

X-ray optics at accelerator-based light sources

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8th EIROforum School on instrumentation

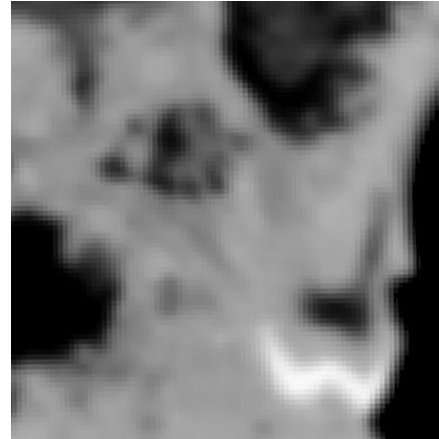
- **Characteristics of Synchrotron Radiation Sources**
- **Why do we need X-ray optics?**
- **X-ray Mirrors**
- **Crystal monochromators**
- **Focusing optics**

ESRF – The European Synchrotron

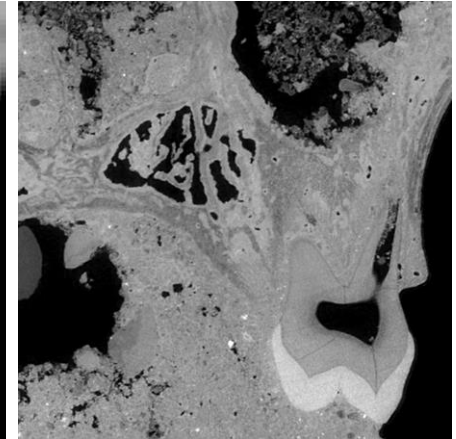


Super microscope producing X-rays

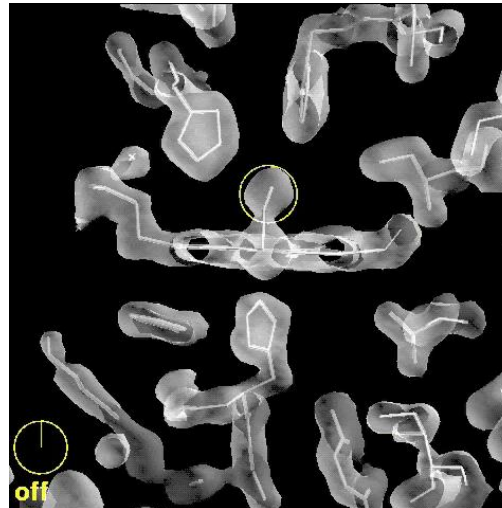
10 trillion times brighter
than in hospitals



In hospital



At the ESRF

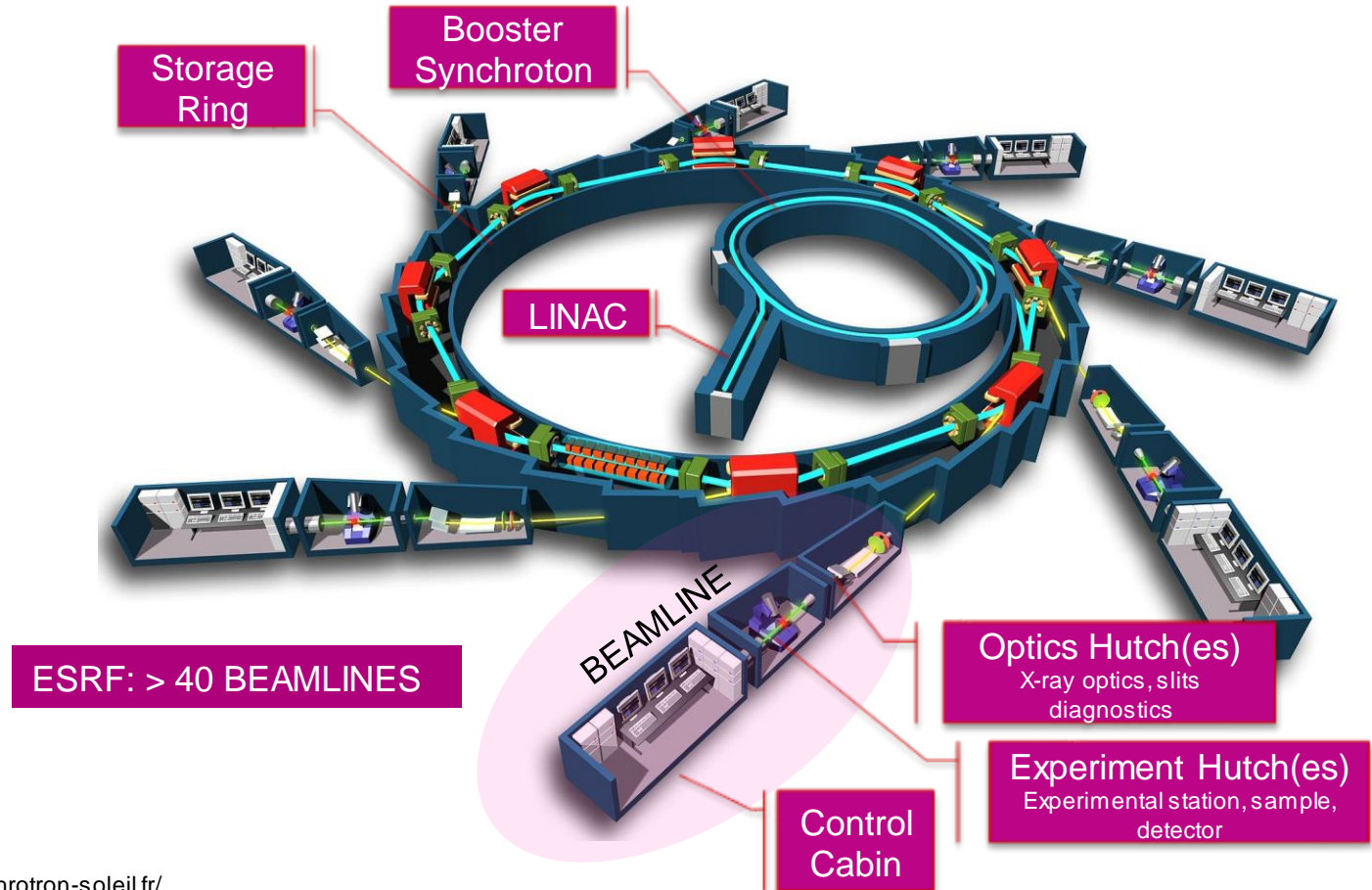


Schofte et al., 2003, Science 300

better resolution

- spatial
- spectral
- time

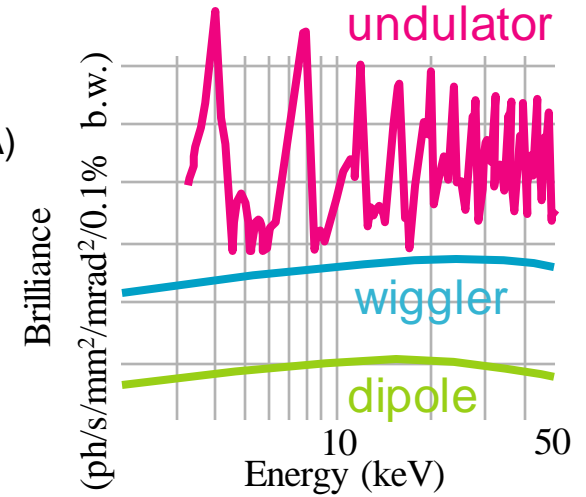
SCHEMATIC OF A SYNCHROTRON RADIATION (SR) LIGHT SOURCE



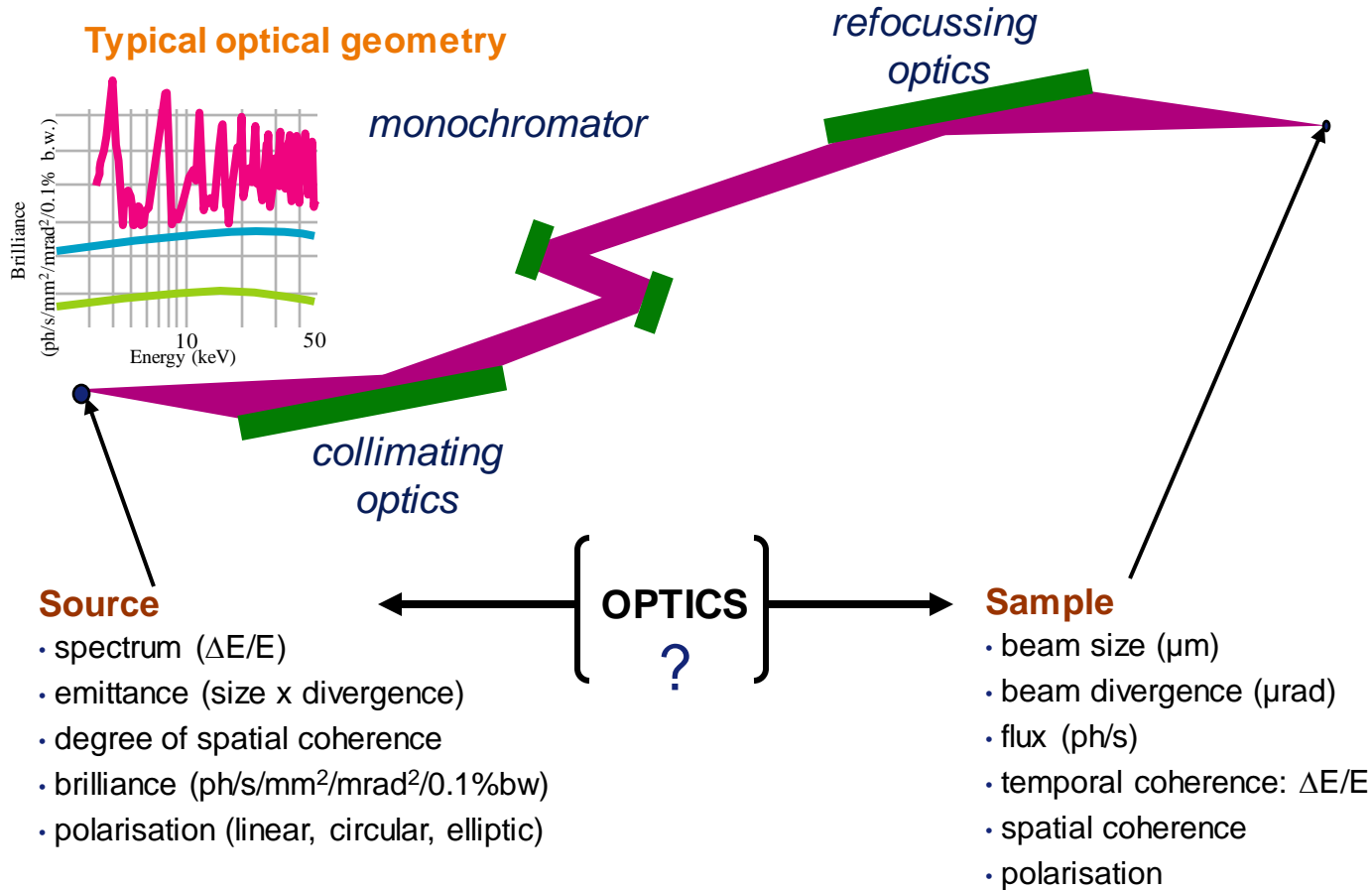
<http://www.synchrotron-soleil.fr/>

X-RAY BEAMS AT 3RD AND 4TH GENERATION SR SOURCES

- Beam size
 - Unfocused: few mm to few cm (source is weakly divergent)
 - Focused beam: < 50 nm to ~10's μm
- Energy range/tunability
 - $0.1 \text{ eV} < E < 0.5 \text{ MeV}$ (at ESRF mostly 3-100 keV \approx 4-0.125 \AA)
- Energy bandwidth ($\Delta E/E$):
 - 10^{-2} to 10^{-8} at sample, typically $\Delta E \sim$ few eV @ 20keV
- Polarized radiation
 - 100% linear or circular or elliptical
- Pulsed radiation
 - Typically 50 ps pulses every ns
- High degree of coherence
- Photon Flux
 - Brilliance: 10^{22} ph/sec/mrad²/mm²/0.1%bw (10^{11} higher than conventional sources) \Rightarrow photon flux (@ $\Delta E/E = 10^{-4}$): 10^9 - 10^{14} ph/s
 - Extremely variable photon rates on detectors (< 1 ph/s to full beam flux)
- Power
 - Several kW total power, several 100 W/mm² power density (white beam)



SYNCHROTRON BEAMLINES



VISIBLE LIGHT OPTICS

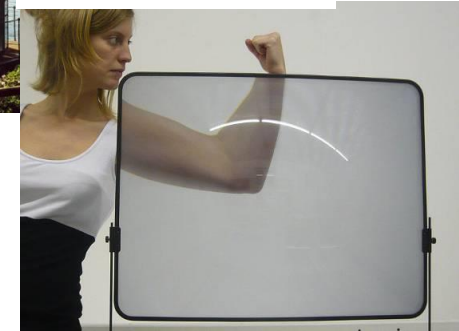
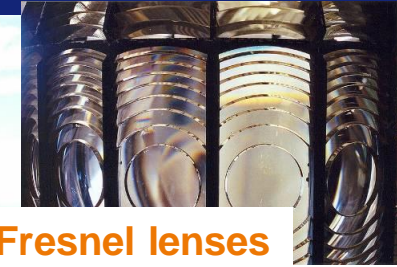
Refractive lenses



Polarising Optics



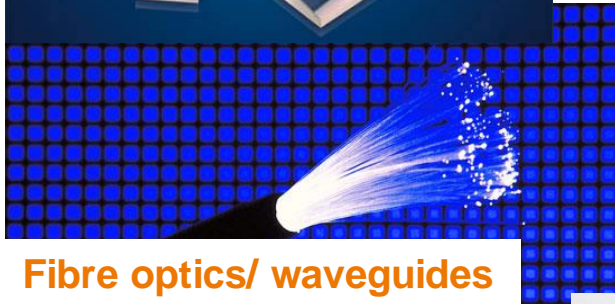
Fresnel lenses



Diffractive optics



Fibre optics/ waveguides



Mirrors

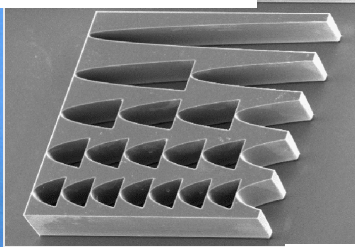
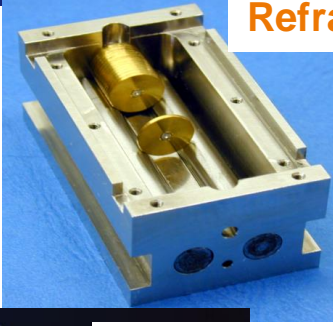


Filters

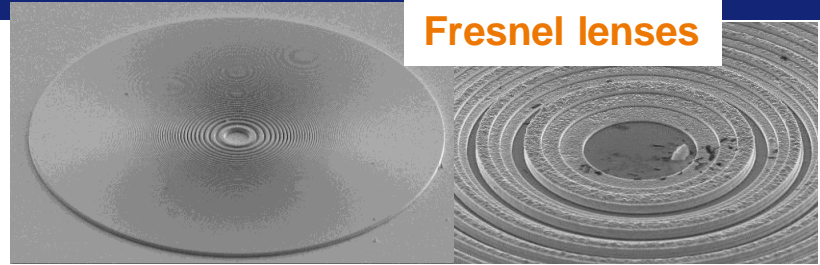
+ interferometers, ...

X-RAY OPTICS

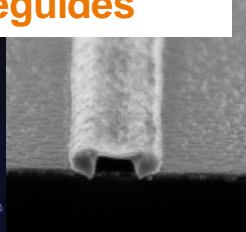
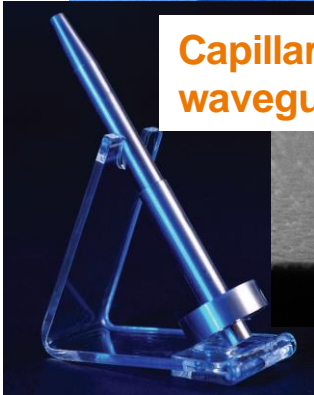
Refractive lenses



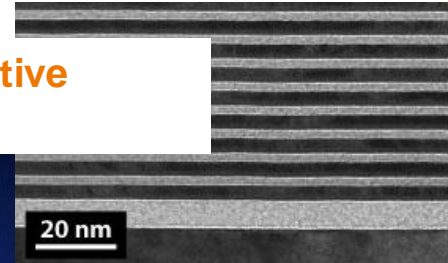
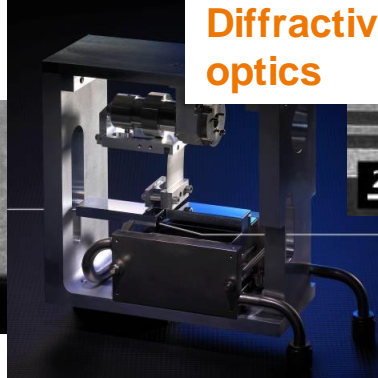
Fresnel lenses



Capillary optics waveguides



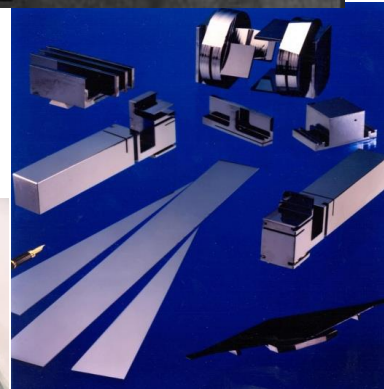
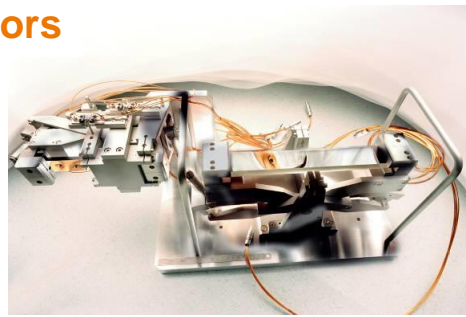
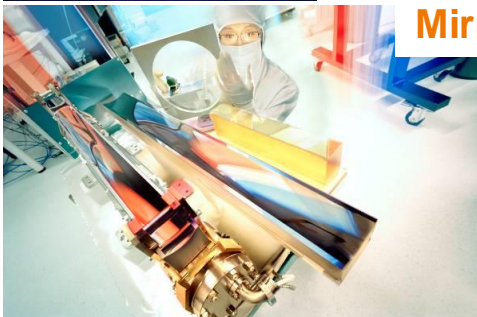
Diffractive optics



Filters



Mirrors



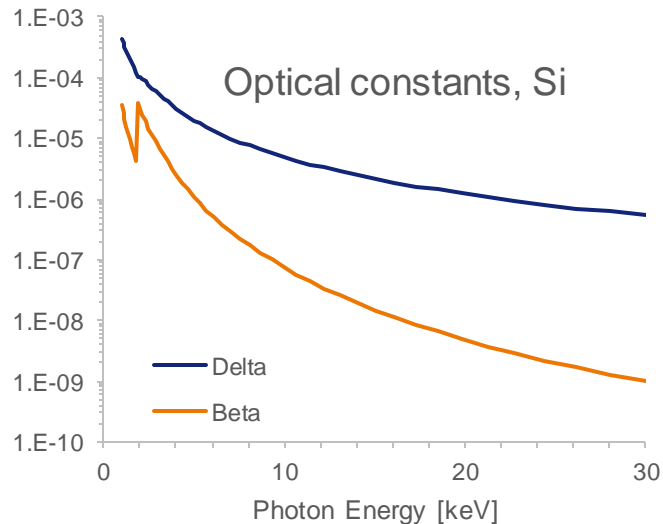
+ polarising optics,
interferometers, ...

REFRACTIVE INDEX FOR X-RAYS: $N < 1$

The refractive index for X-rays for wavelength, λ , is usually written as:

$$n = 1 - \delta - i\beta \quad \text{where } \delta = \left(\frac{n_e r_e f_1}{2\pi} \right) \lambda^2 \quad \text{and } \beta = \lambda\mu/4\pi$$

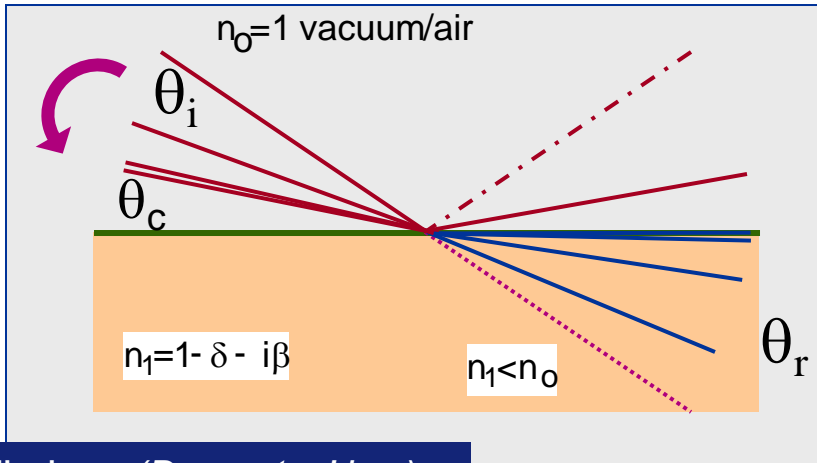
r_e is classical electron radius (2.82×10^{-15} m), n_e is electron density, f_1 is the real part of the atomic scattering factor ($f_0 = f_1 + if_2$), μ is linear absorption coefficient ($I = I_0 e^{-\mu t}$)



So refractive index for X-rays is less than, but close to, 1

- weak refraction c.f. visible light
- consequences for type of optics used for X-rays

TOTAL EXTERNAL REFLECTION OF X-RAYS



θ_c (E=10keV)	• Silicon	3 mrad
	• Nickel	6 mrad
	• Gold	9 mrad

- X-ray mirrors operate in grazing incidence:
- asymmetric apertures
 - long to capture X-ray source emission

Snell's Law (Descartes' law) :

$$n_0 \cos \theta_i = n_1 \cos \theta_r$$

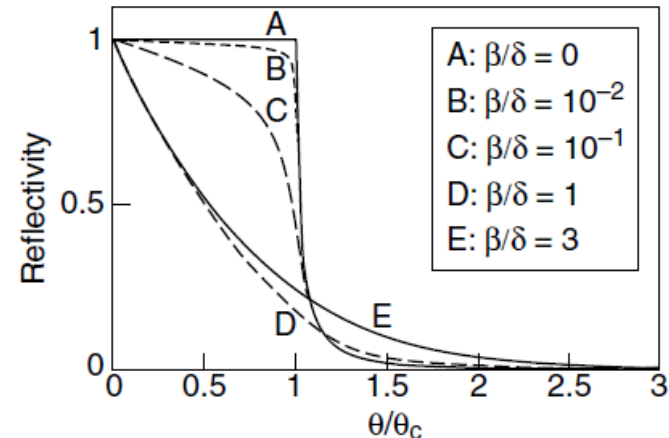
for $\delta \ll 1$ and $\beta \ll \delta$

$$\theta_c \approx \sqrt{2\delta} \propto \lambda \sqrt{Z}$$

θ_c , critical angle for total external reflection.

See also: <http://www.coe.berkeley.edu/AST/sxrev/>

Reflectivity (calculated from Fresnel equations)



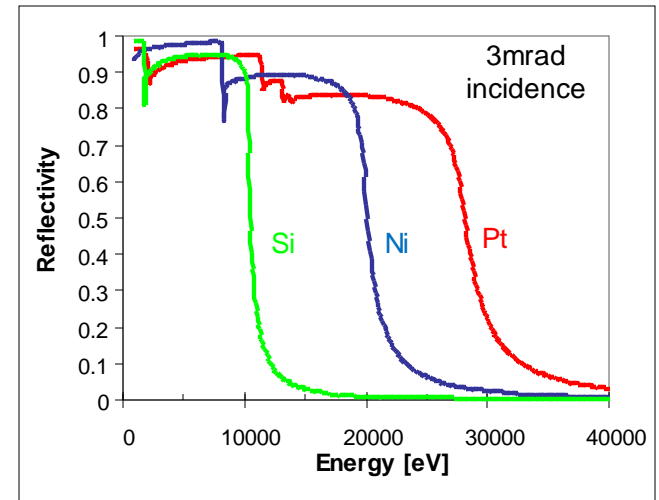
X-RAY MIRRORS: LOW PASS ENERGY FILTERS

X-rays of **fixed energy** will be totally externally reflected at angles below θ_c

Since $\theta_c \approx \sqrt{2\delta} = \lambda \sqrt{\frac{n_e r_e f_1}{\pi}}$ is inversely dependent upon the energy, we can calculate a **critical energy**, E_c , below which X-rays at a **fixed angle** will be totally externally reflected.

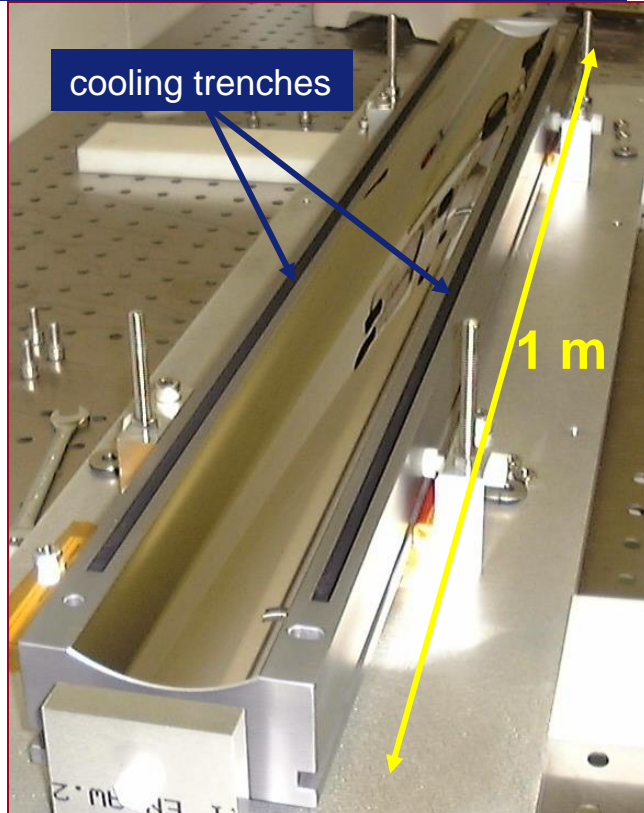
- X-rays above the critical energy will be absorbed, scattered or transmitted through the mirror
- X-ray mirrors are often illuminated with a polychromatic beam and used to filter the high-energies from the beam
- Most mirrors are manufactured from Si
- By reducing the grazing angle, E_c can be increased
- E_c can also be increased by applying a coating on the mirror with higher density

'real' materials



DUE TO SMALL θ_c X-RAY MIRRORS TEND TO BE LONG

Pt-coated single-crystal Si mirror



Mirrors are often curved to provide focusing
Grazing incidence \rightarrow asymmetric curvature

Roughness $\leq 2\text{\AA}$ rms

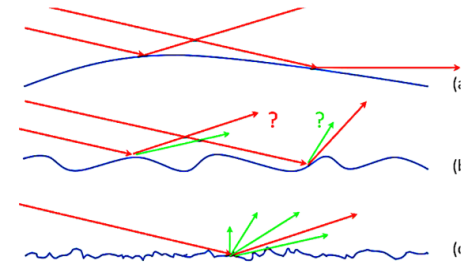
Radii of curvature:

- Sagittal: 71.60 mm
- Meridional: 25 km

Slope error (RMS)

$< 1.0 \mu\text{rad}$ over 900 mm

slope errors & roughness

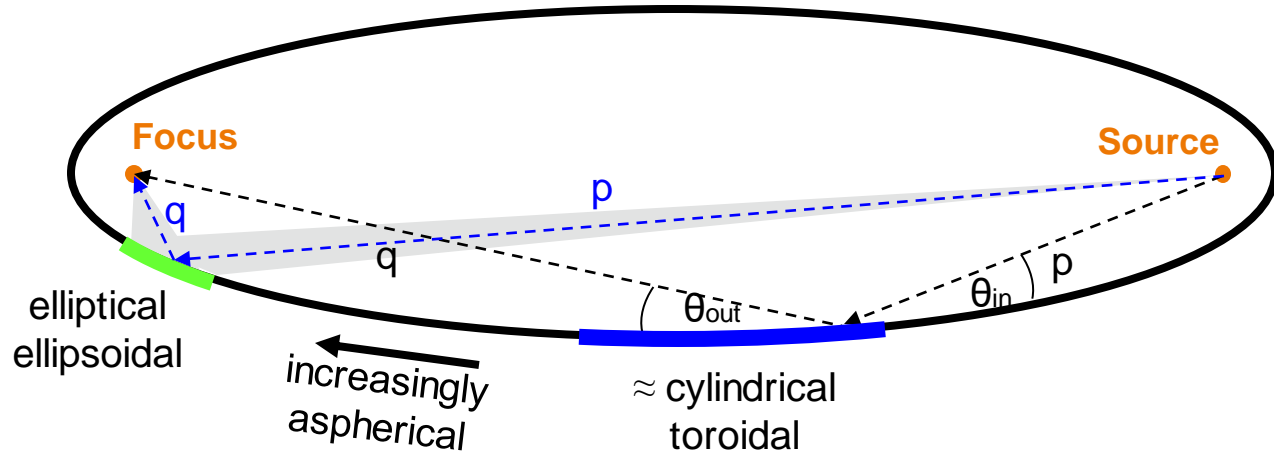


- Mirrors are prone to contamination and are usually operated under vacuum
- They can remove a considerable amount of the power (> 100 's W) of the incident beam and reduce the X-ray power on downstream optics (which may be more beam sensitive)
- Often need to be actively cooled (usually using water cooled heat exchangers)

CURVED MIRRORS FOR FOCUSING

The high reflectivities (and relatively large apertures) of mirror systems makes them attractive for focusing the X-ray beam

For point to point focusing an ellipsoidal surface is ideal – constructing the surface such that the X-ray source is at one focus of the ellipse gives a focused beam at the other. Similarly placing the source at the focus of a paraboloidal mirror will produce a collimated beam after reflection



Local radii of curvature :
incidence angle θ

$$\left. \begin{aligned} R_m &= \frac{2}{\sin \theta_i} \left(\frac{pq}{p+q} \right) \\ R_s &= 2 \sin \theta_i \left(\frac{pq}{p+q} \right) \end{aligned} \right\}$$

Mirror focusing is achromatic – focal length doesn't depend on energy

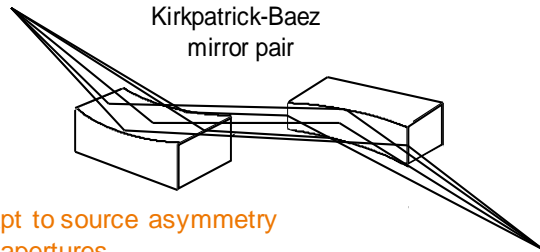
Typically: $R_m \sim \text{km}$ (bending?)
 $R_s \sim \text{cm}$

- If $0.3 < M = q/p < 3$ then generally we can use a simplified cylindrical/toroidal approximation
- For stronger demagnifications aberrations are too strong and technologically challenging aspheric surfaces are required

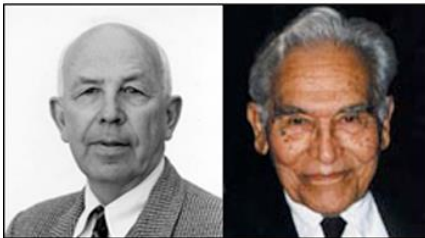
2D FOCUSING: KIRKPATRICK-BAEZ MIRROR SYSTEMS

Short radius ellipsoidal mirrors are particularly difficult to manufacture

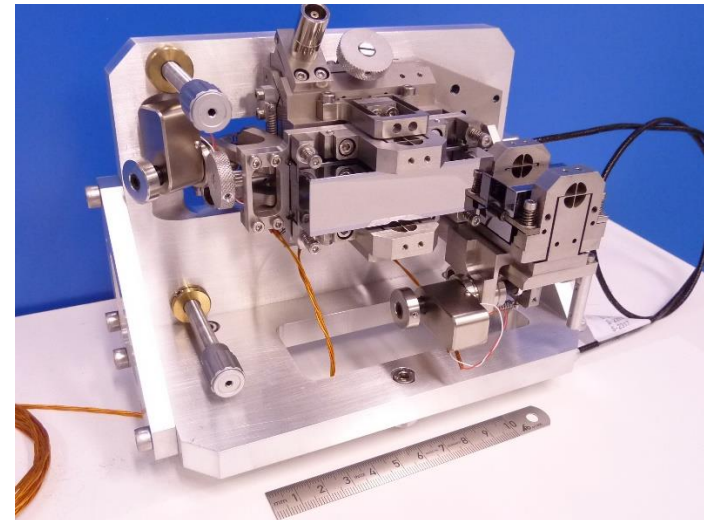
Alternative is to use 2 elliptical cylinder mirrors focusing in orthogonal planes. This was originally proposed in 1948 and is now commonly used for micro- or nano-focusing 'KB-mirrors'



- adapt to source asymmetry
 - apertures
 - demagnification
- mirrors are either dynamically bent or statically figured to create the elliptical surface



Kirkpatrick, Baez

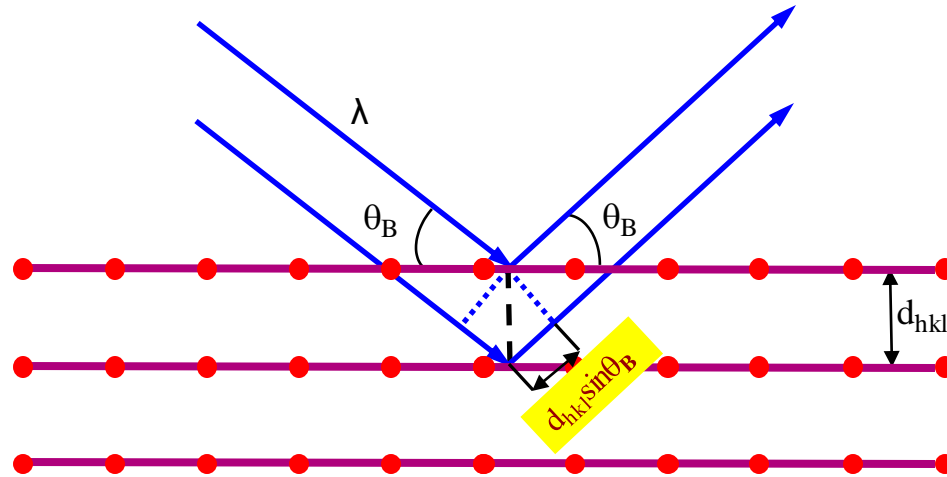


Fixed curvature KB mirror system with surface figure errors ~ 1 nm
Focused beam size at 34 keV: 14×14 nm² with 6×10^{10} ph/s

X-RAY DIFFRACTION

X-ray diffraction results from elastic* scattering of X-rays from structures with long-range order. For X-ray optics we are generally concerned with **highly perfect single crystals** cf **neutron mosaic crystals**

*elastic -> energy unchanged by scattering process



$$\text{Bragg equation: } 2d_{hkl} \sin\theta_B = n\lambda$$

- Incident X-rays are “reflected” at atomic planes in the crystal lattice
- **Path difference** of the rays: $2d_{hkl} \sin\theta_B$
- Constructive interference if the path difference amounts to λ ($n\lambda$)

h k l are usually used, (e.g. 1 1 1, 3 3 3, 4 4 4), these are not Miller indices, but Laue indices, or “general Miller indices”.

CRYSTAL MONOCHROMATORS

A crystal monochromator slices out a narrow energy band from incident beam. Energy, E , determined by incidence angle, θ_B , of X-ray beam onto crystal planes according to Bragg equation:

$$E = \frac{hc}{\lambda} = \frac{hc}{2d_{hkl} \sin \theta_B}$$

c = light velocity
 h = Plancks constant

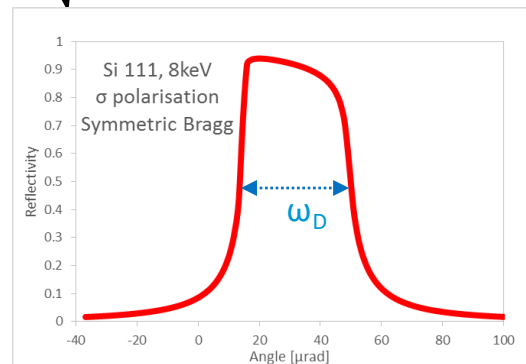
Energy width of beam depends upon type of crystal and reflecting planes used (described by angular Darwin width ω_D) & divergence of incident beam, ψ_0

$$\frac{\Delta E}{E} = \frac{\Delta \lambda}{\lambda} = \sqrt{\omega_D^2 + \psi_0^2} \cot \theta_B$$

If $\psi_0 = 0$, intrinsic (best) energy resolution

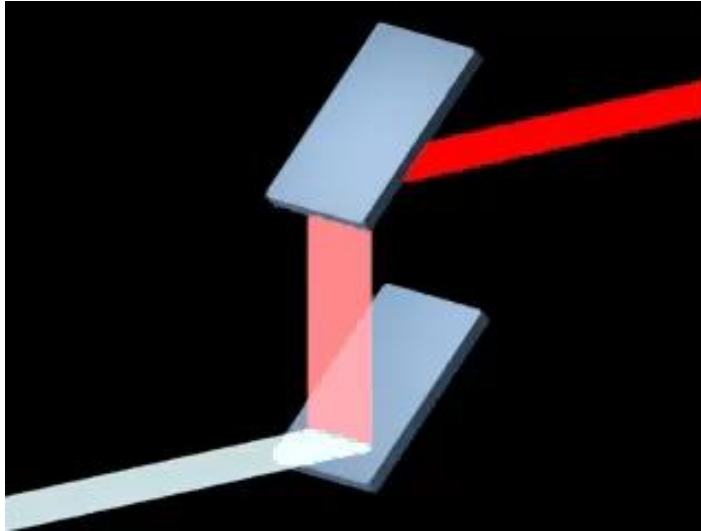
e.g. Si 111 Bragg reflexion,
 $d_{hkl} = 3.1355 \text{ \AA}$
 $\omega_D = 34 \text{ \mu rad}$ (@ 8keV):

$\theta_B = 14^\circ$
with a parallel incident beam:
 $\Delta E/E = 1.4 \cdot 10^{-4}$, $\Delta E = 1.1 \text{ eV}$



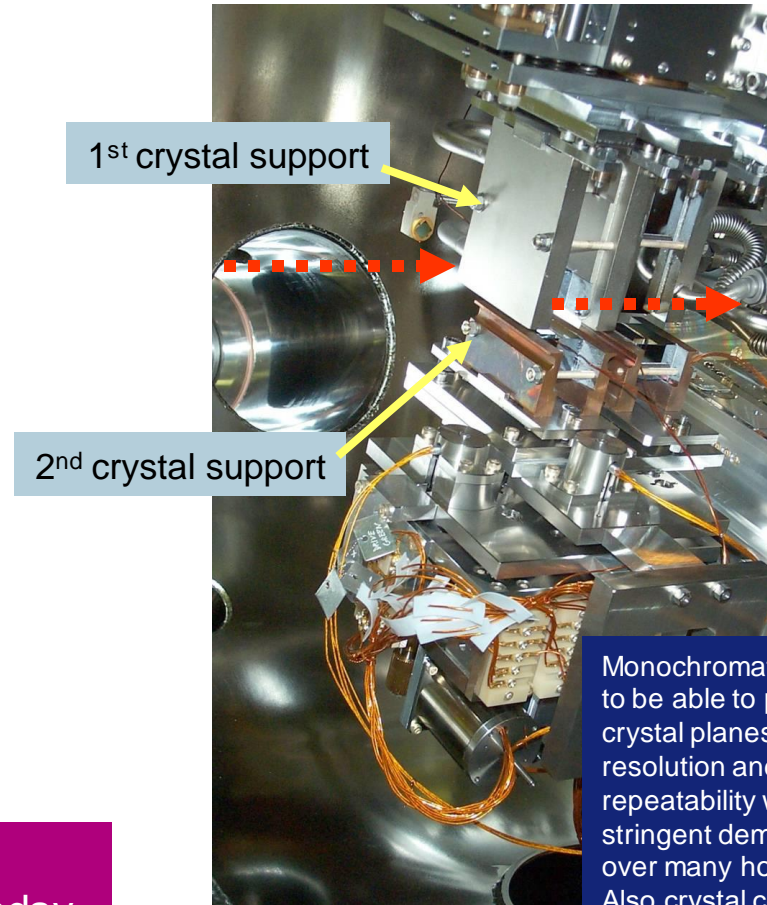
Refl.	θ_B (12keV) [°]	ω_D [μrad]	ΔE [eV]
111	9.48	22.4	1.6
220	15.61	15.9	0.7
311	18.39	8.9	0.3
333	29.62	4.8	0.1

DOUBLE CRYSTAL MONOCHROMATORS



First crystal monochromatizes beam
– the second ensures incident and
exit beams have the same direction

Increasingly mechatronics used for
monochromators – L. Ducotté's talk Tuesday



Monochromators typically need
to be able to position the
crystal planes with μrad
resolution and similar
repeatability with particularly
stringent demands on stability
over many hours.
Also crystal cooling required...

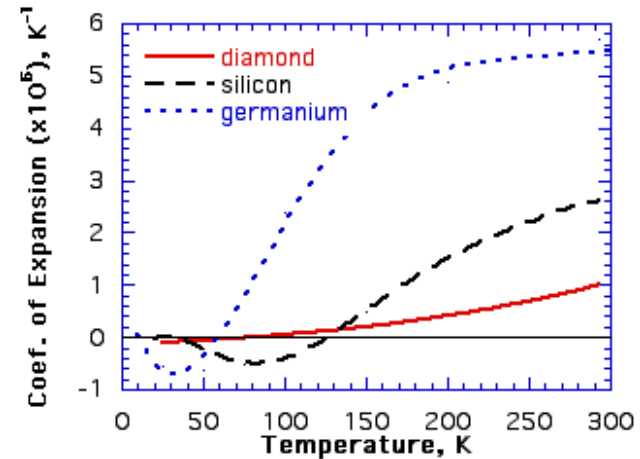
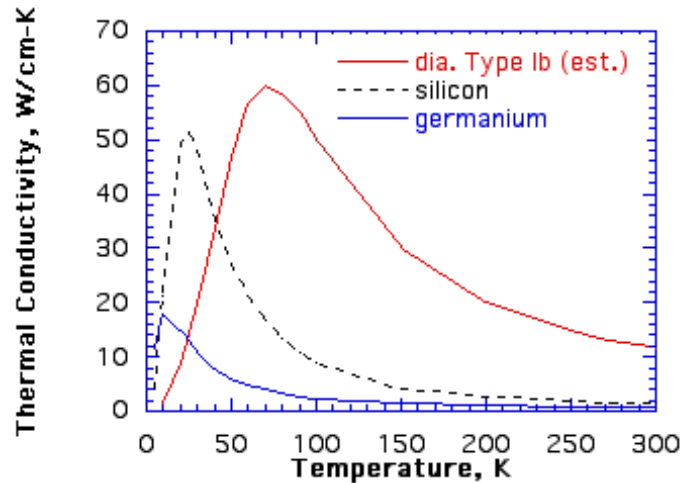
MONOCHROMATOR COOLING

The first monochromator crystal absorbs considerable power

Thermal gradients ΔT , and coefficient of thermal expansion, α , contribute to crystal distortions:

$$\alpha \Delta T = \Delta d/d = \cot(\theta) \Delta\theta < \cot(\theta) \omega_D$$

We therefore need to look for materials that have a very low coefficient of thermal expansion, α , and/or have a very high thermal conductivity, k , so that the material cannot develop large ΔT 's.



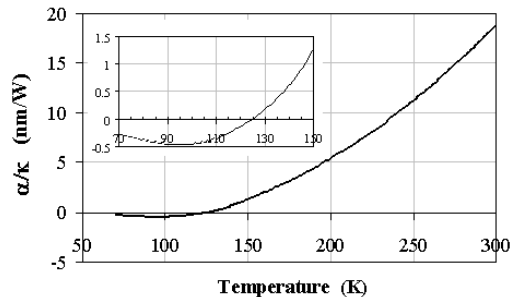
courtesy D. Mills, APS

FIGURE OF MERIT (FOM) FOR CHOICE OF CRYSTAL FOR HIGH HEAT LOADS

These properties incite us to use cryogenically cooled silicon or room temperature diamond as high heat load monochromators

FOM for various crystals –higher is better

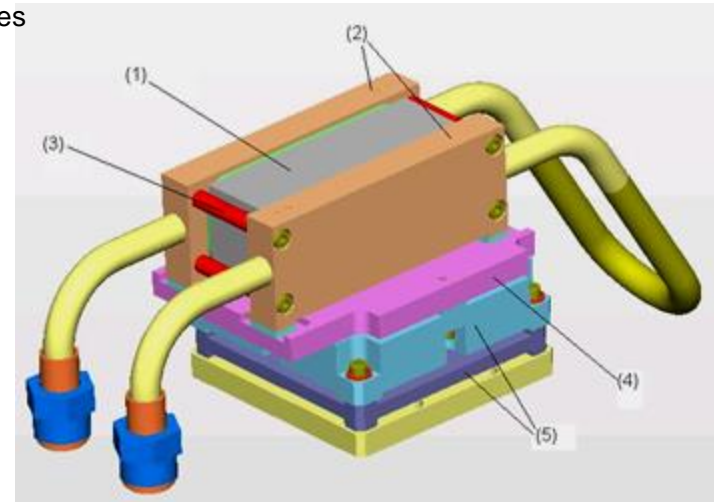
Material	k –thermal conductivity [W/cm .K]	α –coeff thermal expansion [K ⁻¹]	k/ α FOM
Si (300K)	1.2	2.3×10^{-6}	0.5
Si (78 K)	14	-0.5×10^{-6}	28
Diamond (300K)	20	0.8×10^{-6}	25



Ratio of thermal expansion and thermal conductivity of Si α/k vs temperature

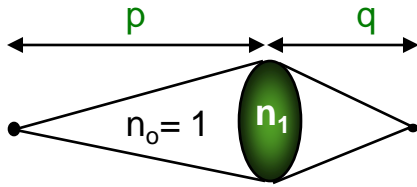
Example of ESRF first crystal assembly:

(1) silicon crystal; (2) copper cooling blocks with internal fins; (3) invar clamping rods; (4) invar base plate; (5) ceramic insulating plates



- Crystal is side cooled with cooling blocks clamped with pressures between 5-10bar
- Deformation of crystal planes due to clamping $<1\mu\text{rad}$
- Most ESRF beamlines using LN2 cooled monochromators

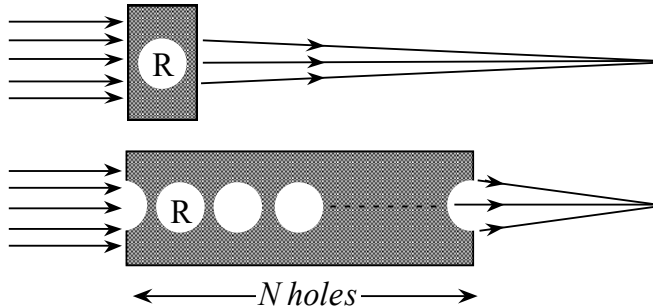
COMPOUND REFRACTIVE LENS



Gaussian lens equation: $\frac{1}{f} = \frac{2(n_1 - 1)}{R}$

Thin lens equation: $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$

R = radius of curvature of lens apex



$$\frac{1}{f} = \frac{2\delta}{R}$$

X-rays: $n = 1 - \delta - i\beta$



$n_1 < 1$: concave lens

$$\frac{1}{f} = N \frac{2\delta}{R}$$

Typically Be or Al lenses –e.g.

Aluminium @ 10keV $\delta = 5.5 \cdot 10^{-6}$

1 hole 100 μm radius : $f = 9 \text{ m}$

15 holes 100 μm radius: $f = 60 \text{ cm}$

A. Snigirev et al. Nature, 384 (1996)

Advantages

- simplicity and relatively low cost
- low sensitivity to heat load
- on-axis focusing

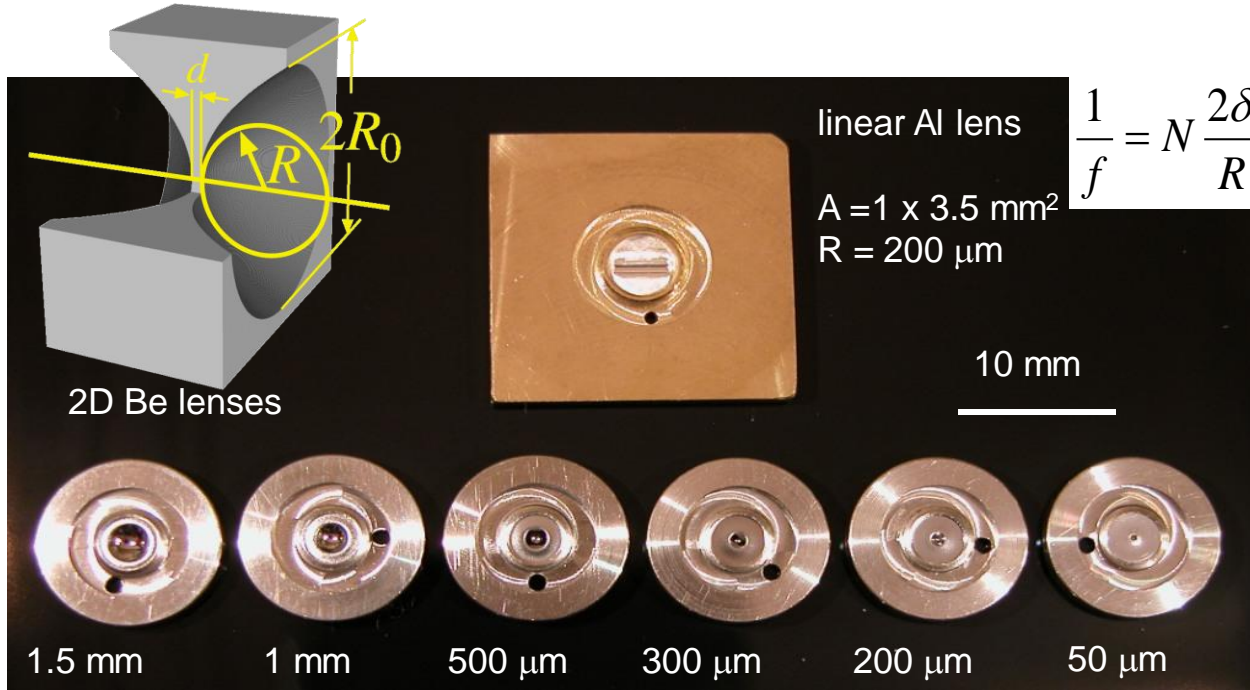
Disadvantages

- efficiency limited by absorption
- small aperture (limited resolution)
- strong chromatic aberrations $f \propto E^2$

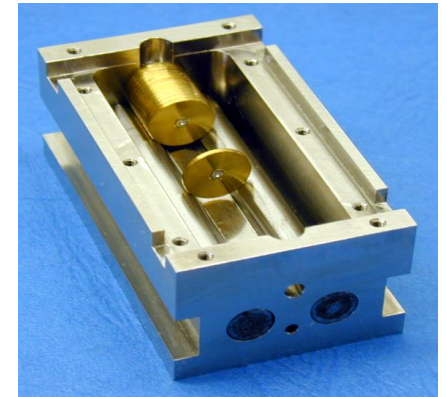
PARABOLOIDAL & PARABOLIC CYLINDER X-RAY LENSES

- Parabolic/oidal profile \Rightarrow no spherical aberration
- Produced by embossing
- Be \sim 2-40 keV \Rightarrow absorption \downarrow
- Al \sim 40-80 keV
- Ni \sim 80-150 keV

Typical parameters :
 $R = 50$ to $1500 \mu\text{m}$
 $2R_0 = 0.45$ to 2.5 mm
 $d < 30 \mu\text{m}$

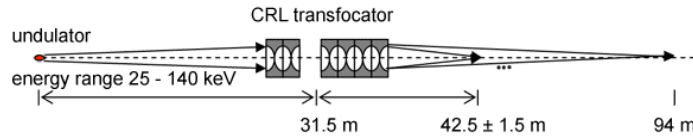


Stack lenses according to required focal length



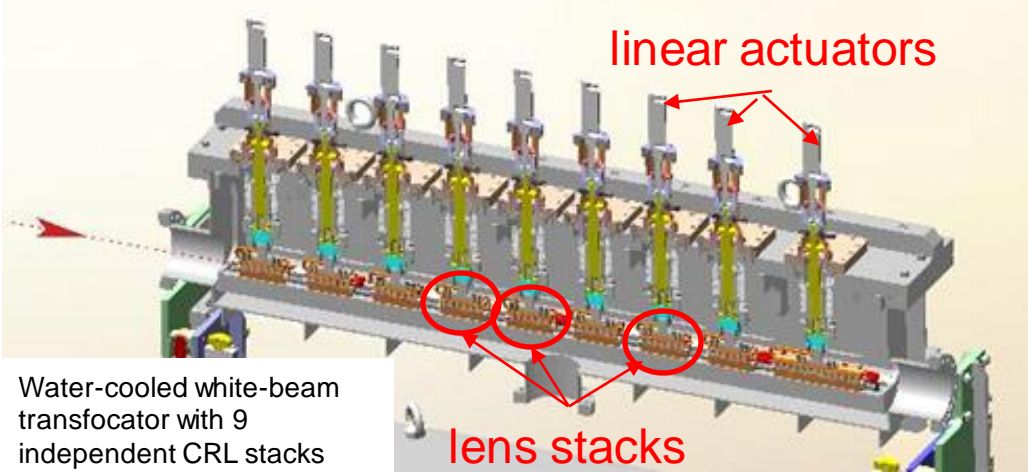
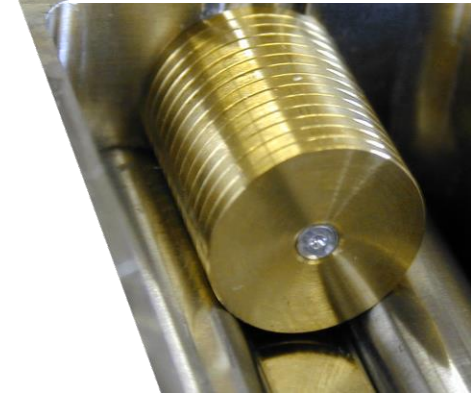
REFRACTIVE LENS 'TRANSFOCATORS'

Need to adapt number of lenses according to required focusing energy and focal length

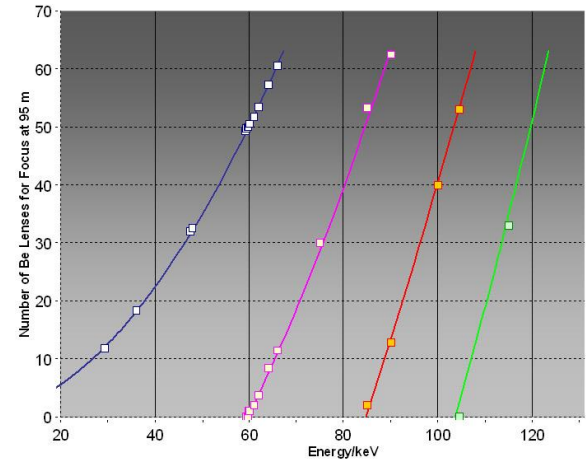


Transfocator scheme allows remote selection of number of lenses to adapt focusing

Stack of 14 CRL lens elements



Water-cooled white-beam translocator with 9 independent CRL stacks



Vaughan et al. J.Synch. Rad. **18** 125-133 (2011)

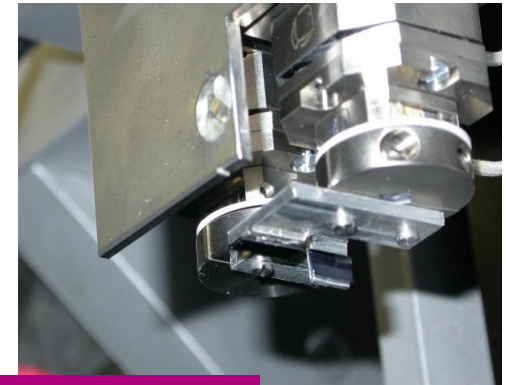
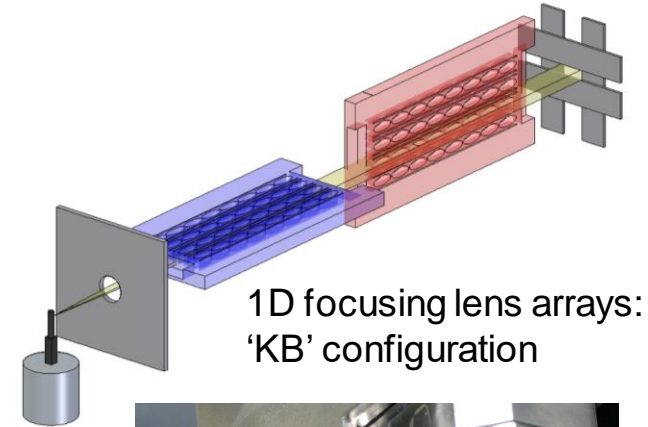
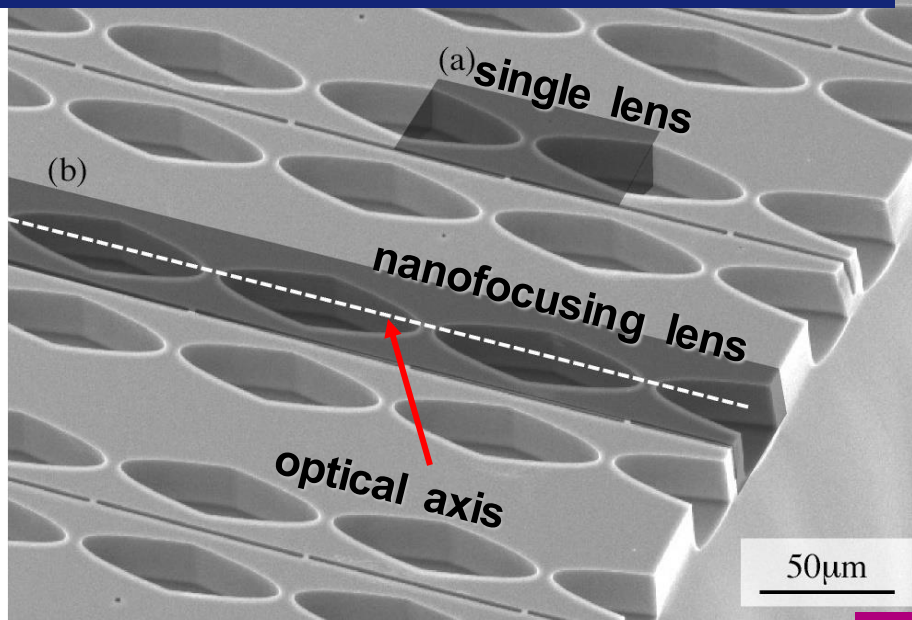
NANOFOCUSING REFRACTIVE LENSES

$$\frac{1}{f} = N \frac{2\delta}{R}$$

Very short focal lengths require large N or small R

extreme curvature: $R = 1\mu\text{m} - 3\mu\text{m}$ $N = 50 - 100$

Si lens arrays made by MEMS technologies



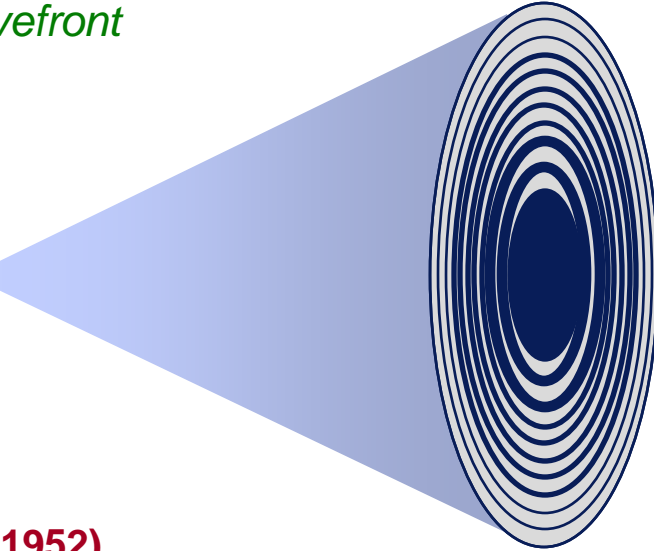
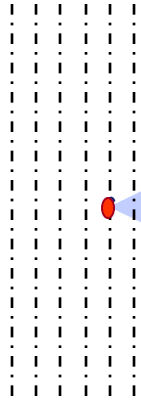
C. Schroer *et al*, *Applied Physics Letters*, **82**(9), 2003

Minimum focus size reported
with refractive lenses 47nm

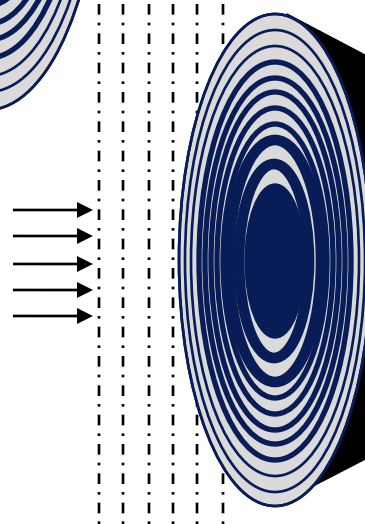
DIFFRACTIVE FOCUSING OPTICS: FRESNEL ZONE PLATES

Planar wavefront

Hologram (Fresnel Zones)



*Reconstruction
by
coherent illumination*



focus

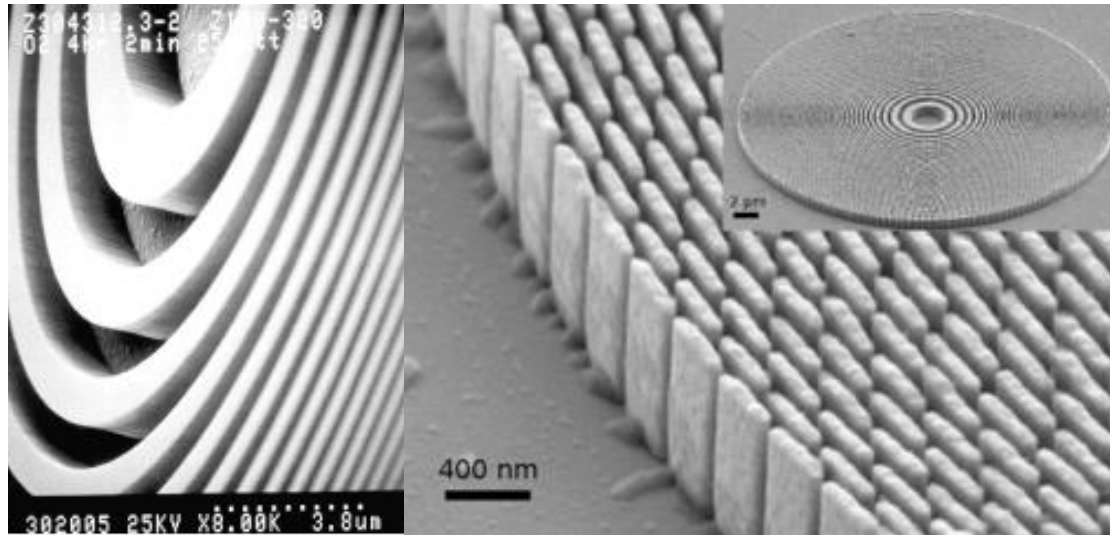
Baez (1952)
Schmahl (1969)
Kirz (1971)
Niemann (1974)

Gabor hologram of a point object

ZONE PLATES

Zone plates are circular diffraction gratings composed of alternating concentric zones

- grating line spacing decreases with the zone radius
- outermost zone has a line width Δr_N (which has to be precisely placed)
- usually manufactured (on thin transmissive membranes) using e-beam lithography
- diffracts/focuses into many orders, m – usually only the first order is used
- filtering the other orders and undiffracted light complicates the implementation



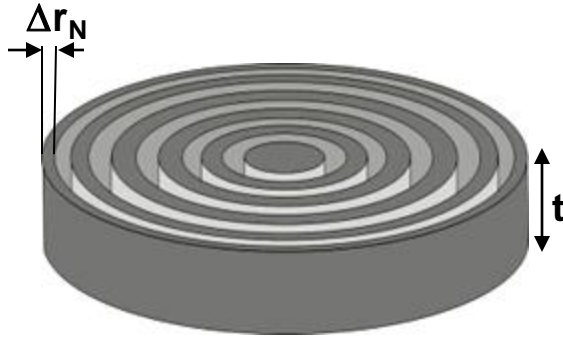
Resolution: $\delta_m = 1.22 \frac{\Delta r_N}{m}$
(smallest $\Delta r_N \sim 10 \text{ nm}$)

Focal length: $f_m = \frac{D \Delta r_N}{m \lambda}$

Consequently zone plates are chromatic with $f \propto E$

DIFFICULT TO FABRICATE EFFICIENT ZONE PLATES!

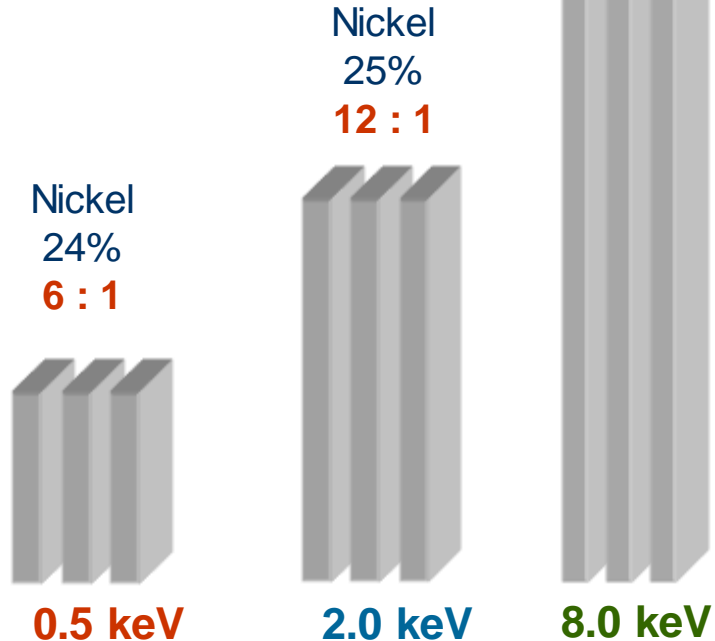
structure height, t , critical for efficiency, Δr_N for resolution



Aspect ratio for $\Delta r_N=50\text{nm}$

Practical limit for small Δr_N is $\sim 10\text{-}15:1$
 \Rightarrow efficiencies $< 10\%$ for tender X-rays (2-10keV)

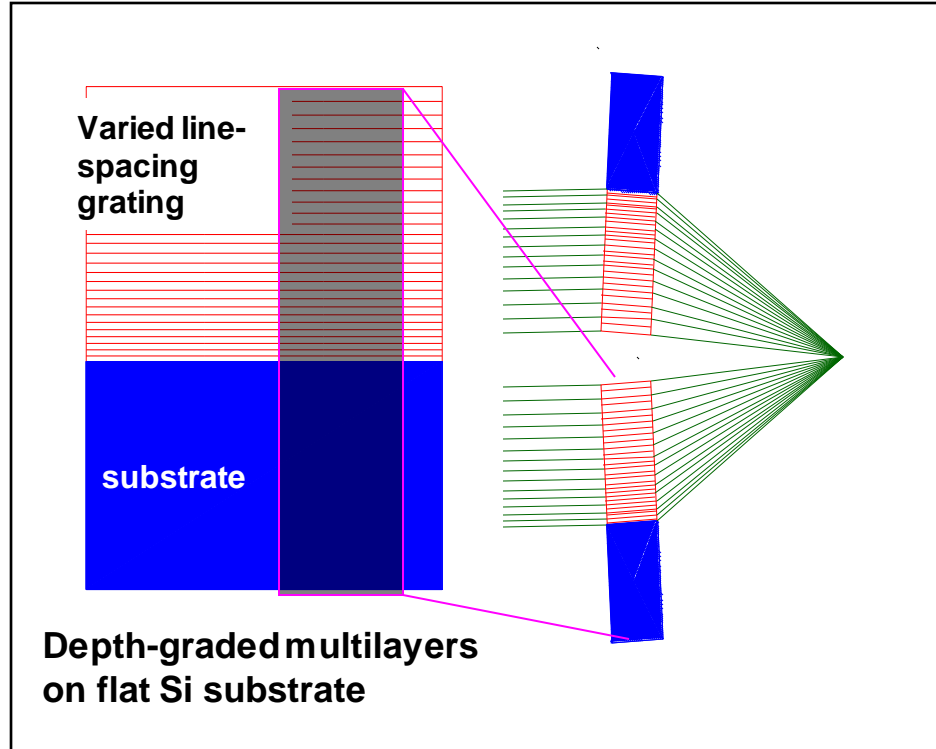
Material	t (μm)	ε (%)
E=0.5keV		
Ge	0.28	16
Ni	0.25	24
E=2.0keV		
Ni	0.60	25
Au	0.45	24
E=8.0keV		
Ta	1.70	32
W	1.50	33



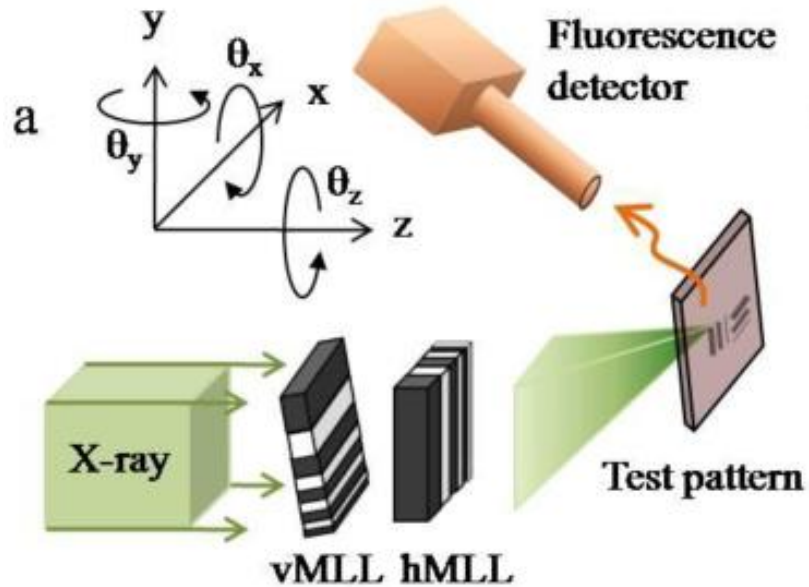
MULTILAYER LAUE LENSES

Initial development at CNM, APS group, Argonne Nat. Lab. USA,
H. C. Kang, J. Maser, G. B. Stephenson, C. Liu, R. Conley, A. T. Macrander, and S. Vogt, Phys. Rev. Lett. **96**, (2006).

- Deposit varied line-spacing grating on flat substrate
(thinnest structures first)
- Section to 5-20 μm thickness
(high aspect ratio structure)
but small aperture ($\sim 40 \mu\text{m}$)
- One dimensionally focusing volume zone plate well adapted to higher photon energies
- Assemble two (four) in orthogonal orientations for 2D focusing (MLL)

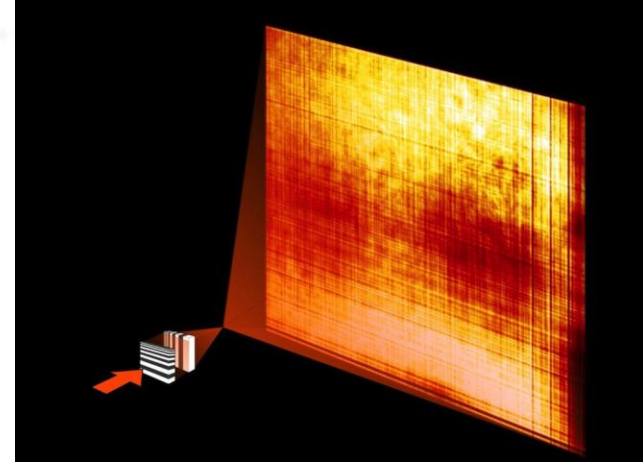


< 10nm focusing @ 22keV



H. Yan et al., *Opt. Express*
19, 15069-15076 (2011)

sub-3 nm 2D focusing at 17.5 keV
Dresselhaus et al. *Opt. Express*
32, (2024): 16004–15

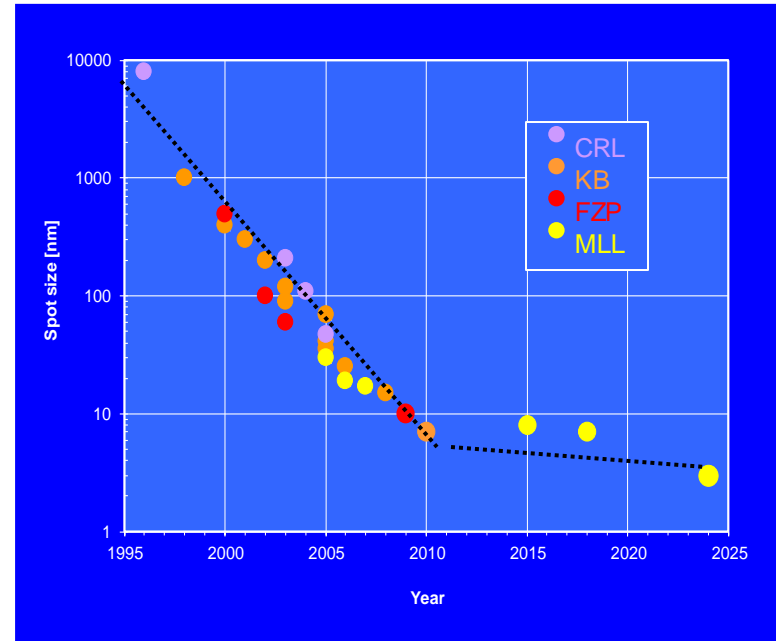


S. Bajt et al., *Light Science & Applications*
(2017); doi: 10.138/lsa2017.162

Optics exist for focusing hard X-rays to sub-10nm dimensions

Routine application of sub-micron beams still complicated

Also many engineering issues in implementing stable, reliable X-ray nanofocusing systems



Historical evolution of the measured spot size for different hard x-ray focusing elements (courtesy C. Morawe)

- A. Morgan et al. *Scientific Reports*, 5, 9892 (2015)
- H. Mimura et al. *Nature Physics*, 6, 122-125 (2010).
- J. Vila-Comamala et al., *Ultramicroscopy*, 109, 1360–1364 (2009)
- J. Dresselhaus et al. *Opt. Express* 32, (2024): 16004–15
- H. Kang et al., *Physical Review Letters*, 96:127401 (2006)
- C. Schroer et al., *Physical Review Letters*, 94:054802 (2005)

Best focus Experiments

Ultimate resolution Theory