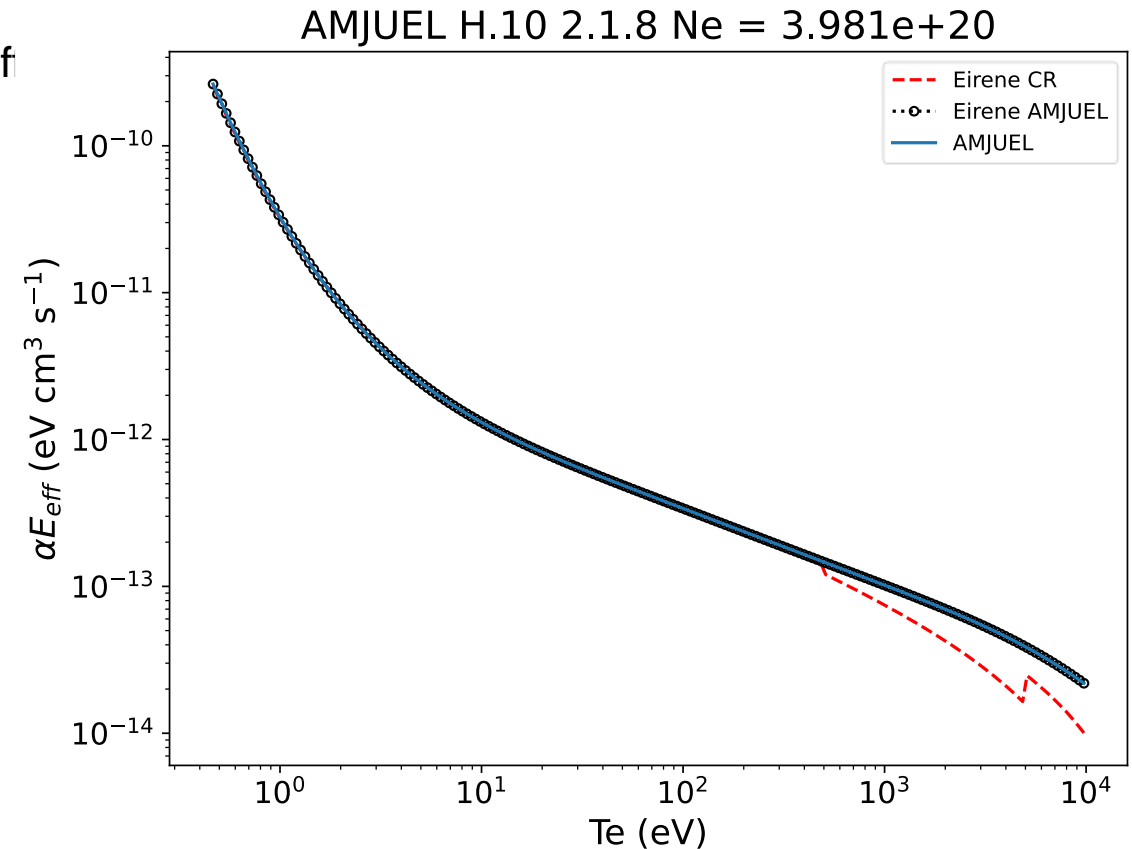


The H colrad recombination cooling rate starts to diverge at $T_e > 400$ eV

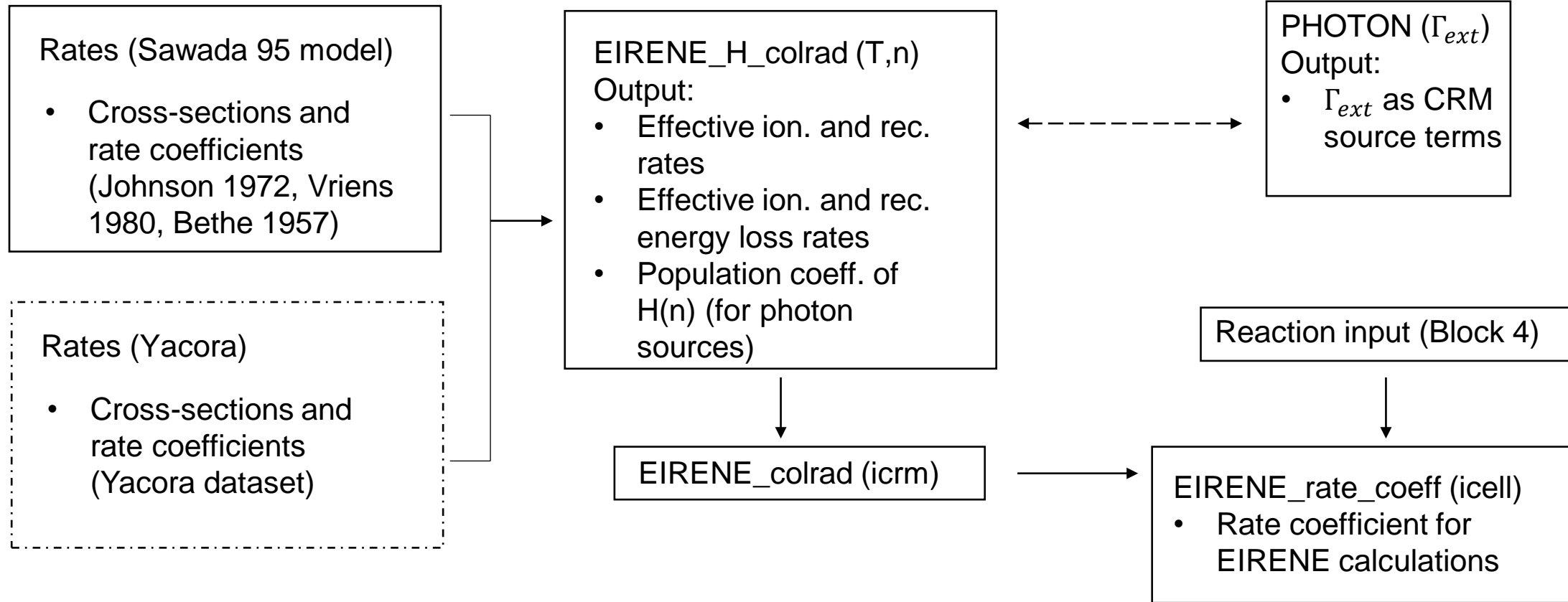
Effective recombination cooling rate αE_{eff}

$$\begin{aligned} \alpha E_{\text{eff}} = & \beta_1 \bar{E}_1 - \alpha_{(1)} n_e E_\alpha \\ & + \sum_p R_{0(p)} (S_p E_{p-\alpha} + \beta_p \bar{E}_p - \alpha_p n_e E_{p-\alpha}) \\ & + \sum_{q>p} (R_{0(p)} C_{p,q} E_{p-q} - R_{0(q)} F_{(p,q)} E_{p-q}) \end{aligned}$$

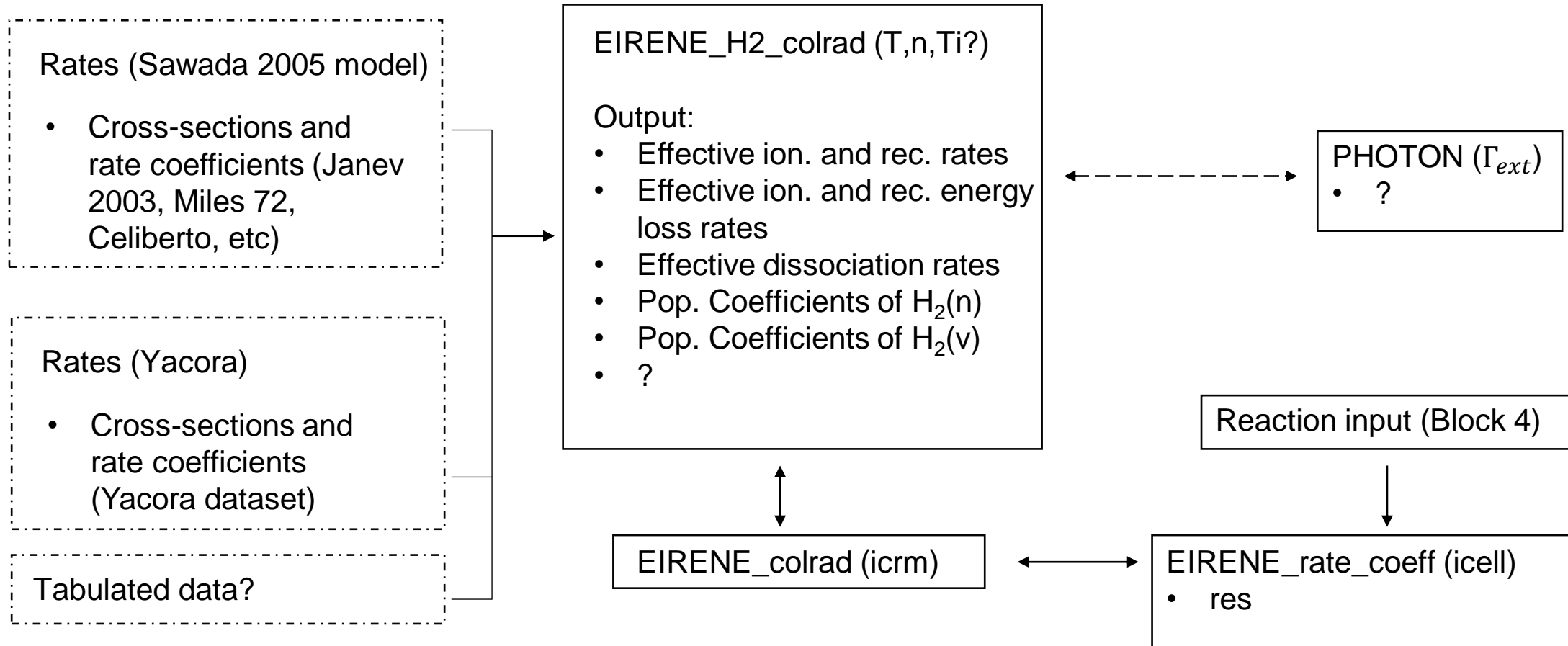
- Directly derived from recombination rate, so same discrepancy occurs



Proposed CRM structure in Eirene (H Colrad)



Proposed *H2* Colrad structure in Eirene (currently non-existent)



The previous (2002) photon tracing model (Reiter et al., PPCF 2002) was revisited

The photon tracing routine in EIRENE is analogous to neutral particle tracing, with $v=c$ and $E=h\nu$, differences:

- Bulk ions are now excited species (photon sources)
- Rate coefficients for sources and photon-background interaction must take into account line shapes (natural, Doppler, Zeeman, etc)

Determine opacity with local population escape factors

Determining the population escape factor

$$\Theta_p = \frac{E - G}{E} = 1 - \frac{G}{E}$$

\longrightarrow absorption
 \longrightarrow emission

$$\Theta_p = 1 - \frac{\int_{\Omega} \int_{line} \alpha(x, \lambda) L_{\lambda}(x, \lambda, \Omega) d\lambda d\Omega}{\int_{\Omega} \int_{line} \epsilon(x, \lambda) d\lambda d\Omega}$$

Behringer K 1998 Escape factors for line emission and population calculations MPI-Garching Report, IPP 10/11

Θ_p in Eirene

$$\Theta_p = \frac{E - G}{E} = 1 - \frac{G}{E}$$

$G \longrightarrow$ absorption
 $E \longrightarrow$ emission

G is simply the number of absorbed photons i.e volume photon sink tallies

E is the volume photon source

Thus Θ_p can be evaluated per cell of Eirene

Photon tracing test case

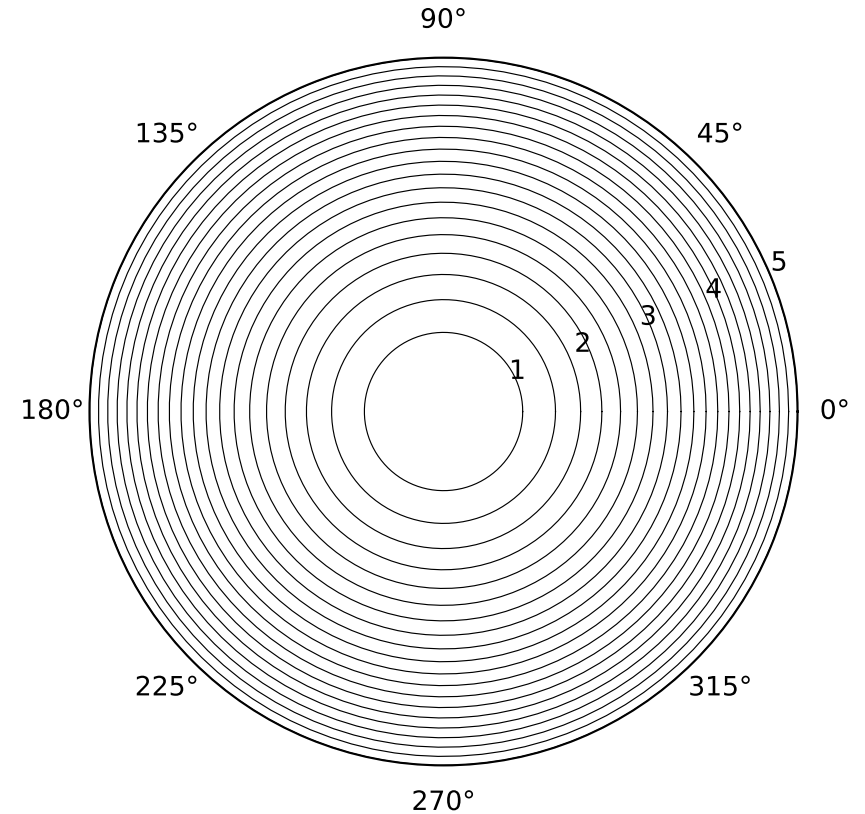
Cylindrical test case, 20 radial points

Homogeneous plasma and atomic density

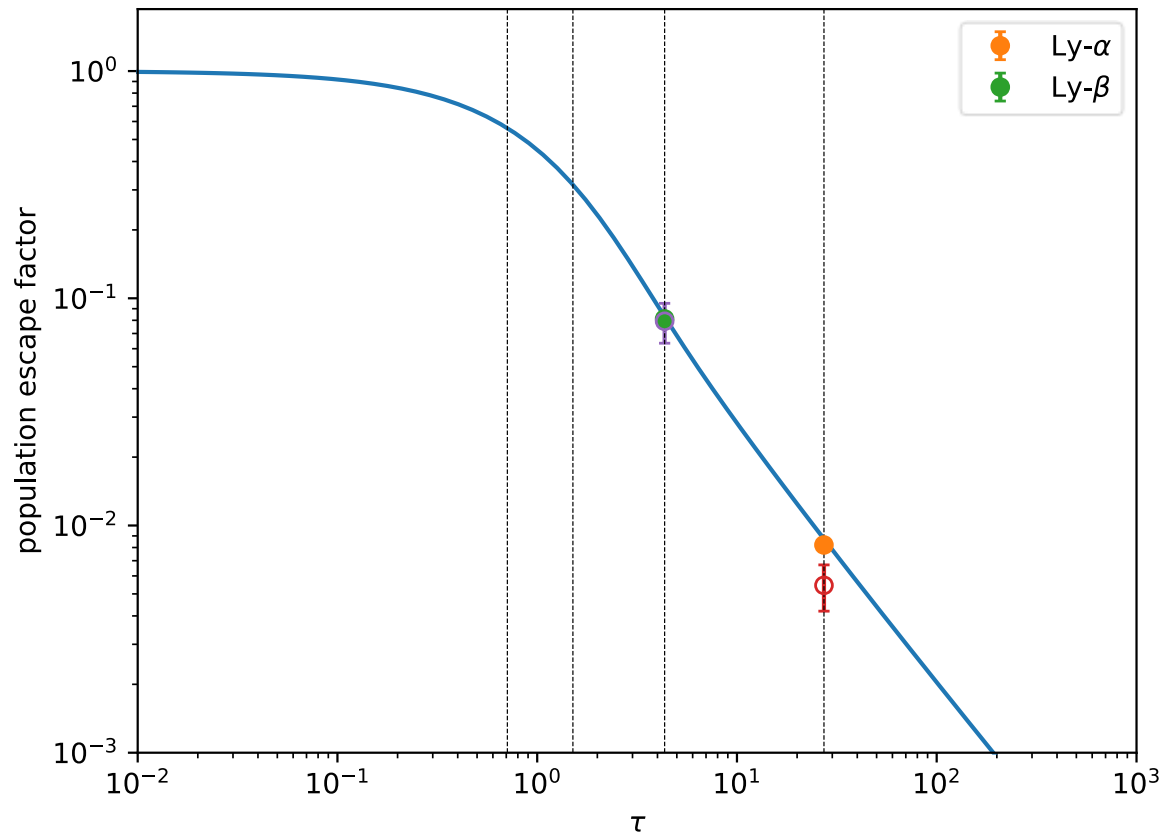
$T_H = 1 \text{ eV}$, $n_H = 10^{14} \text{ cm}^{-3}$, $b = 5 \text{ cm}$

Simulated Ly-a and Ly-b photons ($2e6$) with volumetric sources (H($n=2,3$) as bulk ions)

Line shape only doppler broadening



Population escape factor aligns with analytical function for Ly- α and Ly- β



$$\Theta_p = f(\tau) \text{ (solid blue line)}$$

$$\tau = \alpha(\lambda)b$$

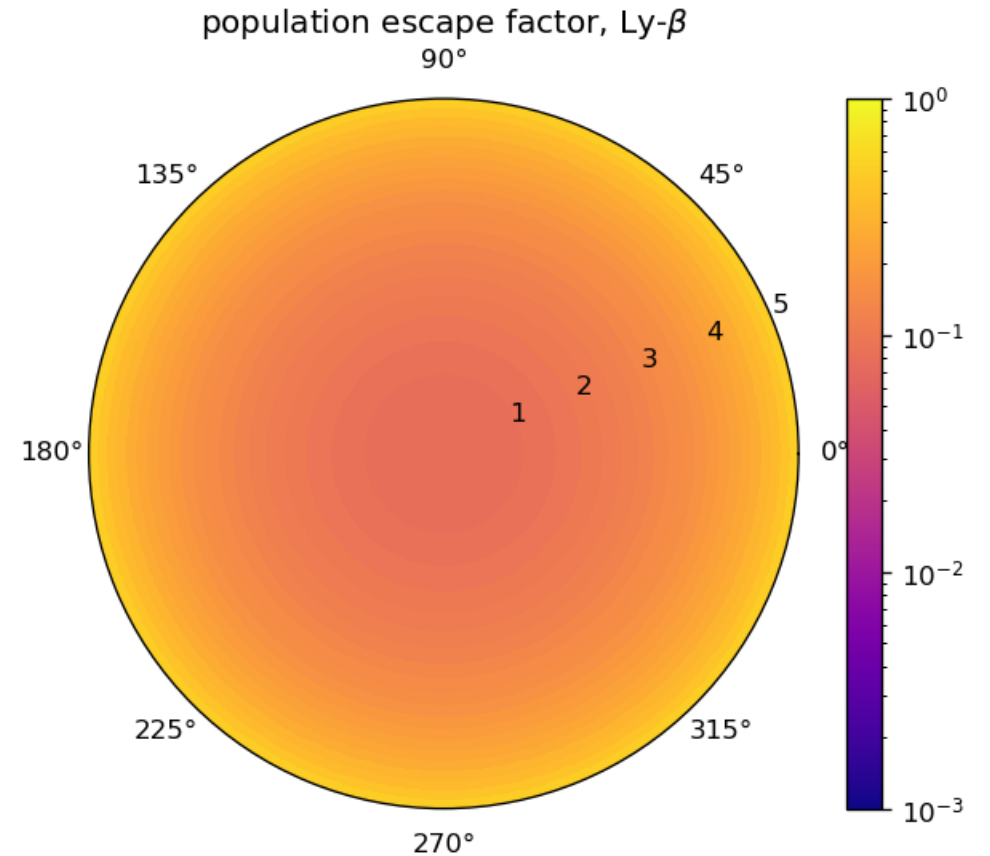
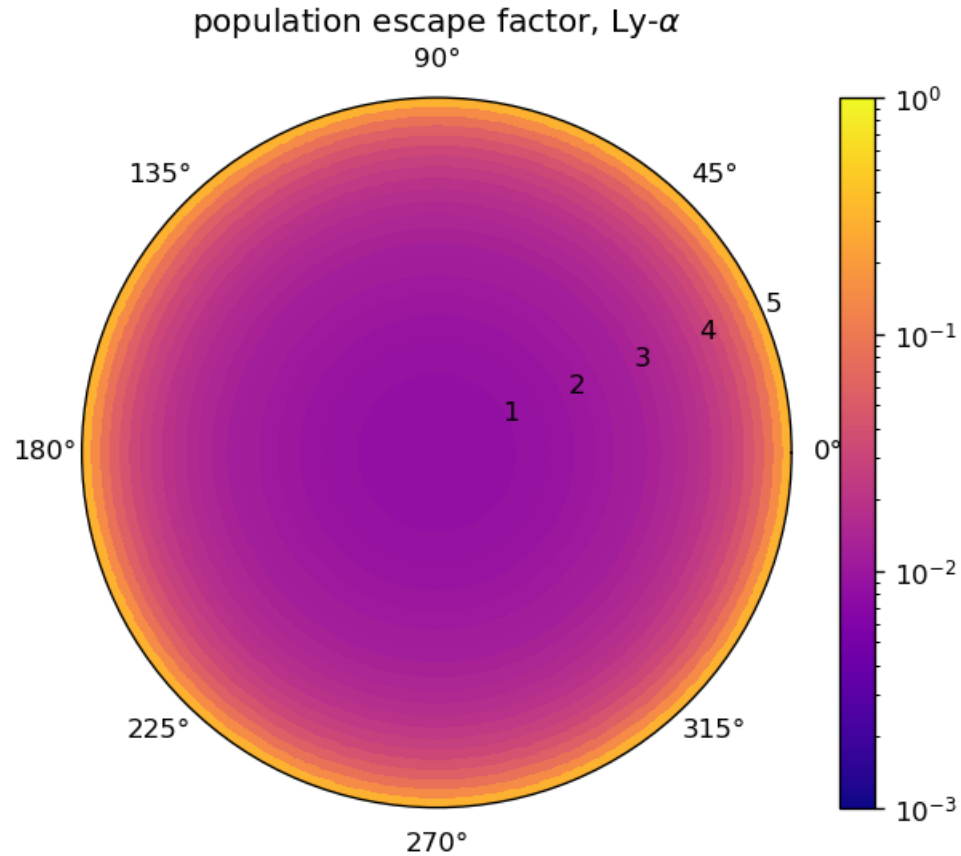
is the optical depth or 'thickness'

Hollow points with $\sim 10^6$ photons

Solid points with $\sim 10^9$ photons

Agreement for Ly- α at better statistics

2D (or 1D) profiles of the population escape factor: Ly- α and Ly- β opaque at the center



Current project state

Whats on hand:

- H colrad, He colrad
- Photon module
- A&M and photon cylinder test cases

Whats planned:

- CRM-photon coupling (for Planck test)
- Application to JET 81472
- H colrad data update
- He colrad testing
- H₂ colrad creation

