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EP stability WF

part 1: LIGKA model

Ph. Lauber, V.-A. Popa, T. Hayward-Schneider

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schedule & organisation

https://indico.euro-fusion.org/event/2729/

EP-Stability-WF Training course - please register before July 10th, 2023

18 Jul 2023, 09:30 → 19 Jul 2023, 15:30 Europe/Berlin

Zoom

Description https://eu01web.zoom.us/j/6025051719?pwd=ZFhRdjdWVjUyMVZVZWVaRkladjFDQT09

Registration *You are registered for this event.*

Philipp Lauber philipp.lauber@ipp.mpg.de

Tuesday, 18 July

09:30 → 11:30 **Introduction: physics model: EP stability WF: introduction**
Convener: Philipp Lauber (IPP)

- 09:30 **EP stability WF, part 1: LIGKA model** — 1h
  Speaker: Philipp Lauber (IPP)
- 10:30 **EP stability WF, part 2: WF design and some results** — 45m
  Speaker: Alin Popa (IPP)

13:30 → 15:00 **Hands-on Session, part 1**

Wednesday, 19 July

09:30 → 11:00 **Hands-on Session, part 2**

13:30 → 15:00 **Troubleshooting, Q&A**

schedule & organisation

- please check your SDCC login (ssh -X.., nomachine)
- or: check gateway access
- register to slack channel: imasusers.slack.com #ep-workflow-training
- we plan to record first session for later use (Q&A not recorded)

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acknowledgements:

- ITER Organisation: S.D. Pinches, M. Schneider, O Hoenen
- Eurofusion ENR Projects MET, ATEP, TSVV10
- to various previous testers:
  A. Bierwage, M. Falessi, M. Vallar, A. Snikker,…

- Lauber PhD Thesis 2003: http://nbn-resolving.de/urn/resolver.pl?urn:nbn:de:bvb:91-diss2003111814131
- LIGKA: Lauber JCP 2007: 10.1016/j.jcp.2007.04.019
- Lauber PPCF 2009: http://stacks.iop.org/0741-3335/51/i=12/a=124009
- Lauber PREP 2013: https://doi.org/10.1016/j.physrep.2013.07.001
- Bierwage NF 2017: https://doi.org/10.1088/1741-4326/aa80fe
- T. Hayward-Schneider NF 2021 https://doi.org/10.1088/1741-4326/abdca2
- Lauber JPC 2018 https://doi.org/10.1088/1742-6596/1125/1/012015
- ITPA EP: https://sharepoint.iter.org/departments/POP/ITPA/EP/Pages/default.aspx
- EP WF confluence page: https://confluence.iter.org/pages/viewpage.action?pageId=289069024
- Ph. Lauber, Energetic particle driven instabilities during the L-H transition in ASDEX Upgrade; Proceedings EPS 2022 paper
- WF: Popa V.A. et al subm. NF 2023

cite:
WF and all actors you use
(Helena, CHEASE, LIGKA)

- more information: Homepage Philipp Lauber
- and git ITER repository: git clone ssh://git@git.iter.org/stab/ligka.git

HELENA: G.T.A. Huysmans, J.P. Goedbloed, and W. Kerner. Isoparametric bicubic Hermite elements for solution of the Grad-Shafranov equation. Proc. CP90 Conf. on Comp. Phys. Proc., page 371, 1991.

CHEASE: H. Luetjens, A. Bondeson, O. Sauter, The CHEASE code for toroidal MHD equilibria, Comput. Phys. Commun. 97 (1996) 219

continuous improvement

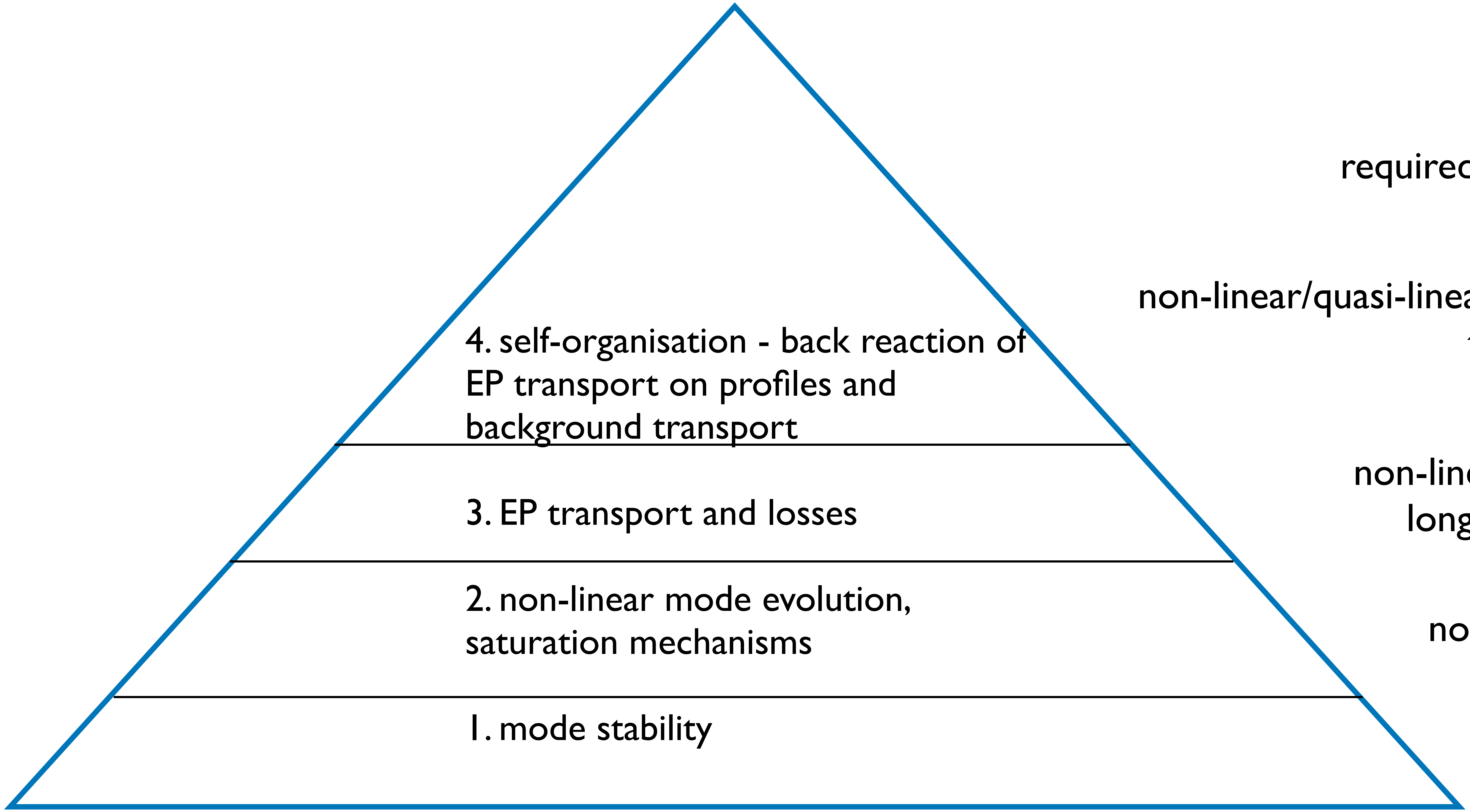
is better than

delayed perfection

please report bugs and shortcomings!

modelling hierarchy for plasmas with significant energetic particle pressure

for scaling from TCV-AUG-JET,… to JT-60SA-DTT-ITER-DEMO-any other other device, we need:



required model:

non-linear/quasi-linear global kinetic + background transport

non-linear/quasi-linear global kinetic + long time scales (source + sink)

non-linear global kinetic e.m.

linear global kinetic e.m.

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modelling hierarchy for plasmas with significant energetic particle pressure

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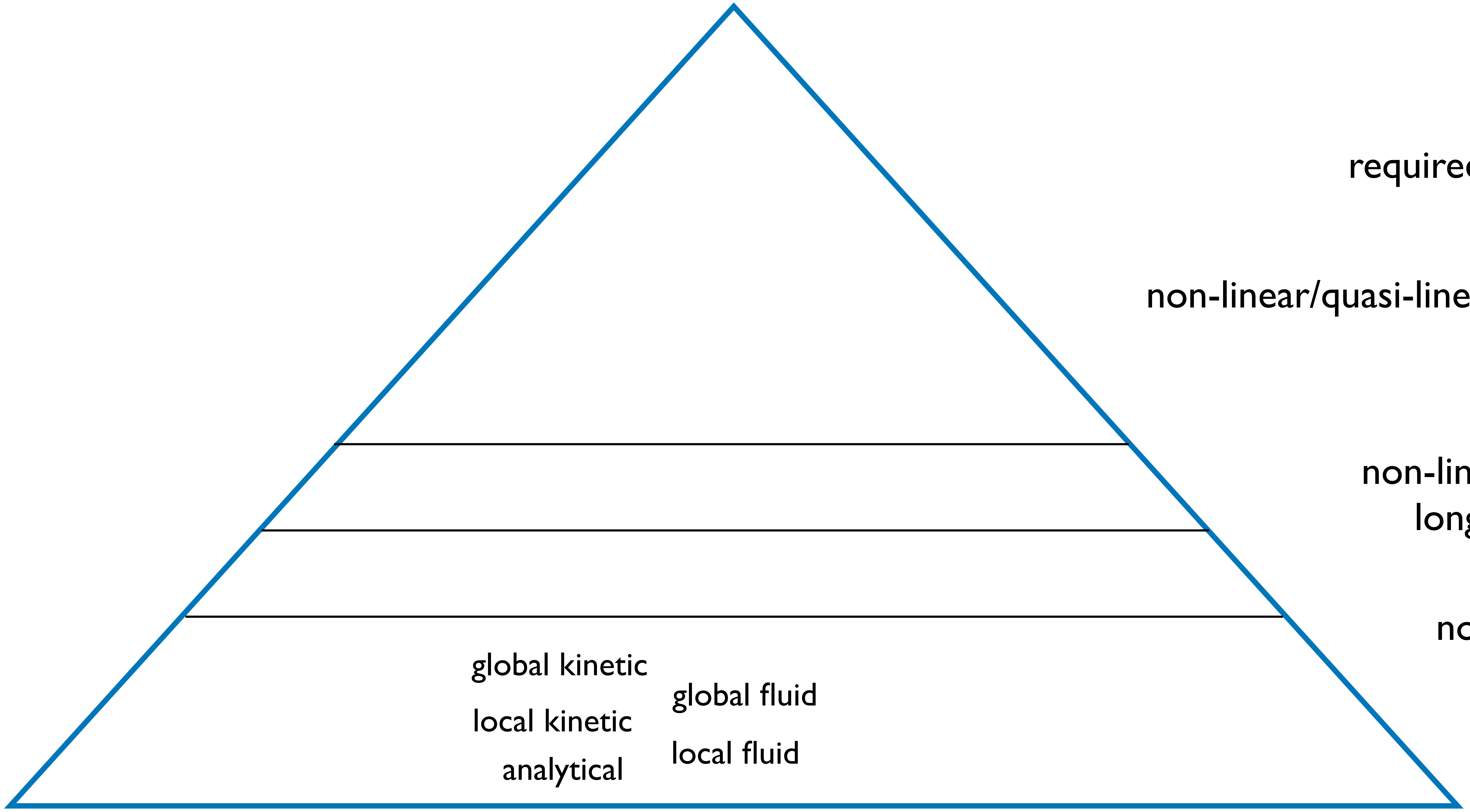
linear modelling hierarchy: linear Alfvénic mode structures are non-linearly robust→ important ingredient for non-linear/ transport models



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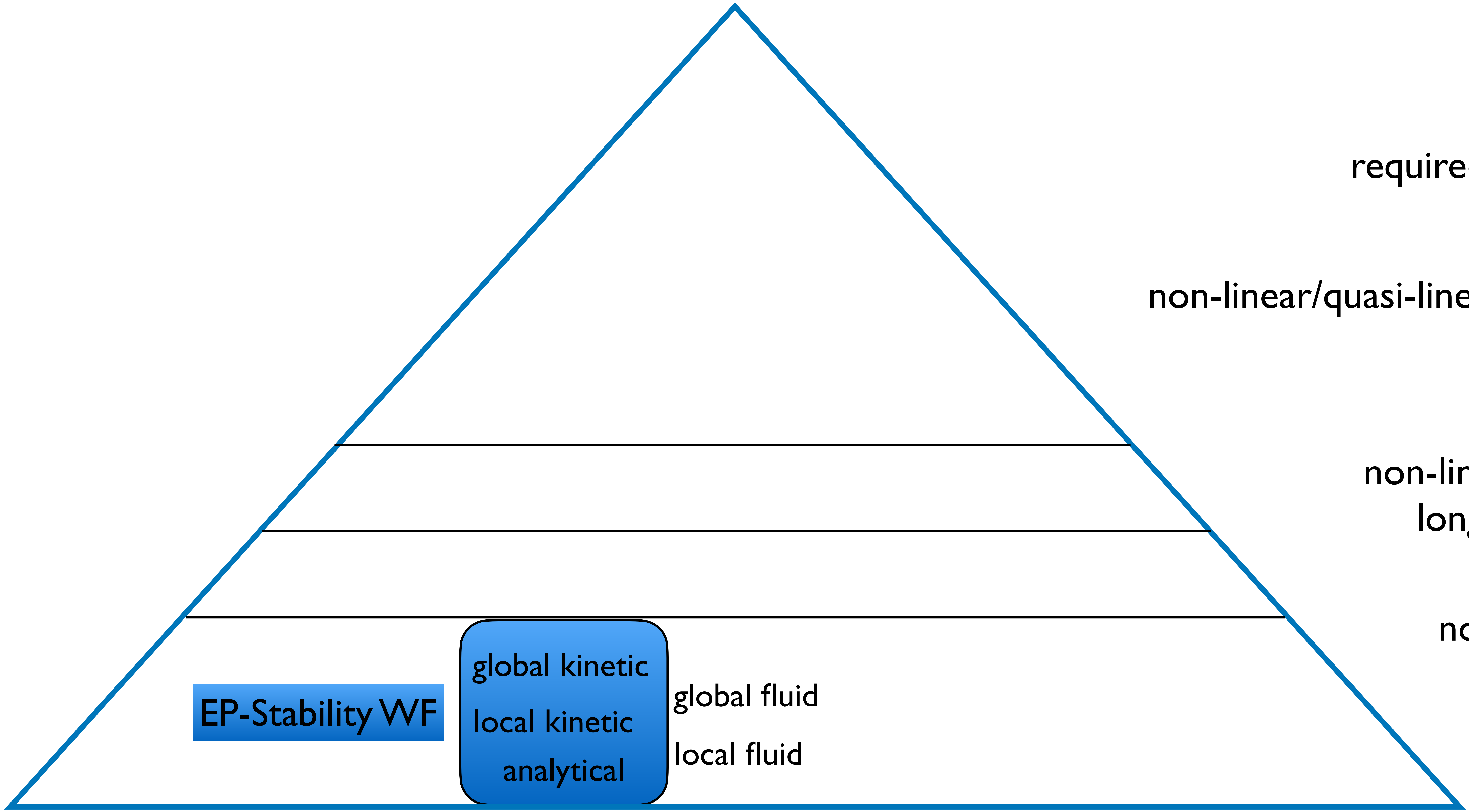
modelling hierarchy for plasmas with significant energetic particle pressure

linear modelling hierarchy: linear Alfvénic mode structures are non-linearly robust→ important ingredient for non-linear/ transport models



required model:

non-linear/quasi-linear global kinetic + background transport

non-linear/quasi-linear global kinetic + long time scales (source + sink)

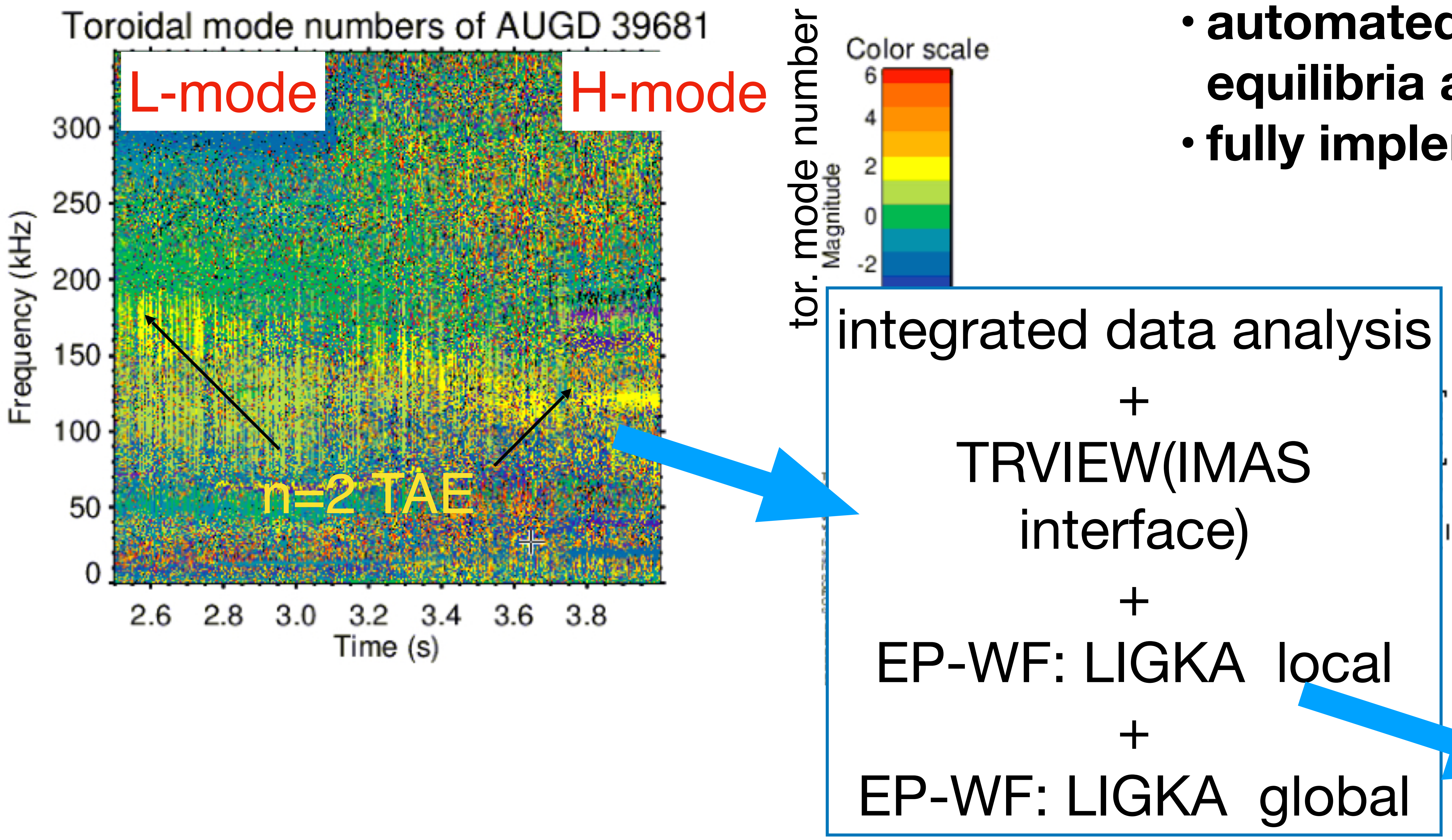
non-linear global kinetic e.m.

linear global kinetic e.m.

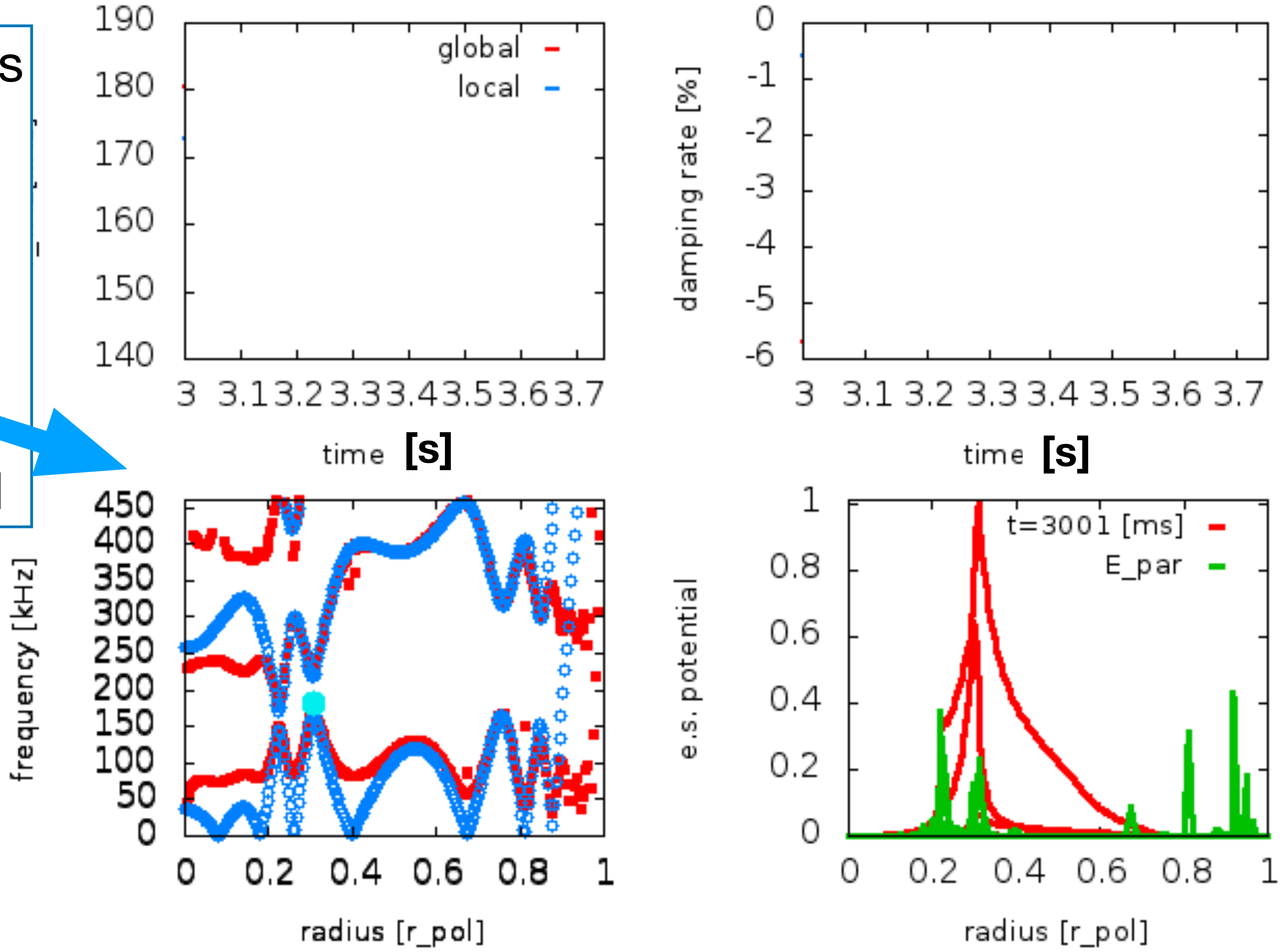
EP stability WF: design criteria

- **automate analysis of stable/unstable Alfvén eigenmodes:**
  - for many equilibrium time slices
  - for many relevant toroidal mode number (Tokamak only, axisymmetry)
  - relevant types of modes
- **use hierarchy:**
  - start with simple, analytical model
  - use local model
  - use global model
- **understand physics and numerical challenges:**
  - determine (kinetic) continuous spectra
  - investigate local vs global damping mechanisms
  - determine resolution requirements for expensive runs
- **determine sensitivity of AEs:** look at series of equilibria, include uncertainties
- **be general:** use IMAS mhd_linear IDS to store results - each model is exchangeable (e.g. spectrum: LIGKA or Falcon)
- **be fast:** use reduced models where possible
- **be robust** enough to use it as fundamental ingredient for transport models

motivation: present-day experiments



- **automated processing of 160 time slices based on IDA equilibria and profiles**
- **fully implemented in IMAS, ensuring reproducibility**



- **analyse L-mode,H-mode and transition phases: beat infamous problem of AE stability sensitivity to profiles - compare trends instead of single time slices**
- **compare local and global models**
- **systematic uncertainty quantification feasible**
- **applied also to TCV, JET, JT-60SA, ITER**

[Lauber, EPS 2022, Popa 2020-2023]

Kinetic Description
Vlasov, Fokker-Planck Equation

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Building Moments by Integration over Velocity Space

↓

Fluid Equations

MHD

↓

perturbative Kinetic MHD Models
[Cheng,1991]

CAS3D-K ,NOVA-K CASTOR-K

Limit

Reduce from 6-D to 5-D

↓

Gyrokinetic Theory
[Littlejohn, Hahm,Brizard]

↓

Gyrokinetic MHD
Self Consistent
non-perturbative
[Qin, 1999]

LIGKA, KIN2DEM GYGLES

Equivalent

Linearization

↓

Kinetic Wave Equations
[Stix, Brambilla]

↓

Dielectric Tensor

TORIC,PENN,LEMAN

non-linear: ORB5,EUTERPE,GENE,GTC

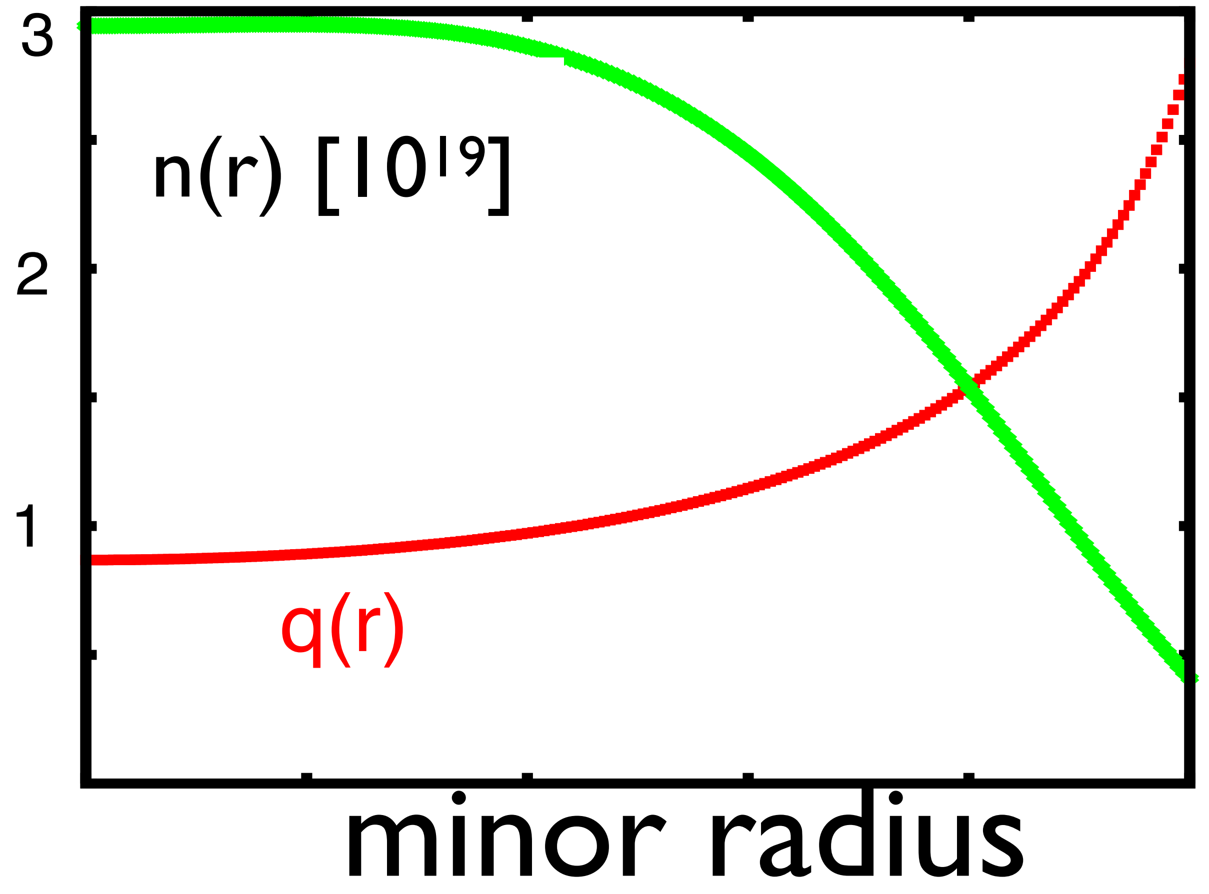
why a kinetic model?

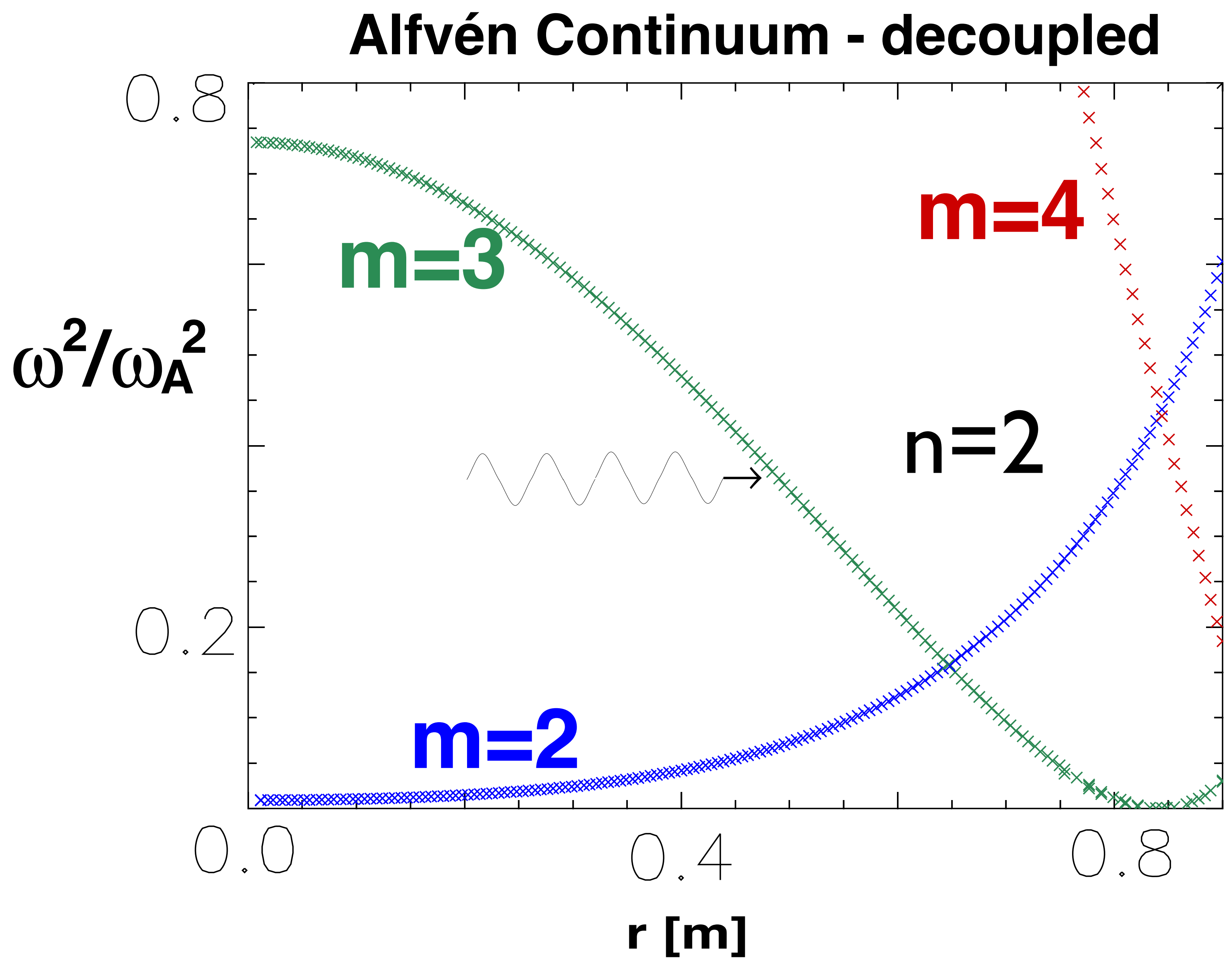
dispersion relation: $\omega = k_{\|} v_A;$

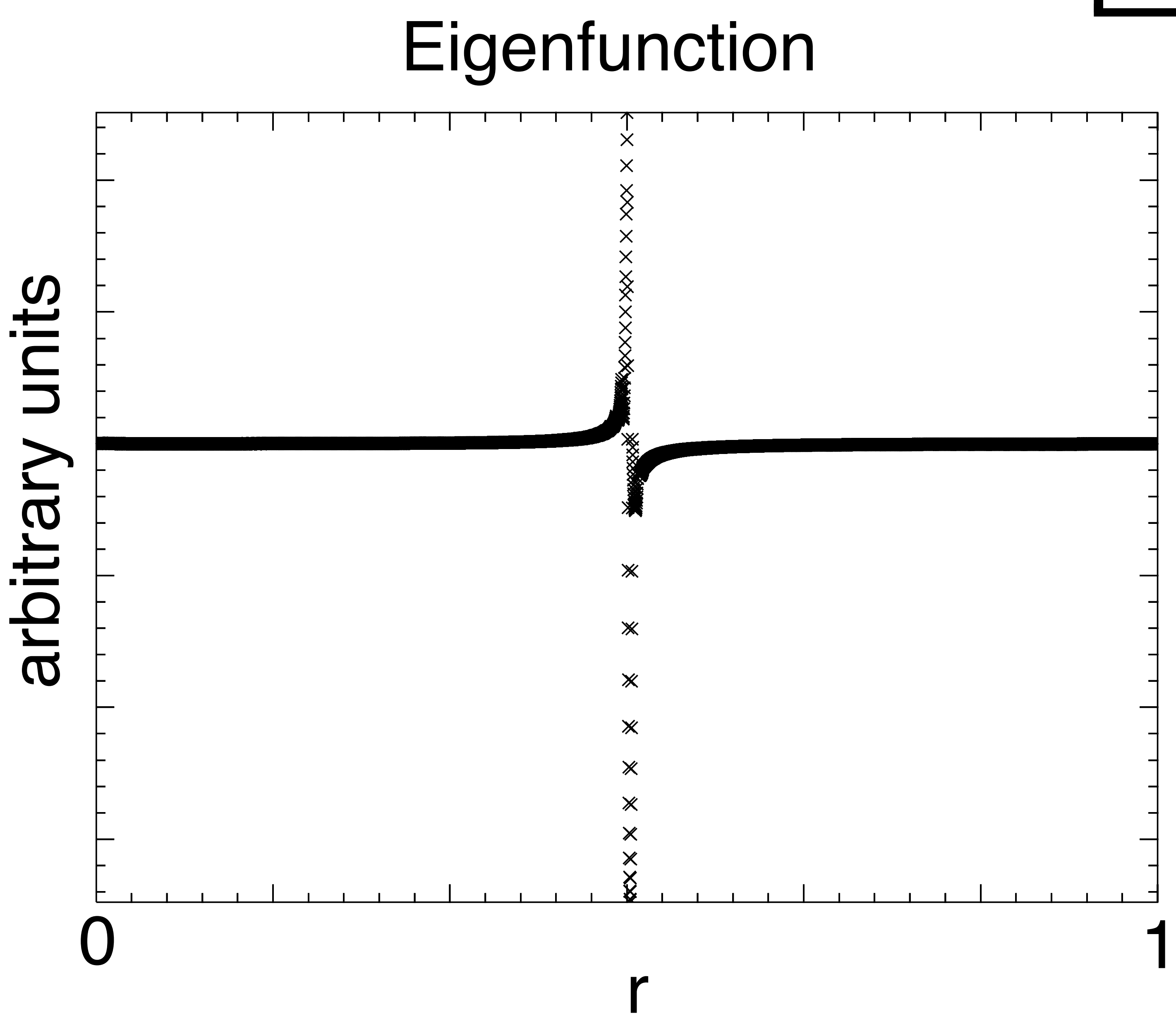
periodic cylinder: phase mixing, i.e. strong damping

$$k_{\|} = \frac{1}{R_0}\left(n - \frac{m}{q(r)}\right); \quad v_A(r) = B(r)/\sqrt{\mu_0 m_i n(r)}; \; q(r) = rB_z/RB_\theta$$

n: toroidal/axial mode number m: poloidal mode number







idea of Alfvén wave heating: efficient absorption of external wave at resonant location

[W. Grossmann, J. Tataronis, Z. Phys. 261, 217 (1973); A. Hasegawa, L. Chen, Phys. Rev. Lett. 35, 370 (1975)]

why a kinetic model?

- it was early recognised that kinetic effects need to be included to understand the absorption mechanism [A. Hasegawa, L. Chen, Phys. Rev. Lett. 35, 370 (1975), A. Hasegawa, L. Chen, Phys. Fluids 19 (1976) 1924]

- use quasi-neutrality and shear-Alfvén law including lowest order finite Larmor radius effects and Landau damping-like terms (LD):

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}; \qquad A_{\|} = \frac{1}{i\omega}(\nabla\psi)_{\|}$$

$$[1 + \xi_e Z(\xi_e) + 1 + \xi_i Z(\xi_i)](\phi - \psi) = T_e/T_i \varrho_i^2 \nabla_\perp^2 \phi$$

$$\nabla_\perp \cdot \frac{\omega^2}{v_A^2}\nabla_\perp\phi + \frac{\partial}{\partial s}\nabla_\perp^2 \frac{\partial\psi}{\partial s} = \frac{3}{4}\varrho_i^2 \frac{\omega^2}{v_A^2}\nabla_\perp^4\phi$$

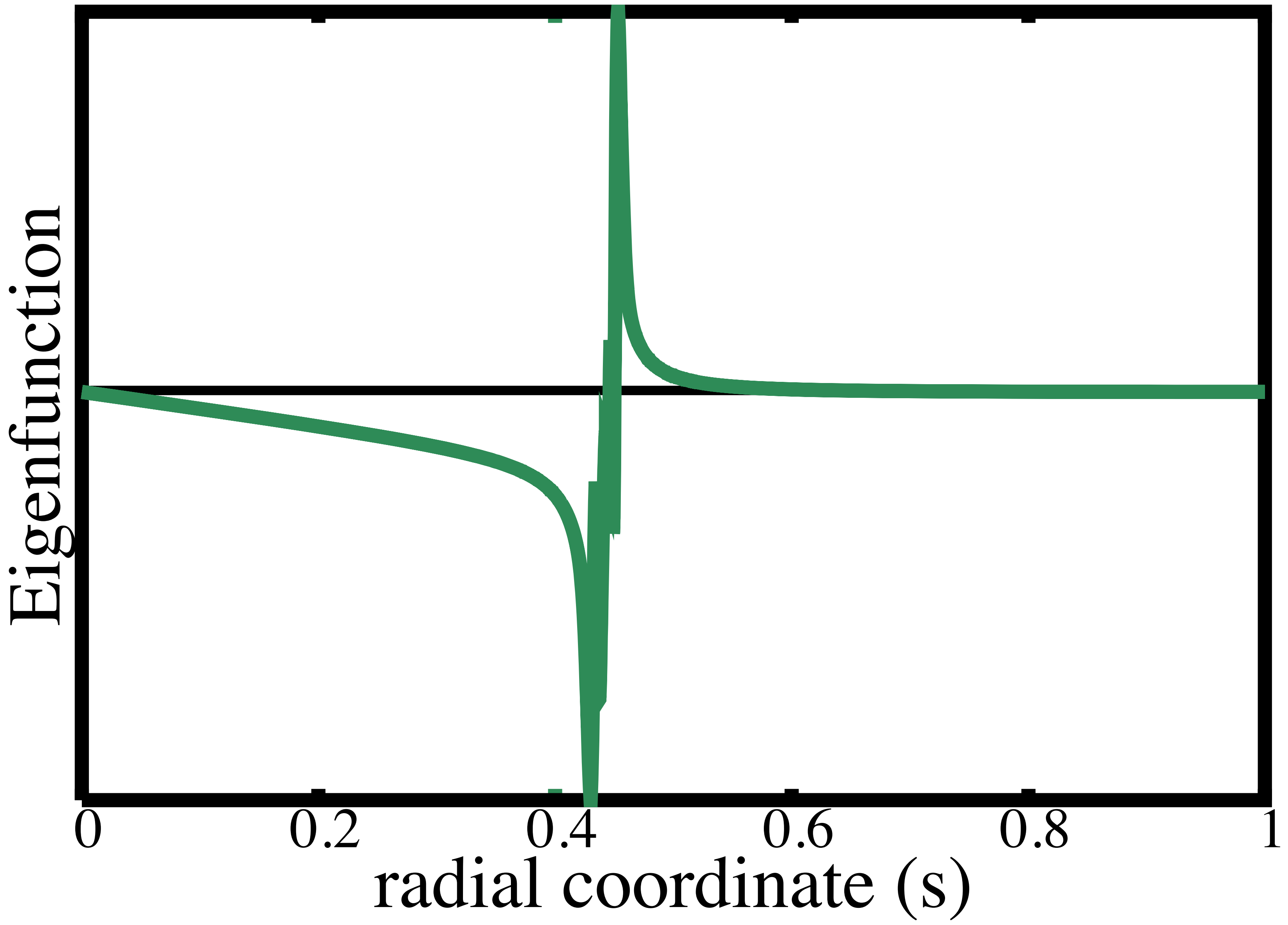
- if mode is purely Alfvénic, $\Phi = \psi$ and $E_{//} = k_{//}(\Phi - \psi) = 0$ *is result of model 1/2*

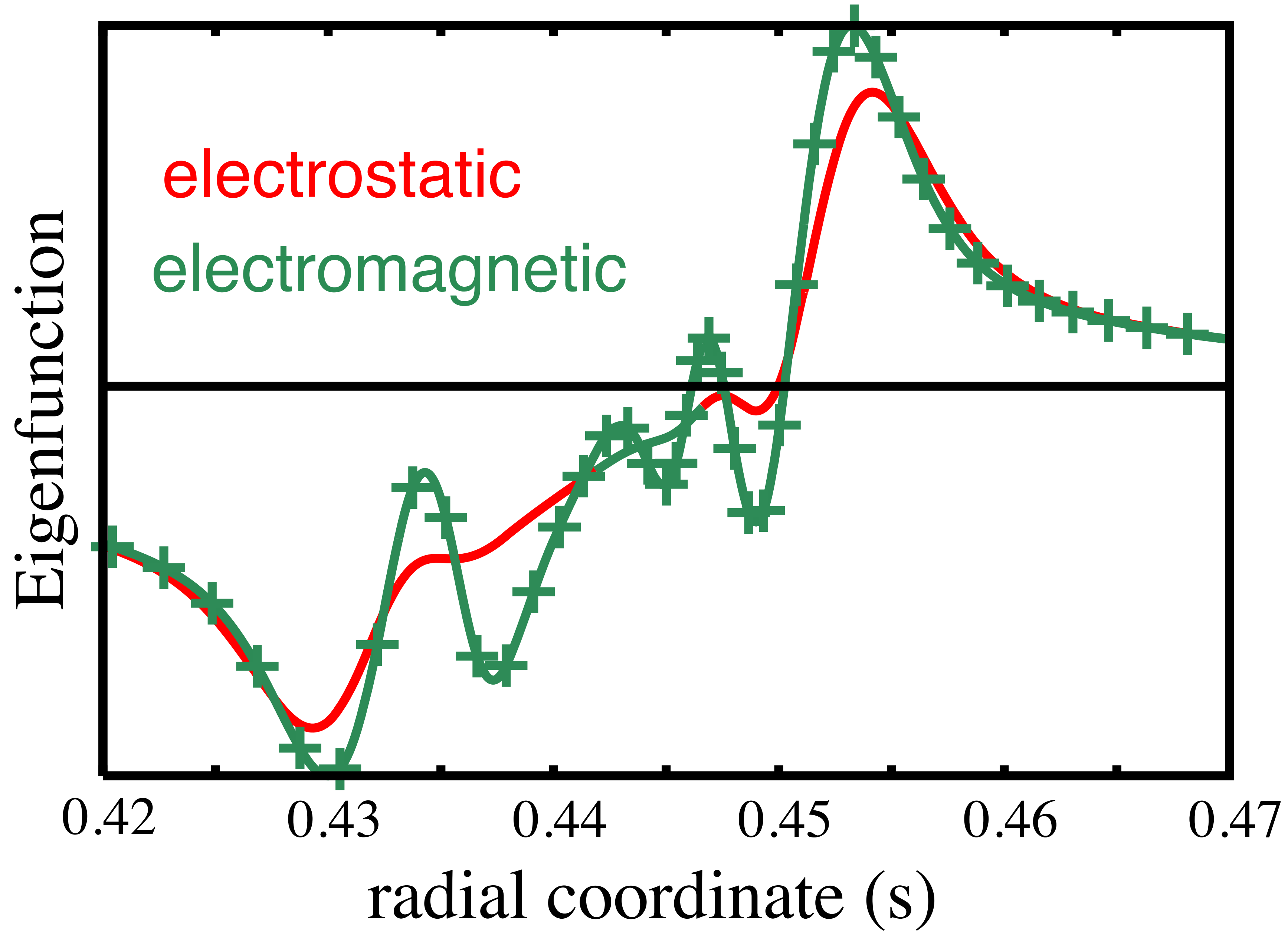
- polarisation gives important information on nature of perturbation: in Tokamaks, predominantly Alfvénic, predominantly electrostatic and mixed polarisation are very common
- Alfvénicity can be determined in MHD limit - FALCON code [M. Falessi]

global solutions change character

$$\left.\begin{array}{r} S(\phi-\psi) = T_e/T_i \varrho_i^2 \nabla_\perp^2 \phi \\ \nabla_\perp^2 \frac{\omega^2}{v_A^2}\phi + \nabla_\perp^2 k_\parallel^2 \psi = \frac{3}{4}\varrho_i^2 \frac{\omega^2}{v_A^2}\nabla_\perp^4 \phi \end{array}\right\} \Rightarrow \omega^2 = k_\parallel^2 v_A^2 \left[1 + k_\perp^2 \varrho_i^2 (3/4 + T_e/T_i)\right]$$

long-wavelength-limit





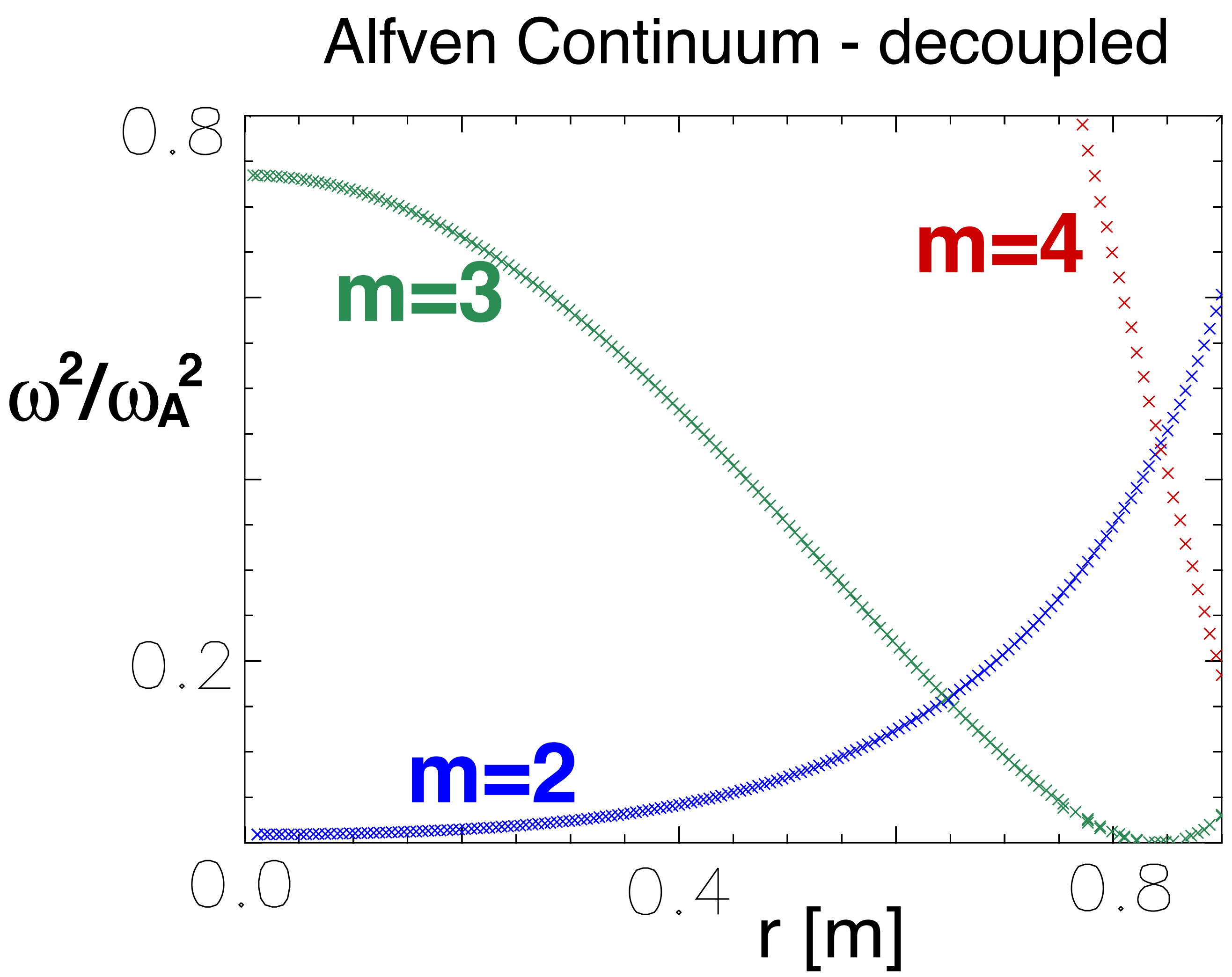
Singularity of the MHD operator is resolved by fourth order terms

deviations due to KAW coupling 'break Alfvénic state' [Walén 1944, Chen&Zonca RMP 2016]

this helps when trying to find least damped modes in presence of dense continuum - automatisation!

toroidal Alfvén eigenmodes (TAE)

[Cheng, Chen & Chance Ann. Phys.1985, Cheng & Chance 1986 Phys. Fluids 29]



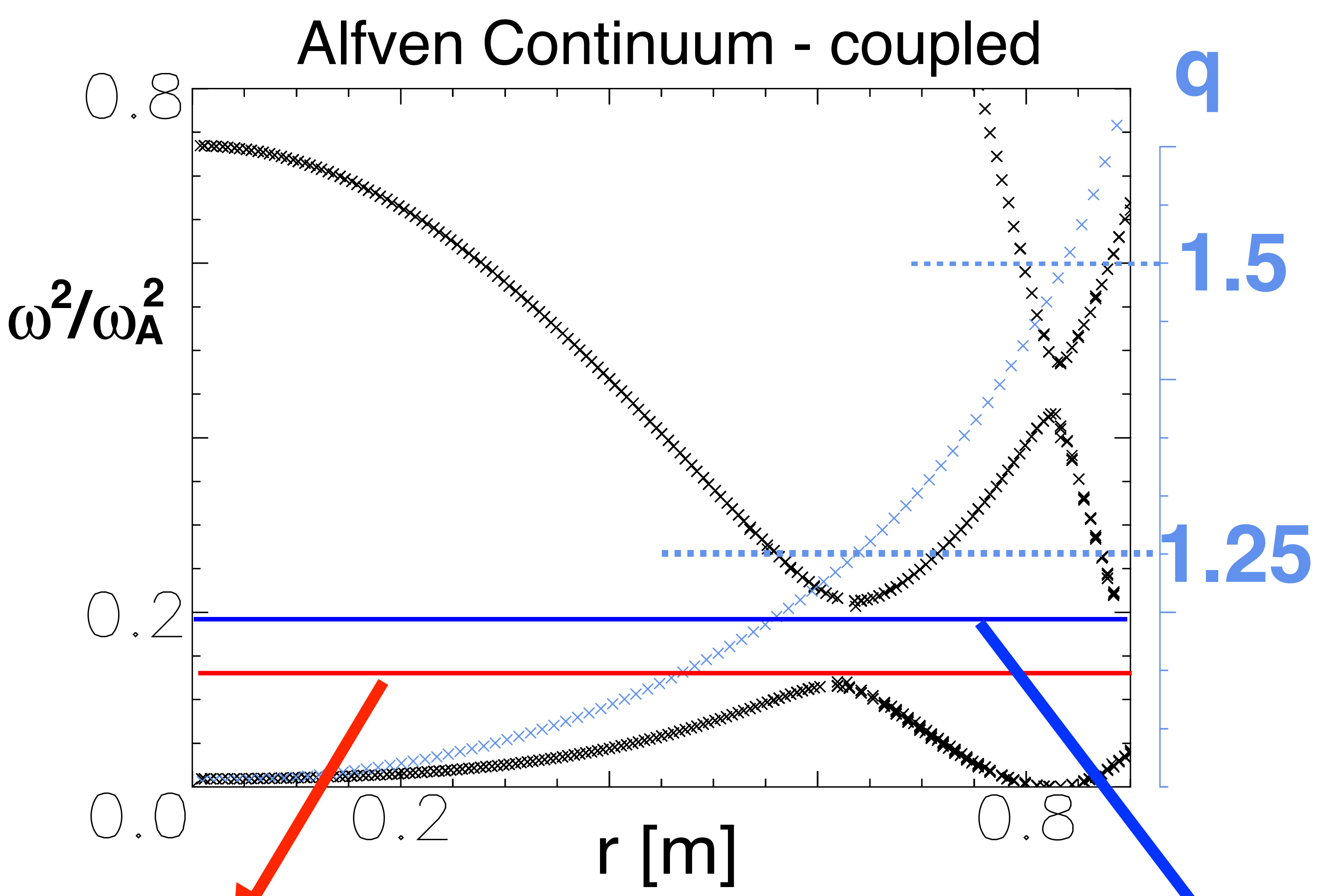
(n=2)

$\Rightarrow$

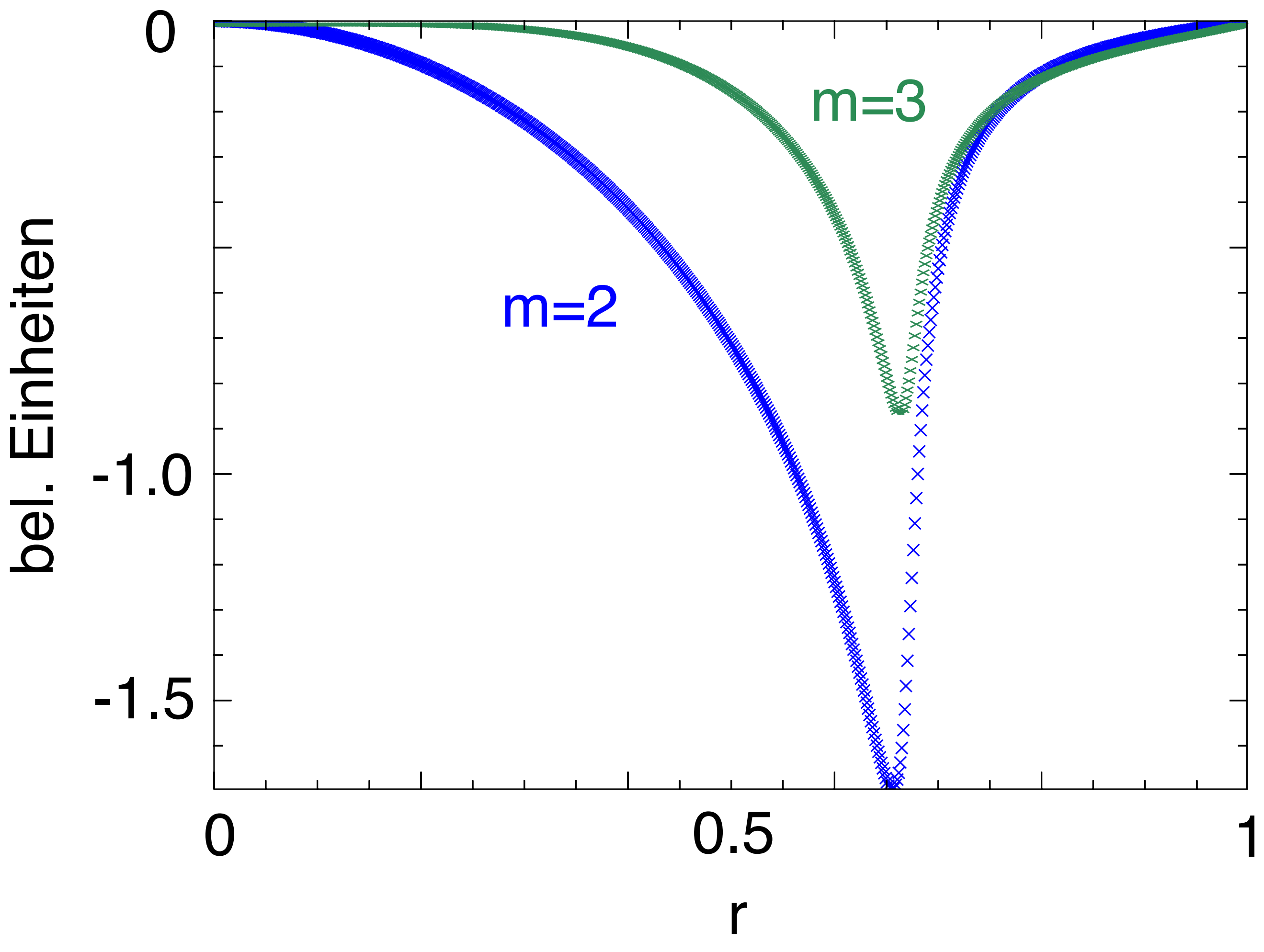
$$R \approx R_0(1+\epsilon\cos\theta)$$

$$B \approx B_0(1-\epsilon\cos\theta)$$

$$\epsilon = r/R_0$$

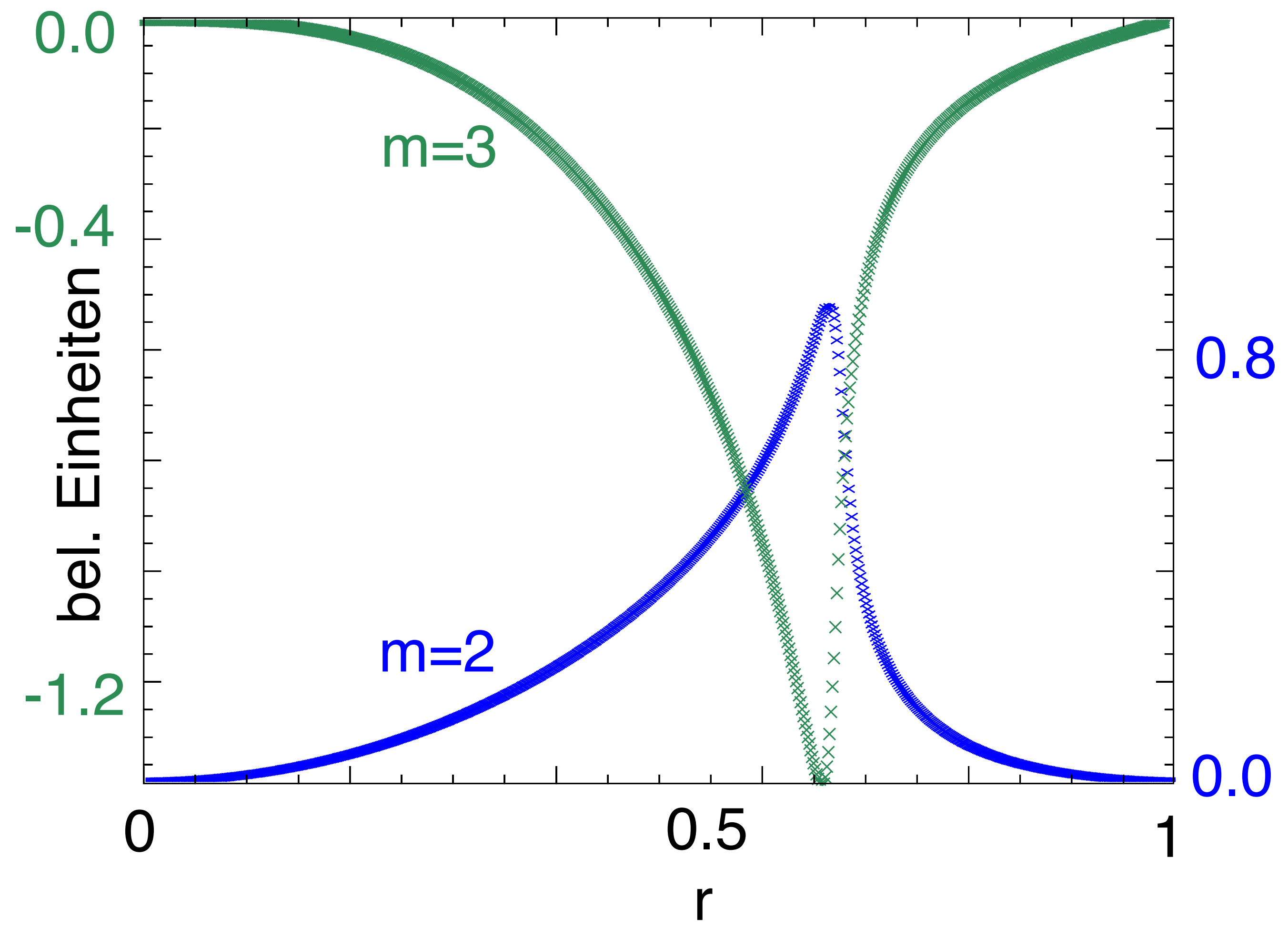


TAE: even mode



niedrigere Frequenz

TAE: odd mode



höhere Frequenz

analogous to electron bands in solid state physics

$q_{TAE}=(m+1/2)/n$

Global analysis:

[e.g. Berk 1992] Physics of Fluids B: Plasma Physics 4, 1806 (1992); https://doi.org/10.1063/1.860455

cylinder:

$$\frac{d}{dr}\left[r^3\left(\frac{\omega^2}{v_A^2}-k_{\|m}^2\right)\frac{dE_m}{dr}\right]+\omega^2 r^2 E_m \frac{d}{dr}\left(\frac{1}{v_A^2}\right) - (m^2-1)\left(\frac{\omega^2}{v_A^2}-k_{\|m}^2\right) r E_m = -iL_k(\omega)E_m.$$

→delta functions, continuum damping

torus:

$$\frac{d}{dr}\left[r^3\left(\frac{\omega^2}{v_A^2}-k_{\|m}^2\right)\frac{dE_m}{dr}\right]+r^2 E_m \frac{d}{dr}\left(\frac{\omega}{v_A}\right)^2 - (m^2-1)\left(\frac{\omega^2}{v_A^2}-k_{\|m}^2\right) r E_m + iL_k(\omega)E_m + \frac{d}{dr}\left[r^3\left(\frac{\omega}{v_A}\right)^2\left(\Delta'+\frac{2r}{R_0}\right)\left(\frac{dE_{m+1}}{dr}+\frac{dE_{m-1}}{dr}\right) - r^3\Delta' k_{\|m}\left(k_{\|m+1}\frac{dE_{m+1}}{dr}+k_{\|m-1}\frac{dE_{m-1}}{dr}\right)\right]=0.$$

coefficient for 2nd order operator does not vanish for certain ω;

- no local, delta-type functions possible for bands of certain ω
- global solutions possible

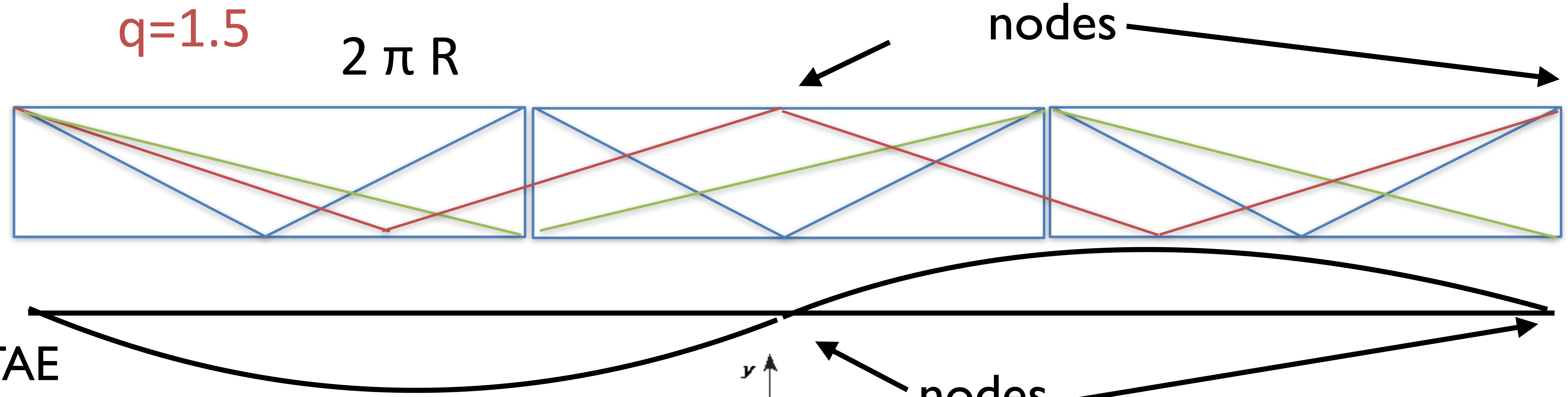
TAE visualisation:

TAE: $k_{m,//}=-k_{m+1,//}$

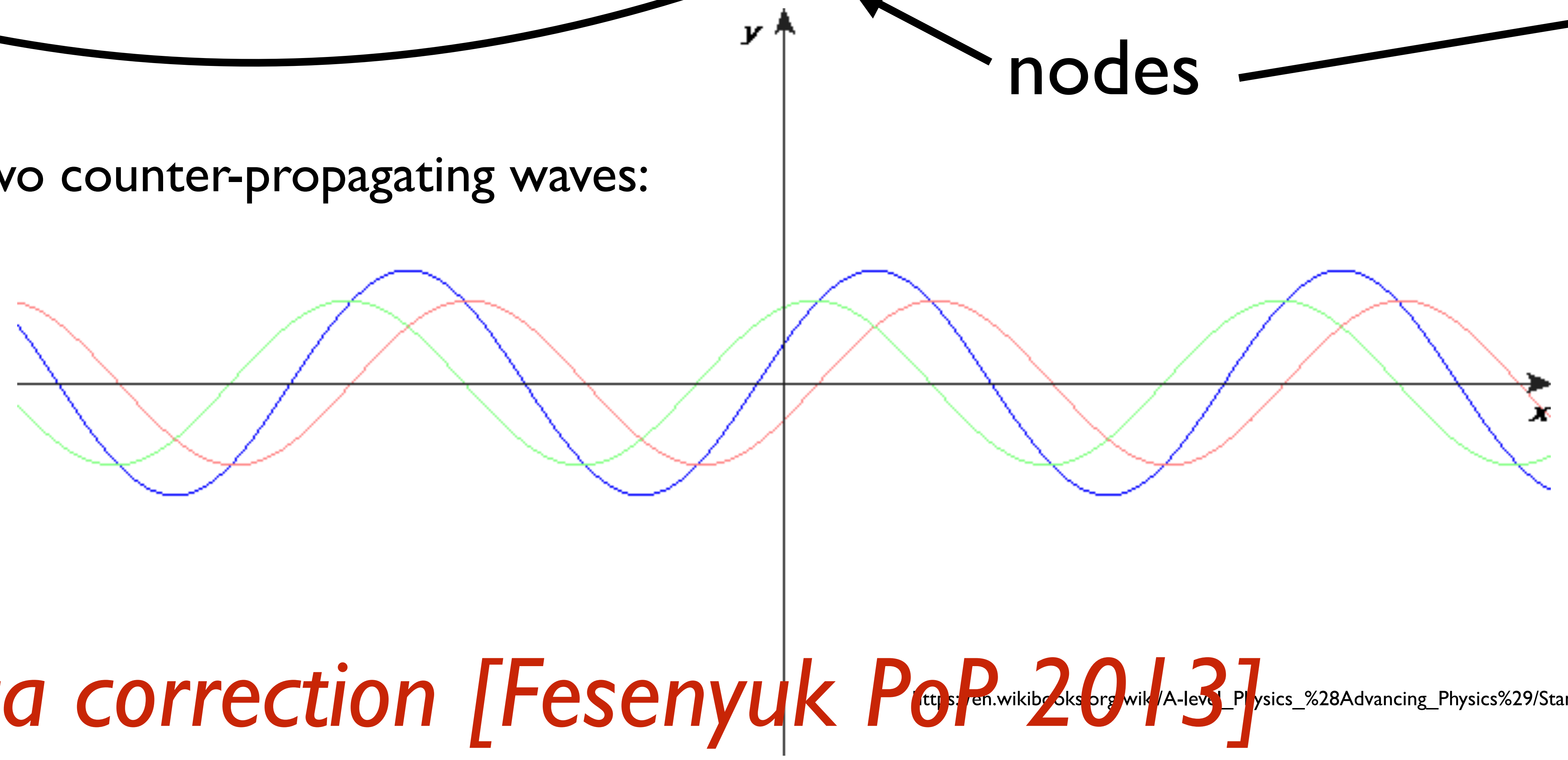
$k_{TAE}=1/(2q_{TAE}R)=(q=1.5, n=1, m=1, m+1=2)=1/(3\,R)$

$\lambda_{TAE}=2\pi/k=2\,(2\,\pi\,R\,q_{TAE})=3\,(2\,\pi\,R)$

q=1
q=2
q=1.5



TAE is formed by two counter-propagating waves:

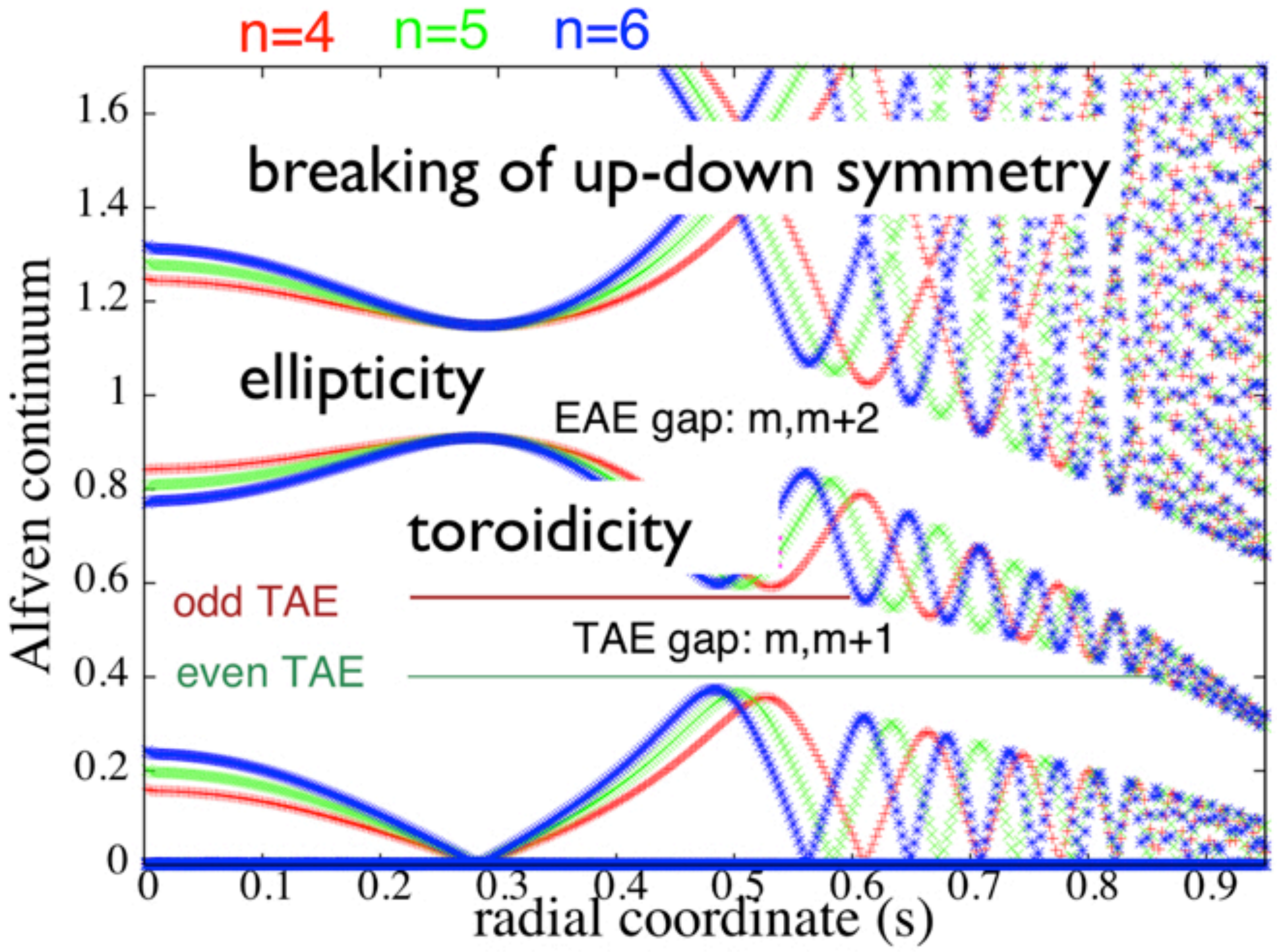


https://en.wikibooks.org/wiki/A-level_Physics_%28Advancing_Physics%29/Standing_Waves#/media/File:Harmonic_Standing_Wave.gif

$\omega_{TAE}=v_A/2q_{TAE}R$

formula implemented in reduced model 5 + beta correction [Fesenyuk PoP 2013]

symmetry-breaking induces more gaps

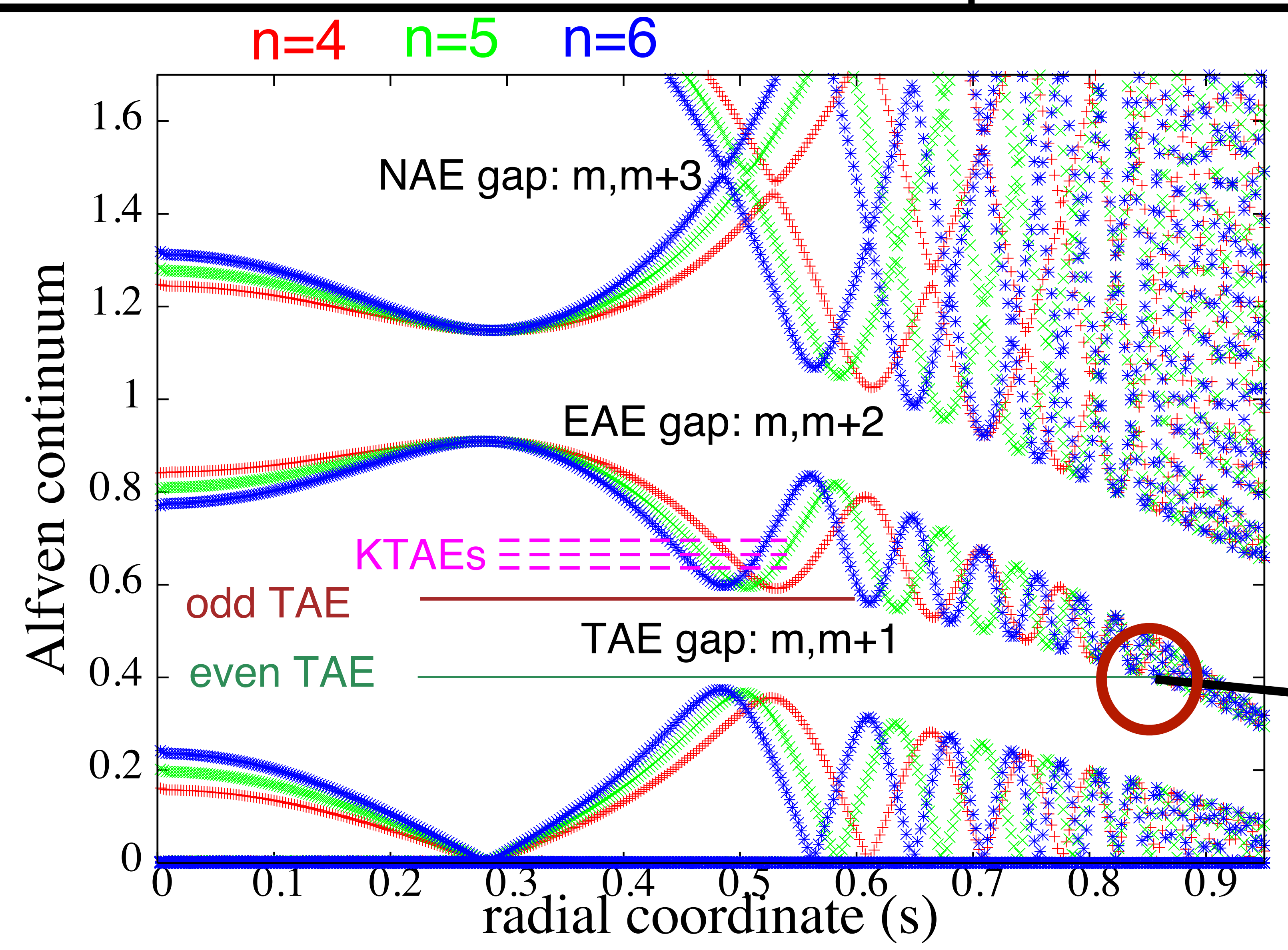


ASDEX Upgrade Alfvén continuum

reduced MHD spectra available using model 6

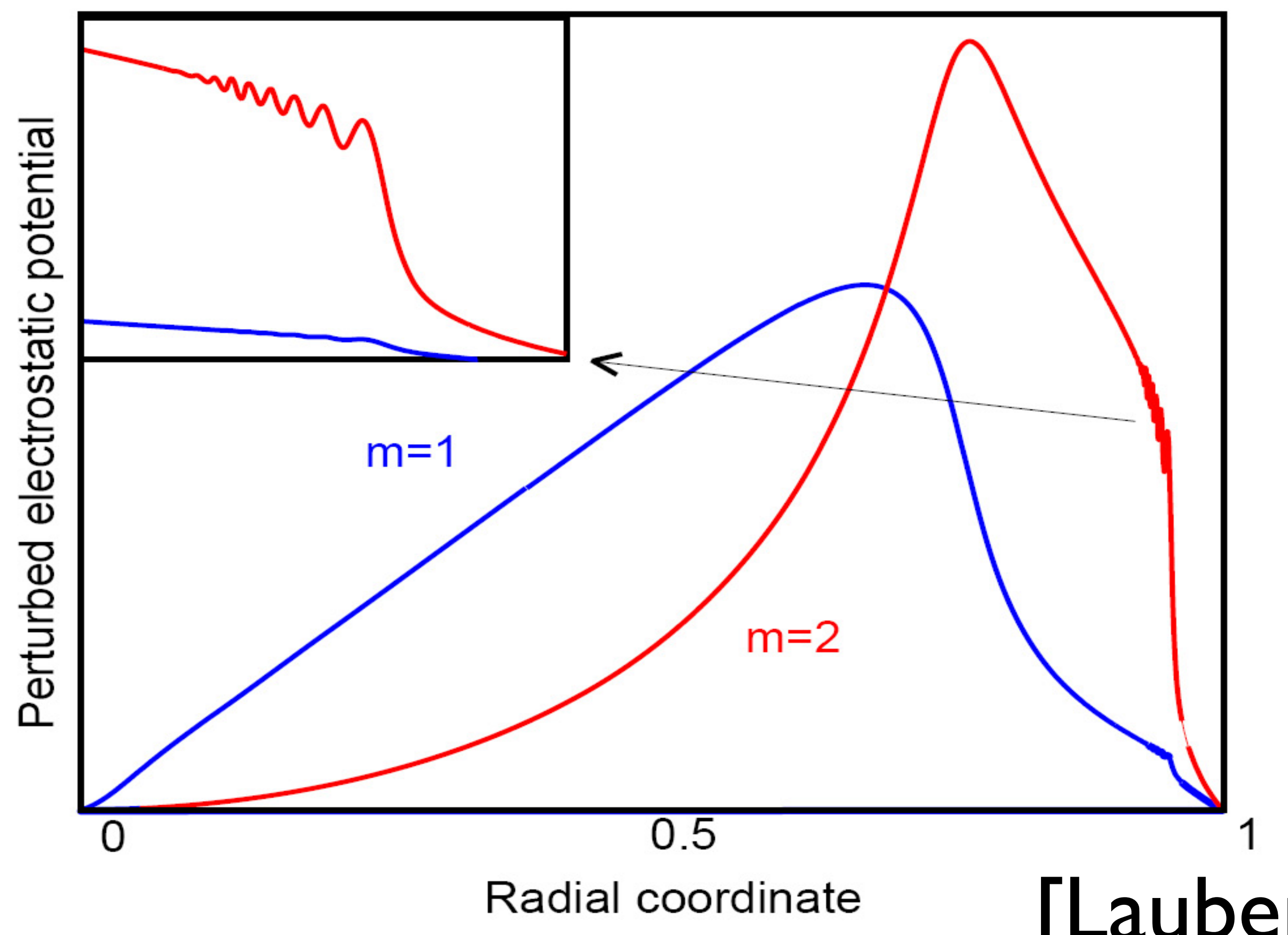
non-local continuum damping

[F. Zonca et al PRL 1992, Rosenbluth et al 1992, Berk et al 1992]



ASDEX Upgrade Alfvén continuum

continuum damping: mode conversion to kinetic Alfvén wave



[Lauber,2005]

radiative damping: cross-scale coupling of global shear Alfvén waves and KAWs

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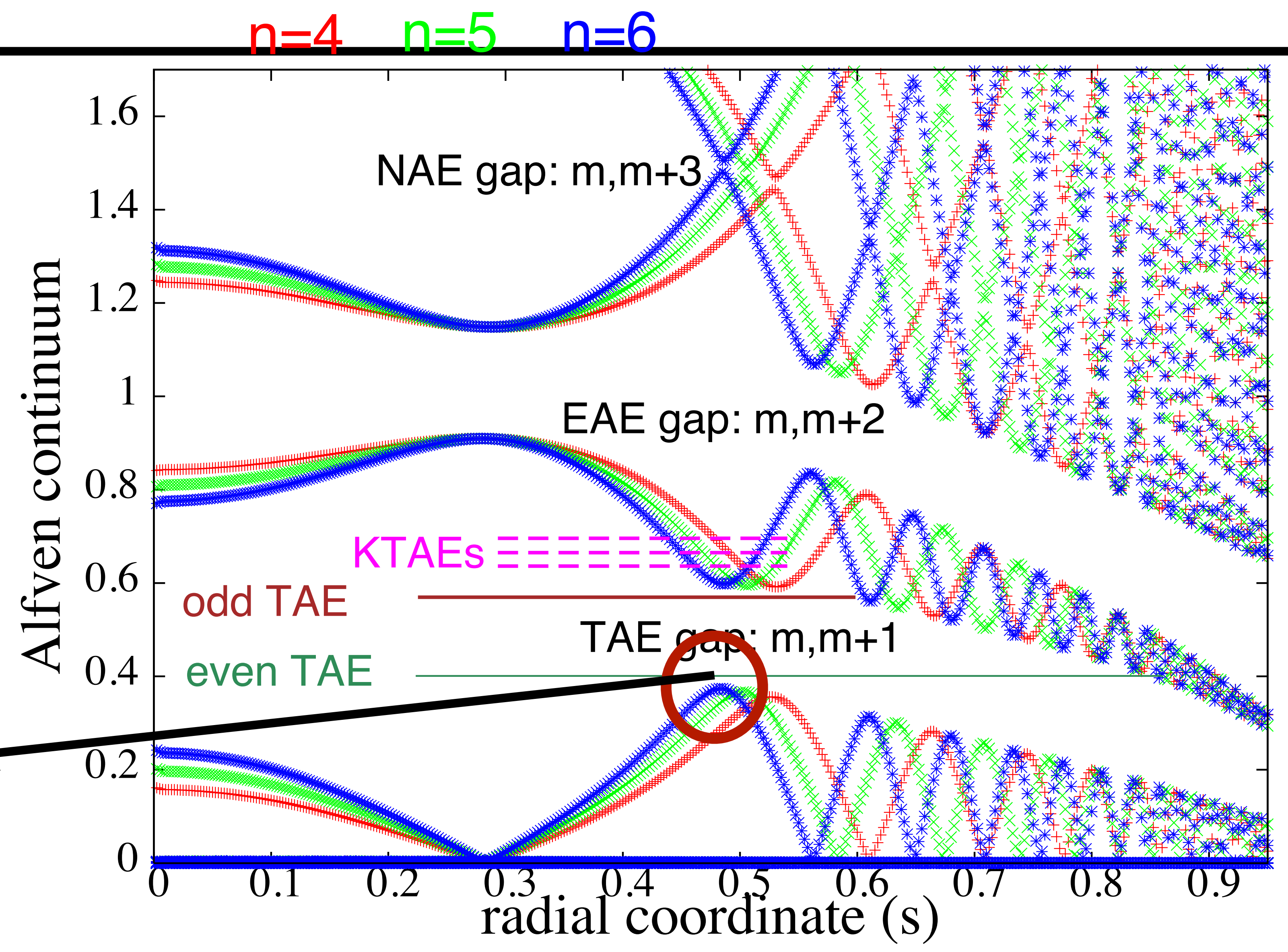


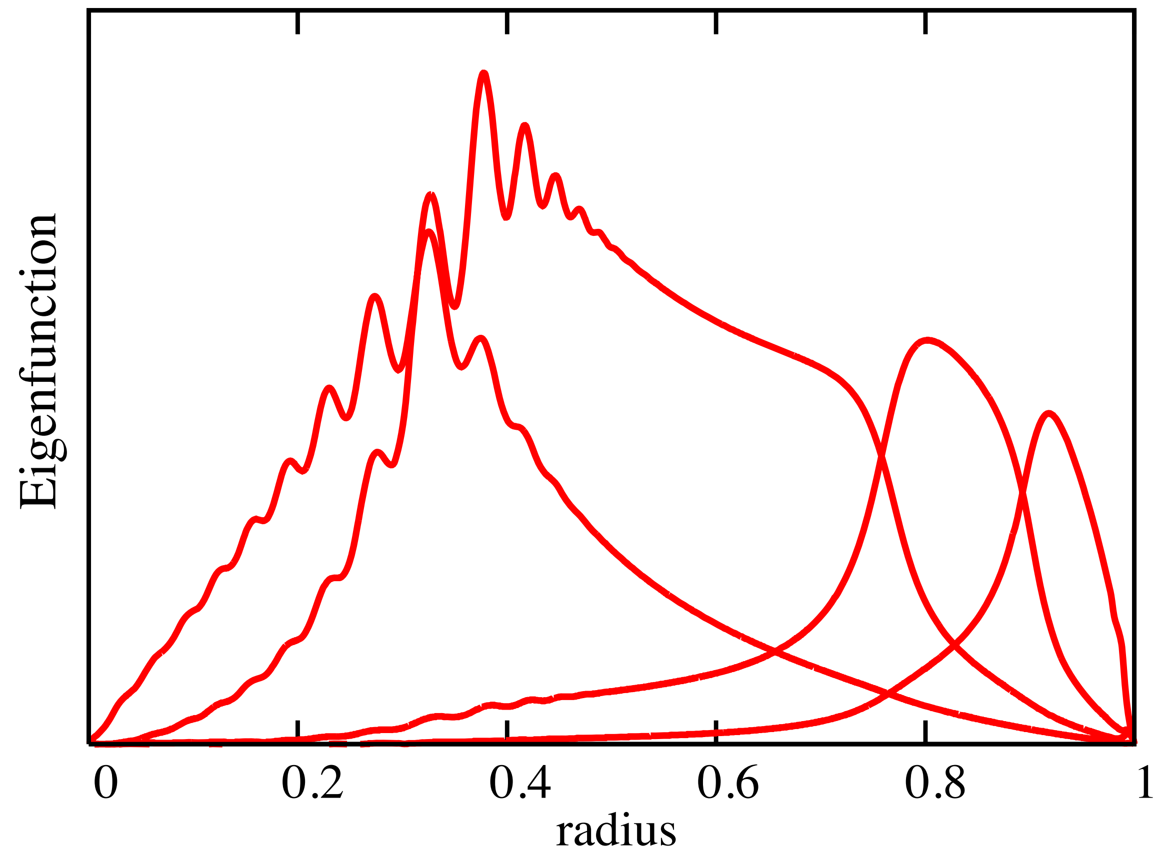
ASDEX Upgrade Alfvén continuum

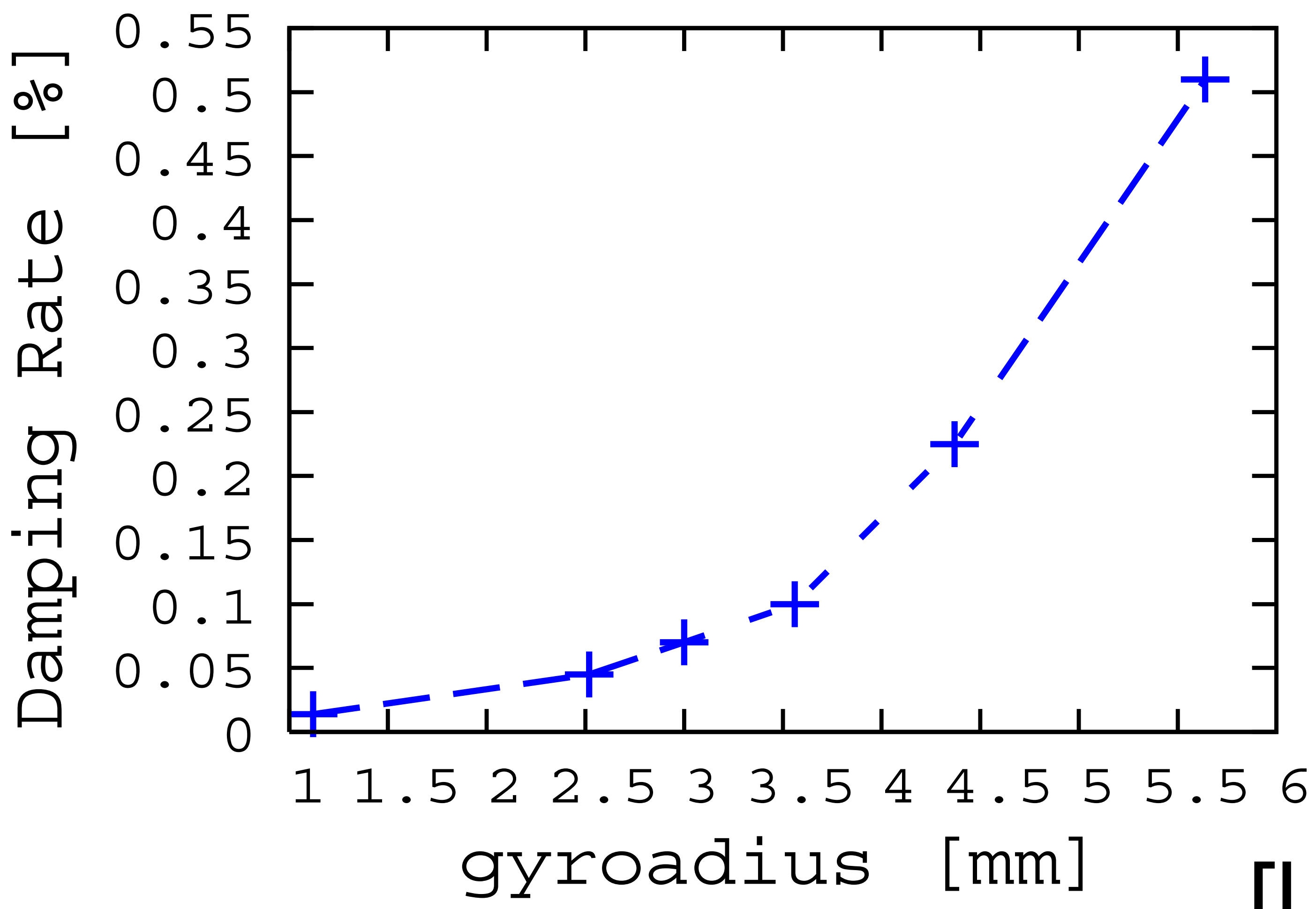
radiative damping: kinetic Alfven waves 'tunnels' into TAE

[Mett, Mahajan, 1992
Berk 1993, Candy & Rosenbluth 1994,
Breizman& Sharapov 1995,Pinches 2015]

coupling strength determined by non-ideal parameter:

$$\lambda \equiv 4\,\frac{\rho_i}{r_m}\frac{mS}{\varepsilon^{3/2}}\left(\frac{3}{4}+\frac{T_e}{T_i}\right)^{1/2}$$





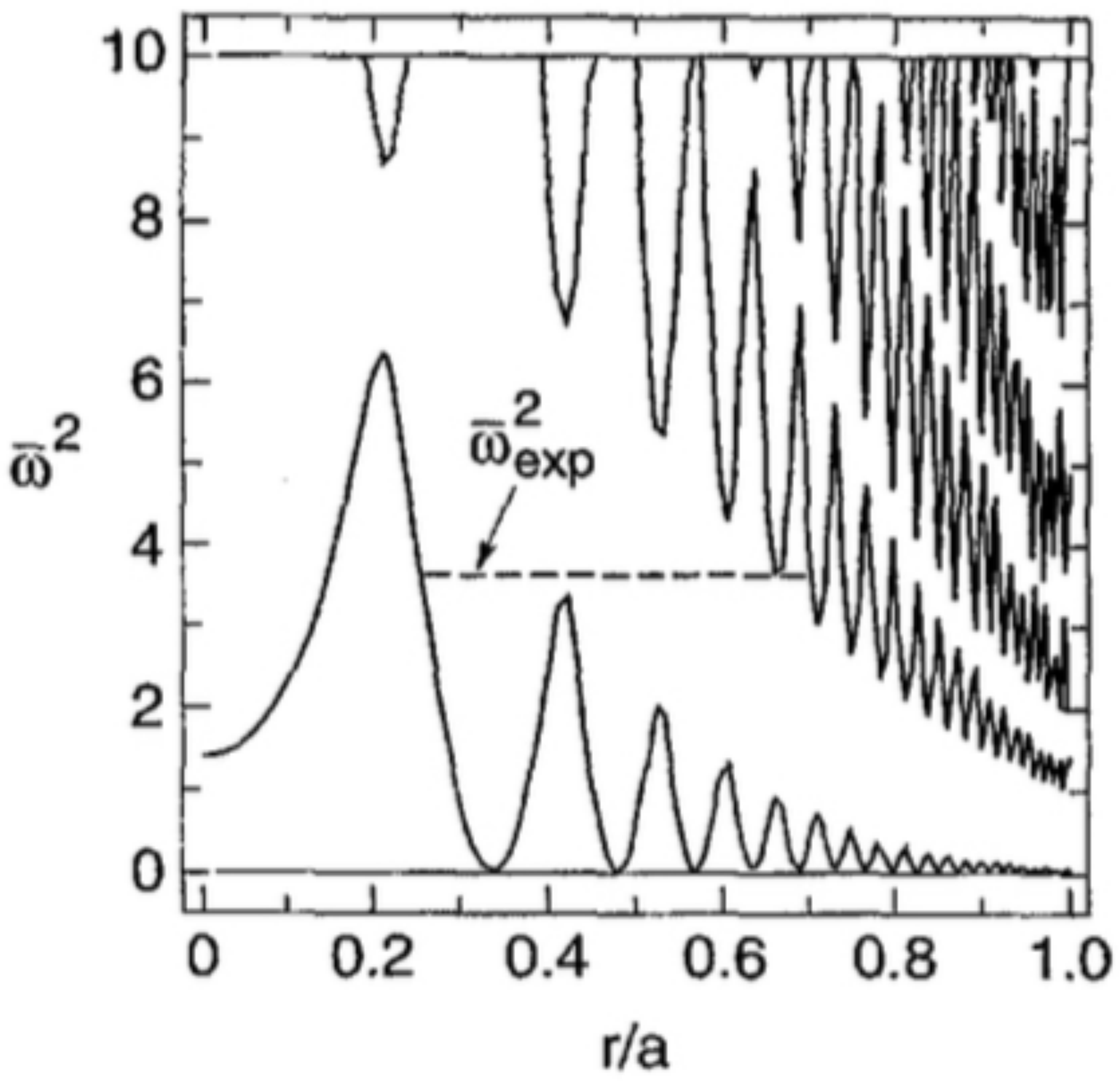
formula implemented in reduced model 5

[Lauber, PoP 2005]

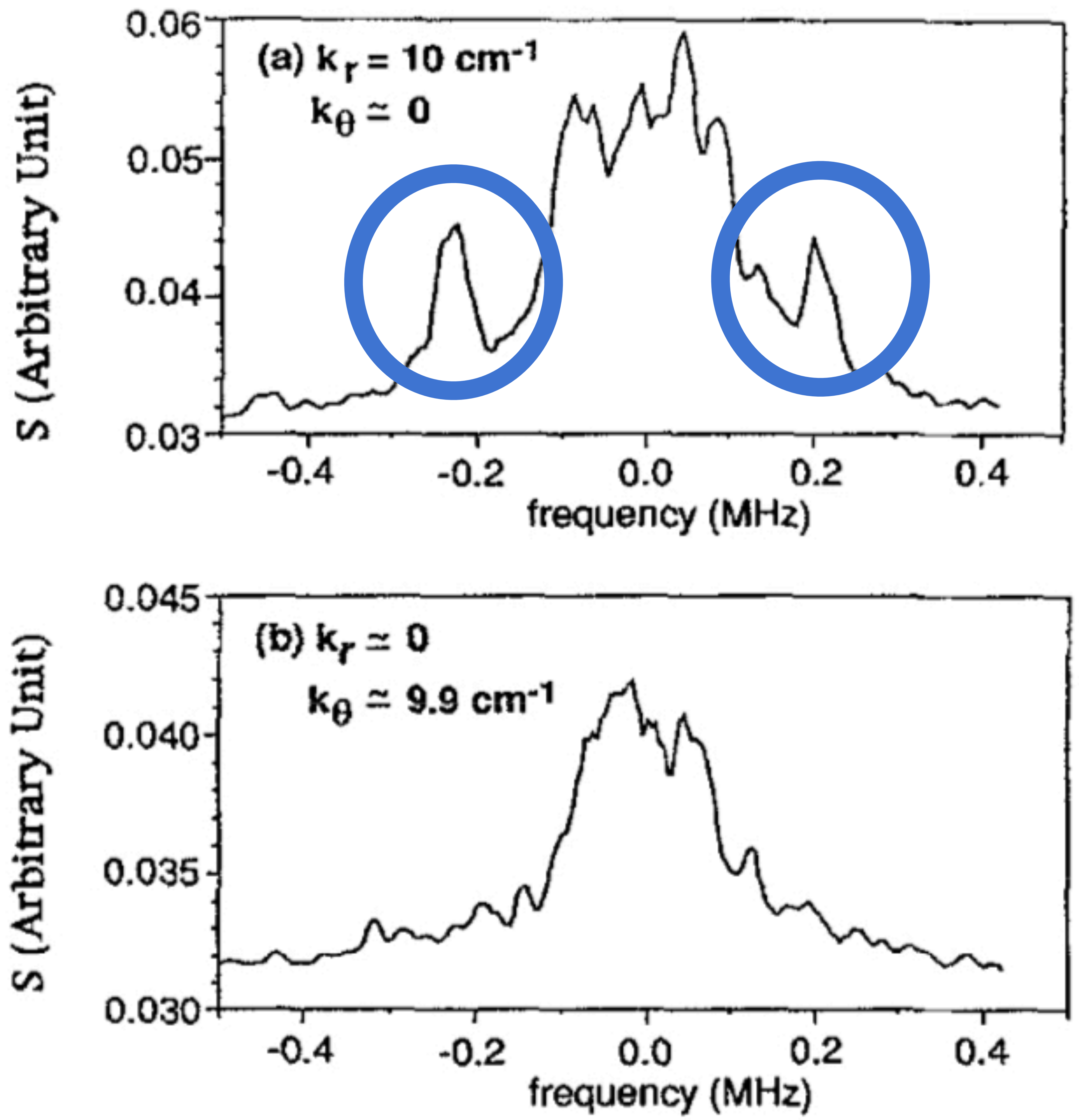
KAWs in the experiment

TFTR: [Wong, Phys. Lett A, 1996]

global mode driven by EPs - short wave-length structure detected at intersection points with continuum



microwave scattering



great confirmation of theory - demonstrating non-local nature of KAW physics

global, linear GK model

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LIGKA [Qin 1998, Lauber 2003, JPC 2007, Lauber PLREP 2013, Bierwage&Lauber 2017, Lauber JPCS 2018]

https://git.iter.org/projects/STAB/repos/ligka/

- gyrokinetic moment equation (GKM):

shear Alfvén law

$$- \frac{\partial}{\partial t}\left[\nabla \cdot \frac{1}{v_A^2}\nabla_\perp \phi\right] + \boldsymbol{B}\cdot\nabla\frac{\nabla\times(\nabla\times(\frac{\nabla\psi}{i\omega})_\parallel \boldsymbol{b})}{B} + (\boldsymbol{b}\times\nabla(\frac{\nabla\psi}{i\omega})_\parallel \boldsymbol{b})\cdot\nabla\frac{\mu_0 j_\parallel}{B}$$

$$= -\sum_a \mu_0 \int \mathrm{d}^2 v\, e_a \{\boldsymbol{v}_\mathrm{d}\cdot\nabla J_0 f\}_a + \sum_a \left[\boldsymbol{b}\times\nabla\left(\frac{\beta_{a\perp}}{2\Omega_a}\right)\right]\cdot\nabla\nabla_\perp^2\phi$$

‘pressure’ tensor - curvature drift coupling

$$+ \sum_a \frac{3v_{th,a}^2}{8v_A^2\Omega_a^2}\nabla_\perp^4\frac{\partial\phi}{\partial t} + \boldsymbol{B}\cdot\nabla\frac{1}{B}\sum_a\frac{\beta_a}{4}\nabla_\perp^2(\frac{\nabla\psi}{i\omega})_\parallel \boldsymbol{b}$$

reduced MHD as limit; contains formally ‘all' electrostatic and electromagnetic instabilities

- quasi neutrality (QN):

$$0 = \sum_a e_a \int \mathrm{d}^2 v\, \{J_0 f\}_a + \nabla_\perp \cdot \frac{m_i n_i \nabla_\perp\phi}{B^2}$$

- non-adiabatic response for perturbed distribution function:

propagator →resonance

$$\hat{h} = ie\sum_m \int_{-\infty}^{t} dt' e^{i[n(\varphi'-\varphi)-m(\theta'-\theta)-\omega(t'-t)]} e^{-im\theta}$$

$$\frac{\partial F_0}{\partial E}[\omega - \hat{\omega}_*]\, J_0^2(k_\perp\varrho_i)\left[\phi_m(r') - (1-\frac{\omega_d(r',\theta')}{\omega})\psi_m(r')\right]$$

free energy

resonances (circ/trapped):

ω_{AE}- ω_{prec} - (nq-m+k) · ω_t = 0

ω_{AE}- ω_{prec} - k · ω_b = 0

for all species , including electrons and energetic particles

reduced MHD limit

GKE: neglect non-adiabatic part: h=0

QN: neglect polarisation, FLR: Φ=Ψ

GKM:

$$-\frac{\omega^2}{\omega_{A0}^2}\nabla_\perp \frac{\hat{n}B_0^2}{\mathbf{B}^2}\nabla_\perp\psi \;+\; \nabla(\nabla_\parallel\psi)\times\mathbf{b}\cdot\nabla(\frac{\nabla\times\mathbf{B}_0}{B}) + (\mathbf{B}\cdot\nabla)\frac{(\nabla\times\nabla\times\nabla_\parallel\psi)\cdot\mathbf{B}}{B^2} +$$

$$+\; \boxed{\mu_0 P_0 \frac{\mathbf{b}}{B}\times\left[(\mathbf{b}\cdot\nabla)\mathbf{b}+\frac{\nabla B}{B}\right]\cdot\nabla\left[\frac{\nabla\hat{P}}{B}(\mathbf{b}\times\nabla)\psi\right] = 0}$$

$$\mu_0\nabla P_1\cdot\nabla\times\frac{\mathbf{B}}{B^2}$$

with P_1= $\frac{\nabla P}{i\omega B}(\mathbf{b}\times\nabla)\psi$

the e.m. kinetic dispersion relation in Tokamak geometry part I: QN

QN: write equations for neighbouring poloidal harmonics, since coupling due to curvature drifts arises:

$$\mathbf{v}_d \cdot \nabla\phi/i = \Big[-\hat{\omega}^r_d \sin\theta/i \frac{\partial}{\partial r} + \omega^n_d \cos\theta + \omega_{prec} \Big]\phi \qquad \phi = \sum_m \phi_m(r) e^{-i\omega t - im\theta + in\varphi}$$

geodesic curvature $\omega^r_d = (\mathbf{v}_d \cdot \nabla)_r \approx \sin(\theta) \frac{v^2_{thi}}{\Omega_{ci} R_0} \frac{\partial}{\partial r}$ (circular geometry)

$$\sum_{m'=m-1}^{m+1} \delta_{m',p} \boxed{D^m}(x_{m'})(\phi_{m'} - \psi_{m'}) =$$

contains electrostatic waves(sound, drift)

polarisation terms

$$\begin{pmatrix} \boxed{P_{m-1}} & \tau N^m(x_{m-1})\omega^+_{di}/\omega & 0 \\ \tau N^{m-1}(x_m)\omega^-_{di}/\omega & \boxed{P_m} & \tau N^{m+1}(x_m)\omega^+_{di}/\omega \\ 0 & \tau N^m(x_{m+1})\omega^-_{di}/\omega & \boxed{P_{m+1}} \end{pmatrix} \begin{pmatrix} \psi_{m-1} \\ \psi_m \\ \psi_{m+1} \end{pmatrix}$$

off-diagonal elements (sidebands)

definitions

with

$$\tilde{D}^m(x) = (1 - \frac{\omega_*^m}{\omega})xZ(x) - \frac{\omega_*^m}{\omega}\eta\Big(x^2 + xZ(x)(x^2 - \frac{1}{2})\Big)$$

$$2\tilde{N}^m(x) = (1 - \frac{\omega_*^m}{\omega})\Big[x^2 + xZ(x)(x^2 + \frac{1}{2})\Big] - \frac{\omega_*^m}{\omega}\eta\Big[x^2(x^2 + \frac{1}{2}) + xZ(x)(\frac{1}{4} + x^4)\Big]$$

$$P = \tau(\Gamma_0 - 1)\Big[1 - \frac{\omega_i^*}{\omega}\Big(1 + \eta_i\frac{\Gamma_0 G_0}{\Gamma_0 - 1}\Big)\Big].$$

$$H^m(x_m) = \tilde{H}^m(x_{m,i}) + \tau\tilde{H}^m(x_{m,e}) \text{ and } \tilde{H}^m(x) = \frac{1}{2}\Big[(1 - \frac{\omega_*^m}{\omega})\tilde{F}(x) - \eta\frac{\omega_*^m}{\omega}\tilde{G}(x)\Big],$$

$$2\tilde{F}(x) = xZ(x)(\frac{1}{2} + x^2 + x^4) + \frac{3x^2}{2} + x^4,$$

$$\omega_d^{\pm} \approx \frac{v_{th,i}^2}{\Omega_i}\frac{1}{R_0}(\frac{m}{r} \pm \frac{\partial}{\partial r}) = \omega_d^n \pm \omega_d^r$$

$$2\tilde{G}(x) = xZ(x)(\frac{3}{4} + x^2 + \frac{x^4}{2} + x^6) + 2x^2 + x^4 + x^6$$

Assuming a Maxwellian F_0 with $\partial F_0/\partial E = -F_0/T$ and using

$$\int_0^\infty \frac{dt\, e^{-t^2}}{x_m^2 - t^2} = \frac{-\sqrt{\pi}Z(x_m)}{2x_m}; \qquad \int_0^\infty \frac{dt\, t^2\, e^{-t^2}}{x_m^2 - t^2} = \frac{-\sqrt{\pi}}{2}(x_m + x_m^2 Z(x_m))$$

where

$$x_m = \frac{\omega}{|k_{\parallel,m}|v_{th}}; \qquad t = \frac{v_\parallel}{v_{th}}; \qquad v_{th} = \sqrt{\frac{2T}{m}}$$

τ=Te/Ti

presently being extended to other distributions

the e.m. kinetic dispersion relation in Tokamak geometry part II: GKM

$$-\quad \omega^2\nabla_\perp\frac{1}{v_A^2}\nabla_\perp\phi+\left[\nabla(\nabla\psi)_\parallel\times\mathbf{b}\right]\cdot\nabla(\frac{\mu_0 j_{0\parallel}}{B})+(\mathbf{B}\cdot\nabla)\frac{(\nabla\times\nabla\times(\nabla\psi)_\parallel)\cdot\mathbf{B}}{B^2}$$

$$=\quad -(i\omega)^2\mu_0\sum_a e_a\int\frac{\mathbf{v}_{d,a}\cdot\nabla}{i\omega}J_0 f_a d^3\mathbf{v}$$

(current equation)

$$\frac{\tau|e|^2 n_e}{\omega^2 T_e}\delta_{m,p}\Big[\Big((\omega_d^n)^2-(\omega_d^r)^2\Big)H^m(x_{m-1})+\Big((\omega_d^n)^2-(\omega_d^r)^2\Big)H^m(x_{m+1})\Big]\psi_m$$

$$+\quad\frac{|e|^2 n_e}{\omega T_e}\begin{pmatrix}0 & \tau N^m(x_m)\omega_{di}^- & 0\\ \tau N^{m-1}(x_{m-1})\omega_{di}^+ & 0 & \tau N^{m+1}(x_{m+1})\omega_{di}^-\\ 0 & \tau N^m(x_m)\omega_{di}^+ & 0\end{pmatrix}\begin{pmatrix}(\phi-\psi)_{m-1}\\ (\phi-\psi)_m\\ (\phi-\psi)_{m+1}\end{pmatrix}$$

combine with QN (Φ-ψ) ⇒ dispersion relation (no fast ions):

$$\Sigma_m\ \omega^2\Big(1-\frac{\omega_{*p}}{\omega}\Big)-k_\parallel^2\omega_A^2R_0^2=2\frac{v_{thi}^2}{R_0^2}\Big(-\Big[H(x_{m-1})+H(x_{m+1})\Big]+\tau\Big[\frac{N^m(x_{m-1})N^{m-1}(x_{m-1})}{D^{m-1}(x_{m-1})}+\frac{N^m(x_{m+1})N^{m+1}(x_{m+1})}{D^{m+1}(x_{m+1})}\Big]\Big)$$

(LIGKA MODEL 3/4):

ballooning formulation [Zonca PPCF 1996,2009, Gotit lectures, Garbet 2006], [Lauber PPCF 2009]
extension trapped particles [I. Chavdarovski et al, 2014…]

the e.m. kinetic dispersion relation in Tokamak geometry part

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$$\sum_m \omega^2\left(1-\frac{\omega_{*p}}{\omega}\right) - k_\parallel^2\omega_A^2 R_0^2 = 2\frac{v_{thi}^2}{R_0^2}\Bigg(-\Big[H(x_{m-1}) + H(x_{m+1})\Big] + \tau\Big[\frac{N^m(x_{m-1})N^{m-1}(x_{m-1})}{D^{m-1}(x_{m-1})} + \frac{N^m(x_{m+1})N^{m+1}(x_{m+1})}{D^{m+1}(x_{m+1})}\Big]\Bigg)$$

[Zonca PPCF 1996,2009, Gotit lectures, Garbet 2006], [Lauber PPCF 2009]

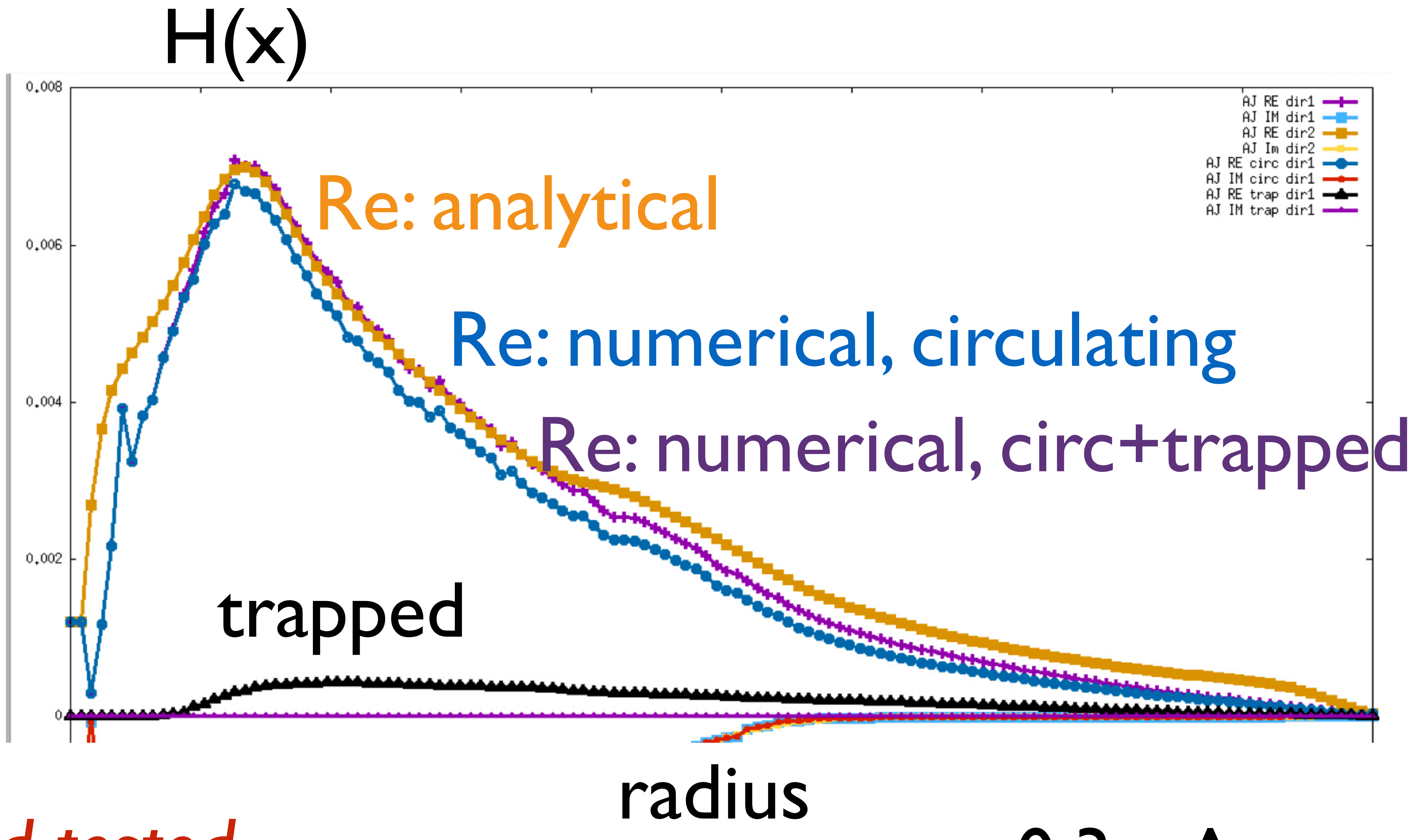
circulating ion appoximation; extension trapped particles [I. Chavdarovski et al, 2014…]

$$h = -e_a \sum_m \sum_k \underbrace{\frac{\frac{\partial F_0}{\partial E}(\omega - \hat{\omega}_*)e^{-im\theta}J_0}{(\omega - \omega_D^0 - H\sigma S_m^0\omega_t - k\omega_{b,t})}}_{=\mathcal{R}_{m,k}}\left[a_{km}\phi_m(r) - \left(a_{km} - \frac{a_{km}^G\bar{\omega}_d(r)}{\omega}\right)\psi_m(r)\right]$$

numerical results are available (LIGKA):

for TAE-range modes often good approximation - deviations for low-f regime!

orbit-based-kinetic version ready - to be packaged and tested



$\omega = 0.3\omega A$

the e.m. kinetic dispersion relation in Tokamak geometry part II: GKM

$$-\quad \omega^2 \nabla_\perp \frac{1}{v_A^2} \nabla_\perp \phi + \left[\nabla(\nabla\psi)_\parallel \times \mathbf{b}\right] \cdot \nabla\left(\frac{\mu_0 j_{0\parallel}}{B}\right) + (\mathbf{B}\cdot\nabla)\frac{(\nabla\times\nabla\times(\nabla\psi)_\parallel)\cdot\mathbf{B}}{B^2}$$

$$= -(i\omega)^2 \mu_0 \sum_a e_a \int \frac{\mathbf{v}_{d,a}\cdot\nabla}{i\omega} J_0 f_a d^3\mathbf{v}$$

(current equation)

$$\frac{\tau |e|^2 n_e}{\omega^2 T_e} \delta_{m,p} \left[\left((\omega_d^n)^2 - (\omega_d^r)^2\right) H^m(x_{m-1}) + \left((\omega_d^n)^2 - (\omega_d^r)^2\right) H^m(x_{m+1})\right]\psi_m$$

$$+ \quad \frac{|e|^2 n_e}{\omega T_e} \begin{pmatrix} 0 & \tau N^m(x_m)\omega_{di}^- & 0 \\ \tau N^{m-1}(x_{m-1})\omega_{di}^+ & 0 & \tau N^{m+1}(x_{m+1})\omega_{di}^- \\ 0 & \tau N^m(x_m)\omega_{di}^+ & 0 \end{pmatrix} \begin{pmatrix} (\phi-\psi)_{m-1} \\ (\phi-\psi)_m \\ (\phi-\psi)_{m+1} \end{pmatrix}$$

combine with QN (Φ-ψ) ⇒ dispersion relation (no fast ions):

$$\Sigma_m\ \omega^2\left(1 - \frac{\omega_{*p}}{\omega}\right) - k_\parallel^2 \omega_A^2 R_0^2 = 2\frac{v_{thi}^2}{R_0^2}\Big(\quad 7/8 \quad)\Big] +$$

$$\tau\Big| \quad 2 \quad \Big)$$

ballooning formulation [Zonca PPCF 1996,2009, Garbet 2006], [Lauber PPCF 2009]

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simplifications:

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1. The fast-circulating approximation is used, trapped particles are not included - may alter the plasma response in the low-frequency domain, $\omega \sim \omega_{ti} \sim \omega_{*pi}$ 0.1 $\cdot\omega_{A0}$
2. Isotropic Maxwellian distributions are used for all species. A generalisation is being implemented.
3. Finite-orbit-width (FOW) effects are not included in local models 3 and 4. Model 9 will include them - presently in packaging stage.
4. Due to the fast-circulating particle approximation geometric coupling in the kinetic ion and electron response is included only up to first order (geodesic). Thus, all modes with poloidal mode number m couple only with neighbouring harmonics m ± 1. This means the kinetic coupling terms for EAEs are not consistent, meaning the EAE damping with the reduced model is not fully consistent (need to use 2nd order coupling since EAEs consist of m,m+2)
5. Electron Landau damping is underestimated (effectively absent) in regions away from rational surfaces because only trapped electrons can satisfy $k_\parallel v_\parallel \sim 1$ for finite $k_\parallel$, but trapped particle motion is not included.
6. The leading-order terms of the compressional magnetic response $\delta B_\parallel$ are known to cancel exactly with a correction of the magnetic drift. Apart from this important self-consistent cancellation effects of $\delta B_\parallel$ are ignored.

simplifications:

1. The fast-circulating approximation is used, trapped particles are not included - may alter the plasma response in the low-frequency domain, $\omega \sim \omega_{ti} \sim \omega_{*pi}$ 0.1 $\cdot\omega_{A0}$
2. Isotropic Maxwellian distributions are used for all species. A generalisation is b
3. Finite-orbit-width (FOW) effects are not included in model 3 and model 4. M presently in packaging stage.
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5. Electron Landau damping is underestimated (effectively absent) in regions awa only trapped electrons can satisfy $k_\parallel v_\parallel \sim 1$ for finite $k_\parallel$, but trapped particle m
6. The leading-order terms of the compressional magnetic response $\delta B_\parallel$ are kno high-beta correction of the magnetic drift. Apart from this important self-cons are ignored.

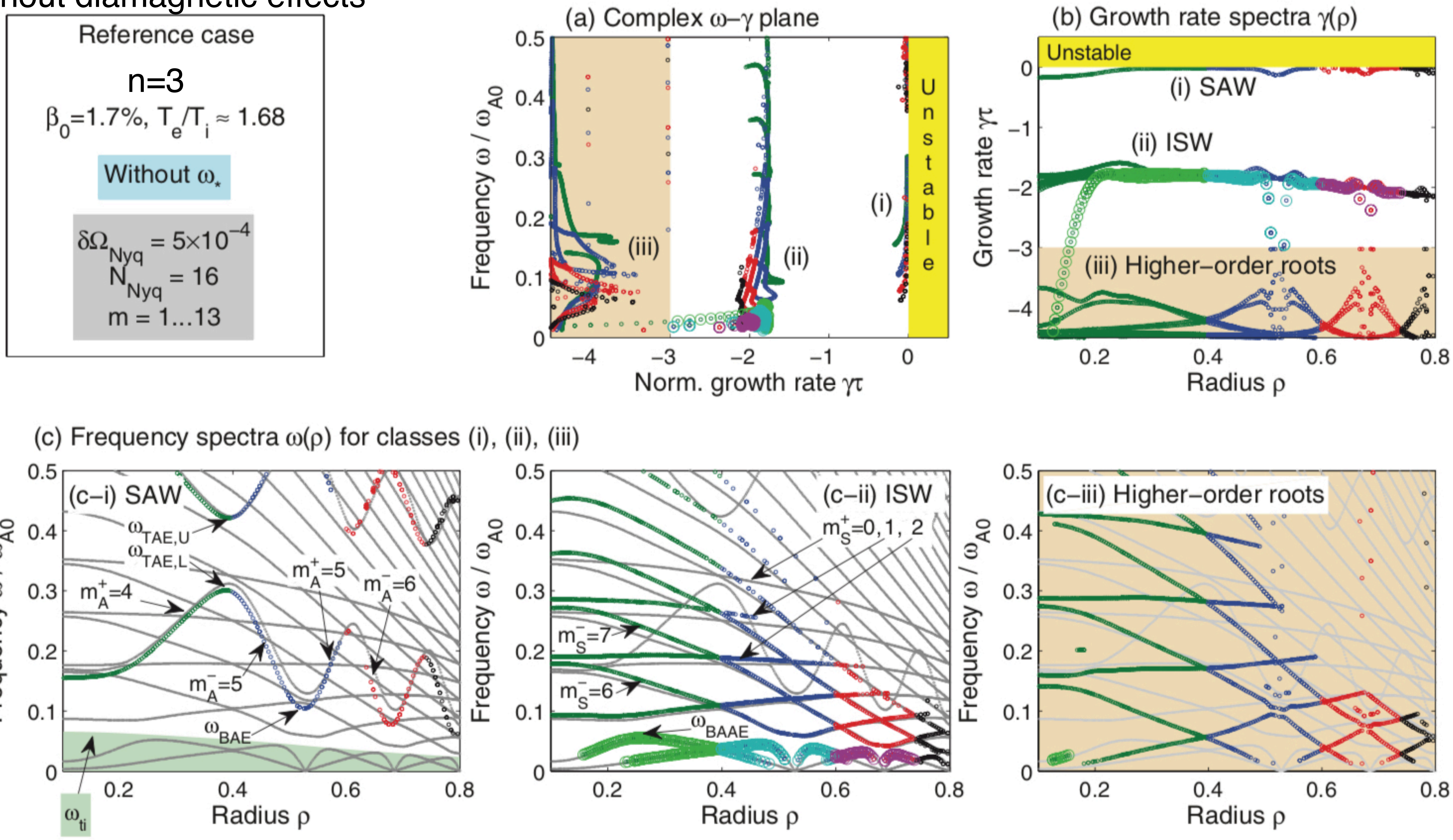
features are available in fully numerical LIGKA; not yet packaged; WF 2.0 to arrive soon

but: will be (much) more expensive - for overview and transport studies, present version is very useful

application example from [A. Bierwage NF 57 2017] (LIGKA MODEL 3/4):

without diamagnetic effects

JT-60U



application example from [Bierwage 2017]

(LIGKA MODEL 3/4):

with diamagnetic effects

JT-60U

Reference case

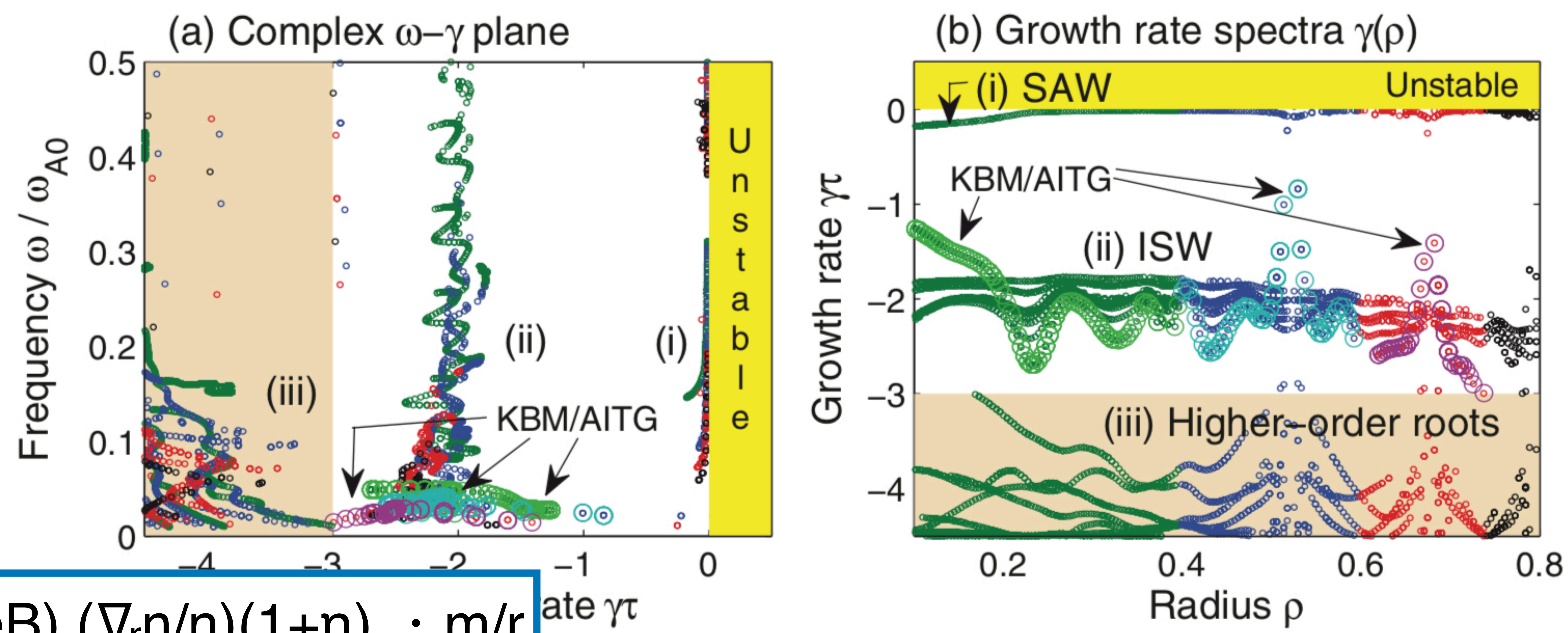
n=3

β_0=1.7%, $T_e/T_i \approx 1.68$

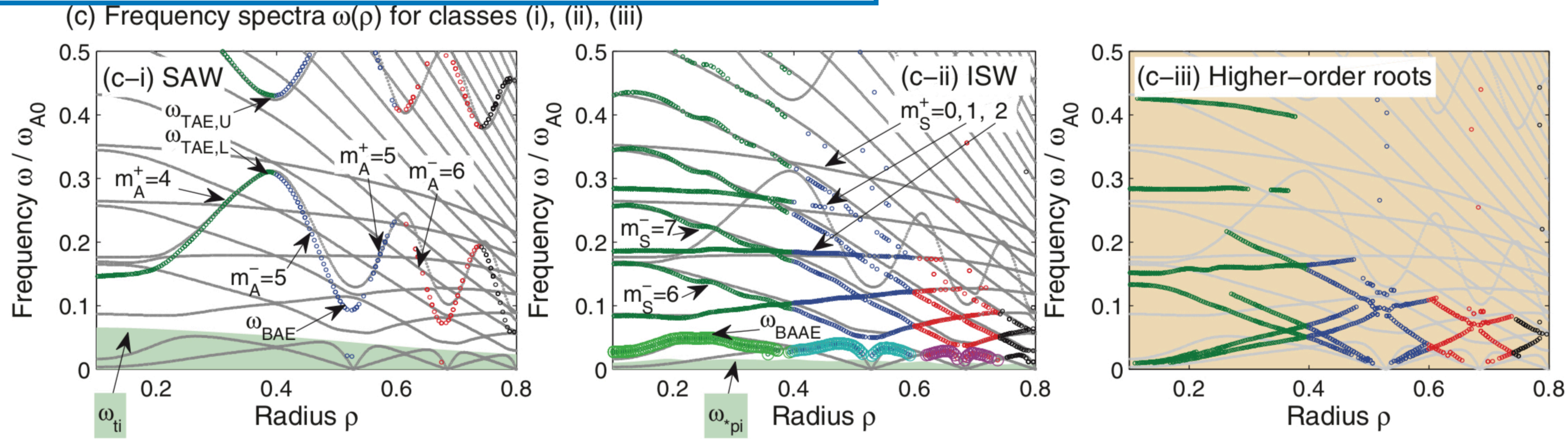
$\delta\Omega_{Nyq} = 5\times10^{-4}$

$N_{Nyq} = 16$

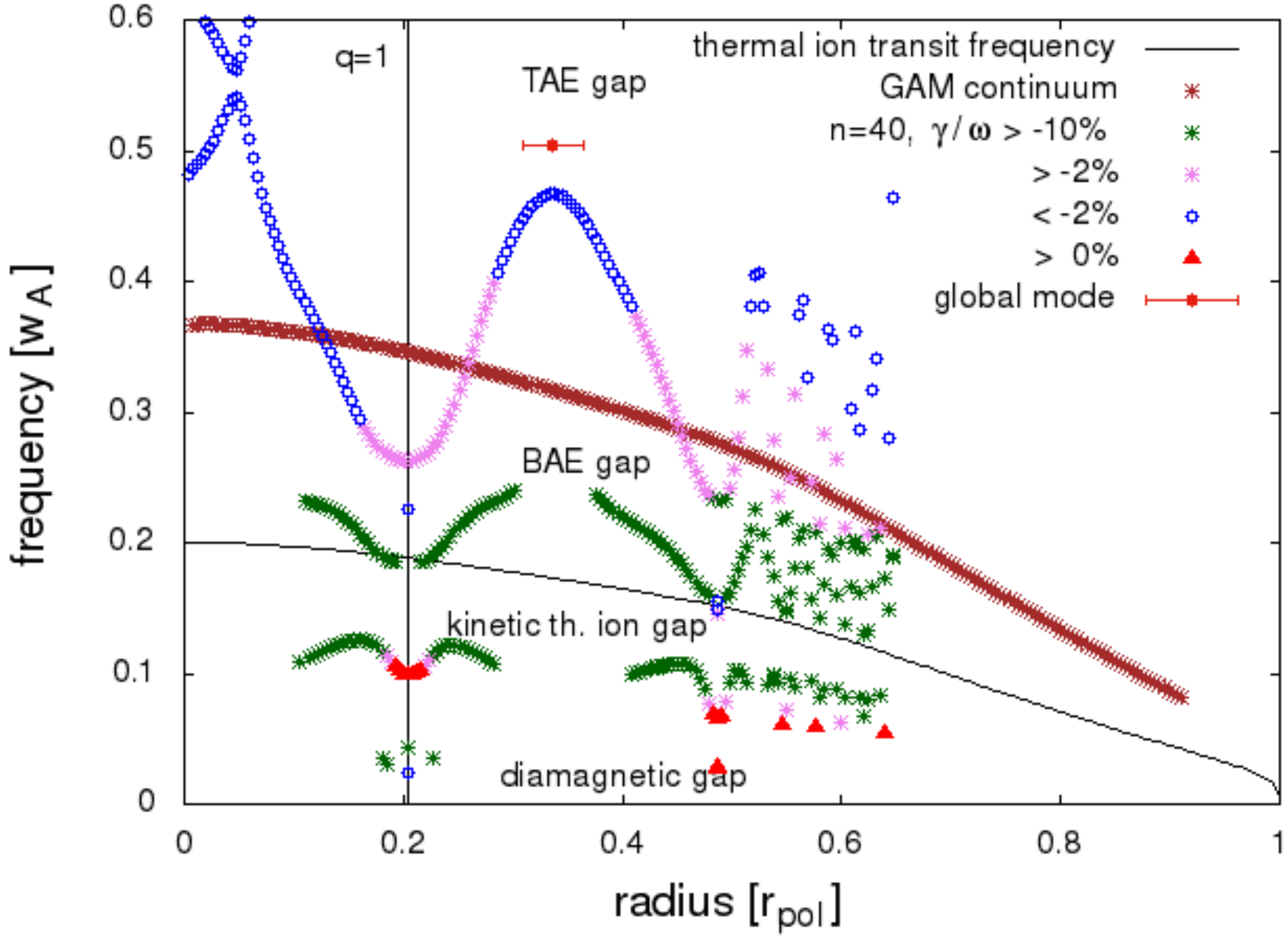
m = 1...13



$$\omega^* = T/(ieB)\ \mathbf{b}\mathrm{x}\boldsymbol{\nabla}\mathbf{n}/n(1+\eta)\ \cdot \boldsymbol{\nabla} \approx -T/(ieB)\ (\nabla_r n/n)(1+\eta)\ \cdot m/r$$



local dispersion relation: ITER 15 MA [details ITPA Seville, 2017]



- use Nyquist contour integrals in complex plane to determine roots of dispersion relation **(LIGKA MODE 3/4)**:
- physics: good estimate of ion LD damping, simplified electron LD
- use values at gaps to estimate f, gamma of mode belonging to this gap,
- runtime: seconds to minutes

local dispersion relation: ITER 15 MA [details ITPA Seville, 2017]

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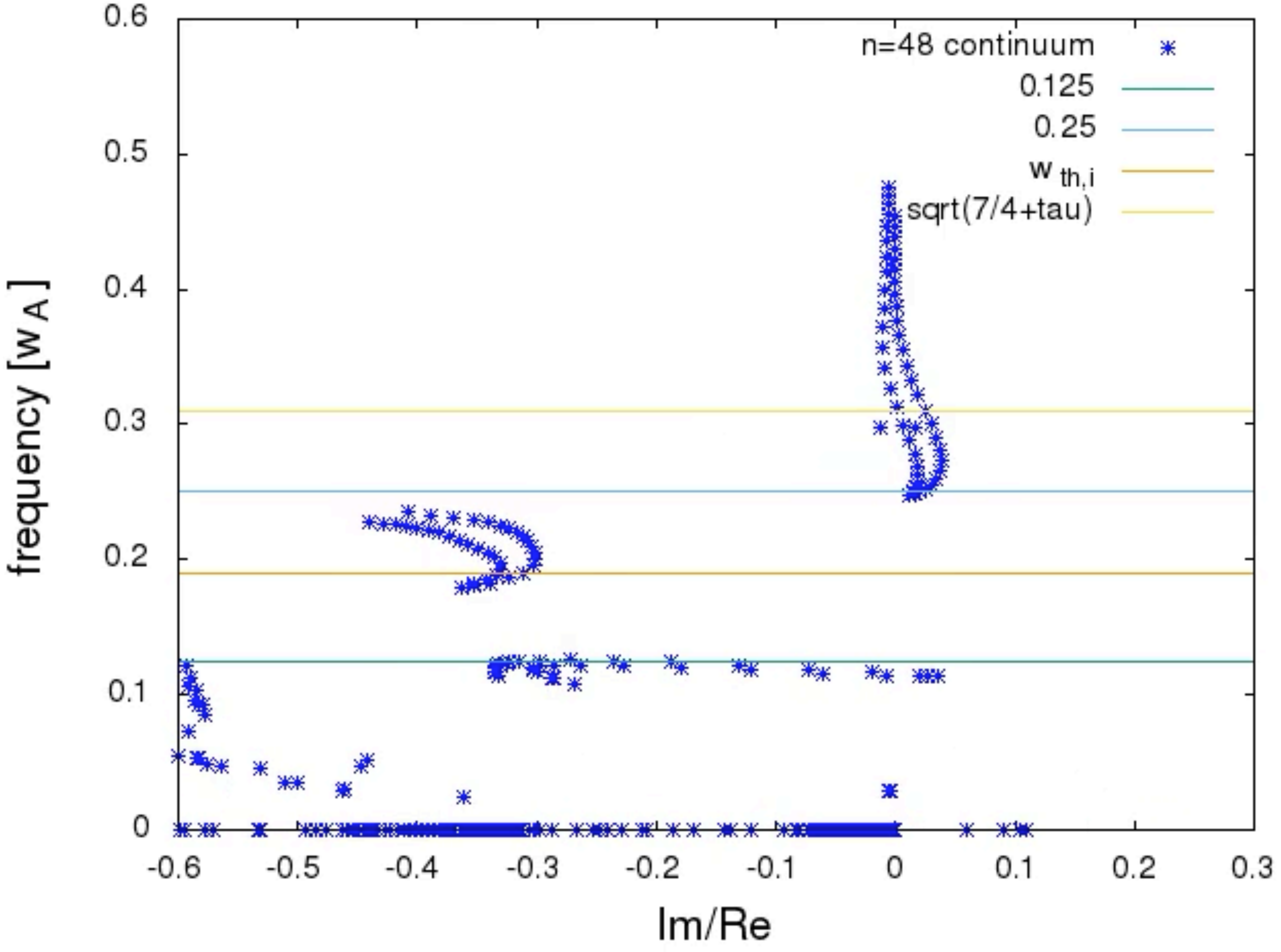
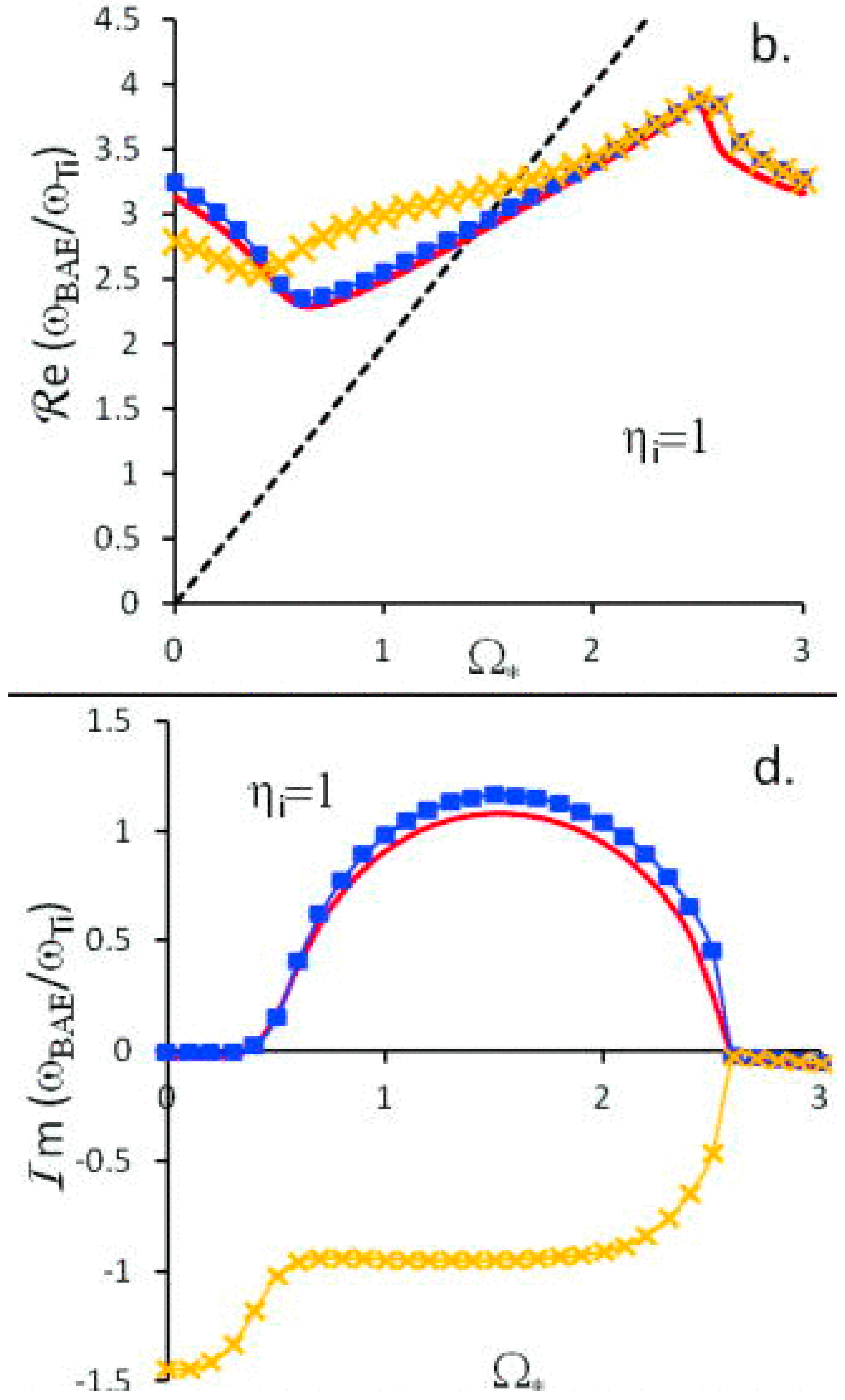
(LIGKA MODEL 3/4):





[Chavdarovski 2014]

$\omega^* = -T/(ieB)\ (\nabla_r n/n)(1+\eta)\ \cdot m/r$

shows qualitative beta stabilisation,AITG/KBM unstable for n>~40, see also ORB5 results [T Hayward-Schneider, 2022
but: global effects, trapped electrons etc need to be considered [e.g.G. Falchetto, PoP 2003]

connection to the generalised fishbone dispersion relation [Chen, Zonca,1984…]

$$\Sigma_m\, \omega^2\left(1-\frac{\omega_{*p}}{\omega}\right) - k_\parallel^2\omega_A^2 R_0^2 = 2\frac{v_{thi}^2}{R_0^2}\Bigg(-\Big[H(x_{m-1})+H(x_{m+1})\Big] + \tau\Big[\frac{N^m(x_{m-1})N^{m-1}(x_{m-1})}{D^{m-1}(x_{m-1})} + \frac{N^m(x_{m+1})N^{m+1}(x_{m+1})}{D^{m+1}(x_{m+1})}\Big]\Bigg)$$

Λ^2

$$-i\Lambda + \delta W_{core} + \delta W_{hot} = 0,$$

$$\delta W_{\mathrm{hot}} \sim \int dE\,d\mu\,dP_\varphi\,d\theta\,d\varphi \sum_{k=-\infty}^{\infty} \frac{\partial F}{\partial E}\frac{(\omega-\bar{\omega}_*)|\mathcal{L}_k|^2}{\omega-\omega_{prec}-(nq-k)\omega_{t,b}}$$

$$\delta \hat{W}_{\mathrm{core}} = 3\pi\Delta q_0\left(13/144-\beta_{ps}^2\right)\left(r_s^2/R_0^2\right)$$

with $\beta_{ps} = -(R_0/r_s^2)^2\int_0^{r_s} r^2(d\beta/dr)dr$, $\Delta q_0 = 1-q(r=0)$ and $\beta = 8\pi P/B_0^2$

the fishbone dispersion relation

[Chen, 1984]

$$-i\Lambda + \delta W_{core} + \delta W_{hot} = 0,$$

Re[Λ^2] <0 : gap modes
Re[Λ^2] >0 : EP modes in continuum

the combined effect of δWcore and Re[δWhot] is to 'move' the mode away from the local continuum solution and determines if the mode can exist -> 'Alfven zoo'

for EPMs, the mode frequency is set by the EPs
the drive has to overcome continuum damping i.e.
Im(δWhot)> Re(Λ)

theory for linear onset well developed [Zonca PoP, 2005, R.-R. Ma, 2019-2023]

Finite orbit width dispersion relation for LIGKA

$$\omega^2\left(1-\frac{\omega_{*p}}{\omega}\right)-k_\parallel^2\omega_A^2R_0^2=2\frac{v_{thi}^2}{R_0^2}\Bigg(-\Big[H(x_{m-1})+H(x_{m+1})\Big]+$$

$$\tau\left[\frac{N^m(x_{m-1})N^{m-1}(x_{m-1})}{D^{m-1}(x_{m-1})}+\frac{N^m(x_{m+1})N^{m+1}(x_{m+1})}{D^{m+1}(x_{m+1})}\right]\Bigg)$$

no FOW,
circulating particle
approximation

[Zonca 1996,2009 Lauber 2009]

$$\left(\frac{N_{-1,0}N^G{}_{0,-1}}{D_{-1,-1}}+\frac{N_{1,0}N^G{}_{0,1}}{D_{1,1}}\right)+$$

$$\rho^2\Bigg[D_{0,0}\Big[D_{-2,-2}D_{1,1}D_{-1,-1}\Big(D_{2,2}\Big(D_{-1,-1}\left(Q_{1,0}N^G{}_{0,1}+N_{1,0}Q^G{}_{0,1}\right)-$$

$$F_{-1,1}\left(N_{1,0}N^G{}_{0,-1}+N_{-1,0}N^G{}_{0,1}\right)\Big)$$

$$-D_{-1,-1}\left(E_{1,2}P_{2,0}N^G{}_{0,1}+E_{2,1}N_{1,0}P^G{}_{0,2}\right)\Big)+$$

$$D_{1,1}^2\Big(D_{2,2}\Big(E_{-2,-1}N_{-1,0}\left(E_{-1,-2}N^G{}_{0,-1}-D_{-1,-1}P^G{}_{0,-2}\right)+$$

$$D_{-1,-1}P_{-2,0}\left(D_{-1,-1}P^G{}_{0,-2}-E_{-1,-2}N^G{}_{0,-1}\right)+$$

$$D_{-2,-2}\Big(D_{-1,-1}\left(Q_{-1,0}N^G{}_{0,-1}+N_{-1,0}Q^G{}_{0,-1}\right)-F_{-1,-1}N_{-1,0}N^G{}_{0,-1}\Big)\Big)+$$

$$D_{-2,-2}D^2{}_{-1,-1}P_{2,0}P^G{}_{0,2}\Big)+D_{-2,-2}D^2{}_{-1,-1}N_{1,0}N^G{}_{0,1}\left(E_{1,2}E_{2,1}-D_{2,2}F_{1,1}\right)\Big]$$

$$+D_{-2,-2}D_{2,2}\Big(D_{1,1}\left(E_{0,-1}N_{-1,0}-D_{-1,-1}P_{0,0}\right)+$$

$$D_{-1,-1}E_{0,1}N_{1,0}\Big)\Big(D_{1,1}\left(E_{-1,0}N^G{}_{0,-1}-D_{-1,-1}P^G{}_{0,0}\right)+D_{-1,-1}E_{1,0}N^G{}_{0,1}\Big)\Bigg]/$$

$$\left(D_{-2,-2}D^2{}_{-1,-1}D_{0,0}D_{1,1}^2D_{2,2}\right)$$

2nd order FOW

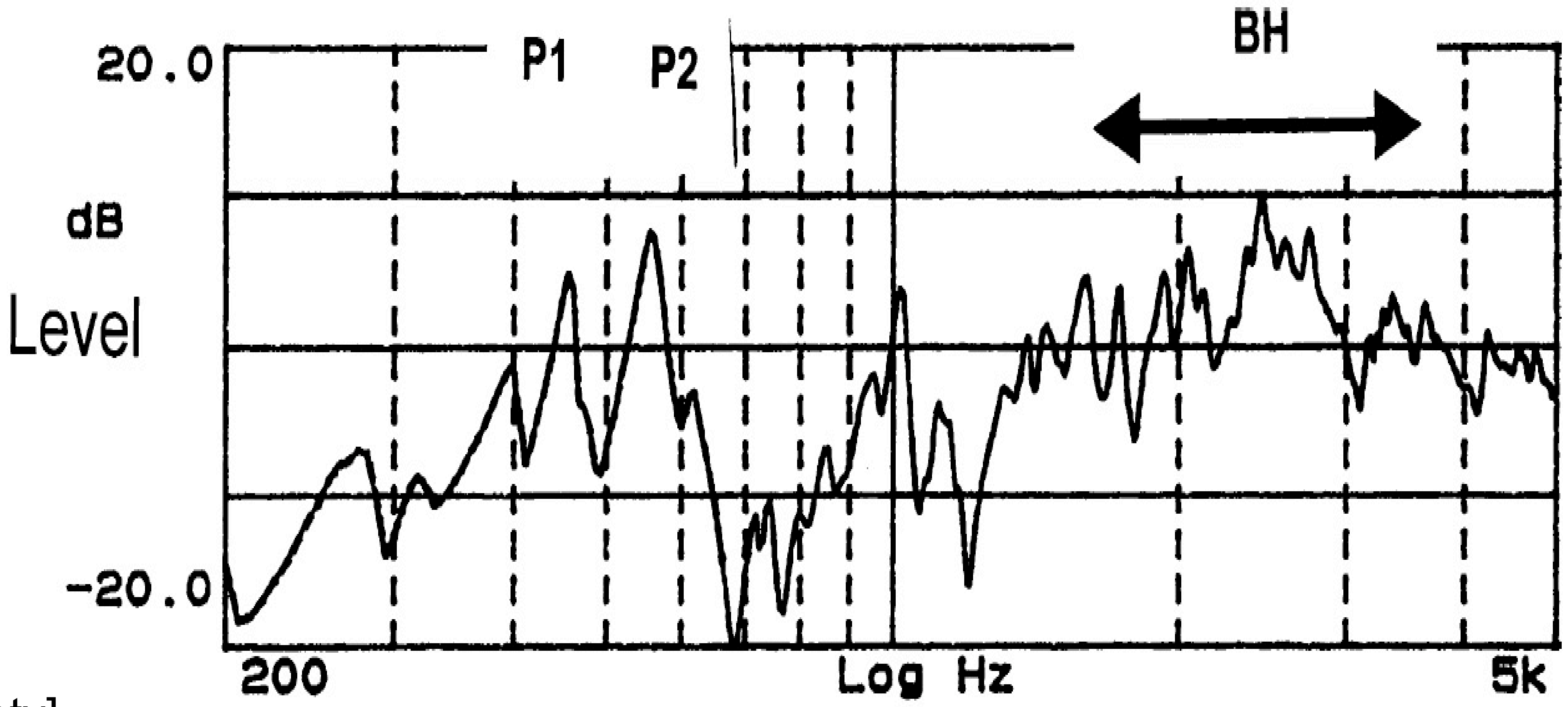
[Zonca 1998, Z.X. Lu 2017,Lauber JPC 2018]

- equivalent to EGAM FOW equations: Qiu [2009], Miki & Idomura [2015]
- fast analytical model for FOW effects: solve equations both locally (scan k_r) and globally
- LIGKA model 9 (specification of kr needed)
- rationale: implement global effects in local model - can be improved by estimating analytically AE mode structures (ongoing…)

global solver - antenna response model

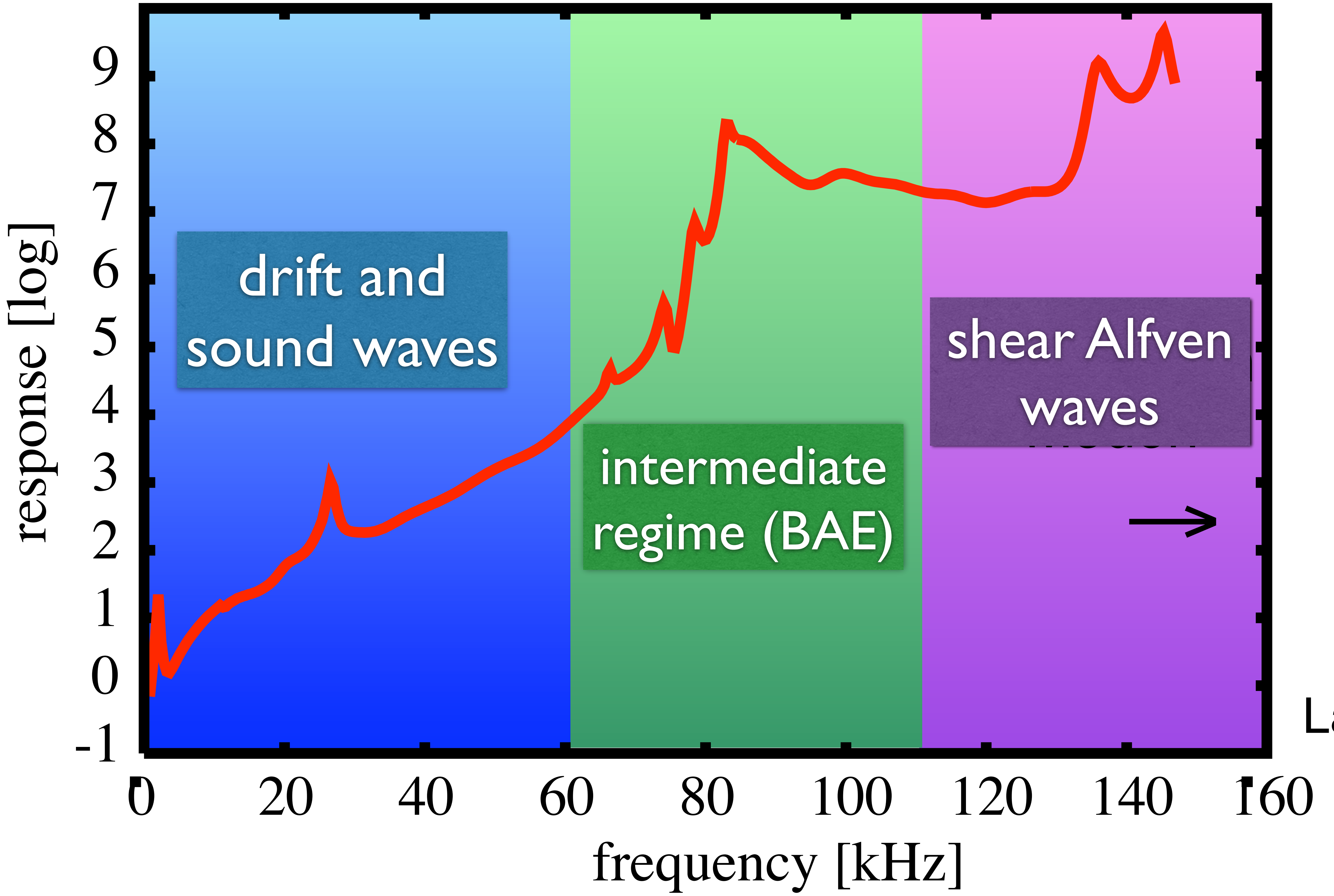
Stradivari frequency response [Jansons,2004]

[Stradivari Society]



global solver - antenna model

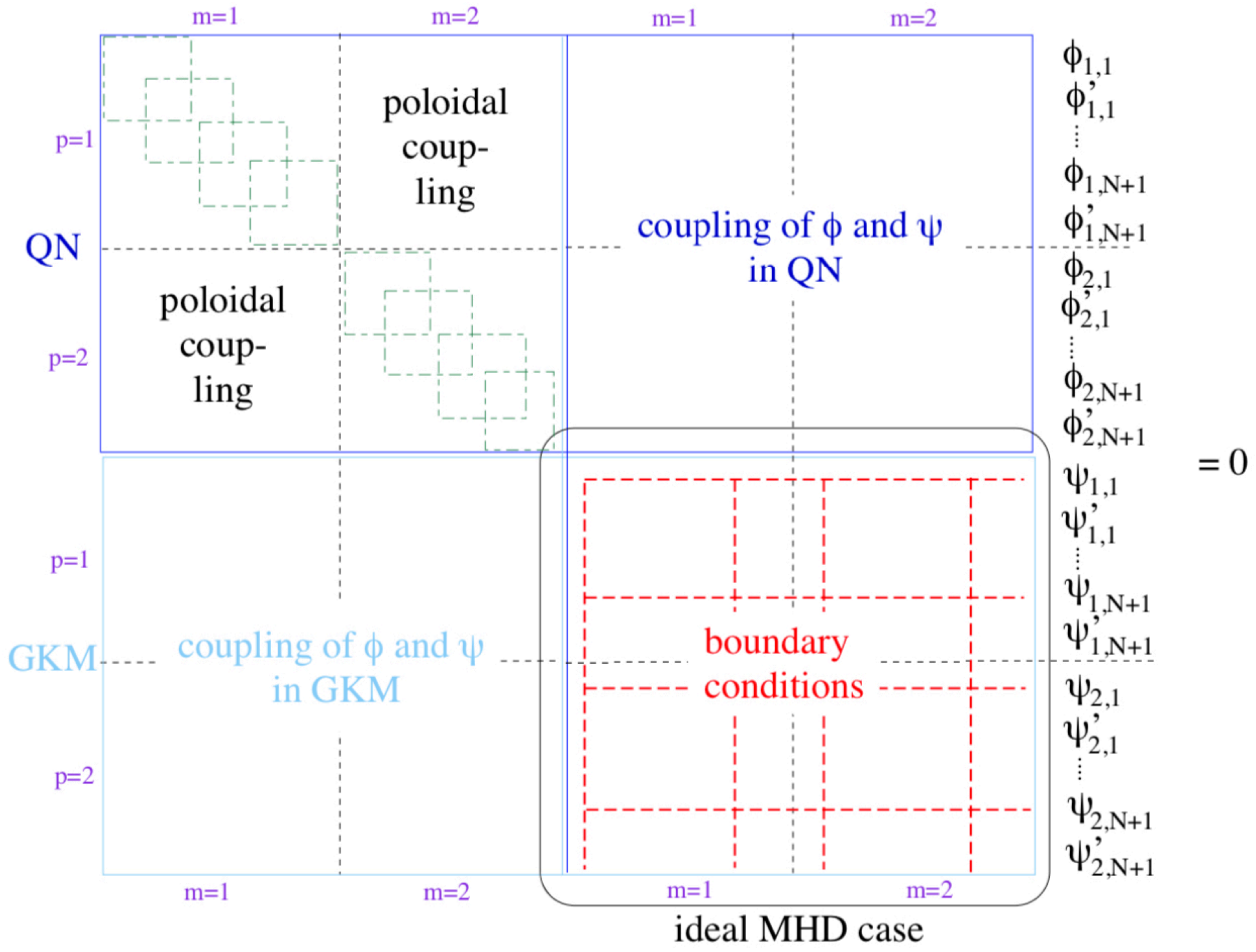
frequency response of ASDEX Upgrade (using linear GK model)



Lauber ~ 2008

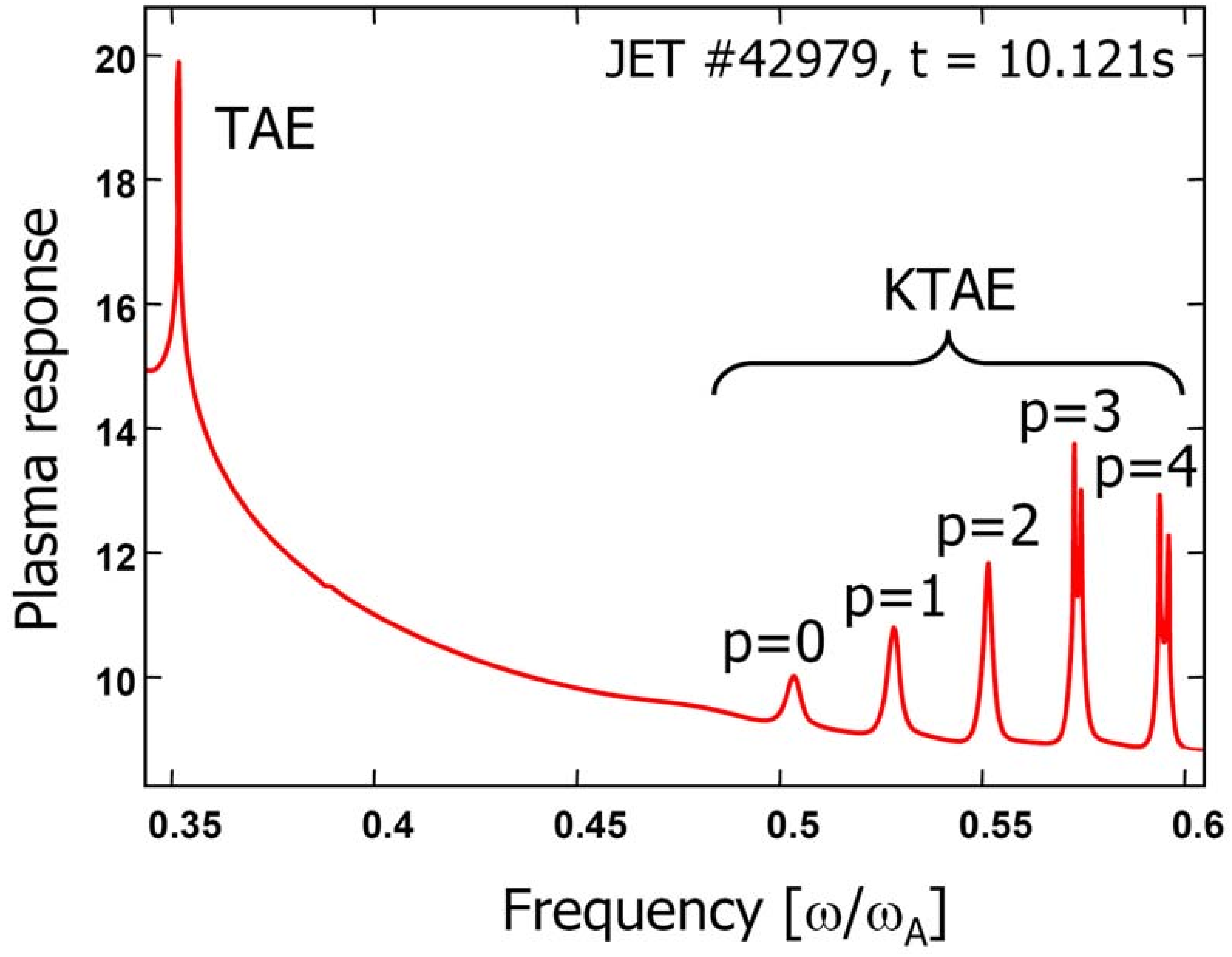
numerics

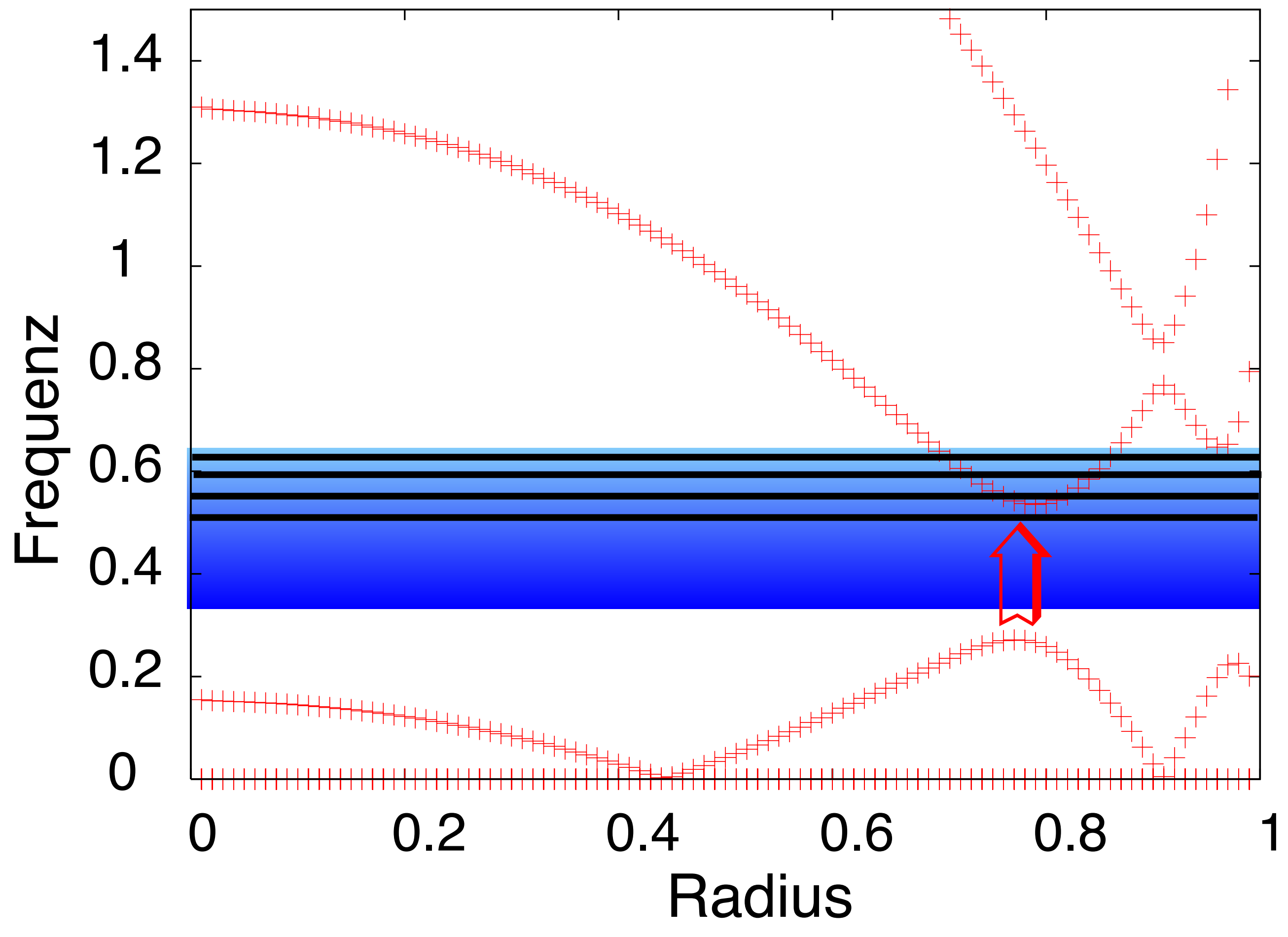
- Fourier in n and m, couple m
- finite cubic Hermite polynomials
- antenna solver
- inverse vector iteration (available for up to 7 pol harms.)
- using the same infrastructure for all models;
- wrapper for IMAS selects, and fills relevant settings automatically



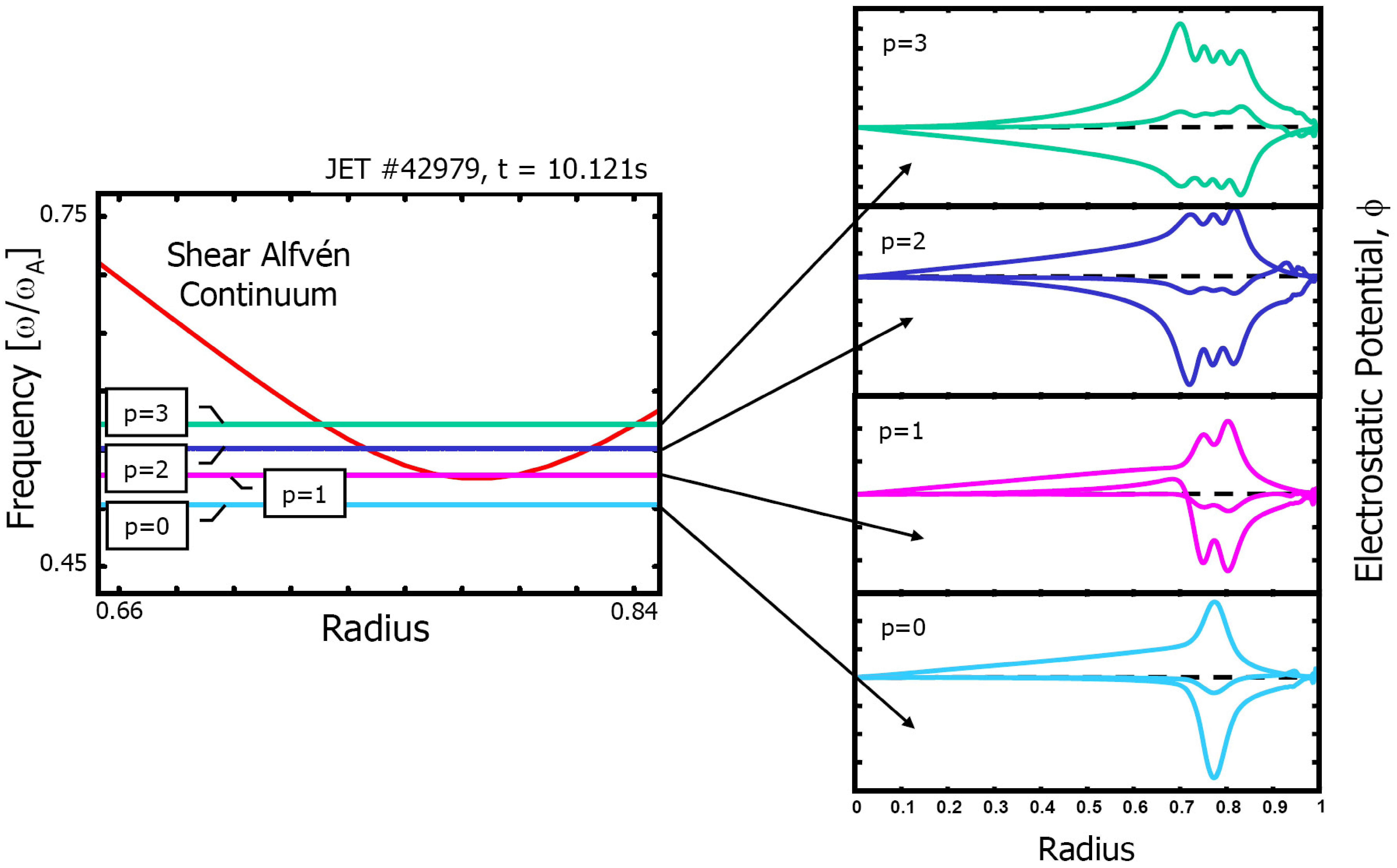
Scan through the gap region

in order to find all the modes in and around the gap: drive perturbation at boundary, or at mode location, sweep frequency and measure plasma response





Kinetic TAEs

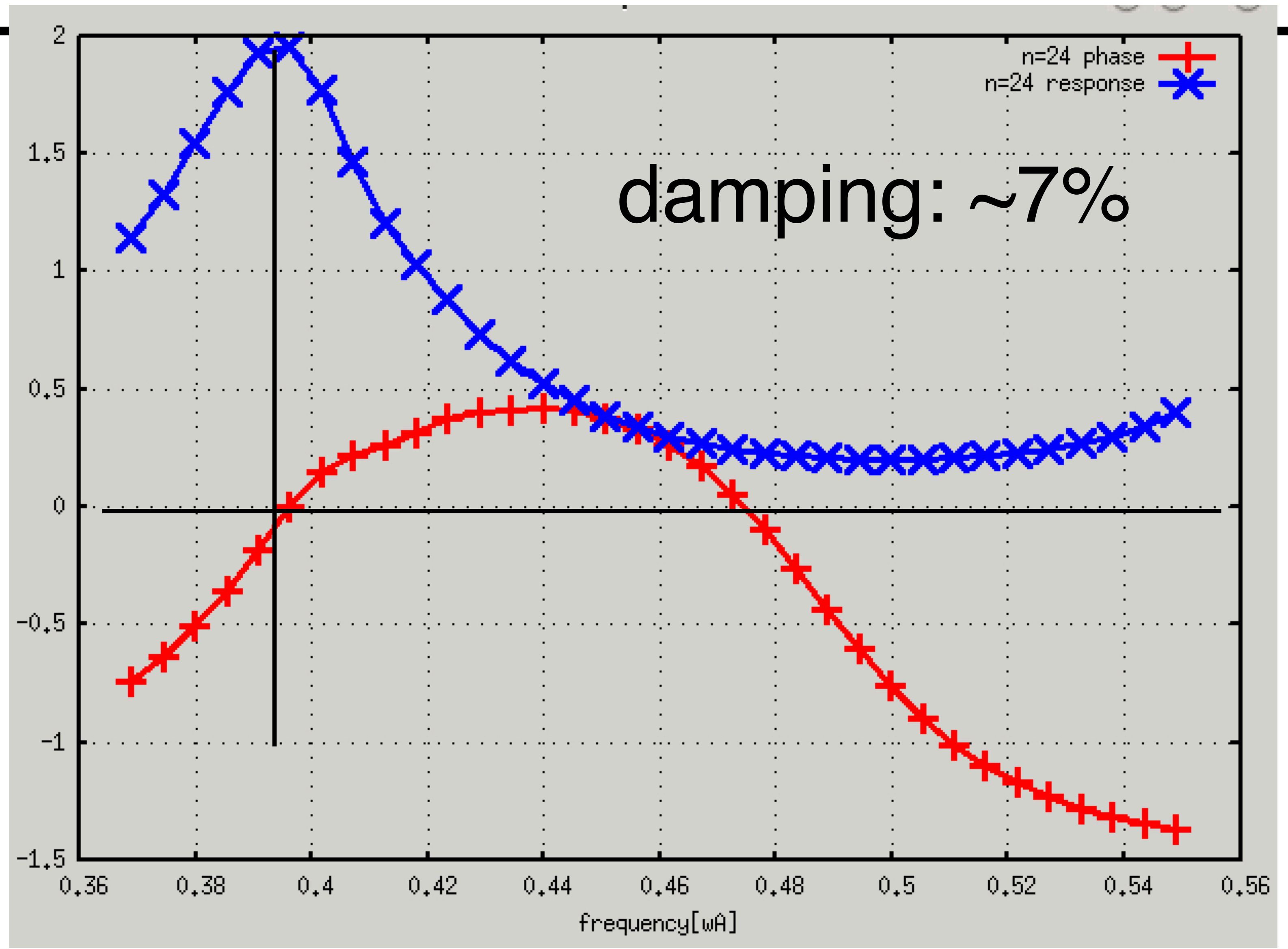


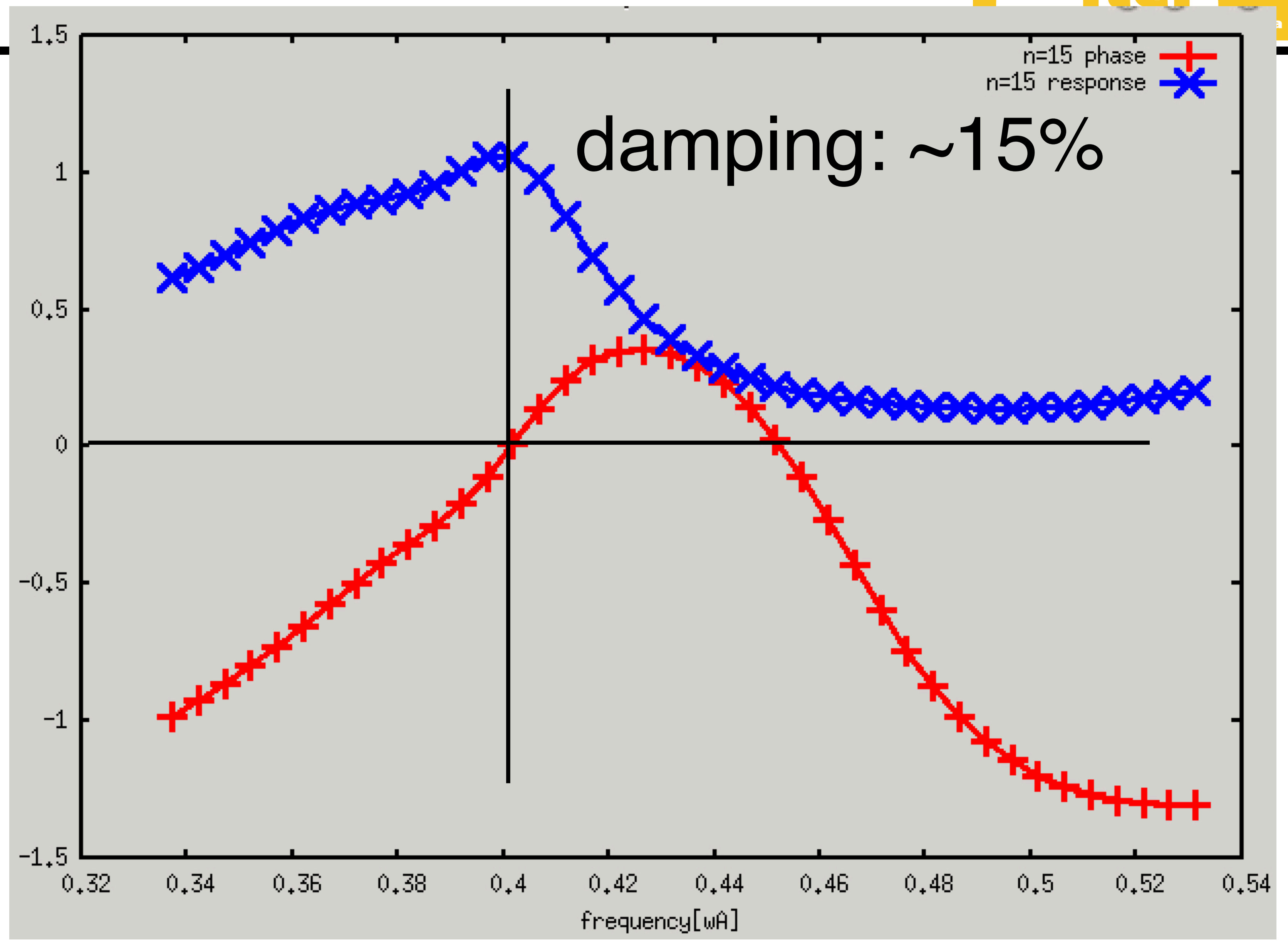
two KAWs propagating in opposite directions form a standing wave: KTAE

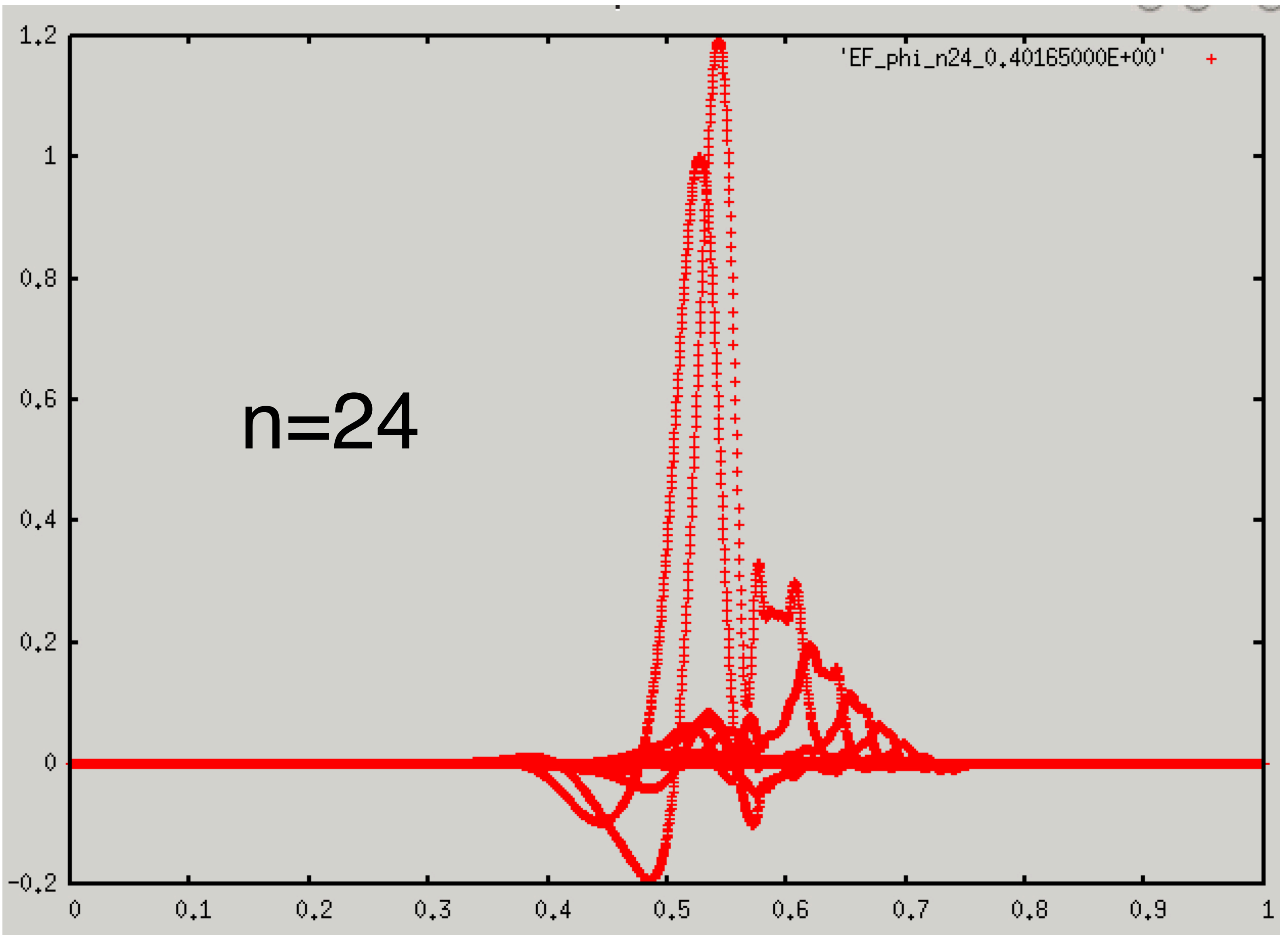
global solver (example antenna solver, LIGKA mode 1)

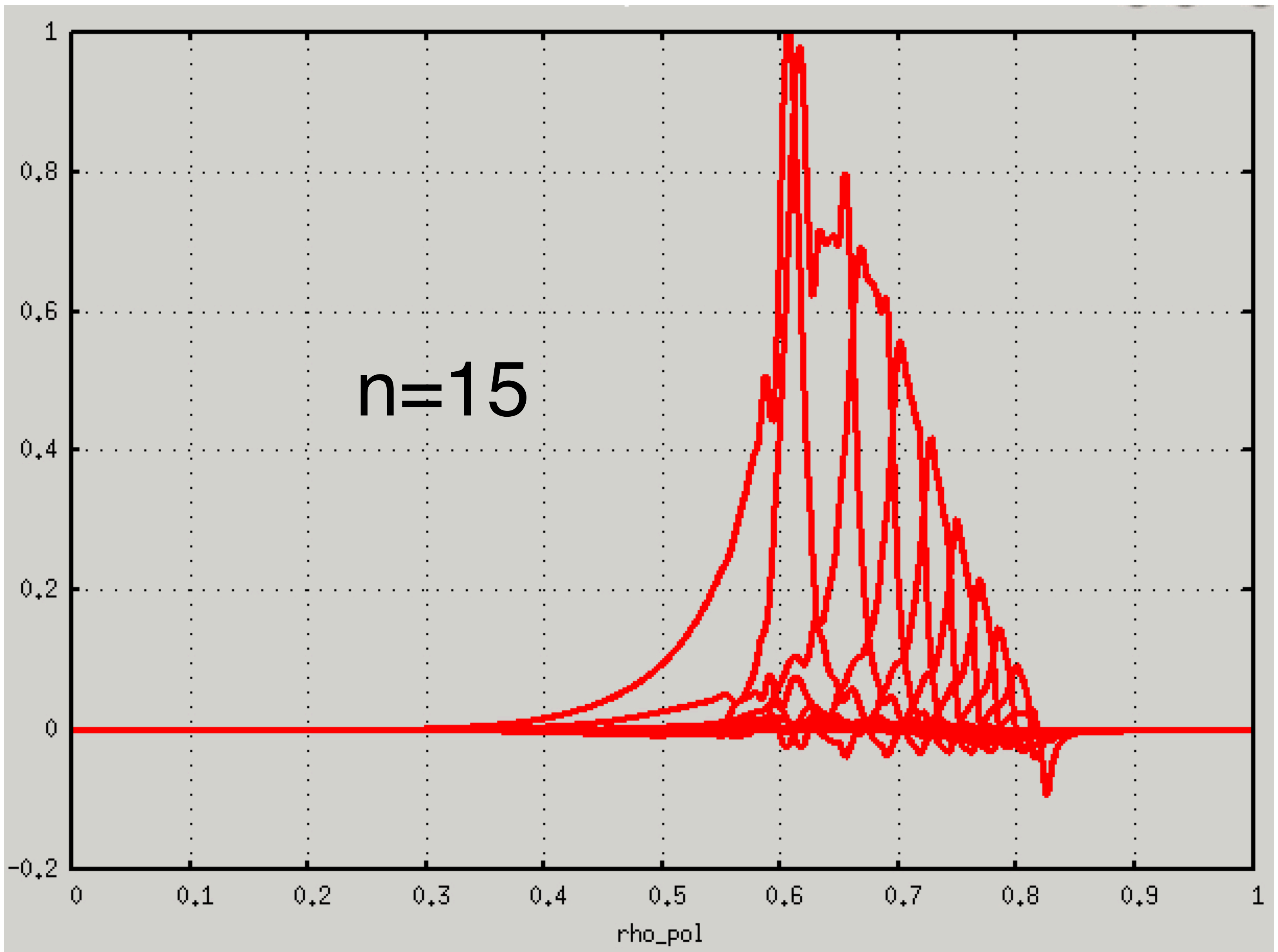
$$M(\omega)\begin{pmatrix}\phi\\ \psi\end{pmatrix} = \mathbf{d}$$

$$\mathcal{R} = \sum_m \int_0^a \phi_m \phi_m^* dr .$$



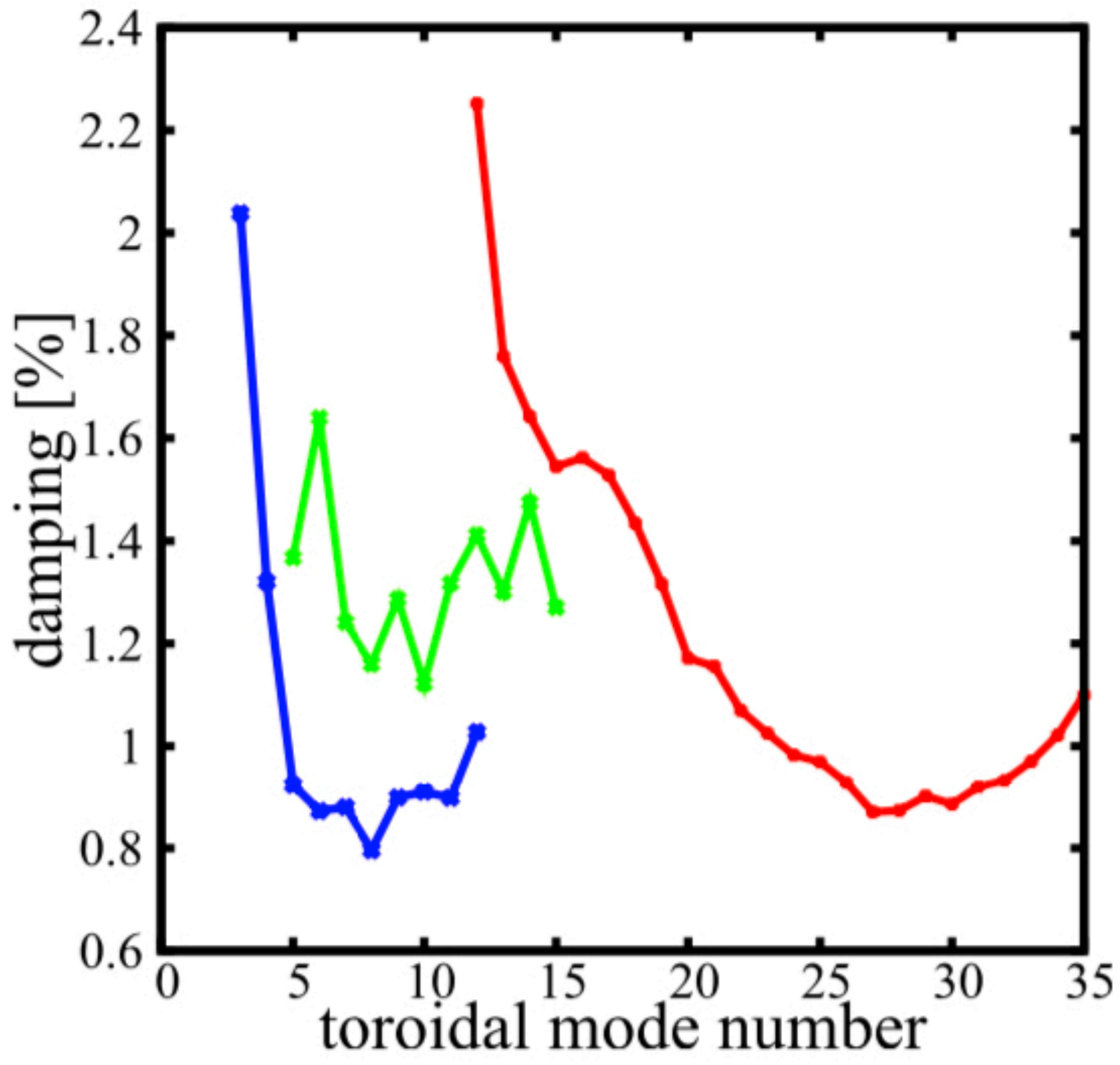


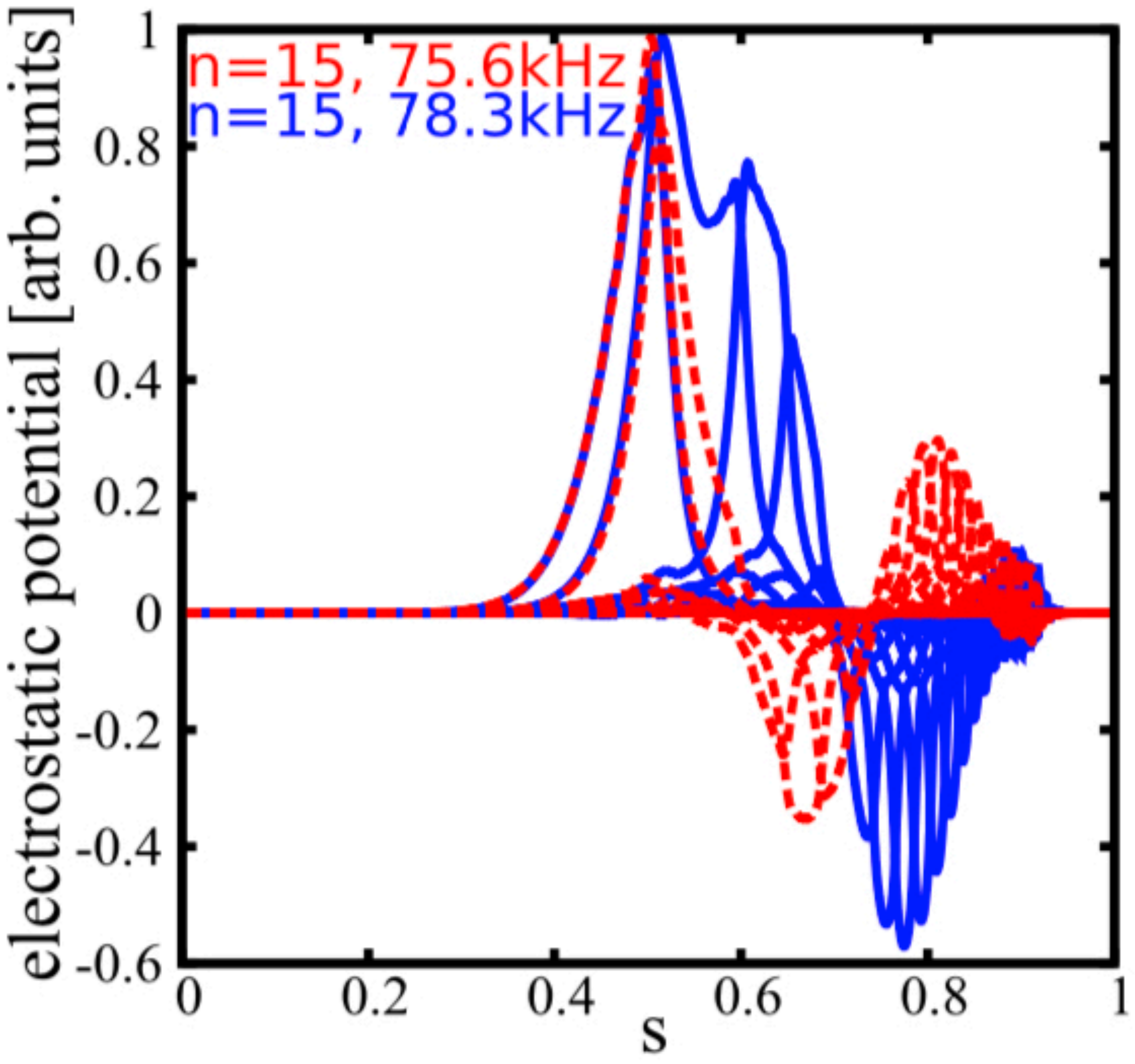




LIGKA mode1 scans entire gap

LIGKA mode 2 can be used to ‘follow’ just one mode as given by mode 1





damping > ~1%

for n<15 more than one TAE branch is found to be weakly damped

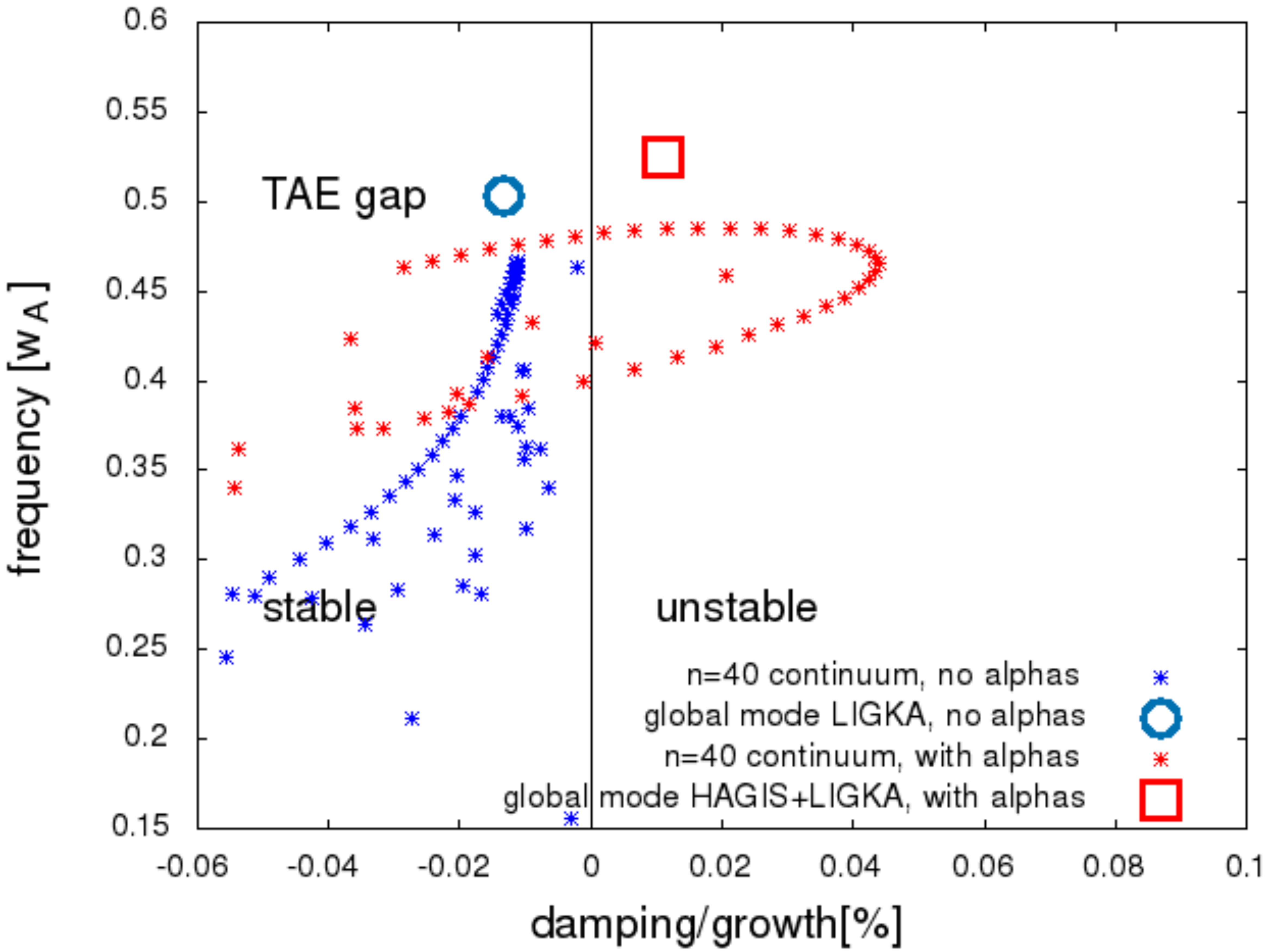
different alignment of TAE gaps from core-edge

may destabilise subdominant modes with lower n in outer core

comparison of local and global results: ITER ITPA case

blue: local continuum (*) and global LIGKA solution (○) <u>without</u> α-particles

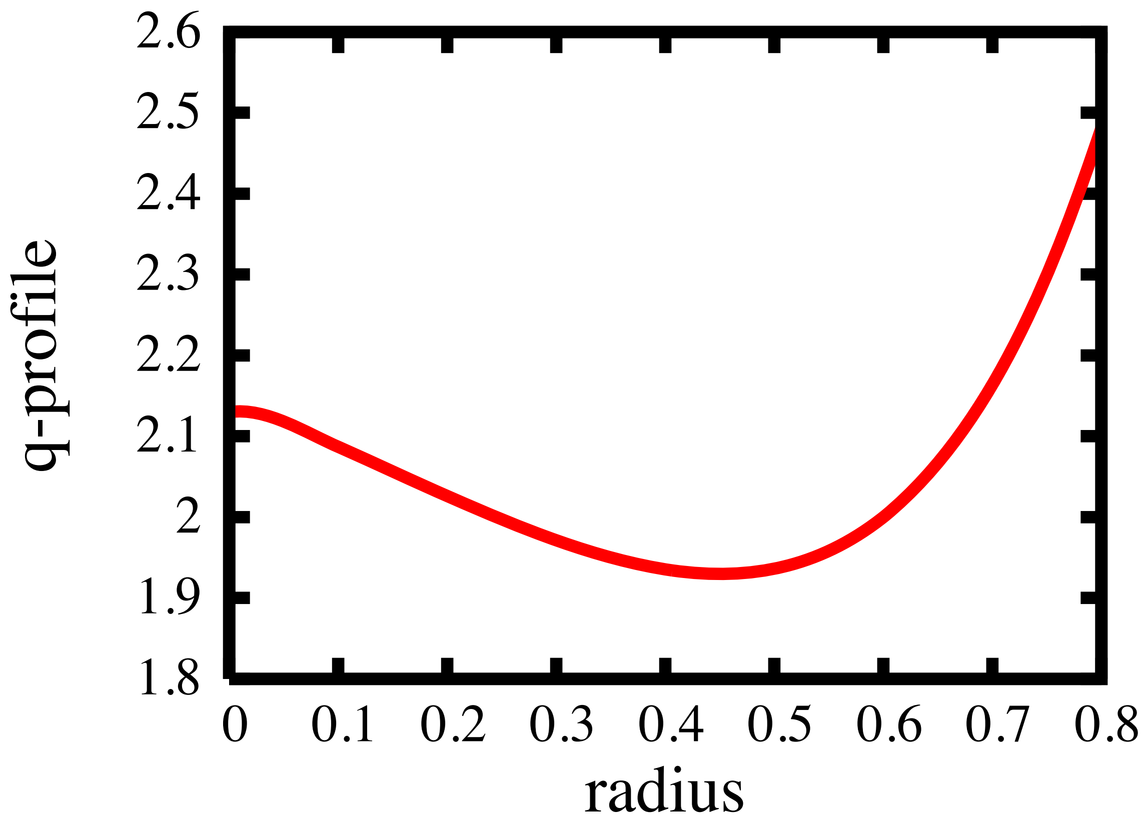
red: local continuum including finite orbit width effects (*) and global LIGKA+HAGIS solution (□)<u>with</u> α particles

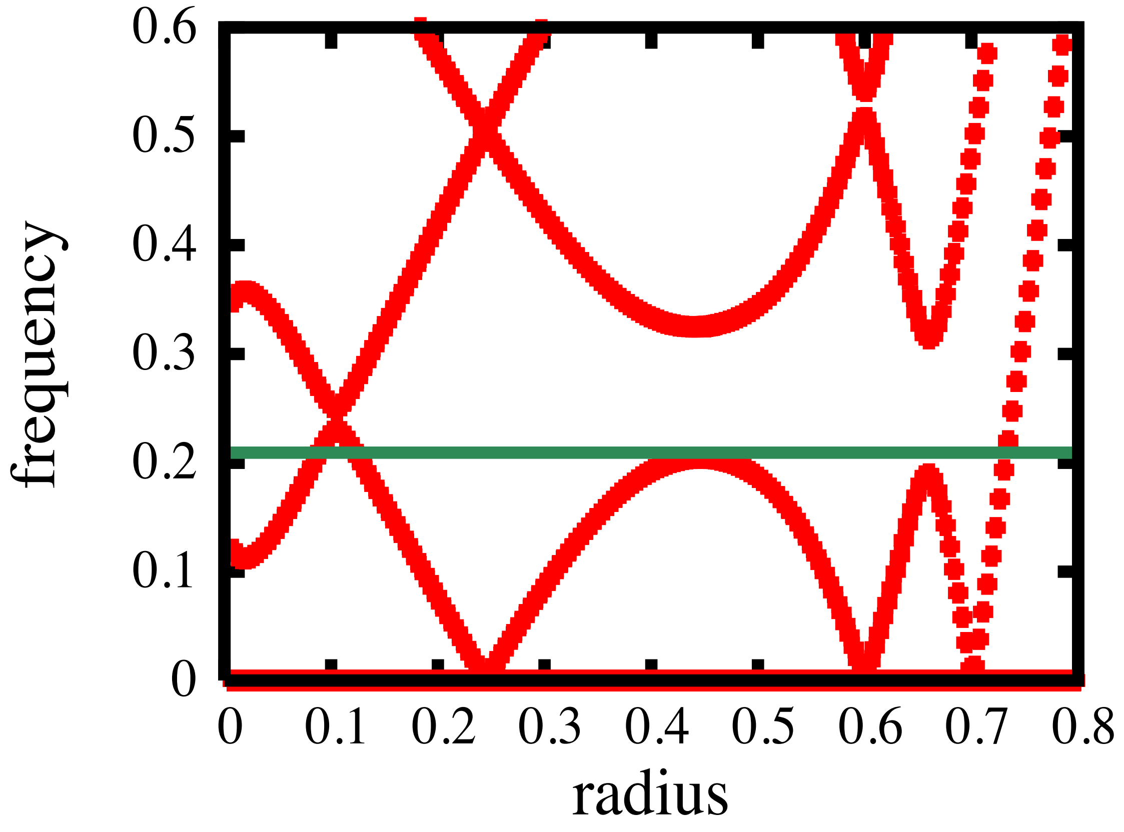


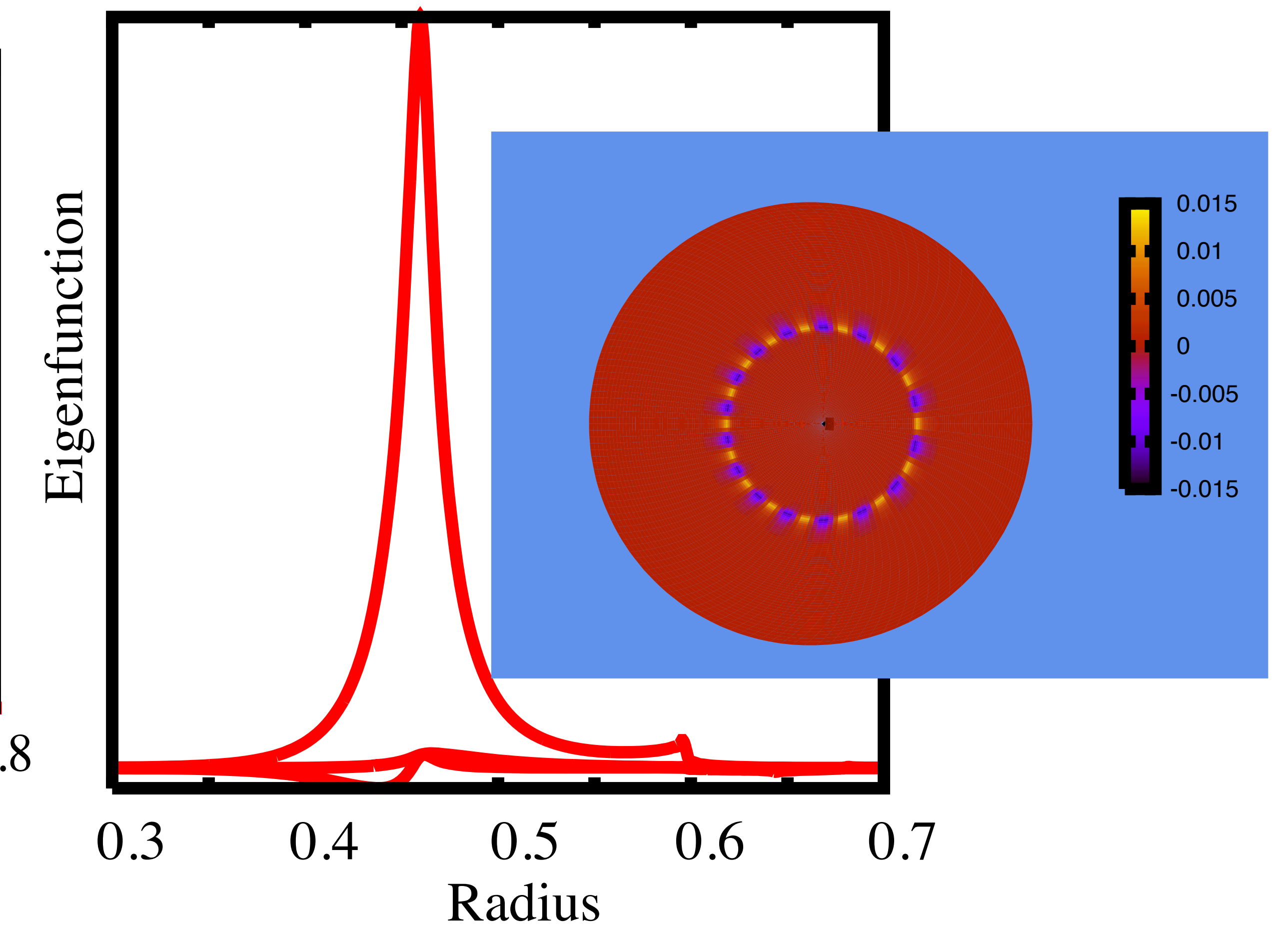
sophisticated local model (~100 times cheaper than global model) predicts instability threshold and linear damping/growth rates reasonably well: deviations low toroidal mode numbers - switch to global solver

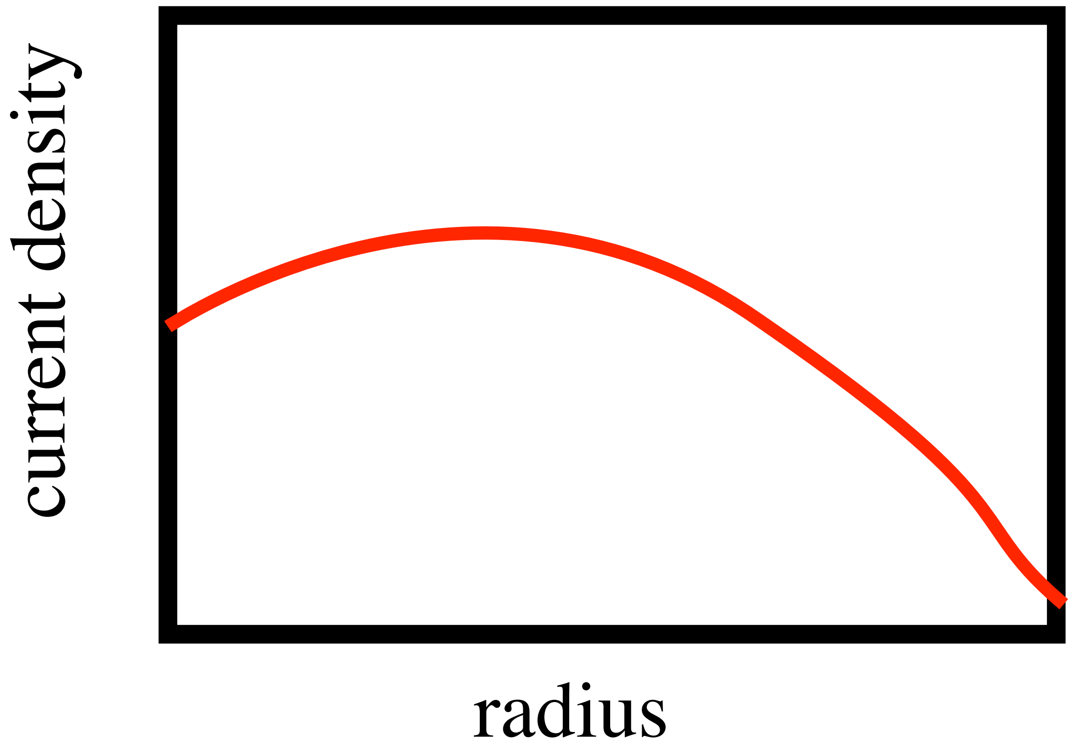
caveat: for global modes with continuum interaction, local solver underestimates damping considerably!

‘Reversed shear’ Alfvén Eigenmodes (RSAE)









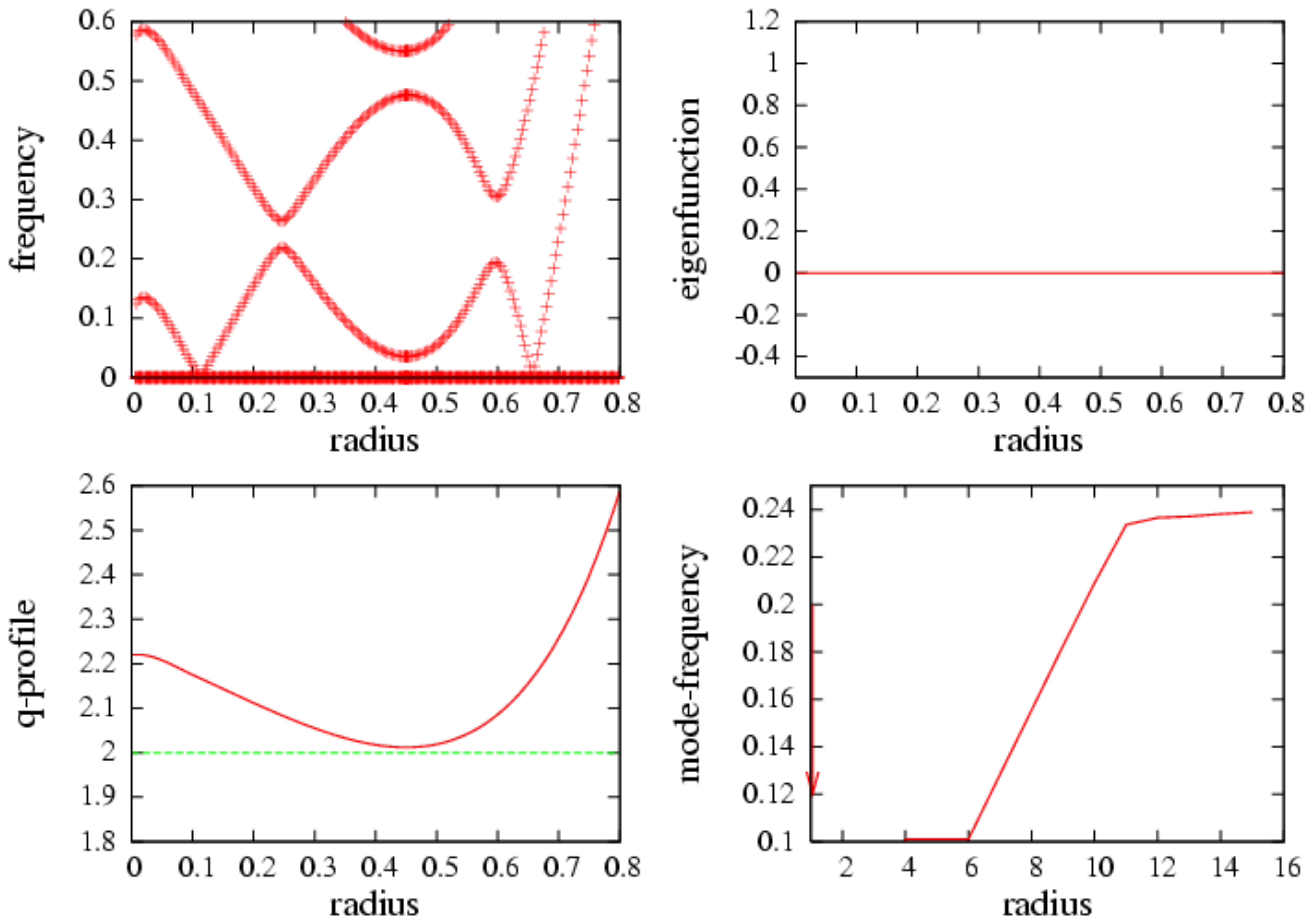
off axis peaked current profile:
“advanced tokamaks” - steady state

⇒ q-profile has minimum

⇒ region without continuum damping

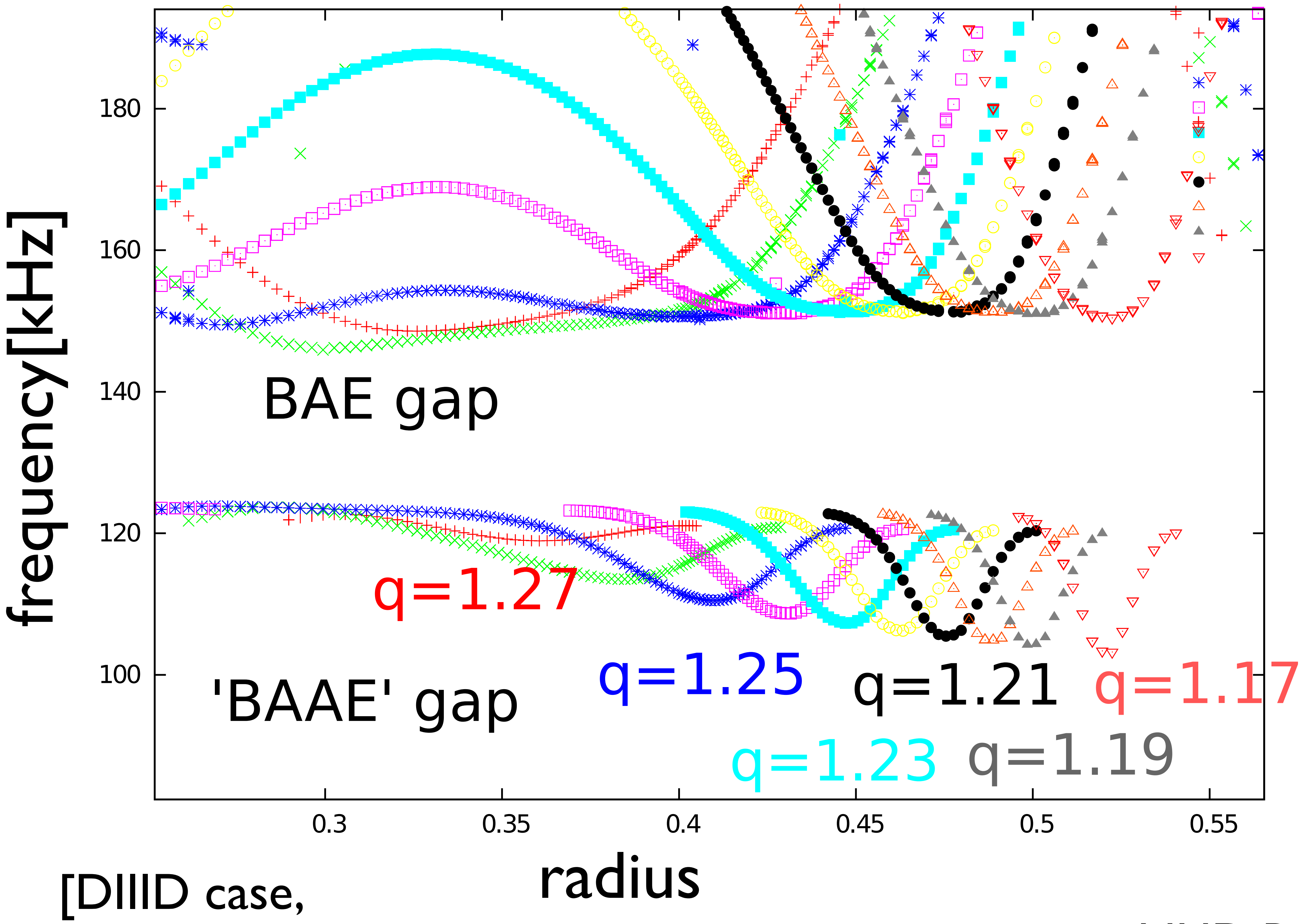
[Berk, Breizman, Fu, Sharapov, Konovalov, Lauber 2000-2006]

RSAE-TAE conversion



further gaps due to geodesic curvature and coupling between Alfvén and acoustic waves (see below)

$$\Sigma_m (\omega/v_A)^2 - k^2_{||m} = \beta * F(\omega^2/c^2_s - k^2_{||m})$$



[DIIID case, Lauber, 2012]

gaps scale with plasma beta:

$$\beta = \frac{\text{kinetic pressure}}{\text{magnetic pressure}}$$

⇒ beta induced Alfvén eigenmode : BAE

⇒ beta induced Alfvén- Acoustic eigenmode : BAAE

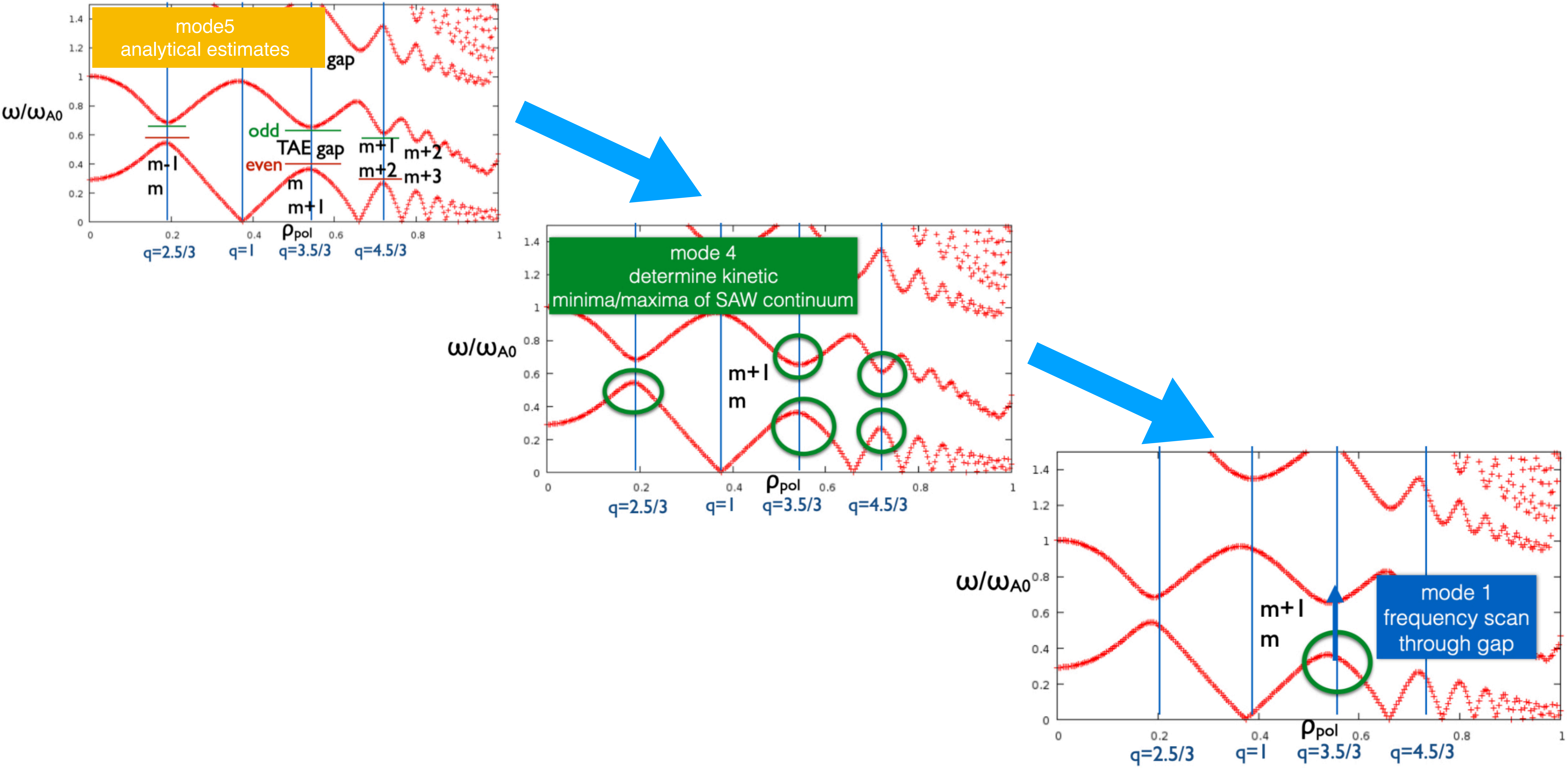
strongly modified in kinetic description! ($\omega \sim \omega_{t,b}$)

MHD BAAE cannot be excited - strongly damped;

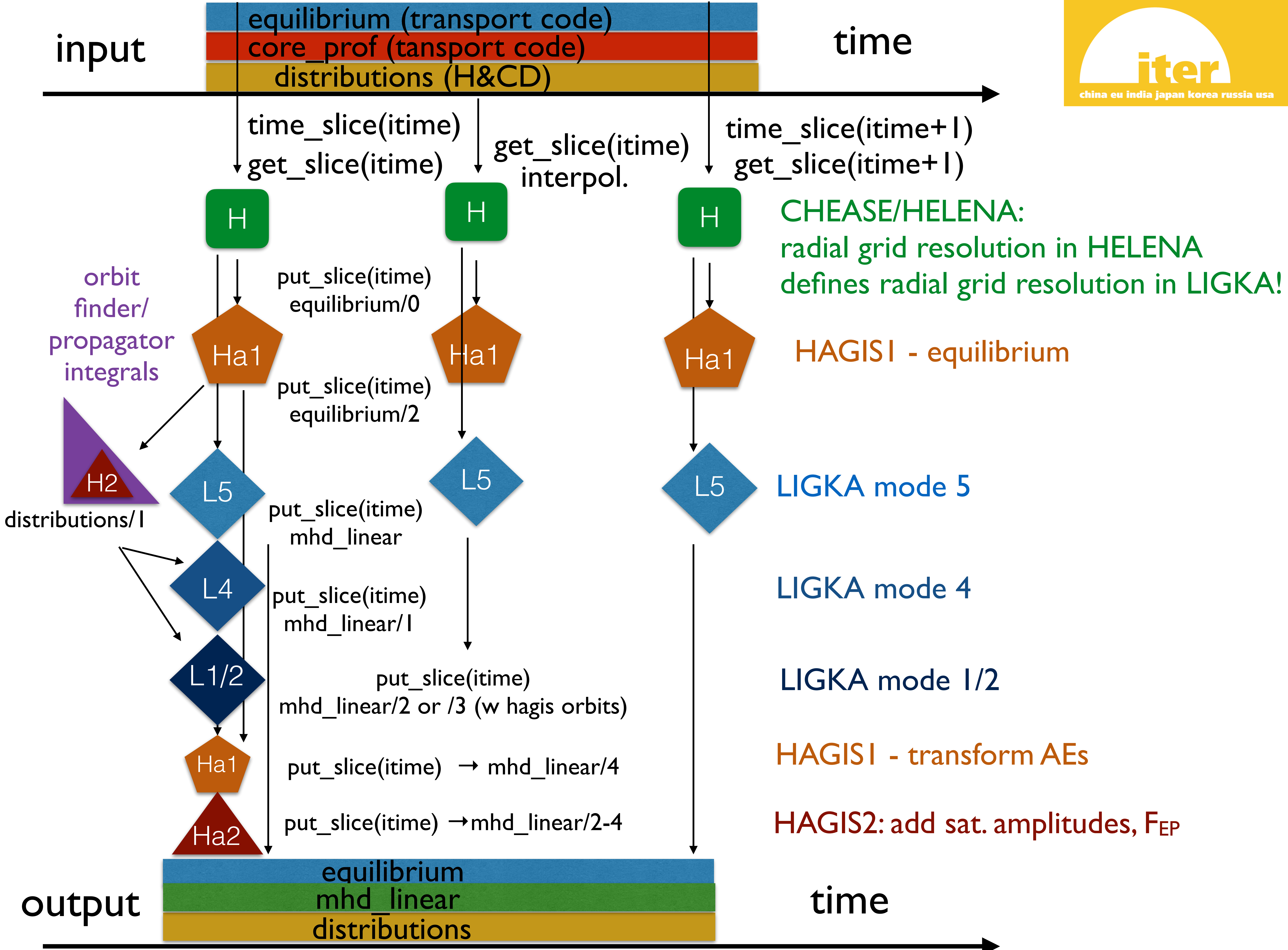
drift-Alfvén-type instabilities at rational surfaces - can be excited by thermal gradients

[Heidbrink 1992, Zonca 1996,Gorelenkov 2006, Lauber 2013, Heidbrink 2020, Ma 2021-23]

go global: use local information for setting up global simulation



EP WORKFLOW SCHEMATICS



LIGKA: linear GK model, multi-species, general 2D geometry

equations are solved in different limits and approximations, sharing same infrastructure:

- local, analytical estimates
- local reduced MHD - shear Alfvén spectra
- local kinetic (w/o numerical coefficients, i.e. orbits given by GC code) [Zonca 1996, Lauber 2009]
- local kinetic with FLR/FOW (w/o numerical coefficients) [Zonca 1998,Lauber JPC 2018]

- global reduced MHD - global eigenfunction
- global kinetic (w/o numerical coefficients): 2 solvers
- global kinetic track mode (w/o numerical coefficients)
- typically modes are called in sequence - to large part automated (workflow, IMAS format)

- for technical details and introduction into various options, please refer to talk by V.-A. Popa

- toolbox ready for the use in various transport models: Eurofusion 'ATEP' Enabling research project [Lauber, Falessi et al 2021]