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Phase Space Zonal Structures and equilibrium distribution functions in ORB5

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Motivation

- **Phase Space Zonal Structures** (**PSZSs**) are obtained by averaging out dependencies on angle-like variables in the **Energetic particle** (**EPs**) distribution function.
- PSZSs play a fundamental role in regulating EP transport induced by shear Alfvén instabilities in burning plasmas, acting as a slowly varying nonlinear equilibrium state [1], see **Falessi I4.104**.
- They are of great interest for the development of reduced models for EP heat and particle transport on long time scales for future burning plasma experiments (Eurofusion Enabling Research **ATEP**). [`https://wiki.euro-fusion.org/wiki/Project_No10`].
- In ORB5 the PSZS is constructed by projecting the marker weights on a B-spline basis [2]. This projection can also be used as **control-variate** (evolving background) for δf PIC simulations.

Theory

- Following [3], the PSZS is defined as the angle average (orbit average) of the gyrocenter distribution function;

$$\hat{F}_{0,sp}(P_\varphi,\mu,\epsilon) = \tau_b^{-1} \oint \frac{\mathrm{d}\theta}{\dot{\theta}} F_{z,sp} \ , \quad \tau_b = \oint \frac{\mathrm{d}\theta}{\dot{\theta}} \ , \quad F_z = \frac{1}{2\pi}\int_0^{2\pi} F_{sp}(P_\varphi,\mu,\epsilon,\theta,\varphi)\,\mathrm{d}\varphi \ ,$$

$$\dot{\theta} = -\frac{v_\parallel}{eB_\parallel^* J}\frac{\partial P_\varphi}{\partial \psi} \ , \quad P_\varphi = m_{sp} v_\parallel \frac{T(\psi)}{q_{sp} B} + \psi \ , \quad \epsilon = \frac{1}{2} m_{sp} v_\parallel^2 + \mu B$$

P_φ constant of motion (toroidal symmetry, for $\mathbf{B} = T(\psi)\nabla\varphi + \nabla\varphi \times \nabla\psi$).

- The finite element (**B-splines**) representation of the PSZS is

$$\hat{F}_{0,sp}(P_\varphi,\mu,\epsilon,t) = \sum_l f_l(t)\Lambda_l(P_\varphi,\mu,\epsilon) \ , \quad \Lambda_l(P_\varphi,\mu,\epsilon) = \Lambda_i^d(P_\varphi)\Lambda_j^d(\epsilon)\Lambda_k^d(\mu)$$

- $\Lambda_l(P_\varphi, \mu, \epsilon)$, tensor product of 3 one dimensional B-splines of order d.
- Set as linear algebra problem (Galerkin method)

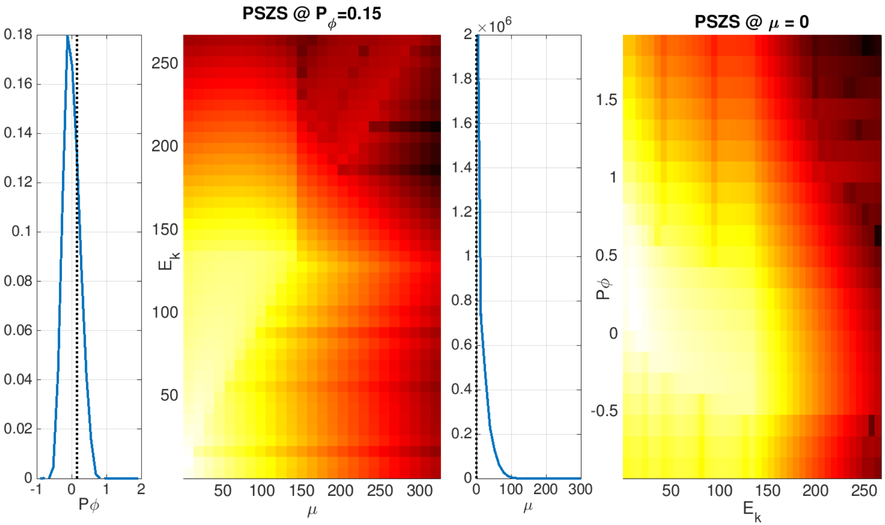
$$\sum_l f_l(t) \int J_{P_\varphi,\mu,\epsilon}\, \mathrm{d}P_\varphi\, \mathrm{d}\epsilon\, \mathrm{d}\mu\, \Lambda_n \Lambda_l = \int J_{P_\varphi,\mu,\epsilon}\, \mathrm{d}P_\varphi\, \mathrm{d}\epsilon\, \mathrm{d}\mu\, \Lambda_n \hat{F}_{0,sp} \ ,$$

$$\sum_l A_{l,m} f_l = b_n$$

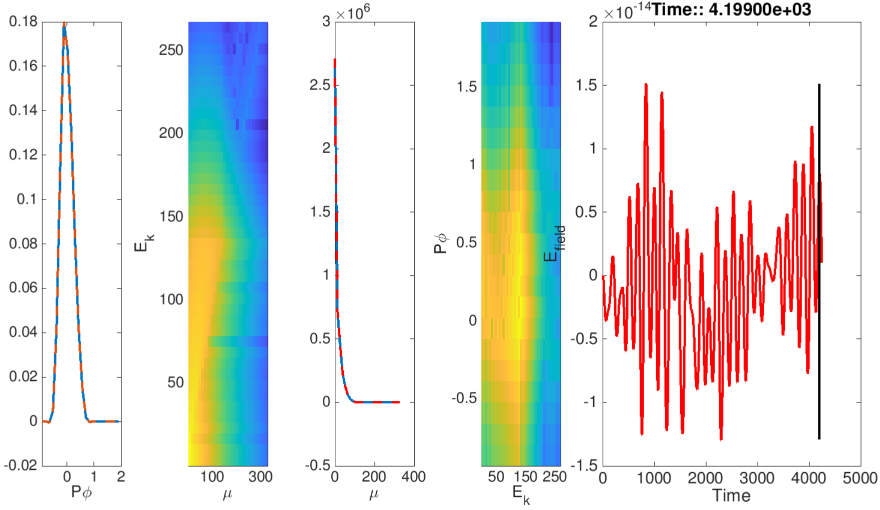
  $A_{l,m}$, elements of a symmetric sparse band matrix of rank $(N_{P_\varphi} + d)(N_\epsilon + d)(N_\mu + d)$.
- The right-hand-side is constructed by projecting the markers on the B-spline space.

$$b_n = \sum_{p=1}^{N} \left(\frac{N_{\mathrm{ph}}}{N} \overline{\delta w}_p + f_0(P_{\varphi,p}, \theta_p, \epsilon_p, \mu_p) \Omega_p \right) \Lambda_\nu(P_{\varphi,p}, \epsilon_p, \mu_p) \ .$$

Example: PSZS for the NLED-AUG case, early nonlinear phase



Example: PSZS for the NLED-AUG case [4]



Control-variate

- Formally, the GK Polarization equation looks like:

$$F(\Phi) = \sum_{\mathrm{sp}} q_{\mathrm{sp}} \int f_{\mathrm{sp}} \mathrm{d}\mathbf{v} \simeq \sum_{\mathrm{sp}} \sum_{\mathrm{p}} w_{\mathrm{p}}$$

- Control-variate:

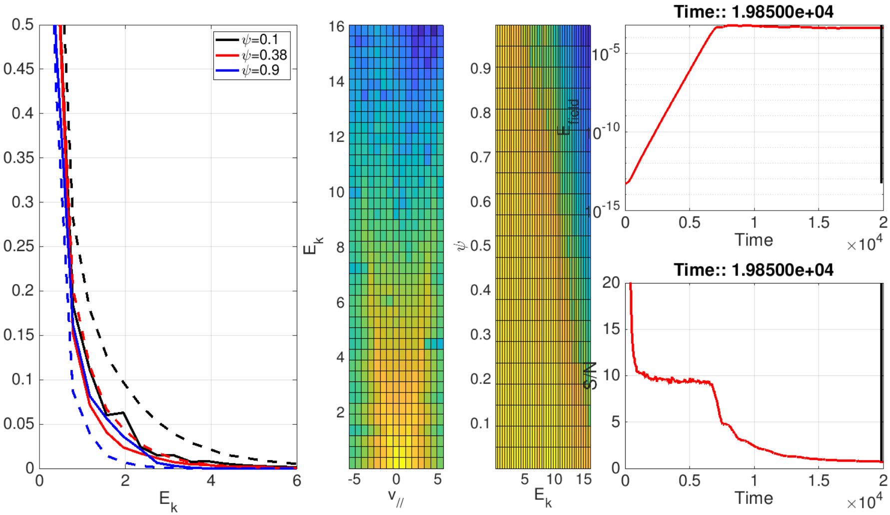
$$F(\Phi) = \sum_{\mathrm{sp}} q_{\mathrm{sp}} \int (f_{\mathrm{sp}} - f_{cv,\mathrm{sp}}) \mathrm{d}\mathbf{v} + \sum_{\mathrm{sp}} q_{\mathrm{sp}} \int f_{cv,\mathrm{sp}} \mathrm{d}\mathbf{v} \simeq \sum_{\mathrm{sp}} \sum_{\mathrm{p}} \hat{w}_{\mathrm{p}} + \sum_{\mathrm{sp}} q_{\mathrm{sp}} \int f_{cv,\mathrm{sp}} \mathrm{d}\mathbf{v}$$

  where $f_{cv,\mathrm{sp}}$ is a known function, e.g. a Maxwellian or a *numerical* distribution function defined on a grid.
- If the variance of the distribution of $\hat{w}_p$ is smaller than the one of w_p (i.e. $f_{cv,\mathrm{sp}} \simeq f_{\mathrm{sp}}$) the statistical noise is reduced.
- The more the distribution function departs from the $f_{cv,\mathrm{sp}}$ the less efficient the control-variate becomes.

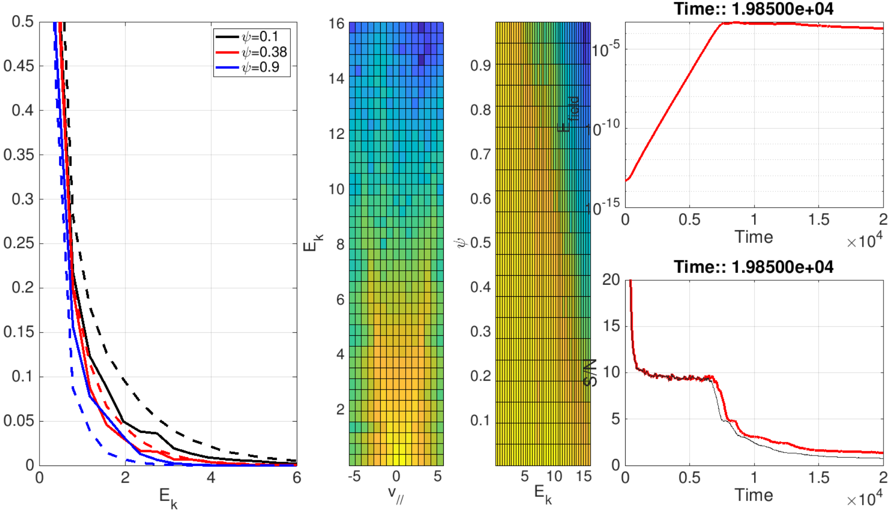
Control-variate using a B-spline projection

- ORB5 can already cope with numerical distribution functions defined on a grid $f(\psi, E_k, v_{||})$ (RABBIT interface, T. Hayward-Schneider, used in Ref. [4]).
- The B-splines coefficients of $f_{\mathrm{fsa}}(\psi, E_k, v_{||})$ can be used to calculate the value of the function (and its derivatives) on the grid points of the numerical background distribution function of ORB5.
- The corresponding numerical distribution function is now used as **time dependent control-variate**.
- Only the particle weights are updated by subtracting the value of the control-variate at the particle position from the (conserved) value of the total distribution function.
- Caveat: if linearized polarization is used, the polarisation density should also be updated (TBD).

Example, the cyclone base case with ORB5, standard simulation



Proof of principle: $f_{\mathrm{fsa}}(\psi, E_k, v_{||})@t \simeq 0.6 \times 10^4$ as control-variate



Experimental Setup: the NLED-AUG case

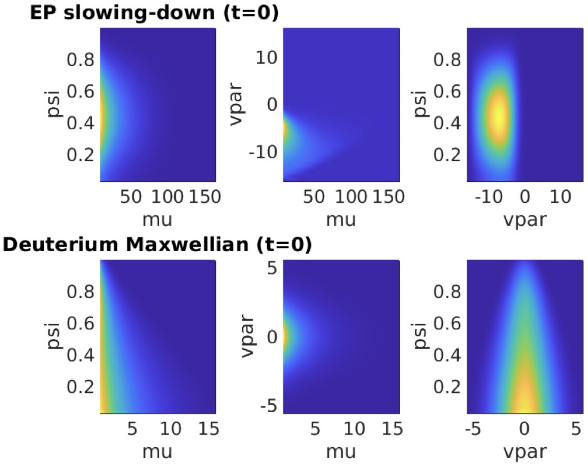
- NLED-AUG case [5], ASDEX Upgrade 31213 at $t = 0.84\ \mathrm{s}$ in the discharge number 31213.
- NBI generated EP; $\beta_{\mathrm{EP}} \sim \beta_{\mathrm{BULK}}$ and EP average kinetic energy $\epsilon_{\mathrm{EP}} \sim 100 \cdot T_{\mathrm{BULK}}$, reactor relevant conditions.
- Intense EP-driven activity is observed (EGAMs, SAWs, EPMs...).
- The NLED-AUG case has been widely studied and simulated by several codes, including ORB5.
- Energetic particles are simulated using an analytical pitch-angle dependent slowing-down equilibrium function (see Eq. (3) of [6]).
- Injection pitch angle $\xi_0 = v_{\parallel}/|v| = -0.5$ and EP concentration $n_{\mathrm{EP}}/n_{\mathrm{e}} = 0.0949$ are used.

Numerical Validation

- Tested on analytical distribution functions, reduced dimensionality and simplified geometry.
- Test: equilibrium F_0 reconstruction on ORB5 coordinates $\hat{F}_{0,sp}(\psi, \mu, v_{\parallel}, t = 0)$ for the NLED-AUG case for a Maxwellian and a slowing-down (with pitch angle dependence).
- B-spline projection of F_0 is calculated at each marker position.

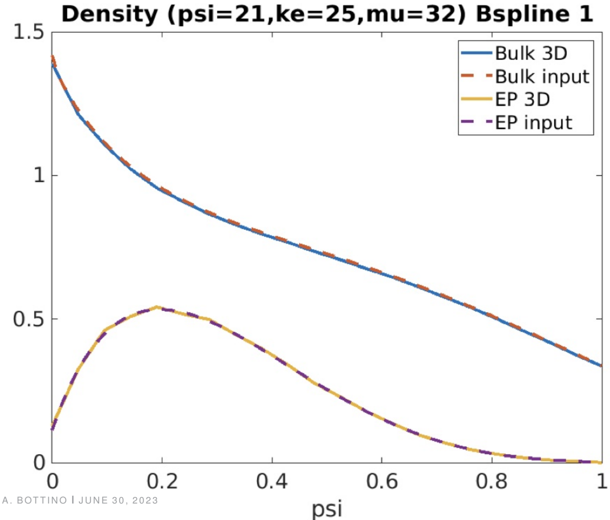
2D projection of the 3D flux-surface averaged F_0

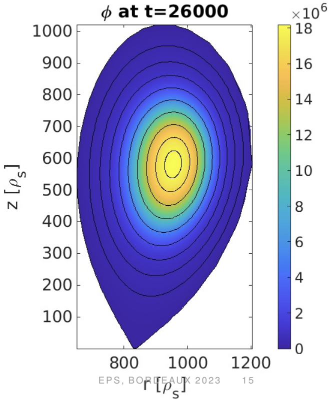
- $(N_\psi = 30, N_\mu = 30, N_{v_{||}} = 30)$ and $d = 1$, integrated along the third direction.



Reconstructed EP and Bulk ion densities

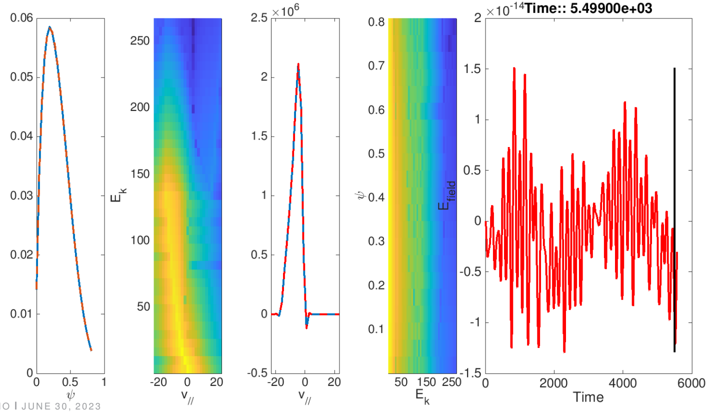
- Comparison with the analytical functions.





Example: $f_{\mathrm{fsa}}(\psi, E_k, v_{||})$ for the NLED-AUG case [4]

- If $(P_\phi, E_k, \mu) \rightarrow (\psi, E_k, v_{||})$ the PSZS numerical infrastructure of ORB5 can be used to get a B-spline projection of the flux-surface averaged distribution function at any time, $f_{\mathrm{fsa}}(\psi, E_k, v_{||})$.



References

- Zonca F, Chen L, Briguglio S, Fogaccia G, Vlad G and Wang X 2015 *New Journal of Physics* **17** 013052
- Bottino A, Falessi M, Hayward-Schneider T, Biancalani A, Briguglio S, Hatzky R, Lauber P, Mishchenko A, Poli E, Rettino B, Vannini F, Wang X and Zonca F 2022 *Journal of Physics Conference Series* **2397** 012019
- Falessi M V and Zonca F 2019 *Physics of Plasmas* **26** 022305 (*Preprint* `https://doi.org/10.1063/1.5063874`)
- Rettino B, Hayward-Schneider T, Biancalani A, Bottino A, Lauber P, Weiland M, Vannini F and Jenko F 2023 *submitted to Nuclear Fusion*
- Lauber P *et al.* 2018 *27th IAEA Fusion Energy Conference (Proc. 27th IAEA FEC)* (https://conferences.iaea.org/event/151/contributions/6094/)
- Rettino B, Hayward-Schneider T, Biancalani A, Bottino A, Lauber P, Chavdarovski I, Weiland M, Vannini F and Jenko F 2022 *Nuclear Fusion* **62** 076027