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Wendelstein 7-X

How to reach a theoretical description of MHD and fast particle suitable for an experimental application

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Introduction

- overview about recent developments with respect to the EUTERPE code
- ongoing related physics projects
- EUTERPE is quasi-universal tool (MHD, micro instabilities, turbulence, neoclassics, ...)
- a bunch of supporting/ supplementing codes CONTI, CAS3D, CKA
- focus of work: W7-X experiment

Recent developments at EUTERPE

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Recent developments at EUTERPE – Code part

Comprehensive refactoring of the code

- improve modularity
- necessary for GPU

GPU version has been developed

- not yet optimized
- kinetic species run on GPU
- linear solver runs on CPU (planned to use GPU capabilities of PETSc)

Full HDF5 support for mapping and diagnostics

Optimization of mapping

- to speed up stellarator optimization for TSVV 13
- coordinate system now follows magnetic axis
- saves memory
- enables investigation of more exotic equilibria

Recent developments at EUTERPE – Physics part:

code capabilities

- model for islands has been implemented
- extended pullback equations
- global neoclassical terms implemented:
  self consistent calculation of radial neoclassical field
- synthetic Mirnov diagnostics has been developed.
- CKA-EUTERPE extended by parallel electric field

some physics projects relevant to TSVV

- investigation of Alfvén physics with islands (with USC)
- build stellarator fast particle transport model with CKA-EUTERPE
- ATEP: local Alfvén sound dispersion relation
- application for the interpretation of experiments
  - low frequency modes/ zonal flow oscillations in W7-X (with J. F. Guerrero Arnaiz)
  - low mode number electromagnetic modes at LHD (with T. Tanaka)

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Recent extensions to the CKA-EUTERPE model

Wendelstein 7-X

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Derivation of the CKA-EUTERPE equations with finite $E_{||}$

CKA-EUTERPE

- CKA-EUTERPE is a HAGIS like code version of EUTERPE
  advancing amplitudes of MHD modes from CKA with EUTERPE
- can move many modes at the same allowing for their interaction by particle non-linearities
- shall be used as fast particle transport code
- estimate for mode damping needed
- inclusion of $E_{||}$ (cf. dissertation of M. Schneller)

Derivation of the CKA-EUTERPE equations with finite $E_{\|}$

CKA-EUTERPE is based on the gyrokinetic density equation

$$\frac{\partial}{\partial t}\nabla\cdot\left[\frac{m_\mathrm{i}n_\mathrm{i}}{B^2}\nabla_\perp\phi\right]=\nabla\cdot\left\{j_\|^{(1)}\mathbf{b}+j_\|^{(0)}\left[\frac{\mathbf{b}\times\boldsymbol{\kappa}}{B}A_\|-\frac{\mathbf{b}\times\nabla A_\|}{B}\right]+p_\|^{(1)}\frac{\mathbf{b}\times\boldsymbol{\kappa}}{B}+p_\perp^{(1)}\frac{\mathbf{b}\times\nabla B}{B^2}+\right.$$
$$\left.+p_{\|,\mathrm{fast}}^{(1)}\frac{\mathbf{b}\times\boldsymbol{\kappa}}{B}+p_{\perp,\mathrm{fast}}^{(1)}\frac{\mathbf{b}\times\nabla B}{B^2}\right\} \tag{1}$$

as well as Ampère's law

$$-\nabla_\perp^2A_\|=\mu_0j_\|^{(1)}, \tag{2}$$

and Ohm's law

$$-\frac{\partial}{\partial t}A_\|-\mathbf{b}\cdot\nabla\phi=-\frac{\mathbf{b}\cdot\nabla}{en_{0,\mathrm{e}}}p_{\|,\mathrm{e}}^{(1)}. \tag{3}$$

We make a complex multi-mode ansatz for ϕ and $A_\|$

$$\phi(\mathbf{r},t)=\frac{1}{2}\sum_j\left[\hat{\phi}_j(t)\,\phi_{0,j}(\mathbf{r})\exp(\mathrm{i}\omega_jt)+\hat{\phi}_j^*(t)\,\phi_{0,j}^*(\mathbf{r})\exp(-\mathrm{i}\omega_jt)\right] \tag{4}$$

$$A_\|(\mathbf{r},t)=\frac{1}{2}\sum_j\left[\hat{A}_j(t)\,A_{0,j}(\mathbf{r})\exp(\mathrm{i}\omega_jt)+\hat{A}_j^*(t)\,A_{0,j}^*(\mathbf{r})\exp(-\mathrm{i}\omega_jt)\right] \tag{5}$$

Derivation of the CKA-EUTERPE equations with finite $E_{\|}$

Insert ansatz into Ohm's law and multiply resulting equation with

$$-\nabla_{\perp}^{2}\left[\hat{A}_{k}^{*}A_{0,k}^{*}\left(\mathbf{r}\right)\exp\left(-\mathrm{i}\omega_{k}t\right)\right] \tag{6}$$

and drop all terms proportional to $\exp\left[-\mathrm{i}\left(\omega_{j}+\omega_{k}\right)t\right]$ (fast oscillations). This yields the first amplitude equation

$$\boxed{\frac{\partial}{\partial t}\hat{A}_{j}+\mathrm{i}\omega_{j}\left(\hat{A}_{j}-\hat{\phi}_{j}\right)=\sum_{k}\hat{\mathbb{N}}_{jk}^{-1}u_{k}\hat{A}_{j}}. \tag{7}$$

where

$$\hat{\mathbb{N}}_{jk}=\hat{A}_{j}\hat{A}_{k}^{*}\exp\left[\mathrm{i}\left(\omega_{j}-\omega_{k}\right)t\right]\int\mathrm{d}^{3}\mathbf{r}\nabla_{\perp}A_{0,j}\cdot\nabla_{\perp}A_{0,k}^{*} \tag{8}$$

and

$$\begin{aligned}u_{k}=&-2\mu_{0}\int\mathrm{d}^{3}\mathbf{r}\left[\frac{\mathbf{B}\cdot\nabla B}{-B^{2}}j_{\|0}^{(1)*}+\mathbf{b}\cdot\nabla\left(j_{\|0}^{(1)*}\right)\right]\frac{\hat{A}_{k}^{*}\exp\left(-\mathrm{i}\omega_{k}t\right)}{en_{0,\mathrm{e}}}\times\\&\times\int\mathrm{d}\mu\mathrm{d}v_{\|}\mathrm{d}\alpha B_{\|}^{\star}m_{\mathrm{e}}v_{\|}^{2}f_{\mathrm{e}}^{(1)}\end{aligned} \tag{9}$$

Derivation of the CKA-EUTERPE equations with finite $E_\parallel$

For the second equation we insert the ansatz into the time derivative of the gyrokinetic density equation, perform a multiplication with

$$\hat{\phi}_k^* \phi_{0,k}^* (\mathbf{r}) \exp(-\mathrm{i}\omega_k t) \tag{10}$$

and again neglect terms proportional to $\exp[-\mathrm{i}(\omega_j + \omega_k) t]$. This yields the second amplitude equation

$$\boxed{\frac{\partial}{\partial t}\hat{\phi}_j + \mathrm{i}\omega_j \left(\hat{\phi}_j - \hat{A}_j\right) = -\sum_k \hat{\mathbb{M}}_{jk}^{-1} T_k \hat{\phi}_j} \tag{11}$$

where

$$\hat{\mathbb{M}}_{jk} = \frac{1}{2}\hat{\phi}_j \hat{\phi}_k^* \exp[\mathrm{i}(\omega_j - \omega_k) t] \int \mathrm{d}^3\mathbf{r} \frac{m_\mathrm{i} n_\mathrm{i}}{B^2} \nabla_\perp \phi_{0,j} \cdot \nabla_\perp \phi_{0,k}^* . \tag{12}$$

and

$$T_k = \int \mathrm{d}^3\mathbf{r} \int \mathrm{d}\mu \mathrm{d}v_\parallel \mathrm{d}\alpha B_\parallel^\star \left\{ \frac{\mathbf{b}\times}{Z\mathrm{e}} \left(\frac{m_\mathrm{f} v_\parallel^2}{B} \boldsymbol{\kappa} + \frac{\mu}{B}\nabla B \right) \cdot \left(-Z\mathrm{e} \nabla \phi_{0,k}^* \hat{\phi}_k^* \exp(-\mathrm{i}\omega_k t) f_\mathrm{fast}^{(1)} \right) \right\} . \tag{13}$$

Caveat: equations of motion in EUTERPE

CKA-EUTERPE uses the $v_\parallel$-formulation of the equations

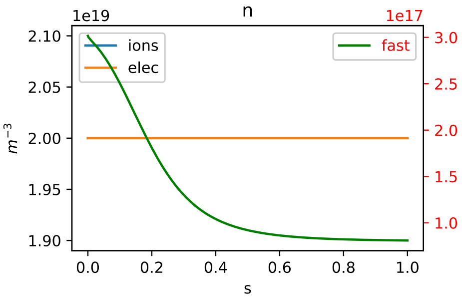
$$\begin{aligned}\dot{v}_\parallel = & -\mu \nabla B \cdot \left[\mathbf{b} + \frac{m_s}{q_s}\frac{v_\parallel}{B B_\parallel^\star}(\nabla \times \mathbf{B})_\perp\right] - \frac{q_s}{m_s}\frac{\partial \left\langle A_\parallel \right\rangle}{\partial t} \\ & - \frac{q_s}{m_s}\left\{\mathbf{b} + \frac{m_s}{q_s}\frac{v_\parallel}{B B_\parallel^\star}\left[\mathbf{b} \times \nabla B + (\nabla \times \mathbf{B})_\perp\right]\right\} \cdot \nabla \left\langle \phi \right\rangle \\ & - \frac{\mu}{B_\parallel^\star}\left[\mathbf{b} \times \nabla B \cdot \nabla \left\langle A_\parallel \right\rangle + \frac{1}{B}\nabla B \cdot (\nabla \times \mathbf{B})_\perp \left\langle A_\parallel \right\rangle\right]\end{aligned} \tag{14}$$

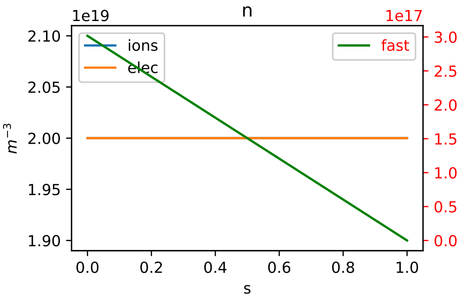
Express $E_\parallel$ with quantities already available in the CKA-EUTERPE model

$$E_\parallel = -\mathbf{b} \cdot \nabla \phi - \frac{\partial A_\parallel}{\partial t} \qquad \longrightarrow \qquad -\sum_j A_{0,j} \exp\left(\mathrm{i}\omega_j t\right) \sum_k \hat{\mathbb{N}}_{jk}^{-1} u_k \hat{A}_j \tag{15}$$

- including $E_\parallel$ in the equations of motion is tricky (see later slides)
- simulation is fine if $E_\parallel$ is kept only in the particle trajectories, but simulation breaks instantly if $E_\parallel$ enters into the equation for the weight evolution (source term)
- reason is unclear at the moment

Cases and profiles

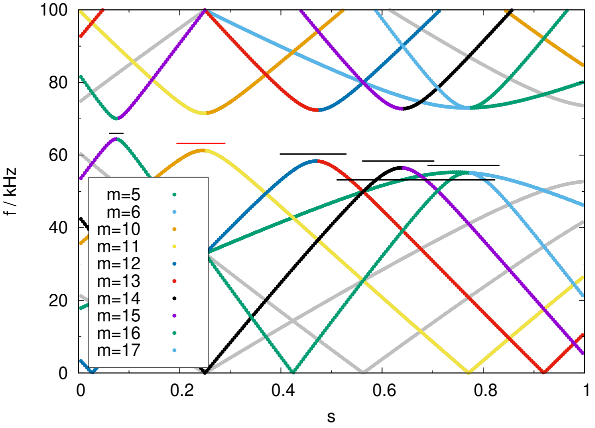




- profiles used for $E_{||}$-studies
- same fast-ion density-profile shape as in ITPA benchmark
- fast-ion density doubled to increase γ and reach saturation faster

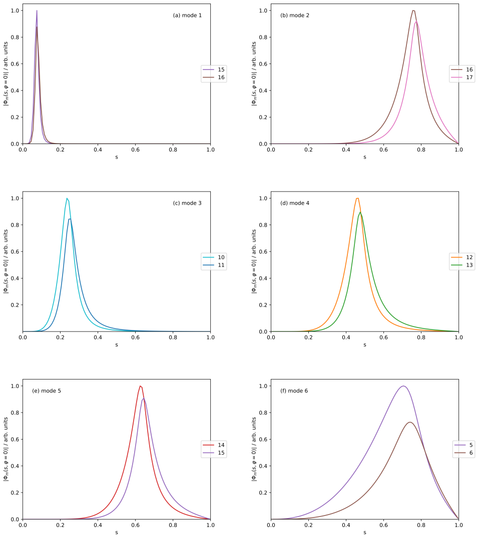
- profiles used for multi-mode studies
- linear fast-ion density profile to provide somewhat uniform drive for all the modes irrespective of their radial position

Shear Alfvén continuum and mode locations



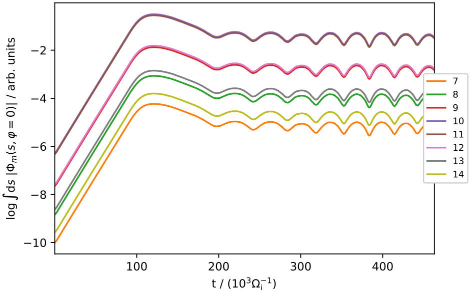
- shear Alfvén continuum for ITPA case with multiple *n*'s (CONTI)
- positions of eigenmodes (found with CKA) indicated by black horizontal lines
- regular ITPA mode also part of this scenario → red line at $s = 0.25$

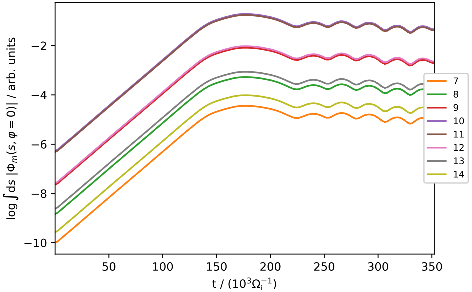
All modes



- for completeness: radial structure of the eigenfunctions found with CKA

Time evolution with and without $E_{\parallel}$

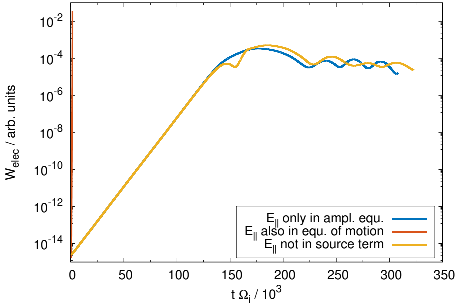




- without $E_{\parallel}$: $f \cong 63.6\ \mathrm{kHz}$ and $\gamma \cong 3.67 \cdot 10^4\ \mathrm{s}^{-1}$
- regular CKA-EUTERPE single-mode simulation

- with $E_{\parallel}$: $f \cong 62.1\ \mathrm{kHz}$ and $\gamma \cong 2.46 \cdot 10^4\ \mathrm{s}^{-1}$
- inclusion of $E_{\parallel}$ reduces γ by about 33%, ω not affected much
- chirping in the nonlinear phase less pronounced with $E_{\parallel}$ present

Problems with $E_{||}$



- CKA-EUTERPE uses $v_{||}$-formulation of gyrokinetic theory

$\Rightarrow$ $\partial A_{||}/\partial t$-term present in equations of motion unless it cancels with $\nabla_{||}\phi$ (ideal Ohm's law)

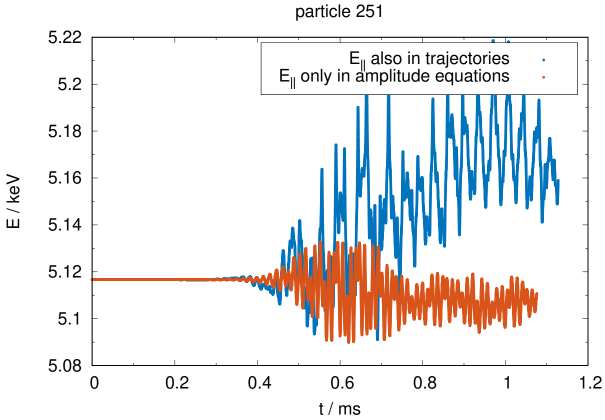
- now: $E_{||}$ should be included in equations of motion
- simulation becomes numerically unstable if $E_{||}$ is included in the source term
- having $E_{||}$ in the trajectories is fine, but does not change the results much

potentially cleanest solution:

- keep $E_{||}$ only in amplitude equations $\rightarrow$ “perturbative” model

Particle energies

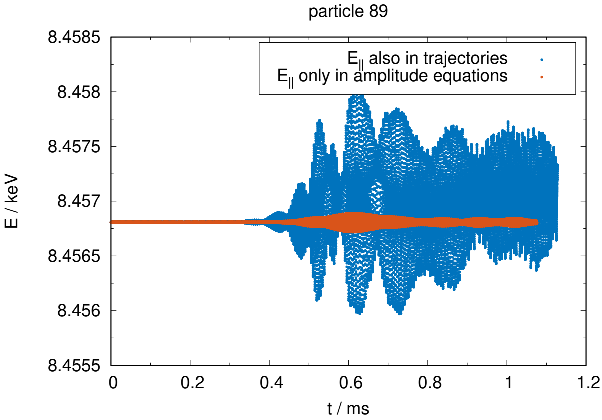
- time evolution of the kinetic energy of a few arbitrary particles



- stronger modifications of E if $E_{||}$ is also present in trajectories (expected)
- relative energy gains can be significant (one particle triples its energy for a short amount of time)

Particle energies

- time evolution of the kinetic energy of a few arbitrary particles



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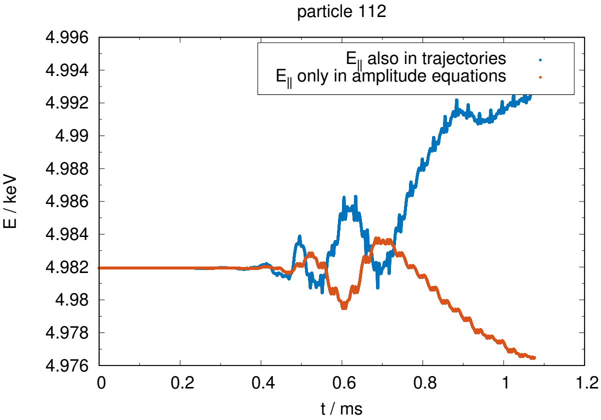

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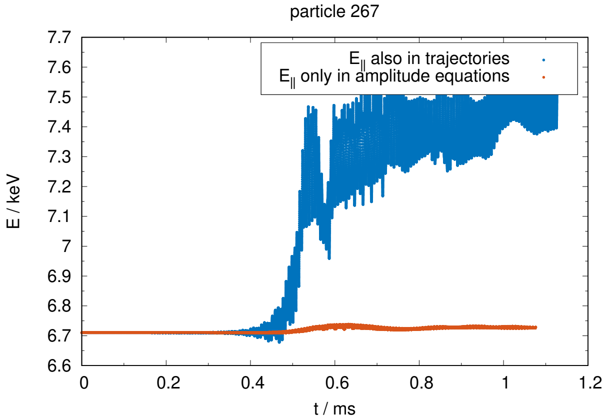

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Particle energies

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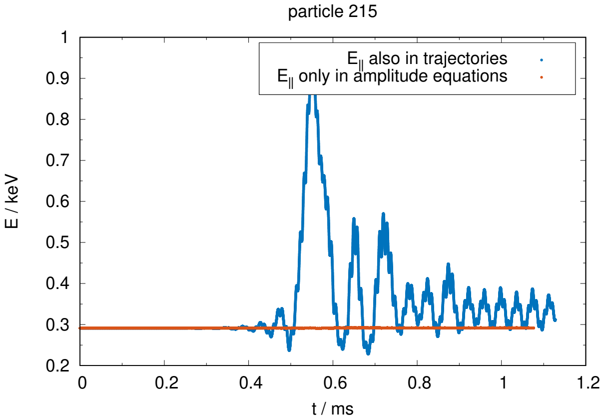

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Particle energies

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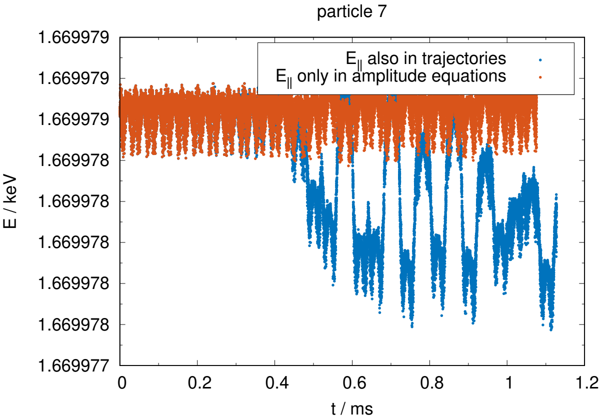
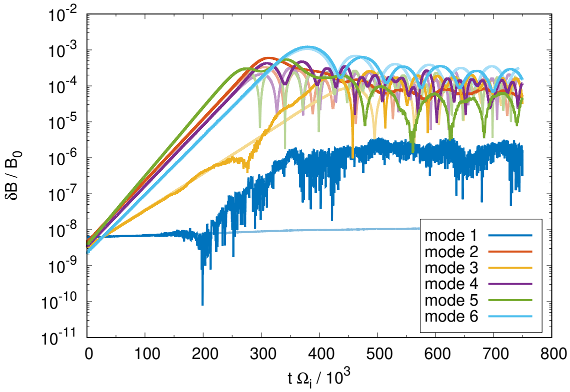


- stronger modifications of E if $E_{||}$ is also present in trajectories (expected)
- relative energy gains can be significant (one particle triples its energy for a short amount of time)

Multi-mode uniform damping

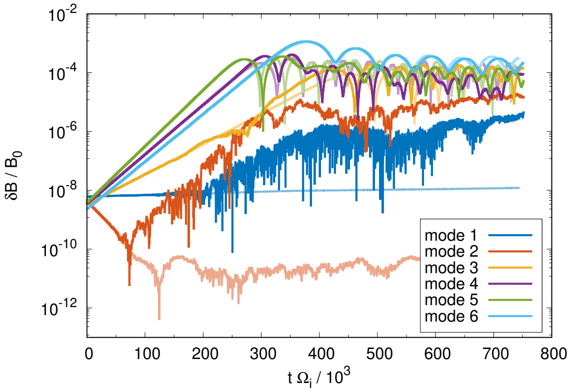
- multi-mode simulations use linear fast-ion-density profile shown in the beginning



- opaque colours: single-mode / full colours: multi-mode
- initially modes grow in the multi-mode simulation with same γ as in single-mode simulation
- later (but still in linear phase) growth rate of slow-growing modes increase to the same value as for fast-growing ones
- nonlinear behaviour in multi-mode simulation can be similar to single-mode case (see mode 6): probably because it has highest saturation level anyway $\rightarrow$ not much affected by other modes
- but nonlinear behaviour can also be quite different (see mode 5) $\rightarrow$ reduced chirping

Multi-mode non-uniform damping

- multi-mode simulations use linear fast-ion-density profile shown in the beginning



- opaque colours: single-mode / full colours: multi-mode
- damping rate of mode 2 increased further → mode is stable in the single-mode case
- interaction with other modes in multi-mode scenario strong enough to destabilize the mode

next steps:

- check the time-evolving fast-ion density profile and investigate profile flattening and chirping in the multi-mode case (using GPU version)
- apply the (now benchmarked model) to a W7-X scenario

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A numerical approach to the calculation of the Alfvén continuum in the presence of magnetic islands

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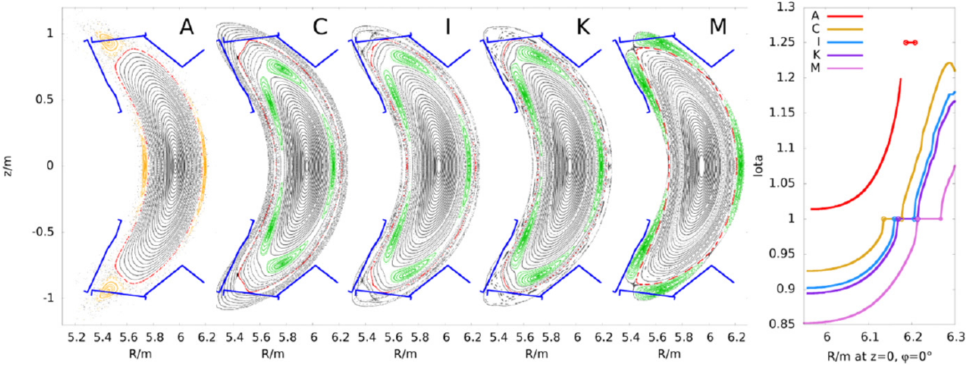
Introduction

experimental: iota scan in W7-X

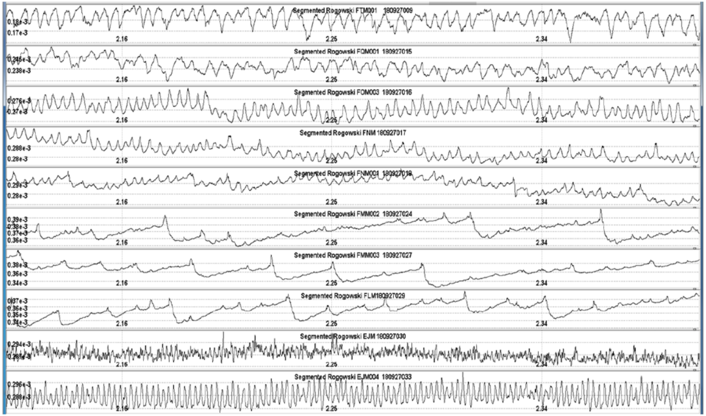


[Andreeva et al. Nuclear Fusion 62, 026032 (2022).]

Introduction

island related mode activity in W7-X

iota scan from high iota (top, FTM) down to standard iota (bottom, EJM) magnetic configuration



Time traces (in seconds) from one segmented Rogowski coil for different shots,
showing the change in the nature of the fluctuations
(in this case, in edge plasma currents)

[G. Wurden et al. EPS P2.1068 (2019)]

New flux surfaces

We solve the latter equation integrating over s and obtain for s^*:

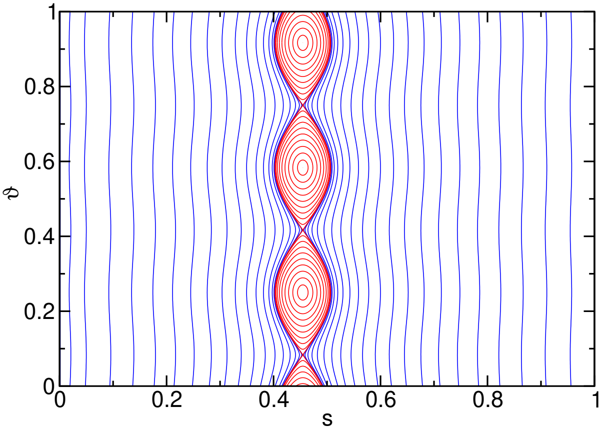
$$s^* = \int_{s_0}^{s} \frac{(F_P' m_l + F_T' \frac{n_l}{N_P})}{(I m_l - J \frac{n_l}{N_P})} \mathrm{d}\bar{s} + A \sin\left[2\pi(m_l \vartheta + \frac{n_l}{N_p}\varphi)\right] ,$$

approximating the integral expression:

$$s^* = \frac{1}{2} \frac{F_T' \iota_0'}{N_P I_0}(s - s_0)^2 + A \sin\left[2\pi(m_l \vartheta + \frac{n_l}{N_p}\varphi)\right] .$$

equation describing the new flux surfaces as $s^* = (s, \vartheta, \varphi)$.

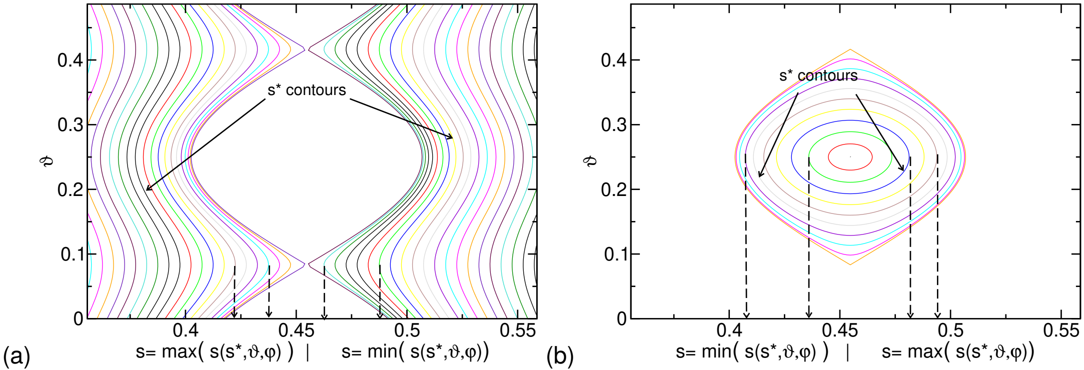
Topology of flux surfaces



contour lines of $s^* = \mathrm{const}$ for $\varphi = 0$

regions inside and outside the island can be distinguished
$s^* = 0$ is the O-point
$s^* = \pm A$ labels the separatrices

Naming conventions for the new flux surfaces



The relevant flux coordinate is s^*.

each $s^* = \mathrm{const.}$ contour is uniquely mapped to a particular value of s, the unperturbed flux function.

Equation for the continuum

The equation for the Alfvén continuum frequency ω in general geometry is [Chen and Chance 1986]:

$$\mu_0 M_i n(s)\,\omega^2 \frac{|\nabla\psi|^2}{B^2}\xi^s \quad = \quad \vec{\nabla}\cdot\left(\vec{B}\,\frac{\vec{B}\cdot\vec{\nabla}}{B^2}|\nabla\psi|^2\xi^s\right)\,,$$

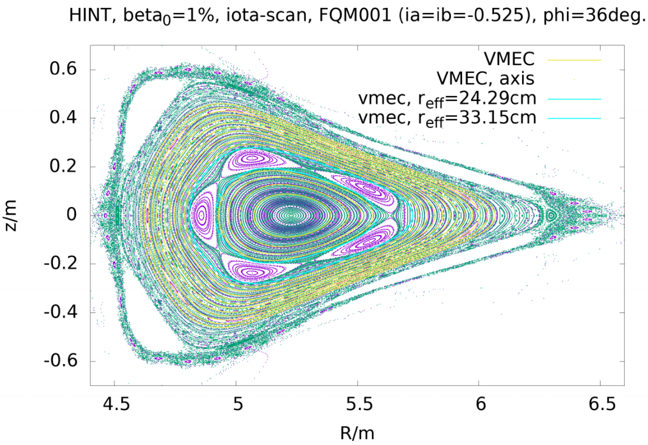
Introducing a Fourier representation for ξ^s

$$\xi^s(s,\vartheta,\varphi) = \sum_{m,n}\xi^s_{mn}e^{2\pi i(m\vartheta+n\varphi)}\,,$$

leads to

$$\mu_0 M_i n(s)\omega^2\sum_{m',n'}A_{m,n;m'n'}\xi^s_{m'n'} = -\sum_{m',n'}B_{m,n;m'n'}\xi^s_{m'n'}$$

Island in W7-X FQM001: HINT vs. VMEC



Comparison of HINT and VMEC calculations

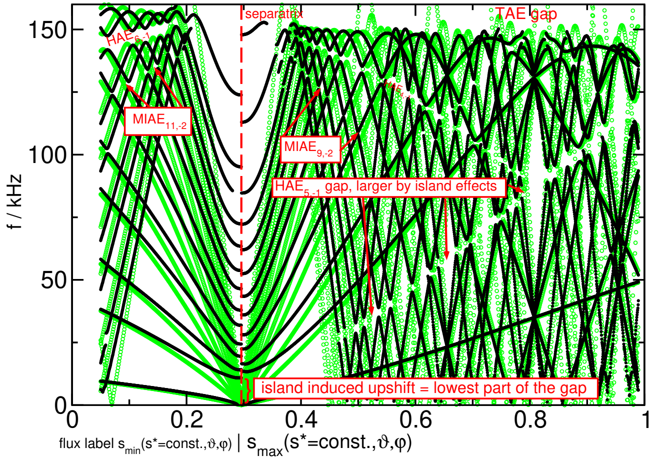
same energy content
same flux surface averaged pressure profile

VMEC flux surfaces limiting the island chain: cyan

r_{eff} envelope of the island chain determines island width

average island width 8.86 cm.

Alfvén continuum of the $N = 1$ mode family for W7-X FQM001



outside the island –

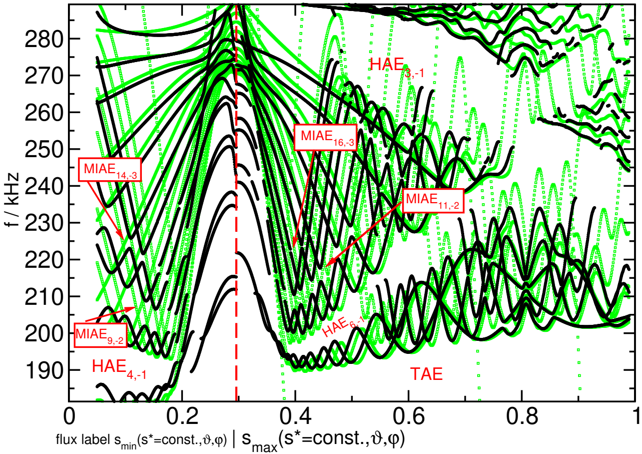
lower frequencies

spectrum without the island
→ green symbols

gaps are named with "MIAE" (Magnetic Island induced Alfvén Eigenmodes)

lowest branch at the island position is $(1, -1)$.

Alfvén continuum of the $N = 1$ mode family for W7-X FQM001



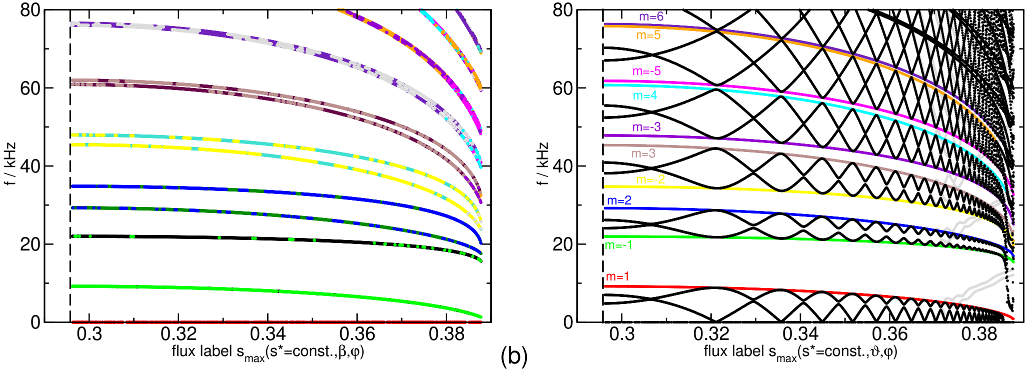
outside the island –

higher frequencies

spectrum without the island
→ green symbols

MIAE gaps close to multiples of the island helicity $(5, -1)$

$n = 0, 1$ continuum inside island for W7-X FQM001



(a) (b)

(a) with $n = 0$ contribution only with coloring to match same parity

(b) with $n = 0, 1$ contributions. $n = 0$ branches: color; $n = 1$ branches: black

convergence demonstrated : (grey: $-15 \leq m \leq 60$, black and colored: $-15 \leq m \leq 235$)

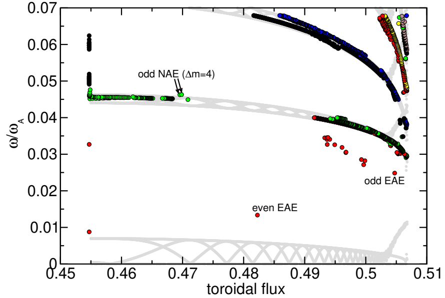
lowest gap: $13 - 14$kHz

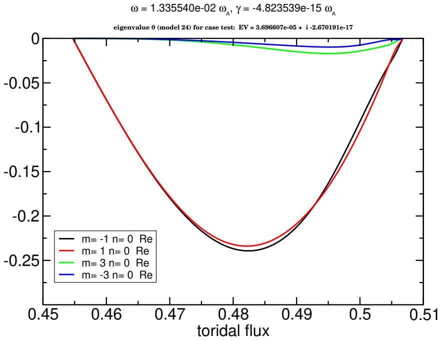
Global Alfvén modes inside an island

(with J. Cao, R. Kleiber, J. Yang)

- Alfvén contiuum inside an island does not strongly depend on surrounding equilibrium
  $\Rightarrow$ start with island in cylindrical equilibrium
- metric with straight filed lines can be expressed analytically using elliptic integrals and Jacobi functions
- **first results** show that global modes (MIAE) do exist

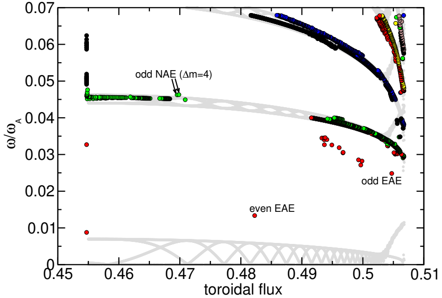
Global Alfvén modes inside an island

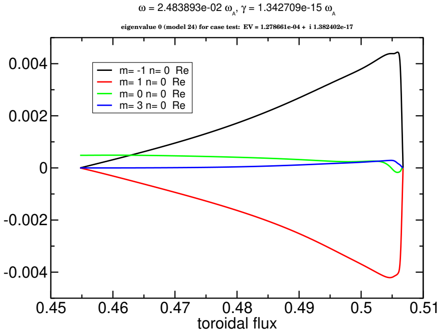




cylindrical equilibrium $m = 3, n = -1$ island: MIAE: even inner island EAE

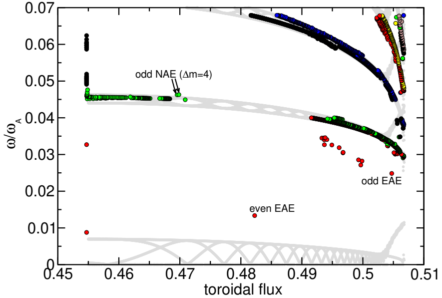
Global Alfvén modes inside an island

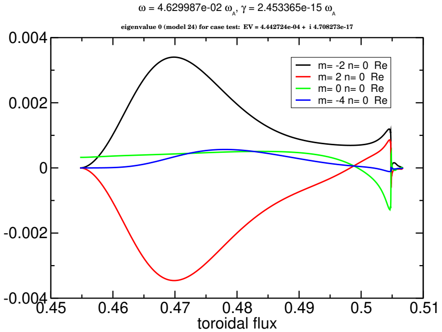




cylindrical equilibrium $m = 3, n = -1$ island: MIAE: odd inner island EAE

Global Alfvén modes inside an island





cylindrical equilibrium $m = 3, n = -1$ island: MIAE: odd inner island $\Delta m = 4$ NAE

Summary of island project

numerical approach to the calculation of the Alfvén continuum

- Alfvén continuum in stellarators in presence of an island
- matching island size with HINT calculations
- up-shift of the Alfén continuum $\rightarrow$ actually lowest part of an island induced gap
- island induced gaps inside and outside the island
- first global Alfvén eigenmodes in a cylinder (work in progress)

estimate of frequency regions where modes may reside possible in spite of convergence issues

- overview about recent developments with respect to the EUTERPE code
- ongoing related physics projects
- EUTERPE is quasi-universal tool (MHD, micro instabilities, turbulence, neoclassics, . . .)
- a bunch of supporting/ supplementing codes CONTI, CAS3D, CKA
- focus of work: W7-X experiment