DTU

nMIHESEL, closure and 3D future

Overview

- Multi-temperature
    - Derivation
    - Implementation
- Simplified polarization
    - Motivation and implementation
- 3D
    - Considerations

Raghunathan closure

- Zhdanov 21-moment closure derived for like-temperature ions
- Raghunathan et al. extends to arbitrary temperature.
- We use the coulomb potential with Debye cut-off
  - This coincides with Zhdanov's closure
- Corrections to collisional terms:
  - Resistivity ($\boldsymbol{R}^{10}$)
  - Heat flux ($\boldsymbol{R}^{11}$)
  - Viscosity ($\boldsymbol{R}^{20}$)

Collisional moments equations

In general:

$$\boldsymbol{R}^{mn} = \sum_l \boldsymbol{R}^{mnl} = \sum_l (1 - \delta_{m0}\delta l0)\left(A^{mnl}_{\alpha\beta} b^{ml}_{\alpha} + B^{mnl}_{\alpha\beta} b^{ml}_{\beta}\right) + \delta_{m0}\delta_{l0} C^{mnl}_{\alpha\beta} \tag{1}$$

Here we are interested in:

- $b^{10}_{\alpha} = m_\alpha u_\alpha,$
- $b^{11}_{\alpha} = \frac{h_\alpha}{n_\alpha},$
- $b^{20}_{\alpha} = \frac{\pi_\alpha}{n_\alpha},$

Generally interested in density, vorticity and pressure equations.
In the following we use: $\boldsymbol{u}_{\alpha,0} = \boldsymbol{u}_{ExB} + \boldsymbol{u}_{b,\alpha}$

Resistivity

The resistivity is given by:

$$\boldsymbol{R}^{10} = \sum_l \boldsymbol{R}^{10l} \tag{2}$$

For $l = 0$ we get with coefficients from Raghunathan et al.:

$$R^{100} = -m_\alpha n_\alpha \nu_{\alpha\beta}(\boldsymbol{u}_\alpha - \boldsymbol{u}_\beta) \approx -m_\alpha n_\alpha \nu_{\alpha\beta}(\boldsymbol{u}_{0,\alpha} - \boldsymbol{u}_{0,\beta}) = -m_\alpha n_\alpha \nu_{\alpha\beta}(\boldsymbol{u}_{d,\alpha} - \boldsymbol{u}_{d,\beta}) \tag{3}$$

$\boldsymbol{u}_{ExB}$ cancels. Identical to Zhdanov.

For $l = 1$:

$$\boldsymbol{R}^{101} \approx \frac{3}{5} \frac{m_\alpha^3 n_\alpha}{(\eta_{\alpha\beta}+1) T_\alpha (n_{\alpha\beta} + \theta_{\alpha\beta}) \mu_{\alpha\beta}} \nu_{\alpha\beta} \left(\theta_{\alpha\beta} \frac{\boldsymbol{h}_{\alpha,0}}{m_\alpha n_\alpha} - \frac{\boldsymbol{h}_{\beta,0}}{m_\beta n_\beta} \right) \tag{4}$$

Heat flux inserted from upcoming results

Heat flux

- Don't want to solve heat flux equation.
- Linearize following same assumptions as Zhdanov:

$$\frac{5}{2}\frac{p_\alpha}{m_\alpha}\nabla T_\alpha - \boldsymbol{h}_\alpha \times \omega_\alpha = \boldsymbol{R}^{11} = \sum_l \boldsymbol{R}^{11l} \quad (5)$$

- Expand similar to drift expansion:

$$\boldsymbol{h} = \boldsymbol{h}_0 + \varepsilon \boldsymbol{h}_1 + \varepsilon^2 \boldsymbol{h}_2 \quad (6)$$

- This yields,
  - $\boldsymbol{h}_{0,\perp,\alpha} = \frac{5}{2}\frac{p_\alpha}{q_\alpha B}\boldsymbol{b} \times \nabla T_\alpha$ which is identical to previous results
  - $\boldsymbol{h}_{1,\perp,\alpha} = -\frac{1}{\omega_\alpha}\boldsymbol{b} \times \boldsymbol{R}^{11}_\alpha$

Heat flux continued

- First part is $l = 0$. I.e. heat flux related to relative drifts $b_\alpha = m_\alpha \boldsymbol{u}_\alpha$:

$$\boldsymbol{h}_{1,0,\perp} = -\frac{1}{\omega_\alpha}\boldsymbol{b} \times \boldsymbol{R}^{110} \tag{7}$$

$$= \frac{3}{2}\frac{p_\alpha \mu_{\alpha\beta} \nu_{\alpha\beta}}{q_\alpha^2 B^2}\left(\frac{\nabla p_\alpha}{n_\alpha} - \frac{q_\alpha}{q_\beta}\frac{\nabla p_\beta}{n_\beta}\right) \tag{8}$$

- Likewise the $l = 1$ part related is related to heat flux. We evaluate at h_0:

$$\boldsymbol{h}_{1,1,\perp} = -\frac{\mu_{\alpha\beta} p_\alpha \nu_{\alpha\beta}}{m_\alpha^2 \omega_\alpha^2}\left[\frac{\left(\eta_{\alpha\beta}^2(52 - 6\theta_{\alpha\beta}) + \eta_{\alpha\beta}\theta_{\alpha\beta}(9\theta_{\alpha\beta} + 20) + 30\eta_{\alpha\beta}^3 + 13\theta_{\alpha\beta}^2\right)}{4(\eta_{\alpha\beta} + \theta_{\alpha\beta})^2}\nabla T_\alpha - \frac{9}{4}\eta_{\alpha\beta}\frac{(\eta_{\alpha\beta}(5\theta_{\alpha\beta} - 2) + 3\theta_{\alpha\beta})}{(\eta_{\alpha\beta} + \theta_{\alpha\beta})^2}\frac{q_\alpha}{q_\beta}\nabla T_\beta\right] \tag{9}$$

Viscosity

- Similar procedure to heat flux.
- Solve linearised equation:

$$-\omega_\alpha \left\{\pi_{\alpha lr} e_{slm} k_m\right\} + p_\alpha W_{rs} = \boldsymbol{R}_{rs}^{20} \tag{10}$$

- Although Viscosity tensor derivation is tedious!
- Focus on coefficients:

Viscosity cont.

- First set of coefficients is:

$$\eta_\alpha^{(4)} = 4\eta_\alpha^{(3)} = \frac{p_\alpha}{\omega_\alpha} \tag{11}$$

- Next set of coefficient are found by substitution:

$$\eta_\alpha^{(2)} = 4\eta_\alpha^{(1)} = \frac{1}{\omega_\alpha}\left(A_{\alpha\beta}^{200}\frac{\eta_\alpha^{(4)}}{n_\alpha} + B_{\alpha\beta}^{200}\frac{\eta_\beta^{(4)}}{n_\beta} \right) \tag{12}$$

$$= -\frac{2}{5}\frac{p_\alpha \nu_{\alpha\beta}\mu_{\alpha\beta}}{m_\alpha \omega_\alpha^2}\left(\frac{2\eta_{\alpha\beta}(\theta_{\alpha\beta}+3)+5\eta_{\alpha\beta}^2+3\theta_{\alpha\beta}}{(\eta_{\alpha\beta}+\theta_{\alpha\beta})} \right. \tag{13}$$

$$\left. - \frac{\eta_{\alpha\beta}(3\theta_{\alpha\beta}-1)+2\theta_{\alpha\beta}}{(\eta_{\alpha\beta}+\theta_{\alpha\beta})}\frac{q_\alpha}{q_\beta}\frac{T_\beta}{T_\alpha} \right) \tag{14}$$

Closure wrap up!

- New expressions for resistive, heat flux and viscosity.
- Only higher order expressions are new.
- Only focused on perpendicular parts
    - Parallel to follow!
- Note: All expressions reduce to Zhdanov for similar temp.
- Further reduces to Braginskii for single ion species.
- Implementation + testing ongoing

Simplified polarisation

MIHESEL has been limited in usage:

- Suffered poor stability
  - requires high diffusion or extreme resolution
- Computationally expensive
  - Linear inversion took 80% time

Question: Can we do anything about it?

Linear problem

- The momentum equation serves as the root for the drift fluid expansion

$$m_\alpha n_\alpha \frac{d}{dt}\boldsymbol{u}_\alpha = n_\alpha q_\alpha (\boldsymbol{E} + \boldsymbol{u}_\alpha \times \boldsymbol{B}) + \boldsymbol{R}_\alpha - \nabla p_\alpha - \nabla \cdot \overleftrightarrow{\boldsymbol{\pi}}_s, \tag{15}$$

- Drift fluid expansion yields the polarization drift:

$$\boldsymbol{u}_{p,\alpha} = \Omega_{c,\alpha}^{-1} \boldsymbol{b} \times \frac{d}{dt}\boldsymbol{u}_{\alpha,0} \tag{16}$$

- Enters ion density, vorticity and pressure equation

Coupled system

- Result is coupled system of equations:

$$
\begin{pmatrix}
-\sum_\alpha a_\alpha \mu_\alpha \nabla^2 & -\frac{\mu_1}{Z_1}\nabla^2 & -\frac{\mu_2}{Z_2}\nabla^2 & \cdots \\
-\frac{\mu_1}{Z_1}\nabla^2 & \frac{3}{2}\frac{1}{p_1} - \frac{\mu_1}{Z_1^2 a_1}\nabla^2 & 0 & \\
-\frac{\mu_2}{Z_2}\nabla^2 & 0 & \frac{3}{2}\frac{1}{p_2} - \frac{\mu_2}{Z_2^2 a_2}\nabla^2 & \\
\vdots & & & \ddots
\end{pmatrix} \cdot \begin{pmatrix} \partial_t \phi \\ \partial_t p_1 \\ \partial_t p_2 \\ \vdots \end{pmatrix} =
\begin{pmatrix}
-\left(\Lambda_w + \mathscr{C}\left(\sum_\alpha p_\alpha + p_e\right) - \sum_\alpha a_\alpha \mu_\alpha \nabla \cdot \{\phi, \nabla_\perp \phi_\alpha^*\}\right) \\
\left(\Lambda_{p_1} - \frac{5}{2} p_1 \mathscr{C}(\phi) - \frac{5}{2}\frac{1}{Z_1}\mathscr{C}\left(\frac{p_1^2}{n_1}\right) + p_1 \frac{\mu_1}{Z_1}\nabla \cdot \{\phi, \nabla_\perp \phi_1^*\}\right)/p_1 \\
\left(\Lambda_{p_2} - \frac{5}{2} p_2 \mathscr{C}(\phi) - \frac{5}{2}\frac{1}{Z_2}\mathscr{C}\left(\frac{p_2^2}{n_2}\right) + p_2 \frac{\mu_2}{Z_2}\nabla \cdot \{\phi, \nabla_\perp \phi_2^*\}\right)/p_2 \\
\vdots
\end{pmatrix} \tag{17}
$$

- Solved using multigrid

Linearization

- Let $\boldsymbol{u}_\alpha = \boldsymbol{u} + \boldsymbol{w}_\alpha$ where $\boldsymbol{u}$ is bulk velocity
- Defined by: $\sum_\alpha m_\alpha n_\alpha \boldsymbol{u} = \sum_\alpha m_\alpha n_\alpha \boldsymbol{u}_\alpha$
- Assuming $\boldsymbol{w}_\alpha < \boldsymbol{u}$ we redefine polarization drift:

$$\nabla \cdot n_\alpha \boldsymbol{u}_{p,\alpha} = \nabla \cdot n_\alpha \Omega_{c,\alpha}^{-1} \boldsymbol{b} \times \frac{d}{dt} \boldsymbol{u}_0 \approx -a_\alpha \frac{\mu_\alpha}{Z_\alpha} \nabla \cdot \left(\frac{d^0}{dt} \nabla_\perp \phi^* \right) \tag{18}$$

- where $\phi^* = \frac{\phi}{B_0} + \frac{\sum_\alpha \Omega_\alpha^- 1 p_\alpha}{\sum_\alpha \rho_\alpha}$ and $\frac{d^0}{dt} = \frac{\partial}{\partial t} + \boldsymbol{u}_{\alpha,0} \cdot \nabla$
- To ensure we keep gyro-viscous cancellation, the stress tensor is evaluated with $\boldsymbol{u}_0$
- Choice of $\boldsymbol{u}_\alpha$ in advection is to ensure the same

New system

Defining $w = \nabla^2 \phi^*$ the vorticity equation becomes:

$$\frac{\partial}{\partial t} w + \{\phi, w\} + \{\partial_x \phi, \partial_x \phi^*\} + \{\partial_y \phi, \partial_y \phi^*\} - \mathscr{C}\left(\sum_\alpha p_\alpha + p_e\right) = \tag{19}$$

$$\sum_\alpha \sum_\beta Z_\alpha \frac{a_\alpha D_{\pi,\beta\to\alpha,0}}{\Omega_{c,0,\alpha}} \nabla^2 \nabla^2 \phi^* \tag{20}$$

- ϕ^* is solved as simple Poisson equation.
- Note: Almost identical to old polarization equation
- Polarization terms in density and pressure equation found by direct substitution. Easy!
  - This is the main effect!

The simpler system

$$
\begin{pmatrix}
\sum_\alpha a_\alpha \mu_\alpha & 0 & 0 & \cdots \\
-\frac{\mu_1}{Z_1} & \frac{3}{2}\frac{1}{p_1} & 0 & \\
-\frac{\mu_2}{Z_2} & 0 & \frac{3}{2}\frac{1}{p_2} & \\
\vdots & & & \ddots
\end{pmatrix}
\cdot
\begin{pmatrix}
\partial_t w \\
\partial_t p_1 \\
\partial_t p_2 \\
\vdots
\end{pmatrix}
=
$$

$$
\begin{pmatrix}
\left(\Lambda_w + \mathscr{C}\left(\sum_\alpha p_\alpha + p_e\right) - \sum_\alpha a_\alpha \mu_\alpha \nabla \cdot \{\phi, \nabla_\perp \phi^*\}\right) \\
\left(\Lambda_{p_1} - \frac{5}{2} p_1 \mathscr{C}(\phi) - \frac{5}{2}\frac{1}{Z_1}\mathscr{C}\left(\frac{p_1^2}{n_1}\right) + p_1 \frac{\mu_1}{Z_1} \nabla \cdot \{\phi, \nabla_\perp \phi^*\}\right)/p_1 \\
\left(\Lambda_{p_2} - \frac{5}{2} p_2 \mathscr{C}(\phi) - \frac{5}{2}\frac{1}{Z_2}\mathscr{C}\left(\frac{p_2^2}{n_2}\right) + p_2 \frac{\mu_2}{Z_2} \nabla \cdot \{\phi, \nabla_\perp \phi^*\}\right)/p_2 \\
\vdots
\end{pmatrix}
\tag{21}
$$

- Triangular system! Easy!
- Linear solve now only takes $\sim 10-20\%$
- Little testing still necessary, but very promising!

3D - into the future

MIHESEL very limited in scope. What are the pros and cons?

- Captures many perpendicular dynamics
- Physics based coefficients etc.
- Fast - slab and 2D

- No parallel dynamics
- Can't easily model parallel effects
  - Triangularity
  - Divertor config.

Considerations

- Roughly same equations.
- What do we want to do?
- Good statistics
  - Full torus?
    - Large computational domaine
  - Big machines?
    - High resolution
  - Easy implementation

Full torus

- Everything including core. Hot core breaks key assumptions
- Similar to Feltor
- Computationally expensive
- Especially with big machines
- Should be relatively easy to implement (working from Feltor)
  - ’Enlarge’ slab to cover all.
  - Small adaption of eq. might be needed.
  - E.g. to account for magnetic field variation

Full edge/SOL

- Similar to Grillix
- Good for what equations are derived for: Edge/sol
- Includes all effects of interest
- Computationally expensive for large machines
- Could be easy to implement. Follow full torus impl.
  - 'Enlarge' slab to cover all.
  - Take out core
  - Small adaption of eq. might be needed.
  - E.g. to account for magnetic field variation

3D-slab-like

- Assumption: All perpendicular needed is already in slab!
- Main progress is in parallel dynamics!
- Follow slab along field lines (bends and twists!)
- Shouldn't be too expensive!
- Implementation more tedious:
  - How to deal with X-point
  - Private Flux region etc.
  - Magnetic field twists in each slab
- Simple option: limiter-plasma...

Similar for all - work to be done

- Implement parallel dynamics:
- Momentum equation
- Resitivity
- Heat flux
- Viscosity
- Can follow Zhdanov updated with Raghunathan et al. closure

Summary

- Rederived MIHESEL equation with new closure
  - Implementation ongoing
- Reduced coupling through polarisation term
  - Significant performance increase
  - Small deviation from previous equations
- Briefly discussed 3D extension which is next major goal!
  - Thoughts and ideas are welcome

See you in January/February