INTRODUCTION TO NEUTRON OPTICS

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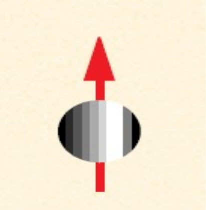
Introduction to Neutron Optics

- The Neutron
- The Neutron Source at ILL
- Reflective Optics - Neutron guides
- Diffractive Optics - Neutron Monochromators
- Polarized Neutrons

Neutron Properties
&
Neutron Source
(ILL)

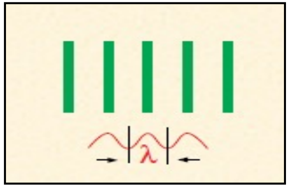
Neutrons have both particle-like and wave-like properties

Particle



- Mass m = $1.675\ 10^{-27}$ Kg
- **No charge**
- Spin 1/2
- Kinetic Energy $E = \frac{1}{2} m v^2 \sim$ meV
  ($v = 2.2$ km/s at $\lambda = 1.8$ Å)
- **Magnetic dipole moment $\mu_n = -1.913\ \mu_N$**
- Life time 886 s

Wave



- Wavelength **$\lambda = h/mv$**
- Wave vector $k = 2\pi/\lambda$
- Moment $p = \hbar k$
- Energy $E = \hbar^2 k^2/2m$

E(meV) = 81.81 / λ^2 (Å)

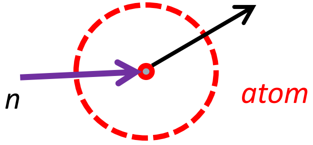
Neutrons	E	λ (Å)
Hot	900 - 80 meV	0.3 - 1 Å
Thermal	80 - 5 meV	1 - 4 Å
cold	5 - 0.03 meV	4 - 50 Å

photons	E	λ
Gamma Ray	~ 1 MeV	< 0.0124 Å
X-Ray	~ 0.5 - 500 keV	25 - 0.025 Å
Visible Light	~ 1.6 - 3 eV	0.4 - 0.8 μm

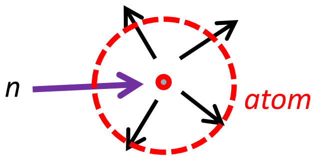
Neutron interactions with matter

***The scattering length** describes the strength and character of the interaction of low-energy neutrons with the individual nuclei and atomic structures.*

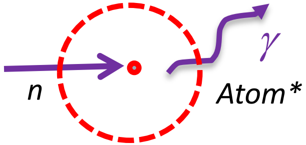
- Neutron Scattering : **Interaction with the individual nuclei via short range forces**
  - **Coherent scattering length b_{coh}** (depends on the direction of scattering vector $Q = k_i - k_f$) → *diffraction, reflection, refraction*
  - Incoherent scattering length $\mathbf{b_{inc}}$ (uniform scattering) → *background …*
  - *Neutron capture (absorption cross section)* → Activation of materials - gamma rays
- Magnetic scattering : **Interaction with unpaired electrons via a dipole interaction**
  - **Magnetic scattering length** $p \approx 0.269\ \boldsymbol{\mu_{at}}\ (10^{-12}\ \mathrm{cm}/\mu_B) \sim b_{coh}$ → *polarized neutron beams*



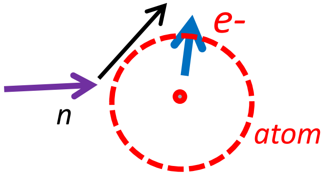
Coherent scattering



Incoherent scattering

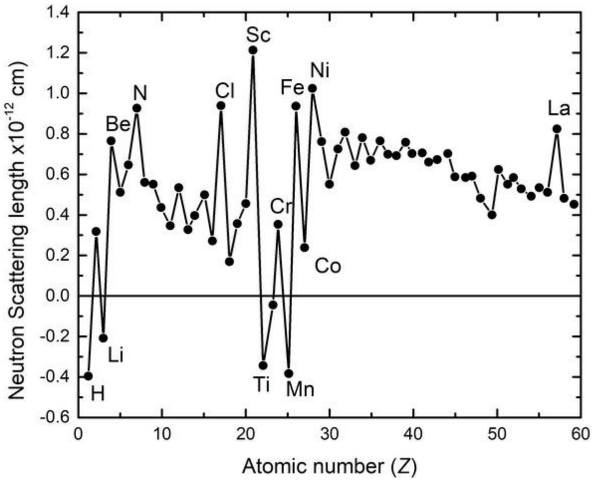


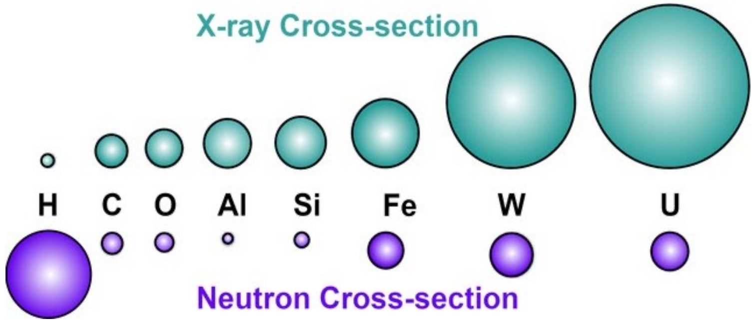
neutron capture



Magnetic scattering

Neutron scattering vs X-Ray scattering





Apparent cross-section σ *of some elements* $\sigma = 4\pi b^2$

Neutron interact with the individual nuclei via short range forces

- The neutron scattering power varies in a quasi-random manner

X-Rays interact with matter via electromagnetic interaction

- The X-Ray scattering power is proportional to the atomic number Z

Scattering lengths of some important elements

Element	b_{coh} (10^{-12} cm)	b_{inc} (10^{-12} cm)	σ_{abs} (10^{-24} cm^2)
H	-3.74	25.2	0.33
D	6.67	4.04	0.0005
B Gd Cd	-	-	767 48890 2520
C	6.64	~ 0	0.0035
Ni	10.3	0.64	4.49
Si	4.14	0.015	0.17
Cu	7.72	0.20	3.78

→ *Background*

→ *Moderator*

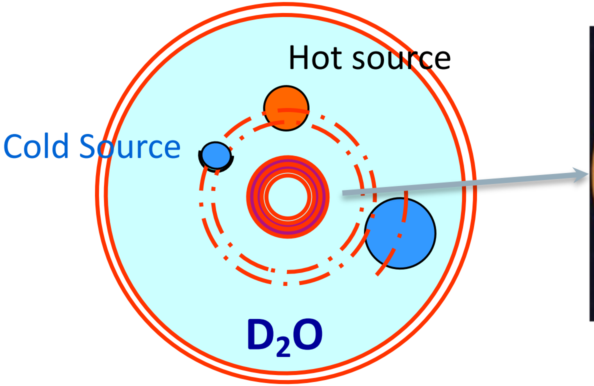
Neutron absorber

Reflective Optics
Diffractive Optics

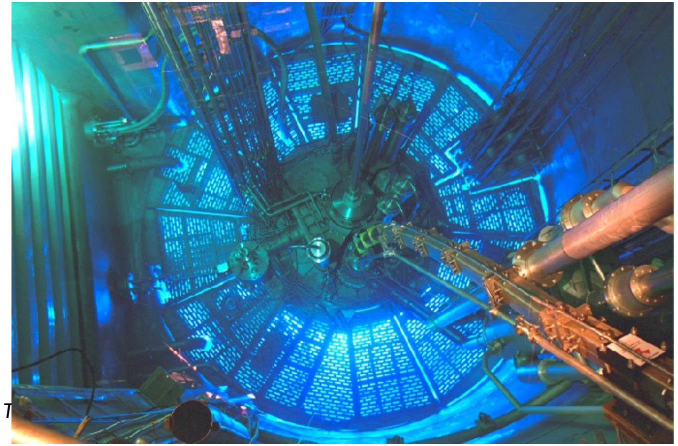
Neutron production

Fission $^{235}U + n\ (meV) \rightarrow F.P. + 2 \text{ or } 3n\ (MeV)$

- *Fast neutrons (~ MeV) are slowed down to meV Energy by collisions in a thermal bath (liquid D_2O) => 1 n sustains the reaction and 1n available*
- **Neutrons must be moderated to give optimized flux distributions** **$\langle E \rangle = k_bT$**
  - ☐ Cold source is liquid Deuterium at T = 20 K $\lambda = 4 \rightarrow 20$ Å
  - ☐ Thermal Source is liquid D_20 at room temperature T = 300 K $\lambda = 1 \rightarrow 4$Å
  - ☐ Hot source is graphite at T = 2000 K (heated by radiation) $\lambda = 0.3 \rightarrow 1$Å



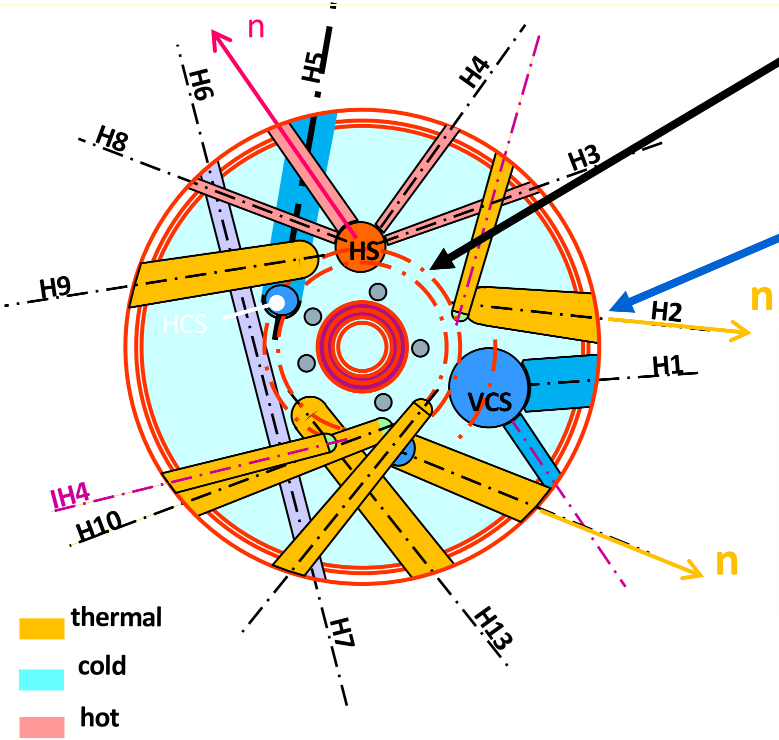
Reactor core



NEUTRONS FOR SOCIETY

Neutron Extraction

The neutrons stream out of the reactor through a series of tubes which conduct beams of **hot**, **cold** or **thermal** neutrons.



Neutron Flux in the moderator region
$1.5\ 10^{15}$ n/cm^2/s

Neutron Flux ~ 10^{11} n/cm^2/s after extraction

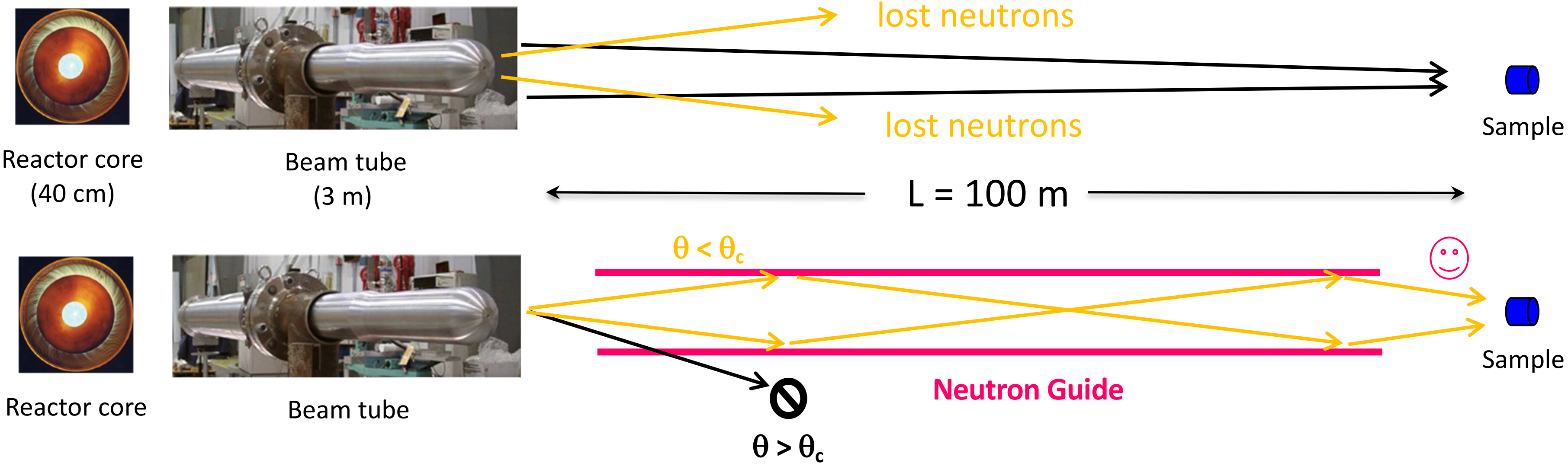
New beam tube assembly

Neutron transport & Neutron Guides

Reflective Optics
Mirrors and Super-Mirrors

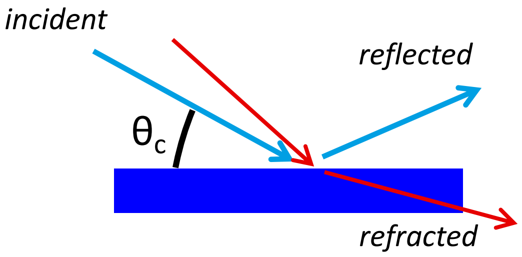
Neutron Transport – Neutron Guides

- Neutron Guides are used to transport neutrons over long distances (L ~ 100 m)
- Guides can increase the neutron flux at the sample by bringing more "divergence" to the sample.



Neutron Mirrors

Reflection/Refraction at Surfaces



Total reflection for $\theta < \theta_c$

Index of Refraction $n = 1 - \frac{\lambda^2 N b_{coh}}{2\pi} - (i\lambda\mu/4\pi)$

n < 1 for most materials, there is a critical angle θc for total external reflection

$$\theta_c = \lambda\sqrt{\frac{Nb_{coh}}{\pi}}$$

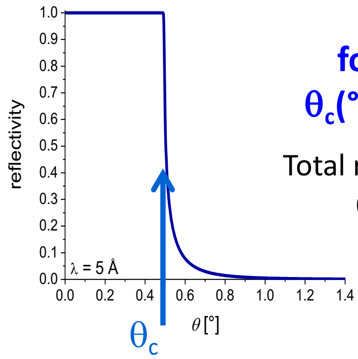
λ = wavelength

Nb_{coh} = Scattering Length Density

(N = atoms/cm^3 ; b_{coh} = coherent scattering length)

μ = linear attenuation factor

Material	Nb (x 10^{38} /m^2)	θ_c (mrad)
^{58}Nickel	**13.31**	**2.03**
Nickel	**9.41**	**1.7**
Iron	8.2	1.62
Copper	6.7	1.39
Silicon	2.08	0.81
Aluminium	2.08	0.81



for natural Ni

$\theta_c(°) = 0.1 \times \lambda$ (Å)

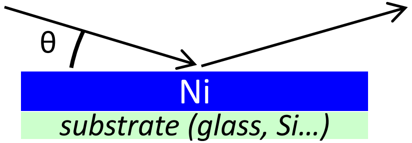
Total reflection for $\theta < \theta_c$

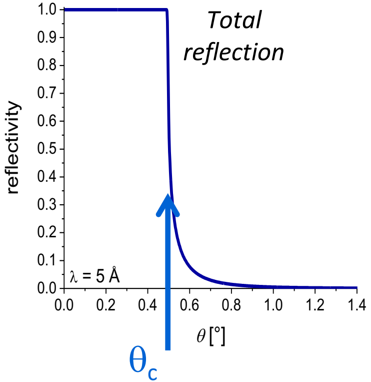
θ_c (5Å) = 0.5°

Neutron Super-Mirrors

Increasing the angular acceptance of Mirrors

Single layer





Total reflection for $\theta < \theta_c$

$\theta_c = (5Å) = 0.5°$

Neutron Super-Mirrors

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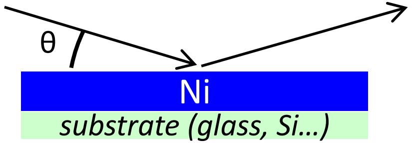
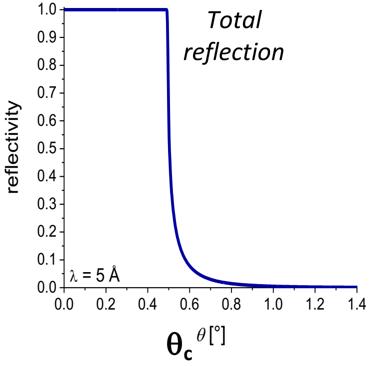
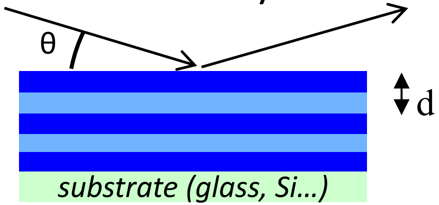


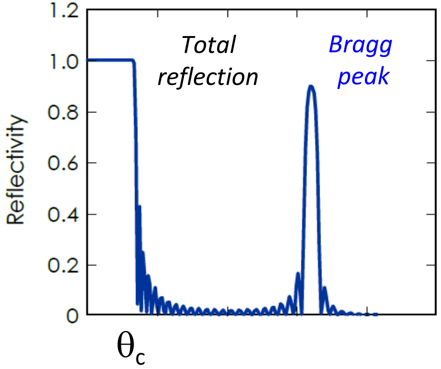


Total reflection for $\theta < \theta_c$

θ_c = (5Å) = 0.5°

Periodic layers





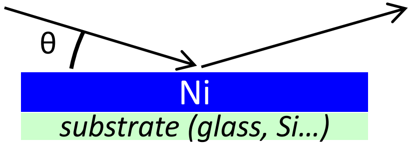
Total reflection for $\theta < \theta_c$

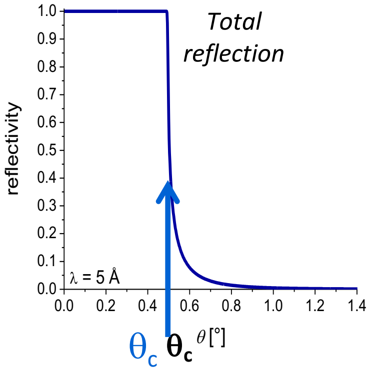
+ additional Bragg peak at $\theta_B \approx \lambda / 2d$

Neutron Super-Mirrors

Increasing the angular acceptance of Mirrors

Single layer





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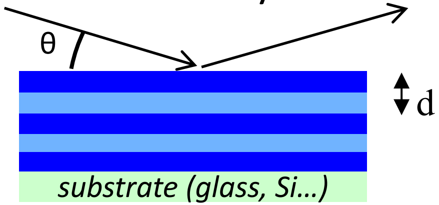

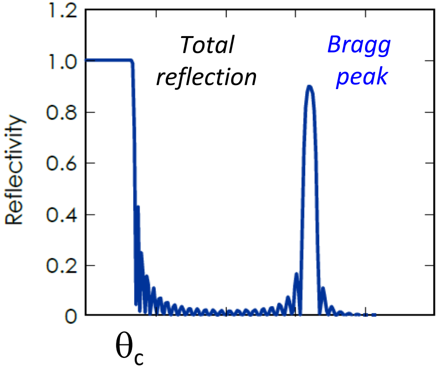

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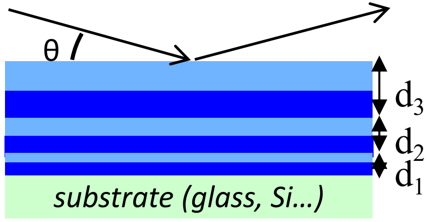
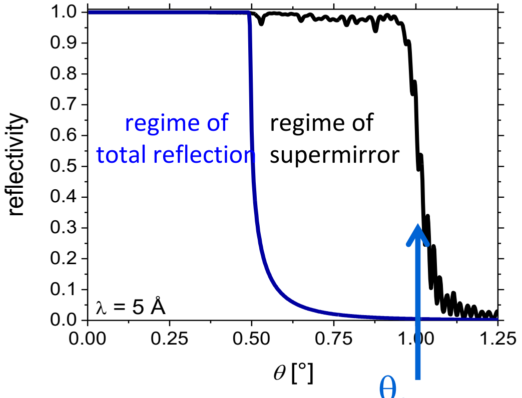
Sequence of bi-layers of variable thicknesses d





Total reflection for $\theta < \theta_c$

+ additional Bragg peaks at $\theta_i \approx \lambda / 2d_i$

Neutron Super-Mirrors

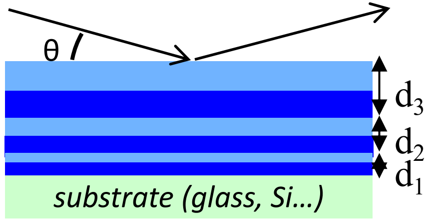
Increasing the angular acceptance of Mirrors

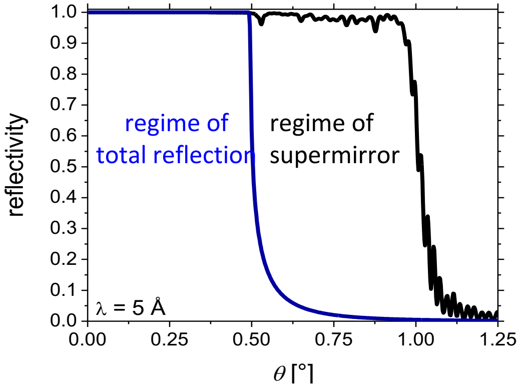
→ **Significant increase in critical angle ($\theta_c = \lambda / 2d_{min}$)**

- m Super-Mirror $$m = \frac{\theta_c^{SM}}{\theta_c^{Ni}}$$
- Gain in neutron flux $$G = \left(\frac{\theta_c^{sm}}{\theta_c^{Ni}}\right)^2 \boxed{\propto m^2}$$
- Neutron Reflectivity $$R \propto \frac{4N^2 d^4 \boxed{(N_1 b_1 - N_2 b_2)^2}}{\pi^2 n^4}$$

High contrast → high reflectivity !

Sequence of bi-layers of variable thicknesses d



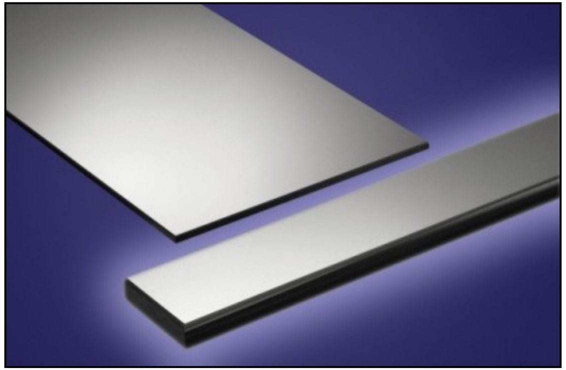


Total reflection for $\theta < \theta_c$

+ additional Bragg peaks at $\theta_\iota \approx \lambda / 2d_i$

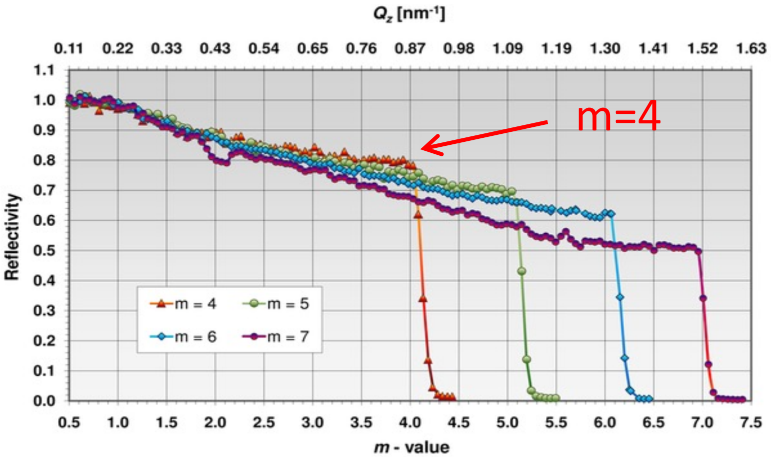
Ni/Ti Super-Mirrors

Still the most efficient ….



Neutron Reflectivity $R \propto (N_1b_1-N_2b_2)^2$

- *Ni* *$Nb = 9.40$ $(10^{-6}Å^{-2})$*
- *Ti* *$Nb = -1.95$ $(10^{-6}Å^{-2})$*



Reflectivity profiles of Ni/Ti super-mirrors for 4 $\leq m \leq$ 7
(sources : www.swissneutronics.ch)

Performances

▪ R > 80% for m = 4 Ni/Ti SM

➢ Gain factor m=4 / Ni mirror = 16 (2D)

➢ but transmission $T \propto R^n$!!

eg : for a 100 m long guide , at least ten reflections

→ Transmission < 10% ...

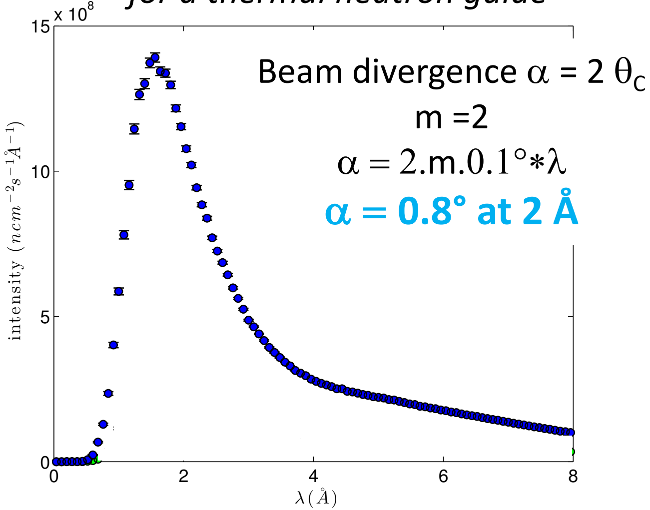
Neutron Guides at I.L.L.

Neutron Guide at ILL

Neutron Guide

- m=2 Ni/Ti Super-Mirrors
- Length > 50 m
- Section ~ 20 x 10 cm^2
- 4 reflective internal surfaces
- Curvature R ~ 1-10 km

Typical spectrum
for a thermal neutron guide



Beam divergence $\alpha = 2\ \theta_C$
m =2
$\alpha = 2.m.0.1° * \lambda$
$\alpha = 0.8°$ at 2 Å

- Neutron guides are typically made by depositing the coating (Ni/Ti supermirror) on smooth glass substrates
- After coating, these substrates are joined together and aligned to make up the desired guide profile
- Curved guides are used in preference to avoid direct line of sight and then fast neutrons are not transmitted (small critical angle)

Production of Neutron Super-Mirrors

Deposition : Reactive DC Magnetron Sputtering

Sputtering machine (ILL) - Production 0.8 m² / day

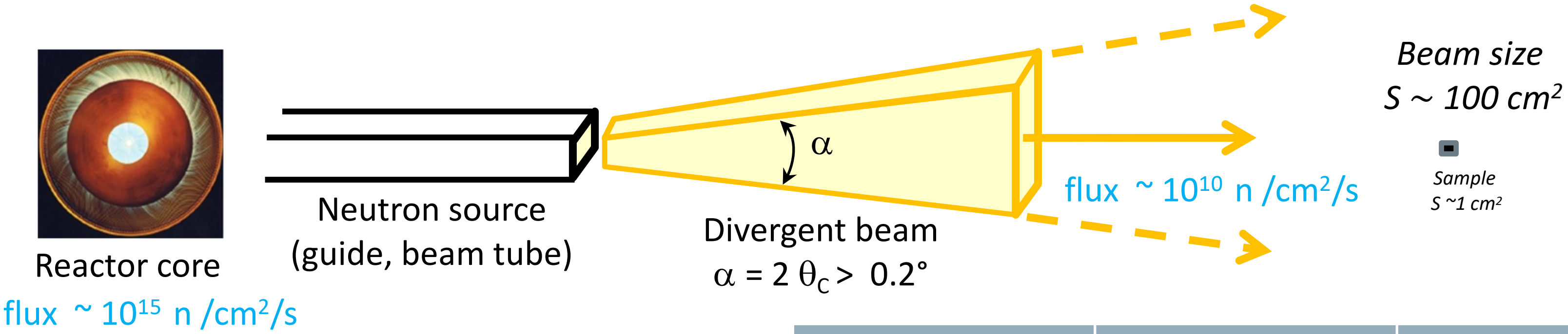
Production

- m= 4 : 1600 layers !
- Substrate : 0.5 cm thick Si wafers or 0.2 cm thick Glass/Si/Sapphire

Diffractive Optics

Crystal Monochromators

Basic Properties of neutron beams



Reactor core

flux ~ 10^{15} n /cm^2/s

Neutron source
(guide, beam tube)

Divergent beam

$\alpha = 2\ \theta_C > 0.2°$

flux ~ 10^{10} n /cm^2/s

Beam size
S ~ 100 cm^2

Sample
S ~1 cm^2

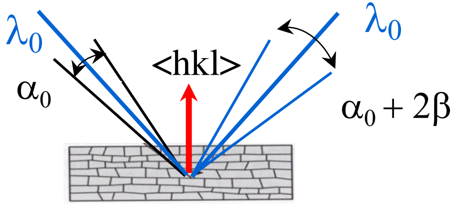
- Neutron sources are **large**
- Neutrons sources are **weak**
- Neutron beams are **divergent**
- Neutron beams are **"polychromatic"**
- Neutron beams are **not polarized**

	Neutrons (ILL)	**X-Rays (ERSF)**
Source	10^{15} n /cm^2/s	> 10^{24} ph /cm^2/s
Flux (white beam)	10^{10} n /cm^2/s	> 10^{10} ph/µm^2/s
Beam Divergence (white beam)	> 3 mRad (H and V)	< 100 µRad Typical 20 µRad
Beam size	20 x 5 cm^2	mm^2 - µm^2

Crystal for neutron monochromator

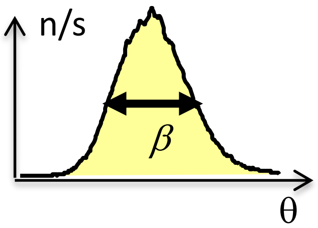
- ➢ To select a given wavelength band according to the Bragg's Law $2\ d_{hkl}\ \sin\theta_B = n\lambda_0$
- ➢ To match the neutron beam divergence α which is typically 0.2°-1°
- Perfect crystal is not suitable since reflection range is in the order of 0.005° << α

Use of mosaic crystals, i.e. crystal with structural defects (dislocations …)



Mosaic Crystal

Reflection range ∽ Beam divergence



$$w(\theta) = \frac{1}{\sqrt{2\pi\eta}}\exp(-\Delta^2/2\eta^2)$$

Selected Bandwidth $\Delta\lambda/\lambda = \cot\theta_B\ .\ \Delta\theta$

$\Delta\theta = f(\alpha, \beta,$ *collimation, sample size…)*

$$\Delta\lambda/\lambda = (\alpha^2 + \beta^2)^{1/2}\cot\theta_B$$

α : incoming beam divergence

β: neutron mosaic spread (FWHM)

$\alpha \approx \beta$ (flux optimization vs resolution)

- The neutron flux is proportional to integrated reflectivity $R(\lambda_0).\beta$ *(R : neutron peak reflectivity at λ_0)*
- The Resolution is given by $\Delta\lambda/\lambda_0 = \cot\theta_B\ .\ \Delta\theta \sim \cot\theta_B\ .\ \beta$
- ➢ ***Choice of mosaic β is always a compromise between flux and resolution !***

Mosaic crystals for neutron monochromators at ILL

- Mosaic crystals should have high neutron reflectivity, low background and small attenuation
- Large single crystals must be available → ILL production !

Crystal	orientation	Crystal Mosaic	Neutron Energy	Application	Supplier
C (graphite) d_{002}= 3.35 Å	HOPG(002)	0.5°- 3°	Cold Thermal	High flux	Momentive
Cu d_{111}= 2.08 Å	(111) (220) (200) (331)	**0.05° - 3°**	Hot Thermal	**High resolution or high flux**	ILL
Si d_{111}= 3.13 Å	(111) (113)	bent 0.2°- 0.5°	Cold Thermal	**High resolution**	ILL
Ge d_{111}= 3.26 Å	(111) (113) (115)	< 0.25°	Cold Thermal	High resolution	ILL
Heusler Cu_2MnAl	(111)	0.2°- 0.6°	Thermal	Polarized Neutrons	ILL

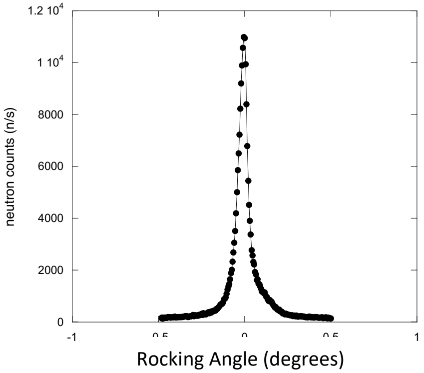
Hot neutrons $\lambda \sim$0.3-1 Å ; thermal $\lambda \sim$1-4 Å; cold λ~ 4-20Å

Production of mosaic crystals at I.L.L.

Control of the mosaic distribution by plastic deformation

Cu single crystal

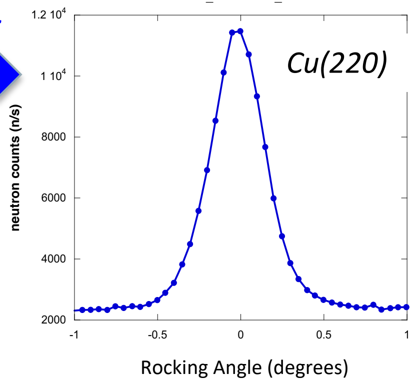
fwhm = 0.05°



Neutron rocking curve
from the as-grown crystal

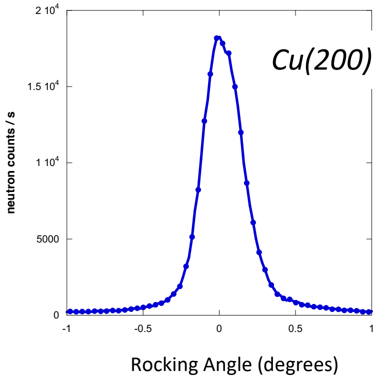
P , T

fwhm = 0.4°



Peak reflectivity
R_{exp} = 35-40 % (at λ = 1.1 Å)

fwhm = 0.25°



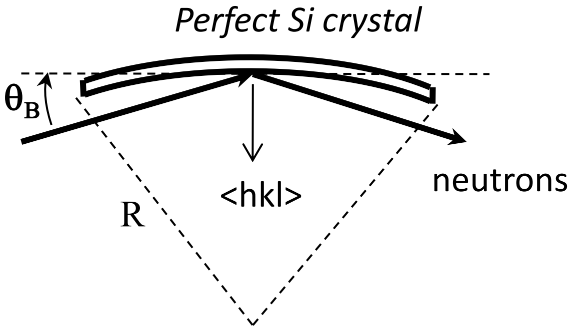
Peak reflectivity
R_{exp} = 40-45% (at λ = 1.1 Å)

- Growth of large Cu single (8 kg) crystals performed at ILL
- Production of high quality Cu(220) and Cu(200) single crystals with a controlled mosaic distribution

Production of “mosaic” Si crystals at I.L.L.

Si crystal exhibits excellent properties for neutrons applications

- No $\lambda/2$ contamination, low attenuation factor, no parasitic scattering
- Use of elastically bent perfect crystals to produce effective mosaic distribution



Effective mosaic δ (rad) $\delta = \cot(\theta_B)\ t / R$

t = total crystal thickness

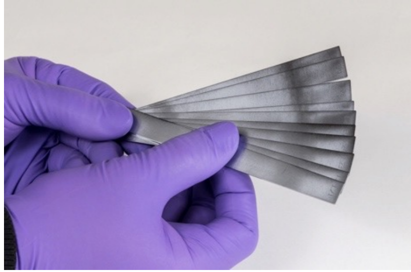
R = radius of curvature

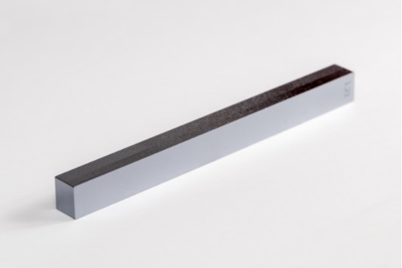
θ_B = Bragg angle

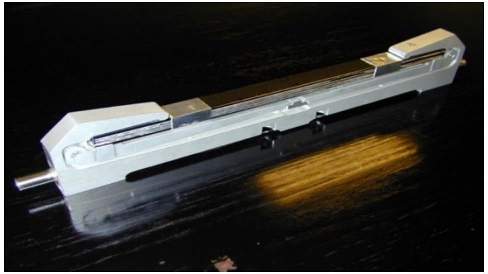
(ex: θ_B= 30°, t=10mm, R=2m -> $\delta = 0.5°$)

Stack of thin Si blades to allow bending

- wafer thickness = 1 mm
- 10 wafers to get t = 10 mm (or more)
- Curvature : flat to $R_H \approx 2$ m



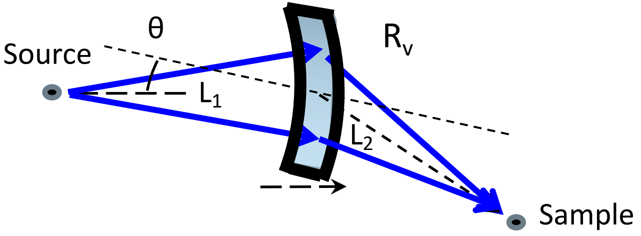




We need also Focusing Devices …

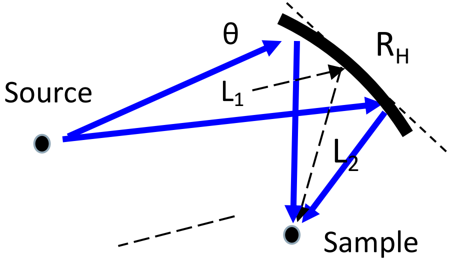
Vertical Focusing $$\frac{1}{R_v}=\frac{1}{2\sin\theta}\left(\frac{1}{L_1}+\frac{1}{L_2}\right)$$

- **Large gain in flux ∝ compression factor**
- Increase of vertical angular divergence

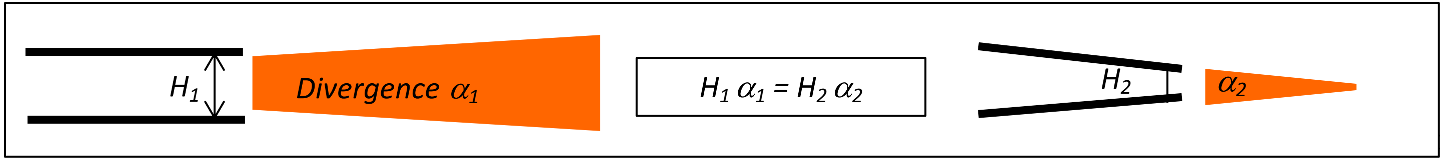


Horizontal Focusing $$\frac{1}{R_H}=\frac{\sin\theta}{2}\left(\frac{1}{L_1}+\frac{1}{L_2}\right)$$

- **Optimization of energy resolution vs flu**
- Horizontal focusing affects Q resolution



➢ Focusing devices are used to increase the neutron flux at the sample position. **However, the increase of neutron flux implies a degradation of the angular resolution (Liouville's theorem !)**

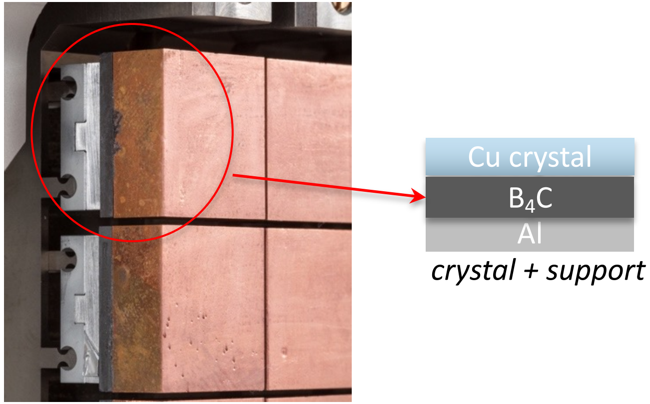


$H_1\,\alpha_1 = H_2\,\alpha_2$

A Neutron Monochromator is a big device…

Double Focusing Cu Monochromator

- Effective Area 300 x 165 mm^2
- 165 Cu mosaic crystals
- Crystal size = 20 x 19.8 mm2
- $^{10}B_4C$ plate is used to reduce background and activation



crystal + support

Monochromator for neutron diffractometers…

Elastic Scattering

Double face monochromator
Cu(200) and HOPG

Cu(200) - λ = 1.2 Å
crystal mosaic = 0.25 °

HOPG (002) - λ = 2.4 Å
crystal mosaic = 0.5 °

HOPG monochromator

- Total Area : 120 x 84 mm^2
- Crystal dimensions : 42 x 8 mm^2
- Neutron flux **5.10^6 n/cm^2/s** (at 2.4 Å)
- Low Resolution **$\Delta\lambda/\lambda$ = 3%**

Cu monochromator

- Total Area : 120 x 84 mm^2
- Crystal dimensions : 42 x 8 mm^2
- Neutron flux **2.10^6 n/cm^2/s** (at 1.2 Å)
- Good Resolution **$\Delta\lambda/\lambda$ = 1%**

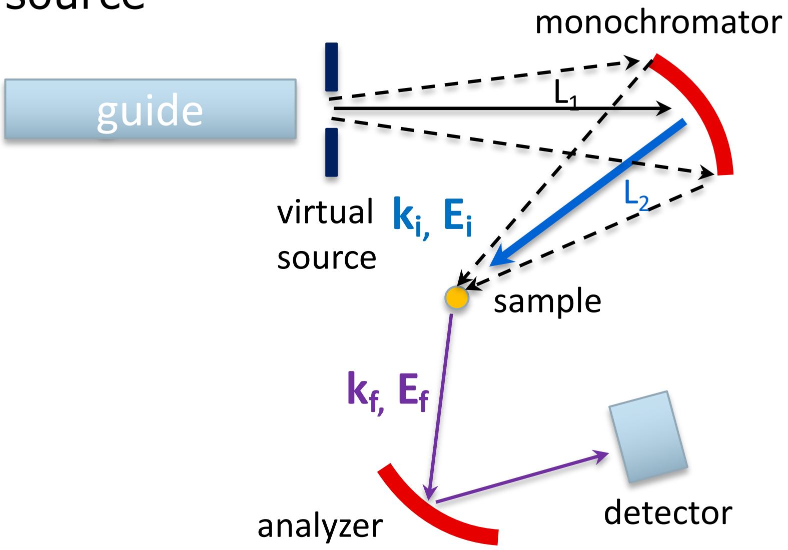
NO Horizontal Focusing – Variable Vertical Focusing

Monochromator for triple axis spectrometers

Inelastic Scattering experiments

- **Optimization of instrument performances for a wide energy range**

➢ Double variable Focusing monochromator is used in combination with virtual source



Monochromator

- Optimization of the neutron flux
- Optimization of the Energy Resolution
- $L_1 = L_2$ (Rowland Geometry)

Analyzer

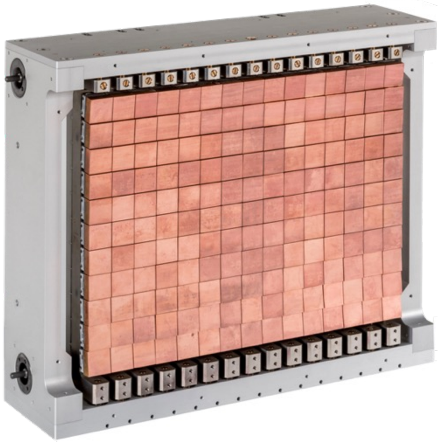
- Energy Analysis after Inelastic Scattering

➢ $\Delta E = E_i - E_f = \hbar\omega$

Monochromator and analyzer should be composed of the same material (i.e. HOPG-HOPG or Cu-Cu)

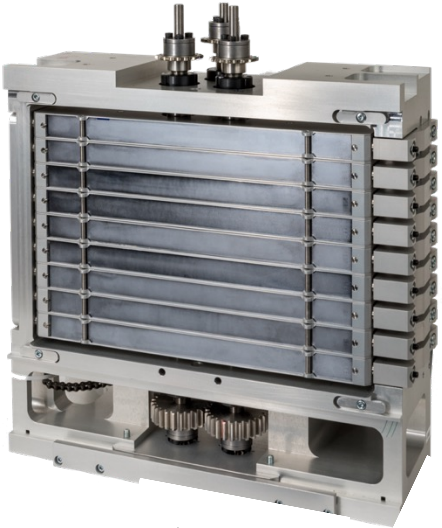
Monochromator for triple axis spectrometers

Double variable Focusing Monochromators



Cu(220) monochromator
Effective Area 300 x 165 mm^2
hot and thermal neutrons
flux : 5.10^8 n/cm^2/s at (at 1.5 Å)

HOPG monochromator
Effective Area 300 x 165 mm^2
cold and thermal neutrons
flux : 10^9 n/cm^2/s at (at 1.5 Å)



Si monochromator
Effective Area 250 x 200 mm^2
cold and thermal neutrons
flux : 10^8 n/cm^2/s at (at 2 Å)

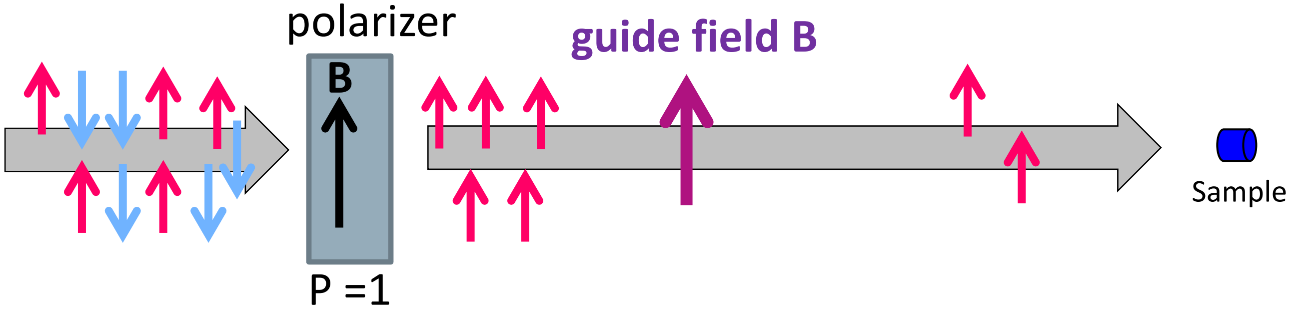
Polarized neutron beams

Polarizing Devices

- Polarization efficiency P
- Good Transmission of the desired spin state
- Angular acceptance, absorption and reflection
- Guide field is necessary to transport the polarization to the sample area

$$P = \frac{N_+ - N_-}{N_+ + N_-}$$

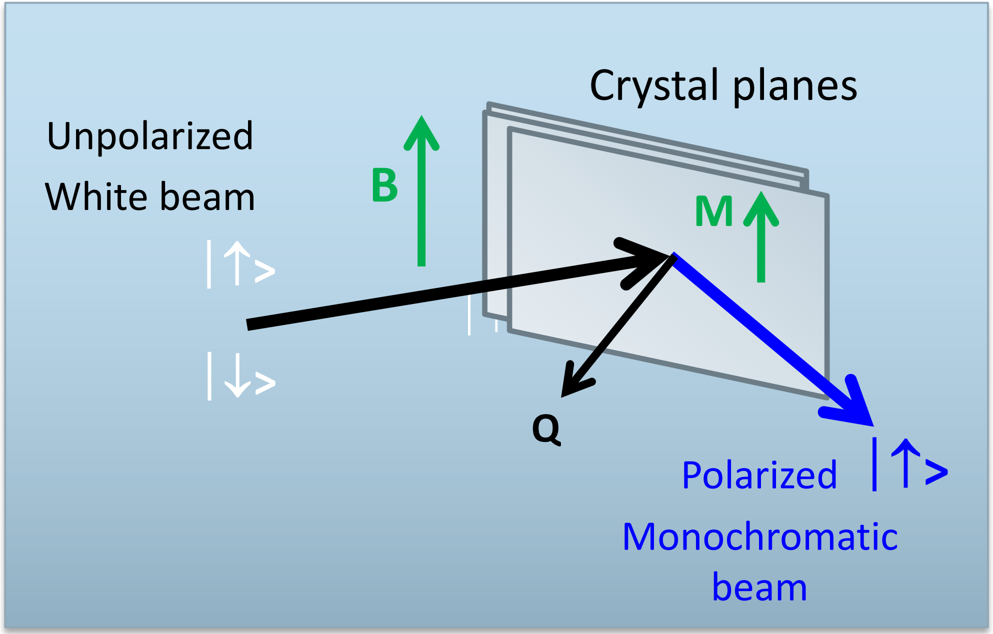
N_+ neutrons with |↑>
N_- neutrons with |↓>



Principal methods

- Fe/Si and Co/Ti super-mirrors (reflection, transmission)
- Heusler Cu_2MnAl crystal (diffraction)
- ^{3}He3 spin filters (absorption by polarized ^{3}He nuclei)

Polarizing crystal monochromators
Heusler Cu_2MnAl single crystal



Bragg reflection

from a ferromagnetic crystal

- $|\uparrow\rangle : I^{+} \propto [F_N(Q) + F_M(Q)]^2$
- $|\downarrow\rangle : I^{-} \propto [F_N(Q) - F_M(Q)]^2$

$F_N(Q)$ nuclear structure factor

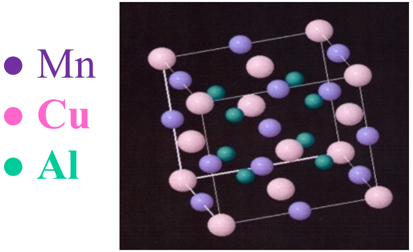
$F_M(Q)$ magnetic structure factor

Polarization

$F_N(Q) = \pm F_M(Q)$

Polarizing crystal monochromators

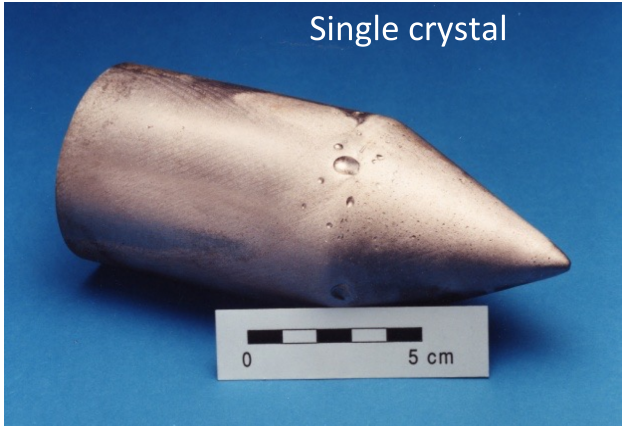
Heusler Cu_2MnAl single crystal

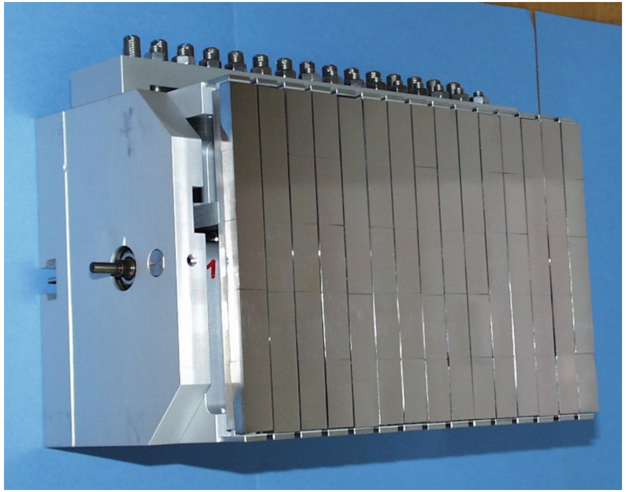


Cubic structure L_{21}

Structures Factors - reflection(111)

- Nuclear: $F_{111N} = 4\,(b_{Mn} - b_{Al})$
  ($F_{111N} = -2.8\ 10^{-12}$ cm)
- Magnetic: $F_{111M} = 4\,p_{Mn}$
  ($F_{111M} = 2.78\ 10^{-12}$ cm)





Heusler Monochromator

• (111) reflection	$F_{111N} = - F_{111M}$
• mosaic	0.2° < fwhm < 0.6°
• Reflectivity	$R_{\text{experimental}} \approx R_{\text{theoretical}}$
• Polarization	P > 92 %

Polarizing Super-Mirrors

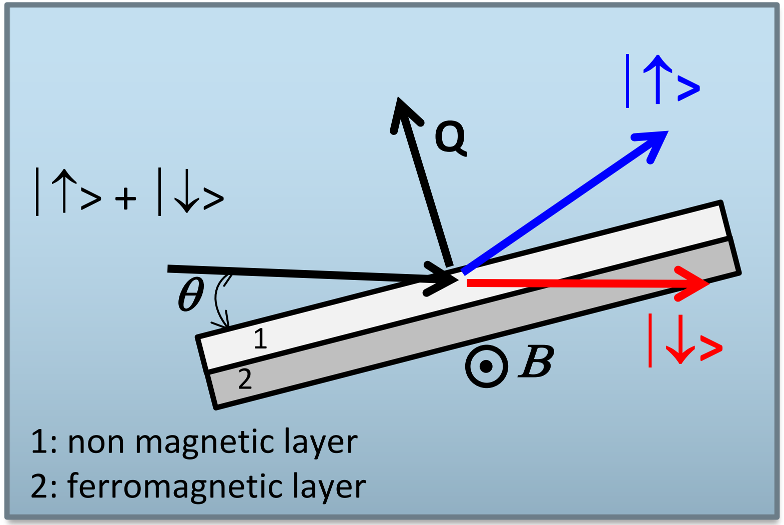
Co/Ti & Fe/Si Super-Mirrors

Refraction index

for a magnetic layer

$$n = 1 - \frac{\lambda^2 N(b \boxed{\pm p})}{2\pi} \longrightarrow$$

Magnetic contribution



1: non magnetic layer

2: ferromagnetic layer

Reflectivity R of a bilayer system

$|\uparrow\rangle$: $R+ \propto [N_1b_1 - N_2(b_2+p_2)]^2$

$|\downarrow\rangle$: $R- \propto [N_1b_1 - N_2(b_2-p_2)]^2$

Polarization

$N_1b_1 = N_2(b_2 \pm p_2)$

→ Reflection / Transmission

of one spin state

Polarizing Super-Mirrors

- **Co/Ti Super-Mirrors**

$|\uparrow>$: $N(b+p)_{Co} = 6.55$;

$|\downarrow>$: $N(b-p)_{Co} = -2.00 \approx Nb_{Ti} = -1.95$

- m= 3.2 Super-Mirrors

but Activation of Cobalt ! (lifetime 5 Years)

➢ can be used only for polarization analysis

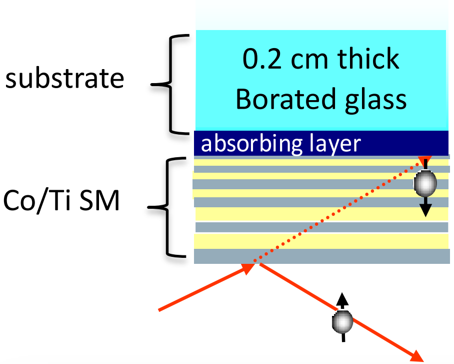
- **Fe/Si Super-mirrors**

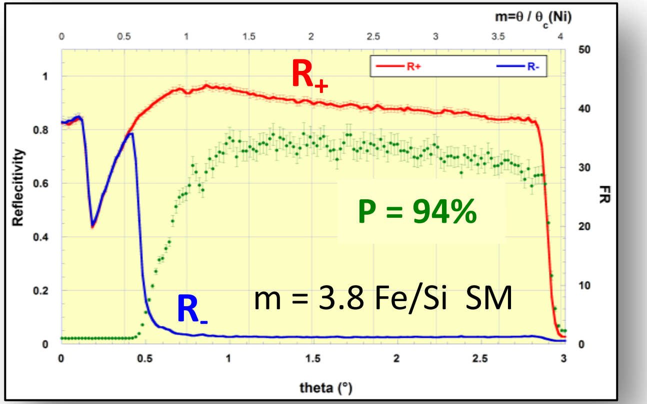
$|\uparrow>$: $N(b+p)_{Fe} = 13.04$;

$|\downarrow>$: $N(b-p)_{Fe} = 3.08 \approx Nb_{Si} = 2.08$

- *m = 4 super-Mirrors*

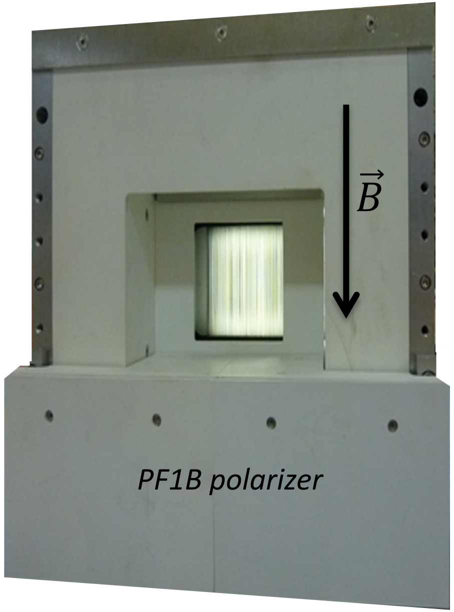
→ Polarizer in reflection / Transmission geometry





Fe/Si Polarizer

An advanced super-mirror solid state polarizer



PF1B polarizer

Sputtering machine at ILL

Stack of Fe/Si SMs

Provides a "perfectly" polarized neutron beam

- Active area : 64 cm^2 (80 x 80 mm^2)
- 800 m=3.2 Fe/Si/Gd Super-Mirrors coated on both sides of Sapphire substrate (t=0.2mm)
- Optimized magnet to provide a strong and homogeneous magnetic field of 0.4 T
- V geometry to ensure at least two reflections on the polarizing SMs (2 stacks of 400 SMs)

➢ **P ≈ 99.7 % for the full bandwidth λ = [3-20 Å]**

➢ **Transmission ≈ 30% (good spin state)**

Neutron spin-Filters

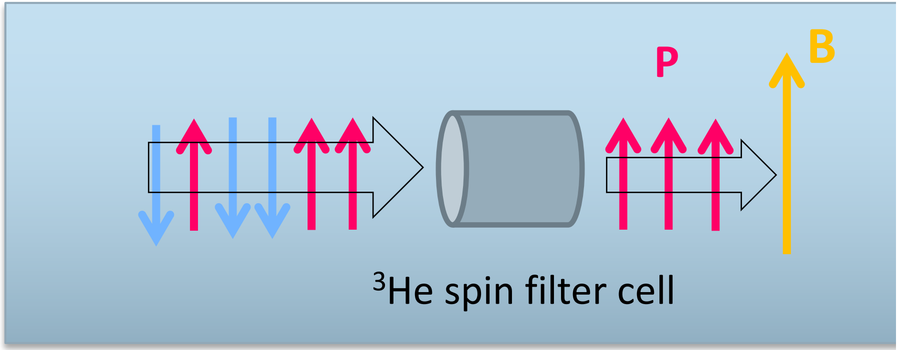
^{3}He spin filters

- **Absorption cross section of ^{3}Helium nuclei**

➢ If the nuclear spin of He and the neutron spin are parallel, **$\sigma_{a\uparrow\uparrow} \approx 0$**

➢ If the nuclear spin of He and the neutron spin are anti-parallel, **$\sigma_{a\uparrow\downarrow} \approx 6000$ barns**

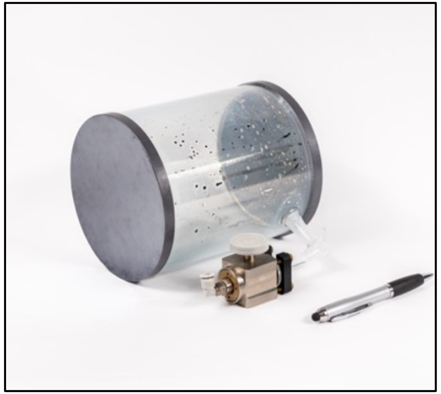
For fully polarized gas ^{3}He ($P_{He} = 1$), *one spin state goes through the filter with zero* absorption. The other spin state is almost fully absorbed since σ = *6000 barns* → ***polarized neutron beam***



^{3}He spin filter cell

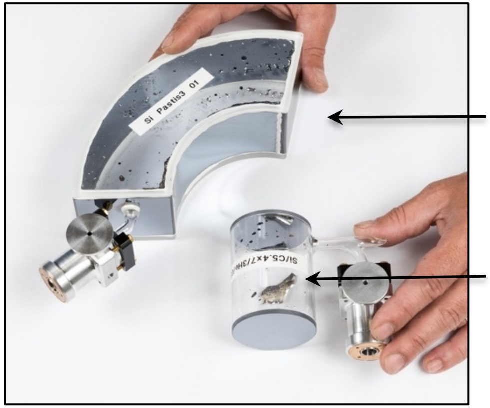
Polarizer and analyzer cells

^{3}He spin filter cell



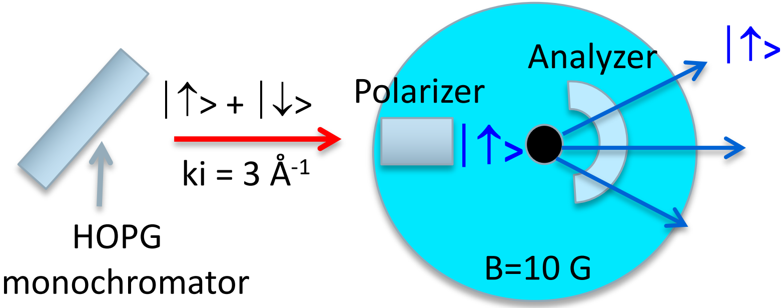
Glass cell with Si windows
(Small angle neutron scattering)

Si cell



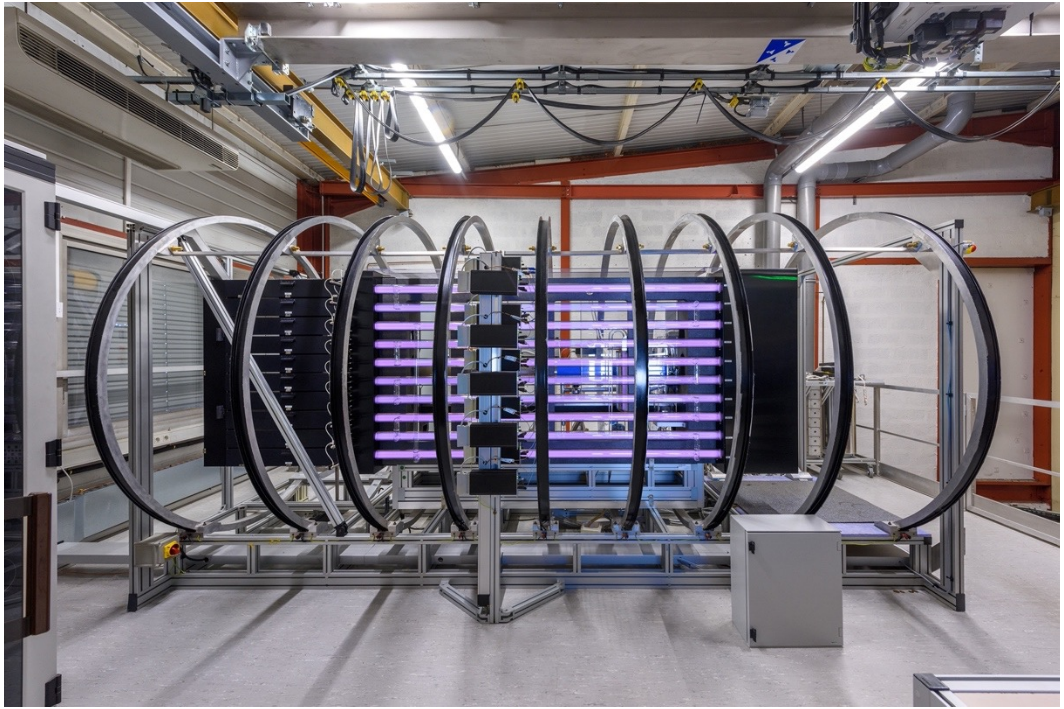
Wide-angle analyzer

Polarizer

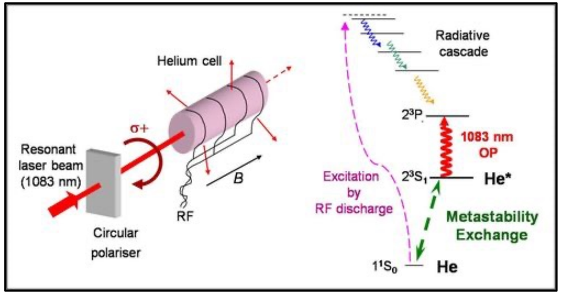


Production of ^{3}He spin filters

Metastability Exchange Optical Pumping (MEOP)



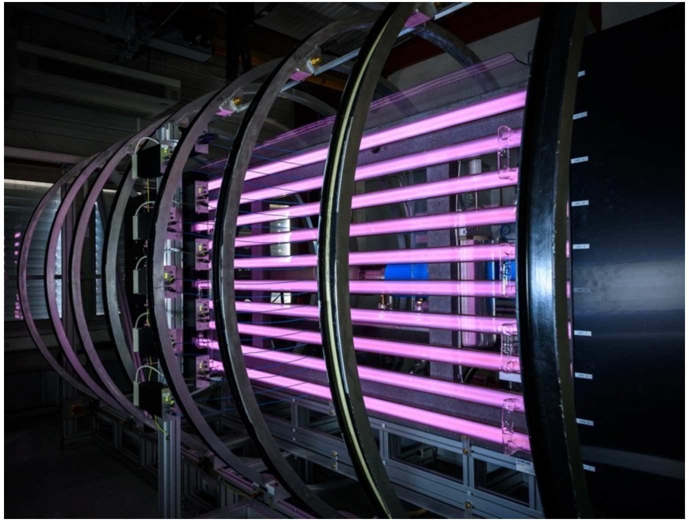
^{3}*He filling station at I.L.L.*



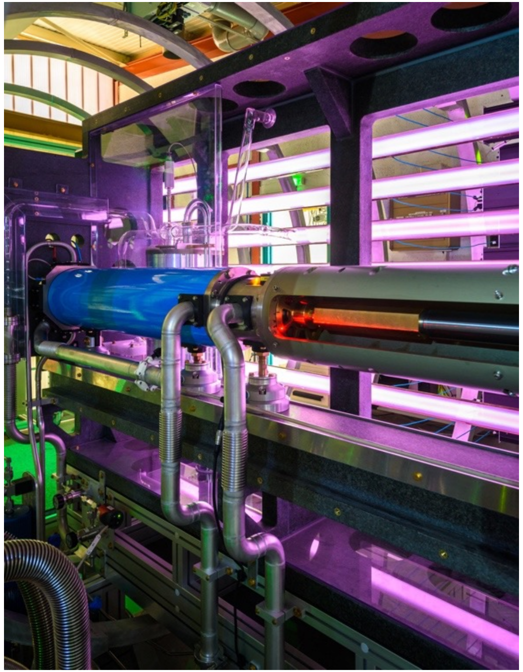
polarization of ^{3}*He nuclei using MEOP*

Production of ^{3}He spin filters

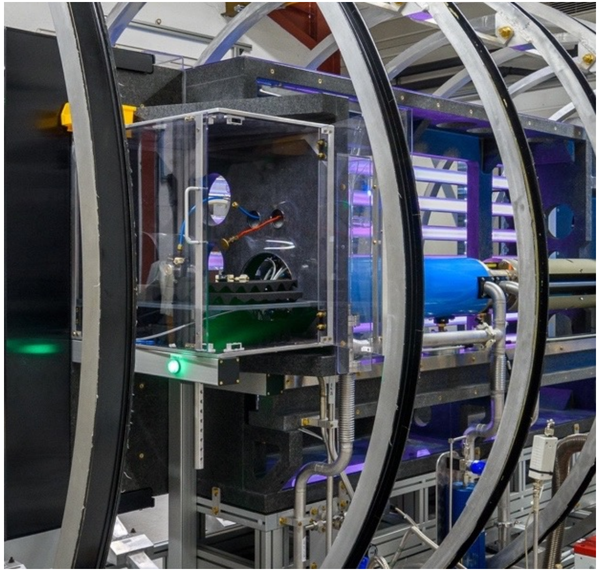
Metastability Exchange Optical Pumping (MEOP)



Optical Pumping Cells



^{3}He *Compressor*



^{3}He *Cell place*

Neutron spin-Filters

^{3}He spin filter cells

- Polarization of ^{3}He is time dependant: $P(t) = \exp(-t/T_1)$
  - *T_w : Relaxation due to interactions with the cell*
  - *T_d : Dipolar relaxation among ^{3}He nuclear spins*
  - *T_m : Relaxation due to field gradients*

$$\frac{1}{\mathrm{T}_1} = \frac{1}{\mathrm{T}_\mathrm{d}} + \frac{1}{\mathrm{T}_\mathrm{w}} + \frac{1}{\mathrm{T}_\mathrm{m}}$$

$$T_m[\mathrm{h}] = \frac{P[\mathrm{b}]}{7000}\left(\frac{\partial B/\partial r[\mathrm{cm}]}{B}\right)^{-2}$$

- The total relaxation time T_1 must be in the order of 100 hours to perform high quality neutron experiments

➢ High quality ^{3}He spin filter cells (Polarization, T_w)

➢ High quality magnetic devices to produce homogeneous magnetic field : $\left(\frac{\partial B/\partial r}{B}\right) < 10^{-3}\,cm^{-1}$

Conclusion

- **Neutron Optics define beam properties**
  - Direction, Divergence, Wavelength, Energy, Polarization
  - Angular Resolution, Wavelength resolution, Energy resolution
- Vertical focusing devices allow the optimization of the neutron flux at the sample position
- Double variable focusing devices allow the optimization of instrument performances for a wide energy range

➢ Since the power of **the source is low and neutron beams are divergent** , **neutron optical components must be of high quality and properly designed**

➢ Neutron Optics obey to Liouville's Theorem **:** It costs flux to increase resolution and it costs resolution to increase flux

The optimization of instrument performances is always

a compromise between flux and resolution

Thank you for your attention