# Gyrokinetic investigation of the Alfvén activity in ASDEX Upgrade

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**Projects:** MET, TSVV10

# Alfvén modes (AMs)

- AMs[1] are global, collective, magnetohydrodynamic (MHD) instabilities driven unstable by energetic particles (EPs). They include:
  1. Alfvén eigenmodes (AEs),
  2. and energetic particle continuum modes (EPMs).
- AMs can redistribute EPs in phase-space.
- **Predict role of AMs in self organization of burning plasma!**
- Need of proper theoretical framework to:
  - treat global MHD modes,
  - retain kinetic effects (**wave-particle resonance interaction**).
    - ⋆ $\Longrightarrow$ **GYROKINETIC THEORY**.

[1] Liu Chen and Fulvio Zonca, Rev. Mod. Phys. **88**, 015008 (2016)

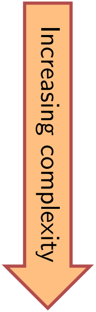
# Investigated physical topics

1. AMs damping mechanisms (*how easily can AMs be destabilized?*) [2],
2. AMs interaction with *zonal structures* (**ZS**) [3]:
   - zonal flows (ZF),
   - geodesic acoustic modes (GAM), EP-driven GAMs (EGAMs).
3. Study of the AM nonlinear frequency modification [4].

[2] F. Vannini et al., Physics of Plasmas **27**, 042501 (2020)
[3] F. Vannini et al., Physics of Plasmas **28**, 072504 (2021)
[4] F. Vannini et al., Nuclear Fusion, **Submitted.**

# Guideline of my studies

1. Newly developed extension of the analytical treatment.
2. Gyrokinetic simulations with **ORB5**:
   - simplified equilibria,
   - experimental ASDEX Upgrade scenario.



# ORB5

- Global, electromagnetic, gyrokinetic, particle-in-cell (PIC), finite-elements code.
- **Gyrokinetic model:**

$$\frac{d}{dt}\delta f_{sp} = -\dot{\boldsymbol{R}} \cdot \left.\frac{\partial F_{0,sp}}{\partial \boldsymbol{R}}\right|_{\mathcal{E},\, v_\parallel} - \dot{\mathcal{E}} \left.\frac{\partial F_{0,sp}}{\partial \mathcal{E}}\right|_{\boldsymbol{R},\, v_\parallel} - \dot{v}_\parallel \left.\frac{\partial F_{0,sp}}{\partial v_\parallel}\right|_{\boldsymbol{R},\, \mathcal{E}}$$

$$\underbrace{\dot{\boldsymbol{R}} = ...,\ \dot{\mathcal{E}} = ...,\ \dot{v}_\parallel = ...}_{\text{Characteristics equations}} \quad + \quad \text{Field equations}$$

- EPs modelled with:
  1. Maxwellian, bi-shifted-Maxwellian (double-bump-on-tail), isotropic slowing-down,
  2. anisotropic slowing-down, experimental distribution[5] from RABBIT[6].

[5] B. Rettino et al., Nucl. Fusion **62**, 076027 (2022)
[6] M. Weiland et al.,Nucl. Fusion **58**, 082032 (2018).

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# How easily can Alfvén modes be driven unstable ?

- Studies on **radiative damping** already presented in [7].
- **Landau damping:** $\gamma = \sum_{sp} \gamma_{sp}$.

- **Phase mixing:**

$$\delta\phi \sim \frac{1}{k_r(t)}, \quad k_r(t) \approx \left|\frac{\partial\omega}{\partial r}\right| t.$$

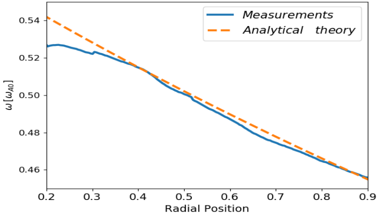


Figure: Continuum spectrum.

- Analytical theory for landau damping and phase mixing validated against ORB5 simulations (simplified equilibria) $\Longrightarrow$ **reasonable agreement!**
- **Next step:** study of an ASDEX Upgrade scenario (*which one?*)

[7] T. Hayward-Schneider et al., IAEA "EPPI" (2019).

# NLED-AUG case [8]

ASDEX Upgrade discharge #31213@0.84s

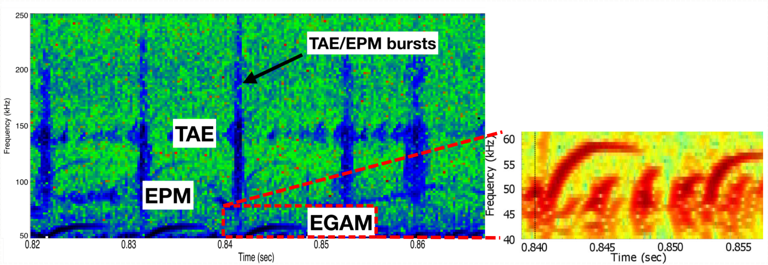


Figure: Spectrogram of the experiment.

- EPs present as injected by a neutral beam.
- Ratio of plasma parameters match those of ITER:

$$\beta_{EP} \sim \beta_{Bulk}, \quad \mathcal{E}_{EP}/T_{Bulk} \approx 93keV/1keV \sim 10^2.$$

- Intense EP-driven activity observed.

[8] P. Lauber et al., proceedings of the 27th IAEA Fusion energy, 2018.

# NLED-AUG case

## ASDEX Upgrade discharge #31213@0.84s

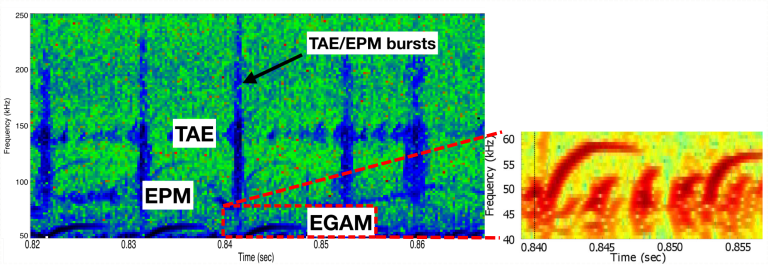


Figure: Spectrogram of the experiment.

- Investigation of the linear AM dynamics and study of the damping mechanisms in this scenario, presented for the first time in [9].
- Linear benchmark between ORB5, MEGA and HYMAGYC [10] $\Longrightarrow$ good agreement among the three codes!

[9] F. Vannini et al., Physics of Plasmas **27**, 042501 (2020)

[10] G. Vlad et al., Nuclear Fusion **61**, (2021).

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# Analytical model

- **Simplifying Assumptions:** Only EP nonlinearity included.
- **Forced-Driven Excitation**[11]
  - **AM** (most unstable mode) $\xrightarrow{\text{TRIGGER}}$ **ZS** (driven mode)

$$AM + (AM)^* \implies ZS; \qquad \gamma_{\mathbf{ZS}} = \mathbf{2}\,\gamma_{\mathbf{AM}}$$

- Extension to **AM-EGAM Interaction** ; ZS $\equiv$ EGAM.
  - **EGAM** (most unstable mode) $\xrightarrow{\text{TRIGGER}}$ **AM** (driven mode)

$$EGAM + AM^{\mathsf{L}} \implies AM^{\mathsf{NL}}; \qquad \gamma^{\mathbf{NL}}_{\mathbf{AM}} = \gamma_{\mathbf{EGAM}} + \gamma^{\mathbf{L}}_{\mathbf{AM}}$$

NL (*nonlinear*) $\equiv$ Many toroidal mode interaction.
L (*linear*) $\equiv$ Absence of many toroidal mode interaction.

[11] Z. Qiu, L. Chen and F.Zonca, Physics of Plasmas **23**, 090702 (2016)

# Wave-wave interaction (mediated by EPs)

The formalism proposed in [12] is followed

- **Nonlinear gyrokinetic Vorticity equation** $\delta\psi_k = \frac{\omega}{ck_\parallel}\delta A_{\parallel,k}$

$$\underbrace{\frac{c^2}{4\pi\omega^2}B\frac{\partial}{\partial l}\frac{k_\perp^2}{B}\frac{\partial}{\partial l}\delta\psi_k}_{\text{Field line bending (FLB)}} + \underbrace{\frac{e^2}{T_i}\langle(1-J_k^2)F_0\rangle_{\boldsymbol{v}}\delta\phi_k}_{\text{Inertial Term (IT)}} - \underbrace{\sum_{sp}\langle\frac{e_{sp}}{\omega}J_k\,\omega_d\,\delta H_k\rangle_{\boldsymbol{v}}}_{\text{Curvature Coupling Term (CCT)}} = 0$$

EPs

- **Nonlinear gyrokinetic Vlasov equation** $\delta L_k = \delta\phi_k - \frac{v_\parallel}{c}\delta A_{\parallel,k}$

$$(-i\omega + v_\parallel\partial_l + i\omega_d)\delta H_k = -i\frac{e_{sp}}{m}QF_0J_k\delta L_k - \frac{c}{B_0}\Lambda_k J_{k'}\delta L_{k'}\delta H_{k''}$$

$$\Lambda_k = \sum_{\boldsymbol{k}=\boldsymbol{k}'+\boldsymbol{k}''}\hat{\boldsymbol{b}}\cdot\boldsymbol{k}'\times\boldsymbol{k}''$$ Mode Coupling Term

[12] E. A. Frieman and L. Chen, The Physics of Fluids **25**, (1982)

# Simulations

- Only EPs treated nonlinearly.
- Anisotropy in velocity space needed to drive unstable EGAM.
- Plasma profiles and parameters of the NLED-AUG case.
- **Magnetic equilibrium:**
  1. circular geometry,
  2. experimental.

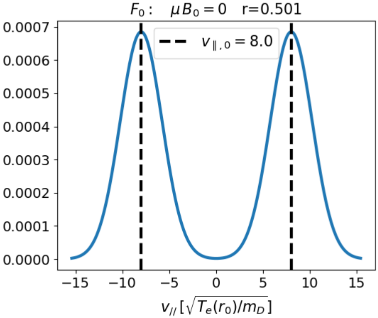
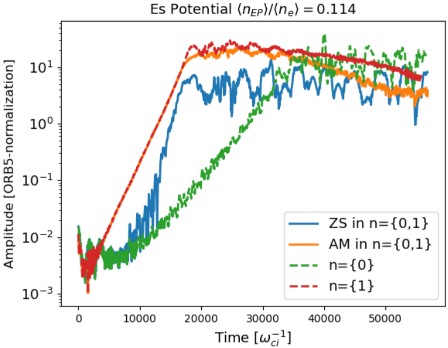



Figure: "Double-bump-on-tail"

- Comparison of the mode dynamics:
  1. **n=$\{0\}$, $\{1\}$:** **only** ZS **or** AM dynamics retained in the simulation.
  2. **n=$\{0,1\}$:** **both** AM **and** ZS retained in the simulations.

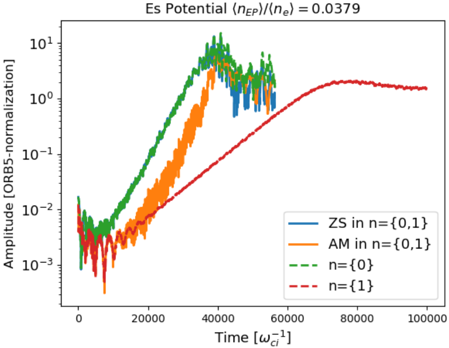
# Circular Geometry: High EP concentration

- **n={1}:** $\gamma^{L}_{AM} = 1.078 \cdot 10^{-2}\omega_{A0}$
- **n={0}:** $\gamma^{L}_{ZS} = 6.3 \cdot 10^{-3}\omega_{A0} \quad \Longrightarrow \quad \gamma^{L}_{AM} > \gamma^{L}_{ZS}$
- **n={0,1}:** $\gamma^{NL}_{ZS} \simeq 2\,\gamma^{L}_{AM}$ **Forced-driven excitation**.



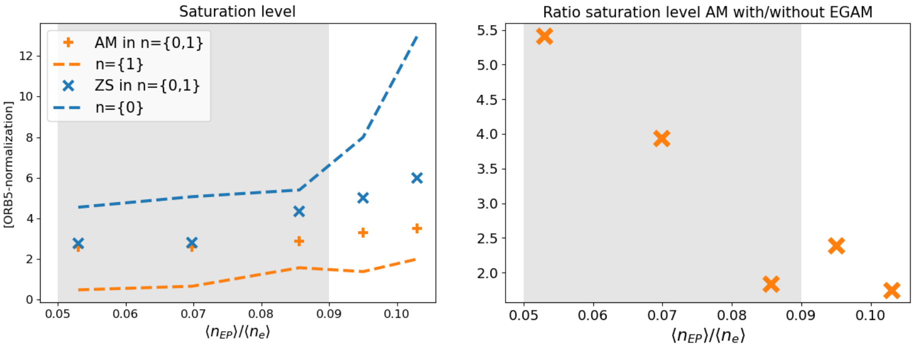
# Circular Geometry: Low EP concentration

- **n={1}:** $\gamma^L_{AM} = 2.43 \cdot 10^{-3} \omega_{A0}$
- **n={0}:** $\gamma^L_{ZS} = 5.4 \cdot 10^{-3} \omega_{A0} \quad \Longrightarrow \quad \gamma^L_{AM} < \gamma^L_{ZS}$
- **n={0,1}:** $\gamma^{NL}_{AM} \simeq \gamma^L_{EGAM} + \gamma^L_{AM}$ **AM-EGAM interaction**



# ASDEX Upgrade magnetic equilibrium

- **Low** EP concentrations: ZS more unstable than AM.
- At **higher** concentrations: AM more unstable than ZS.




- Difference saturation level of AM, w. and w/o ZS, more pronounced at **low** EP concentrations.

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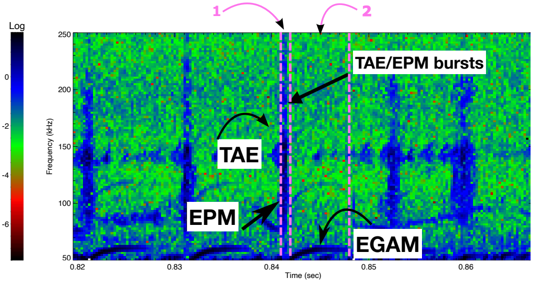
# Study of the TAE-EPM burst in the NLED-AUG case




- Isotropic slowing-down [13] distribution function implemented in ORB5.

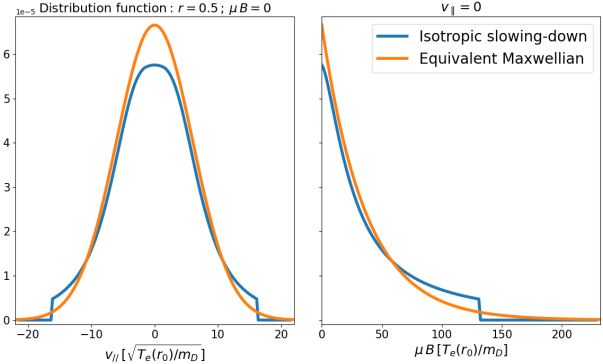
$$F_0 = \frac{n_{EP}(r)}{\frac{4\pi}{3}\log\left[1+\left(\frac{v_{EP}}{v_c}\right)^3\right]} \frac{\theta(v_{EP}-v)}{v_c^3+v^3}$$

- $v_{EP} \equiv$ EP injection energy; $v_c \equiv$ crossover velocity.

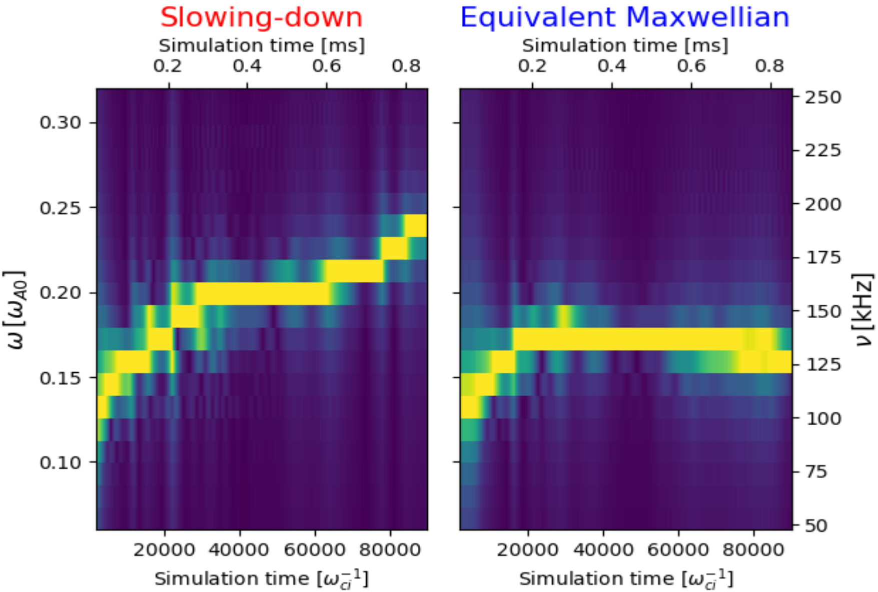
[13] John D. Gaffey Jr.,Journal of Plasma Physics **16**, (1976)

# Isotropic slowing-down vs equivalent Maxwellian

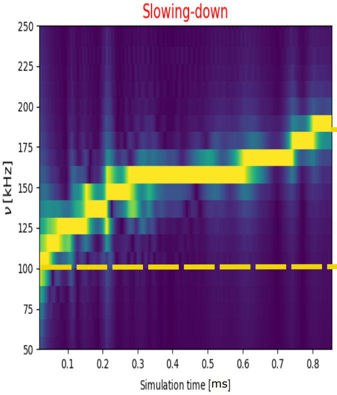
$$\boxed{\int d^3\boldsymbol{v}\, v^2\, F_M = \int d^3\boldsymbol{v}\, v^2\, F_0}, \qquad \int d^3\boldsymbol{v}\, v^2\, F_M = 3\, n_{EP}(r)\frac{T_{EP}(r)}{m_{EP}}\,.$$

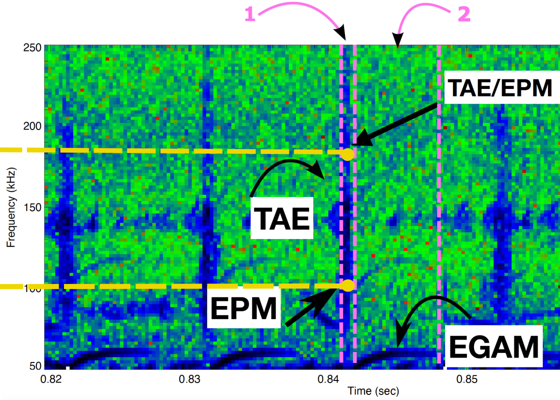


# Simulation spectrograms



# Comparison with the experiment



# Conclusions

- Good understanding of the AM damping mechanisms.
- Good qualitative understanding of the AM/EGAM interaction.
- Good quantitative comparison of the AM frequency with the experiments.
- **Next steps:**
  1. Inclusion of all the plasma nonlinearities,
  2. relaxation of the approximations in the developed analytical models,
  3. investigation of the AM interaction with drift-instabilities.

# ORB5: gyrokinetic model (1)

- *Gyrokineitc Lagrangian of the system* [14]*:* defined on set of coordinates that have **fast gyroangle $\alpha$ ignorable**.

$$L = \underbrace{\sum_{sp} \int d^6\boldsymbol{Z}\, f_{sp}(\boldsymbol{Z}, \dot{\boldsymbol{Z}}, t)\, \mathcal{L}_{sp}(\boldsymbol{Z}, \dot{\boldsymbol{Z}}, t)}_{\text{Charged particle Lagrangian}} + \underbrace{\int dV\, \frac{\delta E^2 - \sigma \delta B^2}{8\pi}}_{\text{Field Lagrangian}} \quad .$$

- $\mathcal{L}_{sp} \equiv$ Lie-Transformed low frequency particle Lagrangian, obtained through *gyrokinetic dynamical reduction:*

  1. *guiding-center transformation:* only effects strong/nonuniform $\boldsymbol{B}_0$.
  2. *gyrocenter transformation:* perturbation through $\delta\phi$ and $\delta\boldsymbol{A}$.

- Action Functional:

$$\mathcal{I}[\delta\phi, \delta A_\parallel, \boldsymbol{Z}] \equiv \int_{t_1}^{t_2} dt\, L[\delta\phi, \delta A_\parallel, \boldsymbol{Z}] \quad .$$

[14] H. Sugama, Physics of Plasmas **7** (2000)

# ORB5: gyrokinetic model (2)

- **Equations of motion** + **field equations**: *variational principles.*

$$\frac{\partial \mathcal{I}}{\partial \Psi} \circ \hat{\Psi} = \int d\Lambda \, \frac{\partial L}{\partial \Psi} \circ \hat{\Psi} + \int d\Lambda \, \frac{\partial L}{\partial \nabla \Psi} \circ \nabla \hat{\Psi} = 0 \quad .$$

1. Equations of motion of the gyrocenter characteristic.

$$\frac{\delta \mathcal{I}}{\delta \boldsymbol{Z}} = 0 \implies \frac{\delta L}{\delta \boldsymbol{Z}} = 0 \quad \implies \quad \textit{gyrokinetic Euler-Lagrange equations.}$$

2. Polarisation/gyrokinetic Poisson equation.

$$\frac{\delta I}{\delta \phi} \circ \delta \hat{\phi} = 0$$

3. Ampère equation.

$$\frac{\delta I}{\delta A_\parallel} \circ \delta \hat{A}_\parallel = 0$$

# ORB5: numerical implementation (1) [15]

- Phase-space sampled by $N_{sp}$ super-particles (*markers*), distributed according to $f_{sp}$:

$$\boldsymbol{z}_{sp,i} \quad \text{with} \quad i = 1, ..., N_{sp}$$

1. $N_{sp}$ realizations of the **same random variable** distributed with $f_{sp}$
2. *or* realizations of $N_{sp}$ random variables **identically** distributed.

$$\underbrace{\mathbb{E}[\psi_{sp}(\boldsymbol{Z})] = \int d^6\boldsymbol{Z}\, f_{sp}(\boldsymbol{Z})\psi(\boldsymbol{Z})}_{\text{Expected values}} \longrightarrow \underbrace{M_N = \frac{1}{N_{sp}}\sum_{i}^{N_{sp}} \psi(\boldsymbol{z}_{sp,i})}_{\text{Estimator}} \quad .$$

- From continuum to statistical (Monte Carlo) description:

$$L \to L_{MC} = \sum_{sp} \frac{1}{N_{sp}} \sum_{i=1}^{N_{sp}} \mathcal{L}_{sp}(\boldsymbol{z}_{sp,i}(t), \dot{\boldsymbol{z}}_{sp,i}(t)) \quad .$$

[15] A. Bottino and E. Sonnendrücker, Journal of Plasma Physics **81** (2015)

# ORB5: numerical implementation (1)

- Error: $\sigma/\sqrt{N_{sp}}$, $\sigma \equiv$ standard deviation random variable.
- Reduce Monet Carlo error $\implies$ construct new random variables $\boldsymbol{Z}^*$:
  - $\mathbb{E}[\boldsymbol{Z}^*] = \mathbb{E}[\boldsymbol{Z}]$,
  - $\sigma_{\boldsymbol{Z}^*} \leq \sigma_{\boldsymbol{Z}}$.

1. **Control variate:** $\psi_{sp}(\widetilde{\boldsymbol{Z}})$ distributed with known $\widetilde{f}_{sp}(\widetilde{\boldsymbol{Z}})$

$$\psi_{sp}(\boldsymbol{Z}^*) = \psi_{sp}(\boldsymbol{Z}) - \beta(\psi_{sp}(\widetilde{\boldsymbol{Z}}) - \mathbb{E}[\psi_{sp}(\widetilde{\boldsymbol{Z}})]) \quad , \quad \beta \in \mathbb{R}$$

2. **Importance sampling:** known $g_{sp}(\boldsymbol{Z}^*) \, : \, dg_{sp}/dt = 0$

$$\mathbb{E}[\psi_{sp}(\boldsymbol{Z}^*)] = \int d\boldsymbol{Z} \, \psi_{sp}(\boldsymbol{Z}) \, \frac{f_{sp}(\boldsymbol{Z})}{g_{sp}(\boldsymbol{Z})} g_{sp}(\boldsymbol{Z})$$

# ORB5: numerical implementation (1)

$$L_{MC} = \sum_{sp} \frac{1}{N_{sp}} \sum_{i=1}^{N_{sp}} W_{sp,i}^{\beta} \, \mathcal{L}_{sp}(\boldsymbol{z}_{sp,i}(t), \dot{\boldsymbol{z}}_{sp,i}(t)) + \beta \mathbb{E}[\mathcal{L}_{sp}(\widetilde{\boldsymbol{z}}_{sp,i}(t), \dot{\widetilde{\boldsymbol{z}}}_{sp,i}(t))]$$

- In "control variate": $\beta = 1$, $\widetilde{f}_{sp} \equiv F_{0,sp}$.

$$W_{sp,i}(t) = \frac{f_{sp}(\boldsymbol{z}_{sp,i}(t_0), t_0)}{g_{sp}(\boldsymbol{z}_{sp,i}(t_0), t_0)} - \frac{F_{0,sp}(\boldsymbol{z}_{sp,i}(t), t)}{g_{sp}(\boldsymbol{z}_{sp,i}(t_0), t_0)} \quad , \quad \delta f_{sp} = f_{sp} - F_{0,sp} \, .$$

$$N_{sp}^{physical\,particles} = \Omega_k \, \delta f_{sp}(\boldsymbol{Z}_k(t), t) = W_{sp,k}(t) \, g_{sp}(\boldsymbol{Z}_{sp,k}(t_0), t_0)$$

- $5 \times N_{sp}$ ODE solved through RK $4th$-order.

# ORB5: numerical implementation (2)

- fields $\Psi = \{\delta\phi, \delta A_{\|}\}$ represented as:

$$\Psi(\boldsymbol{x}, t) = \sum_{\mu} \Psi_{\mu}(t) \quad \underbrace{\Lambda_{\mu}(\boldsymbol{x})}_{\textit{Basis functions}} \qquad \text{with} \quad \Psi_{\mu}(t) \in \mathbb{R} \quad .$$

- $\Lambda_{\mu}(\boldsymbol{x})$: tensor products of $1D$ polynomials (B-splines) of degree $p = \{1, 2, 3\}$:

$$\Lambda_{\mu}(\boldsymbol{x}) = \Lambda_{j}^{p}(s)\Lambda_{k}^{p}(\theta^{*})\Lambda_{l}^{p}(\varphi) \quad \text{with} \quad \mu = (j, k, l) \quad .$$

- Field equations $\implies$ set of linear equations:

$$\sum_{\mu} A_{\mu,\nu}\Psi_{\mu}(t) = b_{\nu}(t)$$

$$\sum_{\mu} \underbrace{\mathcal{F}A_{\mu,\nu}\mathcal{F}^{-1}}_{\hat{A}_{j,j'}^{m,n,m',n}} \underbrace{\mathcal{F}\Psi_{\mu}(t)}_{\hat{\Psi}_{j}^{m,n}(t)} = \underbrace{\mathcal{F}b_{\nu}(t)}_{\hat{b}_{j'}^{m',n}(t)}$$

- $\forall n$ Fourier coefficients of the fields are obtained inverting previous system of equations.