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FÜR PLASMAPHYSIK

A model for AE-induced transport of fast ions in stellarators

Wendelstein 7-X

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EUROfusion

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Contents

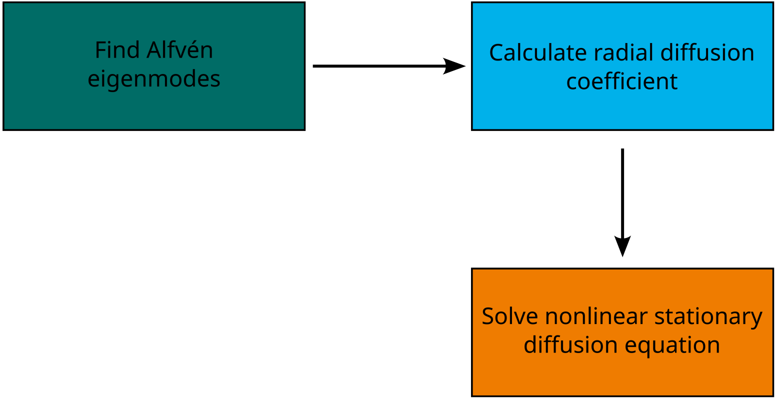
Overview of existing tools

- reduced transport models are desirable for their high computing speeds while still capturing (most of) the relevant physics
- such models could potentially be included in optimization loops
- several reduced models for Alfvén-eigenmode-induced fast-ion transport available for tokamaks
  - Kick-model[1] $\rightarrow$ relies on P_φ as conserved quantity
  - RBQ model[2] $\rightarrow$ relies on P_φ as conserved quantity
  - TGLF-EP[3] $\rightarrow$ simple critical gradient model
  - ATEP code[4] $\rightarrow$ relies on P_φ as conserved quantity, LIGKA-HAGIS solves for phase-space zonal structure
- no such model did exist for stellarators
- similar information always had to be extracted from gyrokinetic simulations or MHD-kinetic simulations $\rightarrow$ much more expensive
- this work: develop a model also suitable for stellarators $\rightarrow$ W7-X, reactors,...

[1] M. Podestà et al., *Plasma Phys. Control. Fusion* **56** 055003 (2014)
[2] N. Gorelenkov et al., *Nucl. Fusion* **58** 082016 (2018)
[3] E. Bass et al., *Nucl. Fusion* **60** 016032 (2020)
[4] Ph. Lauber et al., 29th IAEA FEC, London (2023)

Basic ingredients of the model

Conceptually, the model can be split into 3 main components



[5]Ya. Kolesnichenko et al., *Phys. Plasmas* **9** 517-528 (2002)

Basic ingredients of the model

Conceptually, the model can be split into 3 main components

Find Alfvén eigenmodes → **Calculate radial diffusion coefficient** → **Solve nonlinear stationary diffusion equation**

- needed are their radial position as well as ω and γ
- already existing LGRO code (writen by Axel; based on theory by Kolesnichenko[5]) can be adapted for this purpose
- needs to be re-written in julia

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Calculate radial diffusion coefficient

- mixing-length estimate to link ω and γ of the mode to the diffusion it causes
- stronger growing mode should cause more transport

Solve nonlinear stationary diffusion equation

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Calculate radial diffusion coefficient ↓

- mixing-length estimate to link ω and γ of the mode to the diffusion it causes
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Solve nonlinear stationary diffusion equation

- diffusion equation becomes nonlinear as D itself will depend on fast-ion density and gradient (via γ)
- standard solvers for boundary-value problems unsuitable because of stiffness of the problem

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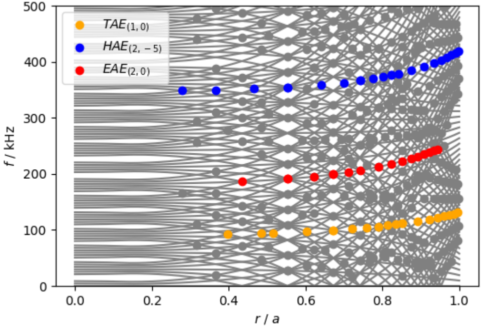
[5] Ya. Kolesnichenko et al., *Phys. Plasmas* **9** 517-528 (2002)

Finding Alfvén eigenmodes I

- start with the cylindrical Alfvén continuum
- assumption: at any crossing point of continuum branches a potential mode can be located
- no continuum gaps calculated (too expensive)



W7-X standard configuration

- crossing points $s_\star$ of any two continuum branches defined by relation

$$|k_{\parallel,1}| = |k_{\parallel,2}| \quad (1)$$

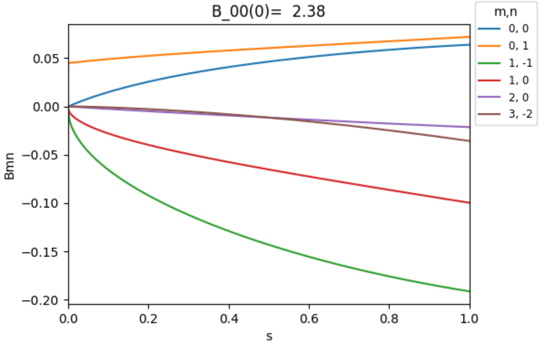
- hence

$$|m_1 \iota_\star + n_1| = |m_2 \iota_\star + n_2| \quad (2)$$

- gives local $\iota_\star$ (and therefore also $s_\star$) of the mode
- knowing the v_A profile, also $\omega = k_\parallel v_A$ is calculated easily

Finding Alfvén eigenmodes II

- couplings and the resulting drive determined by the inhomogeneities of the equilibrium magnetic field



W7-X standard configuration

- $\epsilon_{\mu,\nu}$ defined as

$$\epsilon_{\mu,\nu}(s) = \frac{B_{m,n}(s)}{B_{0,0}(0)} \tag{3}$$

- important to pick the major couplings for a given magnetic equilibrium
- in W7-X usually $B_{1,0}$ and $B_{1,-1}$ have the highest amplitudes (noting that $B_{0,1}$ does not contribute to γ)

Finding Alfvén eigenmodes III

- still needed are the growth rates of the modes → use Kolesnichenko model[5]
- growth rates in local approximation (drive comes from $\partial_r n_\mathrm{f}$ and $\partial_r T_\mathrm{f}$)

$$\gamma = \left(\frac{A}{\sqrt{s}}\frac{1}{|m\iota + n|}\right)^2 \frac{3\pi}{64}\frac{\beta_\star}{\int_0^\infty \mathrm{d}u\, f_\mathrm{press}} g \tag{4}$$

$$g = \sum_{\text{all } w} |w|\,\mu^2\epsilon_{\mu,\nu}^2 \int_{|w|}^\infty \mathrm{d}u\, f_\mathrm{grow} \tag{5}$$

- the w encode the resonances with the inhomogeneities of the magnetic field

$$w = \left[\left(1 \pm \frac{\mu\iota_\star + \nu N_\mathrm{p}}{m\iota + n}\right)\frac{v_{\mathrm{th},\star}}{v_{\mathrm{A},\star}}\right]^{-1} \tag{6}$$

- f_press and f_grow incude integrals over the equilibrium distribution function F

$$f_\mathrm{grow} = 4\pi\left(u^2 + w^2\right)^2\left(\frac{\omega}{2}\frac{\partial F}{\partial u} + u\omega_\star\frac{\partial F}{\partial s}\right) \tag{7}$$

$$f_\mathrm{press} = 4\pi u^4 F \tag{8}$$

[5] Ya. Kolesnichenko et al., *Phys. Plasmas* **9** 517-528, 2002

The radial diffusion coefficient

- we define the diffusion coefficient based on a mixing-length approximation[6,7] as

$$D\left(s\right) = \bar{D}\pi^2 \sum_{i=1}^{N_{\mathrm{modes}}} H\left(\gamma_i\right) \frac{\gamma_i}{k_{\perp,i}^2} \frac{\gamma_i^2}{\omega_i^2 + \gamma_i^2} \exp\left[-\frac{\left(s - s_i\right)^2}{\Delta_i^2}\right] E\left(s\right) + D_{\mathrm{turb}} \quad (9)$$

- each mode has its own mode width that goes as

$$\Delta_i = \bar{\Delta} \frac{\sqrt{\bar{\iota}'}}{\sqrt{m_i \iota_i'}} \quad (10)$$

  (scaling similar to that of a magnetic island)
- $E\left(s\right)$ is an envelope function that ensures that D goes to zero at $s = 0$ and $s = 1$
- $\bar{\Delta}$ and $\bar{D}$ are external scaling factors and D_{turb} accounts for turbulent transport (assumed to be small) / H is Heaviside function
- some calibration needed as mixing-length approximation has uncertain factor $\mathcal{O}\left(10\right)$

[6] J. Connor et al., *Plasma Phys. Control. Fusion* **43** 155–175 (2001)
[7] J. Weiland, *Plasma Physics Reports* **42** 502–513 (2016)

The radial diffusion equation

- we solve the stationary radial diffusion equation for the fast-ion density

$$\frac{\partial n_\mathrm{f}}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(Dr\frac{\partial n_\mathrm{f}}{\partial r}\right) - S_0\left(1 - \frac{n_\mathrm{f}}{n_\mathrm{f}^\mathrm{SD}}\right) = 0 \tag{11}$$

- n_f^SD is the slowing-down profile for the fast-ion density that would develop without AEs
- S_0 is the source profile (given by e.g. NBI or ICRH heating in W7-X)
- boundary-value problem cannot be solved directly because $D = D\left(r, n_\mathrm{f}, \partial_r n_\mathrm{f}\right)$ makes the equation nonlinear and very stiff
- instead: integrate once

$$D\frac{\partial n_\mathrm{f}}{\partial r} = \frac{1}{r}\int \mathrm{d}r\, rS_0\left(1 - \frac{n_\mathrm{f}}{n_\mathrm{f}^\mathrm{SD}}\right) \tag{12}$$

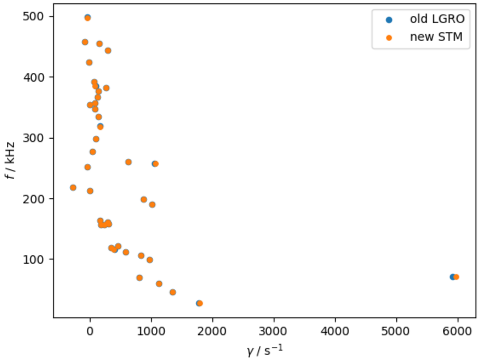
  and transform to s

$$D\frac{\partial n_\mathrm{f}}{\partial s} = \frac{a^2}{4s}\int \mathrm{d}s\, S_0\left(1 - \frac{n_\mathrm{f}}{n_\mathrm{f}^\mathrm{SD}}\right) \tag{13}$$

- solve as nonlinear root-finding problem with the Newton method (iteratively)
- boundary conditions: $\partial_r n_\mathrm{f}(0) = \partial_r n_\mathrm{f}(a) = 0$

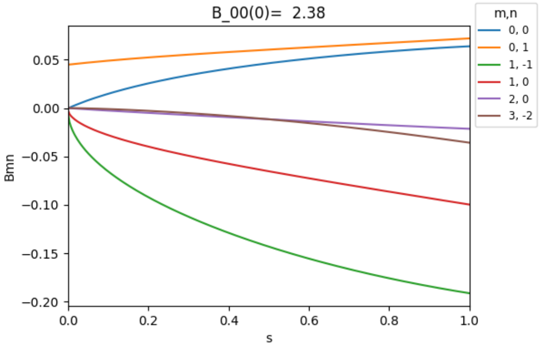
Benchmark with LGRO

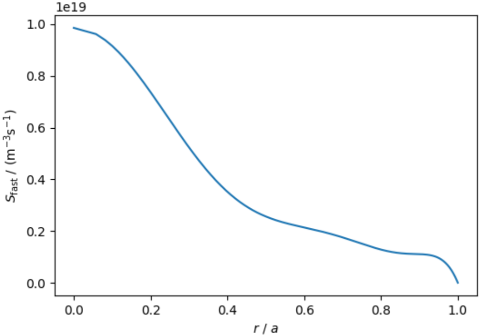
- first part of the new model (frequencies and growth rates of the modes) benchmarked with LGRO code



- good agreement of the new stellarator transport model (STM) and LGRO
- both codes solve exactly the same equations
- small differences most likely caused by minor numerical differences (e.g. interpolations, integration boundaries)

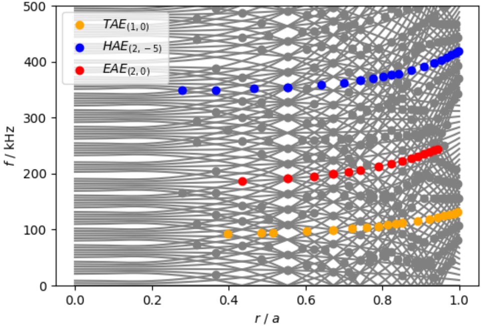
Input of the model

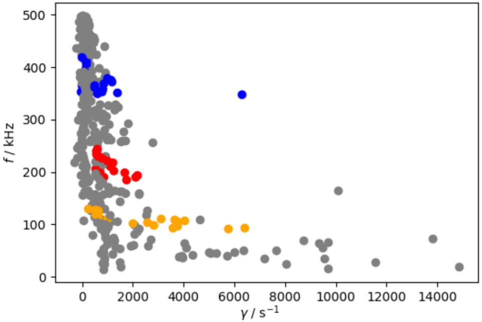




- the model takes as input the magnetic equilibrium (B_{mn} spectrum and ι) as well as profiles for the bulk plasma and for the fast ions
- major B_{mn} components enter the calculation of the kinetic drive
- fast-ion source profile taken from SCENIC simulations done for 4 NBI sources at W7-X

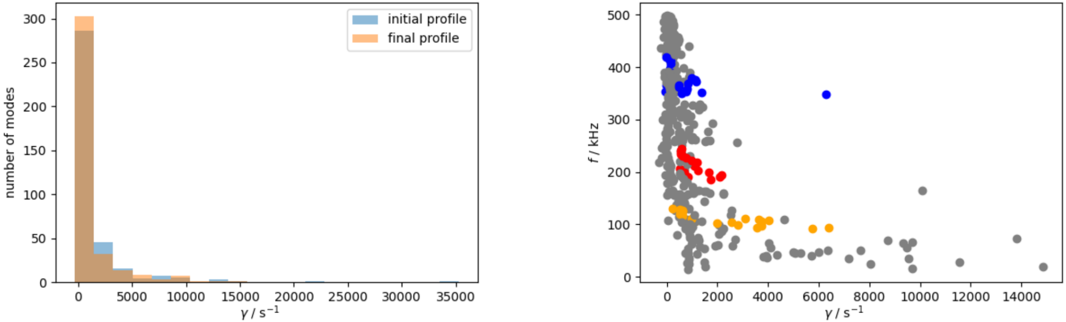
Output of the model I





- growth rates of the previously identified modes are calculated
- unstable ones will contribute to transport, stable ones are neglected in the calculation of D
- plot alreay shows that profile flattening (transport) leads to a more stable situation overall $\rightarrow$ expected behaviour
- not a critical-gradient model $\rightarrow$ growth rates of the modes don't go all the way to zero
- diffusion still balances the fast-ion source

Output of the model I



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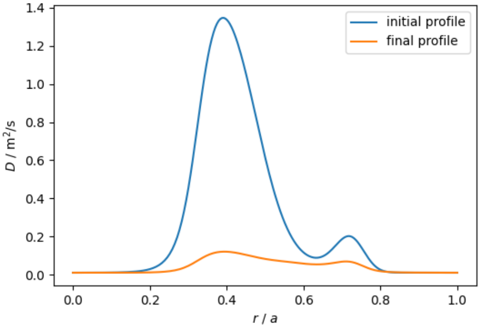
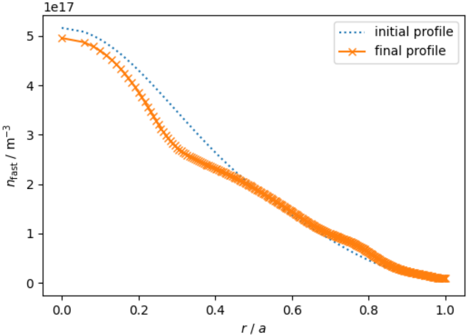
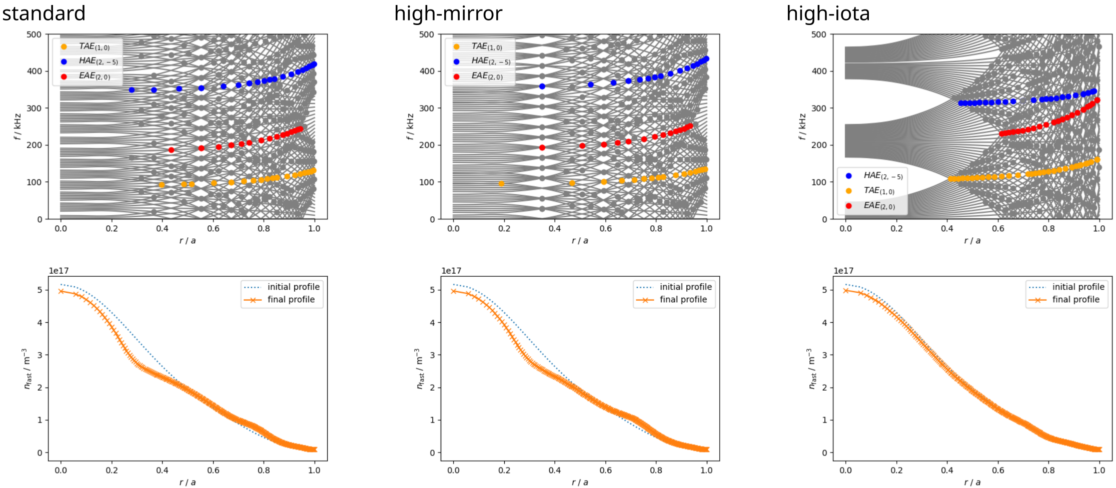
Output of the model II





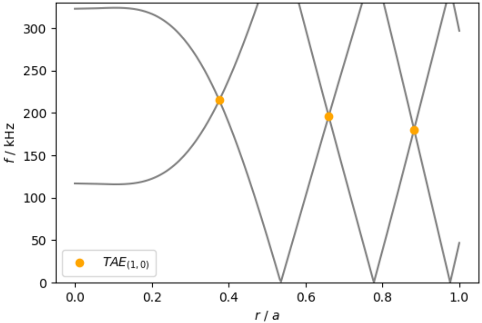
- diffusion coefficient defined using mixing length approximation as shown previously
- strongest diffusion in regions where modes have the highest growth rate
- diffusion leads to flattening of the fast-ion density profile → local flattening reduces the diffusion coefficient
- steady-state reached when source is balanced

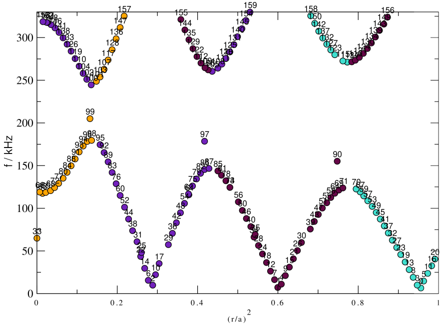
Comparison of configurations



- fast-ion transport depends on magnetic configuration (all other parameters of the model equal)
- in this example: EIM and KJM behave similarly, but lower transport in FTM

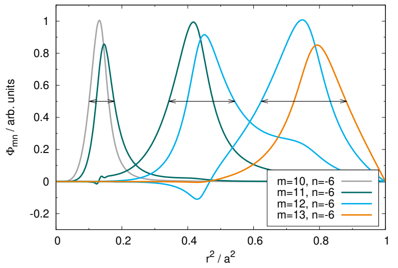
Benchmark: Compare with more complete CKA-EUTERPE model

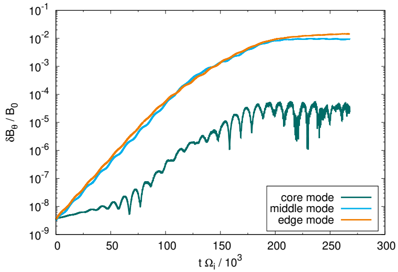




- comparison of the new model with CKA-EUTERPE performed (tokamak – 3 modes)
  - adjust mode widths in transport model to match CKA results (FWHM)
- goal: dial-in the free parameters of the transport model and assess its general quality (i.e. is the flattening similar or not)

Reminder: The CKA-EUTERPE model





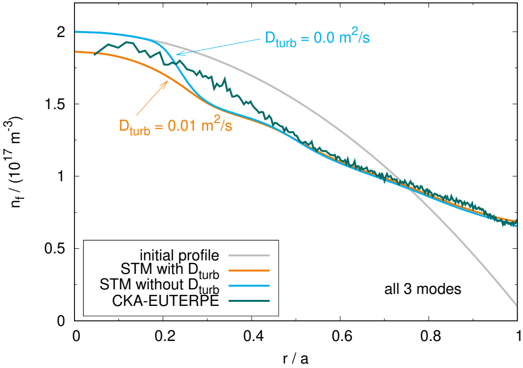
- perturbative model to study linear and nonlinear interaction of Alfvénic modes and fast particles
- spatial eigenfunctions given by ideal-MHD theory (CKA-code) remain fixed in time
- only amplitudes of ϕ and $A_{\|}$ allowed to evolve due to kinetic interactions

$$\frac{\partial \hat{A}_j}{\partial t} + \mathrm{i}\omega_j \left(\hat{\phi}_j - \hat{A}_j\right) = \sum_k \hat{\mathbb{N}}_{jk}^{-1} u_k \hat{A}_j \tag{14}$$

$$\frac{\partial \hat{\phi}_j}{\partial t} + \mathrm{i}\omega_j \left(\hat{A}_j - \hat{\phi}_j\right) = \sum_k \hat{\mathbb{M}}_{jk}^{-1} T_k \hat{\phi}_j - 2\gamma_{\mathrm{d}} \hat{\phi}_j \tag{15}$$

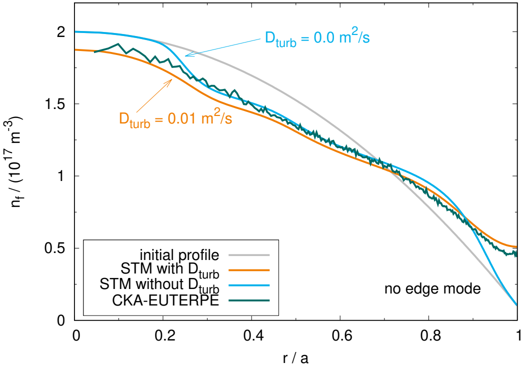
- multi-mode interactions (i.e. mode coupling) included
- markers more along nonlinear trajectories; u_k and T_k are moments of $f^{(1)}$
- cheap compared to fully gyrokinetic simulations, but much more expensive than reduced transport model

CKA-EUTERPE comparison I



- clear profile modifications due to AEs (compare to unperturbed grey profile)
- very good agreement of the reduced model and CKA-EUTERPE in terms of profile flattening over a wide radial range
- some disagreement in the core of the plasma
- choice of D_{turb} has effect on solution in regions where $D_{\mathrm{AE}} = 0$ (no mode)
- Is the agreement just coincidence? What happens when the mode at the edge is removed from the simulations?

CKA-EUTERPE comparison II



- same case as before, but now without the mode at the edge
- still very good agreement of both models in the middle of the radial domain
- slightly stronger deviations now close to $r/a = 1$ where the simplified model underestimates the transport (due to the lack of a mode there) and shows profile steepening
- width of the "middle" mode from CKA larger than in transport model $\rightarrow$ explains stronger profile flattening at the edge in the CKA-EUTERPE case

Summary and conculsions

- new reduced model for AE-induced fast-ion transport in stellarators has been developed
- model based on local stability calculation (LGRO), mixing-length estimate for D, and a radial diffusion equation with realistic fast-ion source profile
- AEs cause flattening of the fast-ion density profile
- not a critical gradient model $\rightarrow \gamma > 0$ still for flattened profile $\rightarrow$ transport balanced by fast-ion source
- as reduced model: comes with free parameters (e.g. $\bar{D}, \bar{\Delta}$) $\rightarrow$ comparison to more complete models necessary to estimate validity
- sucessful benchmark with CKA-EUTERPE in tokamak geometry for just a few modes
- currently in progress: applying the model to a stellarator reactor with α-particles

Back-up slides

All equations of the stability part

$$\gamma = \left(\frac{A}{\sqrt{s}} \frac{1}{|m\iota + n|} \right)^2 \frac{3\pi}{64} \frac{\beta_\star}{\int_0^\infty \mathrm{d}u\, f_{\mathrm{press}}} g \tag{16}$$

$$g = \sum_{\text{all } w} |w|\, \mu^2 \epsilon^2_{\mu,\nu} \int_{|w|}^\infty \mathrm{d}u\, f_{\mathrm{grow}} \tag{17}$$

$$w = \left[\left(1 \pm \frac{\mu\iota_\star + \nu N_{\mathrm{p}}}{m\iota + n} \right) \frac{v_{\mathrm{th},\star}}{v_{\mathrm{A},\star}} \right]^{-1} \tag{18}$$

$$f_{\mathrm{grow}} = 4\pi \left(u^2 + w^2 \right)^2 \left(\frac{\omega}{2} \frac{\partial F}{\partial u} + u\omega_\star \frac{\partial F}{\partial s} \right) \tag{19}$$

$$f_{\mathrm{norm}} = 4\pi u^2 F \tag{20}$$

$$f_{\mathrm{press}} = 4\pi u^4 F \tag{21}$$

$$\beta_\star = 2\mu_0 \frac{m_{\mathrm{f}} n_{\mathrm{f}}}{3B^2} v^2_{\mathrm{th},\star} \frac{\int_0^\infty \mathrm{d}u\, f_{\mathrm{press}}}{\int_0^\infty \mathrm{d}u\, f_{\mathrm{norm}}} \tag{22}$$

$$\omega_\star = -\left(m + \frac{\mu}{2} \right) \left(\frac{1}{2} \frac{m_{\mathrm{f}}}{Z_{\mathrm{f}} \mathrm{e}} v^2_{\mathrm{th},\star} \frac{2\pi}{F'_{\mathrm{T}}} \right) \tag{23}$$