Self-consistent, global, neoclassical radial-electric-field calculations of electron-ion-root transitions in the W7-X stellarator

Paper rehearsal

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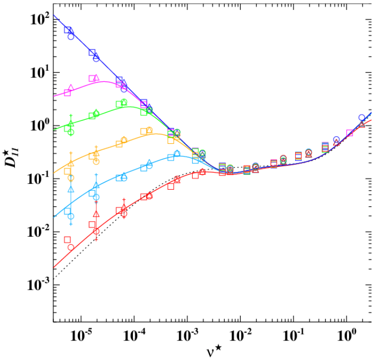
Review of the local theory 1/4

Calculate numerically the mono-energetic transport coefficients, (for example using DKES).

$$D_{11} = D_{11}\left(r, \frac{\nu_\alpha}{v_\alpha}, \frac{E_r}{v_\alpha B_0}\right)$$



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Review of the local theory 2/4

For given n and T profiles, calculate the particle fluxes as a function of the unknown E_r

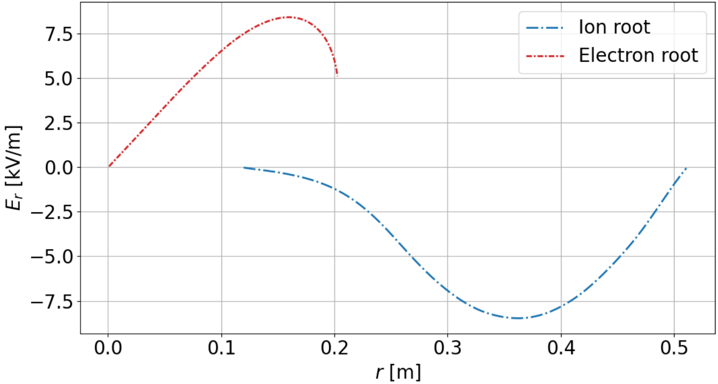
$$\Gamma = -nL_{11}\left(\frac{n'}{n} - \frac{ZeE_r}{T} - \frac{3}{2}\frac{T'}{T}\right) - nL_{12}\frac{T'}{T}, \tag{1}$$

$$L_{1j} = \frac{2}{\sqrt{\pi}}\int_0^\infty K^{3/2-j}e^{-K}D_{11}\mathrm{d}K, \tag{2}$$

with $i = 1, 2$, $\delta_{12} = L_{12}/L_{11}$. E_r is then found by satisfying the ambipolarity condition on each flux surface.

$$\Gamma_e = Z_i\Gamma_i. \tag{3}$$

Review of the local theory 3/4



- In the transition region, **multiple solutions exist**.

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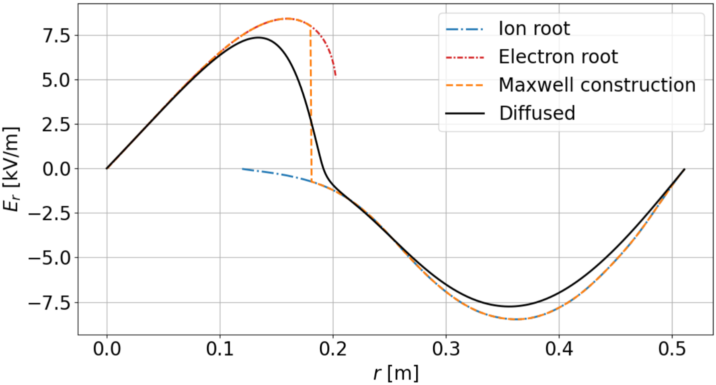
Review of the local theory 4/4



- Maxwell construction results in an **infinitely steep transition**.
- Diffusion model includes a **free parameter**.

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Comparison between local and global theories

Drift kinetic equation solved in EUTERPE:

$$\left(\frac{\partial}{\partial t} + \dot{\boldsymbol{R}}^0 \cdot \frac{\partial}{\partial \mathbf{R}} + \dot{v}_{\parallel}^0 \frac{\partial}{\partial v_{\parallel}}\right) \delta f - \nu \mathcal{L}(\delta f) = -\left(\dot{\boldsymbol{R}}^1 \cdot \frac{\partial}{\partial \mathbf{R}} + \dot{v}_{\parallel}^1 \frac{\partial}{\partial v_{\parallel}}\right) f_0$$

Transport	E_r *calculation*	$\dot{\boldsymbol{R}}^1$	$\dot{\boldsymbol{R}}^0$
Local	Ambipolarity	$\mathbf{v}_{\parallel} + \mathbf{v}_{\mathbf{B}} + \mathbf{v}_{\boldsymbol{E}_r \times \mathbf{B}}$	$\mathbf{v}_{\parallel} + \mathbf{v}_{\boldsymbol{E}_r \times \mathbf{B}}$
Global	Quasi-neutrality eq.	$\mathbf{v}_{\parallel} + \mathbf{v}_{\mathbf{B}} + \mathbf{v}_{\boldsymbol{E} \times \mathbf{B}}$	$\mathbf{v}_{\parallel} + \mathbf{v}_{\mathbf{B}} + \mathbf{v}_{\boldsymbol{E} \times \mathbf{B}}$

	$\dot{v}_{\parallel}^1, \dot{v}_{\parallel}^0$
...	$a_{\mathbf{B}} + a_{\boldsymbol{E}_r \times \mathbf{B}}$
...	$a_{\mathbf{B}} + a_{\boldsymbol{E} \times \mathbf{B}}$

- $\mathbf{v}_{\mathbf{B}}$ drift acts on δf.
- Includes poloidal and toroidal components of the electric potential.

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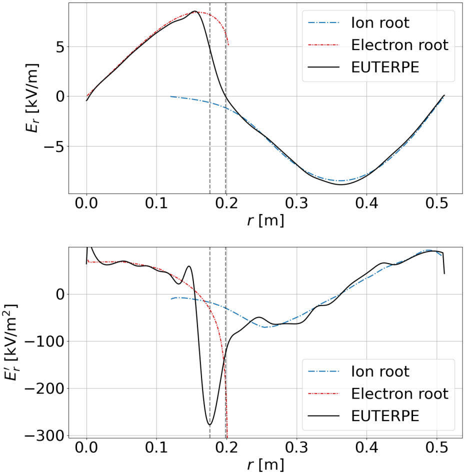
Self-consistent global simulations

Benchmarking of the global, neoclassical radial electric field calculated with EUTERPE against the local neoclassical code Neotransp for the W7-X standard configuration.



Vertical lines denote the inflection point of E_r, r_i, and its zero, r_0 correspondingly.

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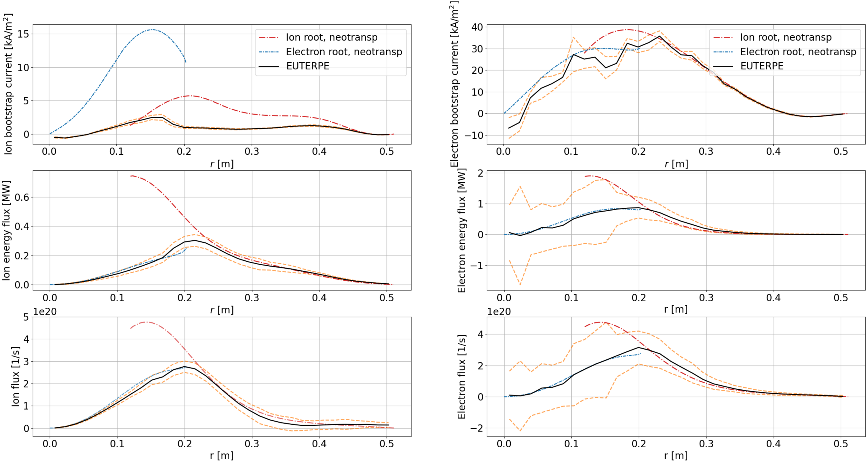
Neoclassical fluxes



Orange lines represent 1-σ fluctuations in the time series.

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Study: scaling of the plasma parameters

The system depends on three dimensionless plasma parameters.

$$\nu_i^* = \frac{a\nu_{\mathrm{th},i}}{v_{\mathrm{th},i}}, \quad \rho_i^* = \frac{v_{\mathrm{th},i}}{\Omega_i a}, \quad \tau = \frac{T_e}{T_i}, \tag{4}$$

Explore the solution space by changing one parameter, while keeping others fixed. Example, τ scaling:

$$T_i = T_{\mathrm{exp},i}, \quad n = n_{\mathrm{exp}}, \quad T_e = \alpha_\tau \left(T_{\mathrm{exp},e} - T_{\mathrm{exp},i} \right) + T_{\mathrm{exp},i}. \tag{5}$$

Local theory expectations

Necessary (but not sufficient!) requirement for electron roots existence:

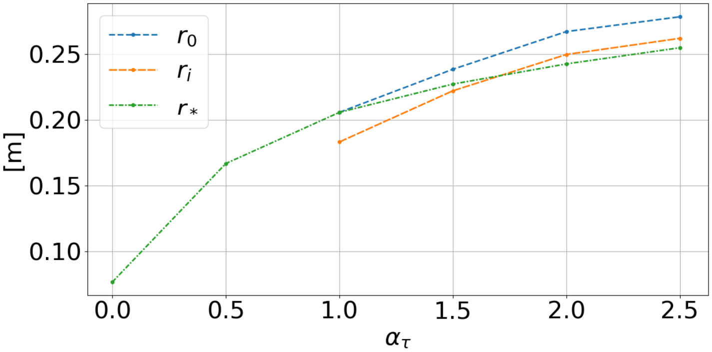
$$\frac{L_{11,e}^{1/\nu}}{L_{11,i}^{\sqrt{\nu}}} > 1 \tag{6}$$

It follows that:

$$\tau > \tau_* \sim \mu_{i/e}^{1/7} \left(\frac{r}{a} \frac{\nu_i^*}{\epsilon_{\text{eff}} \rho_i^*} \right)^{3/7} \left(\frac{b_{10}}{\epsilon_t} \right)^{4/7} . \tag{7}$$

The unique radial location that satisfies $\tau = c\tau_*, c \in \mathbb{R}$ is denoted as r_*

τ scaling of transition position



$$T_i = T_{\exp,i}, \quad n = n_{\exp}, \quad T_e = \alpha_\tau \left(T_{\exp,e} - T_{\exp,i} \right) + T_{\exp,i}. \quad (8)$$

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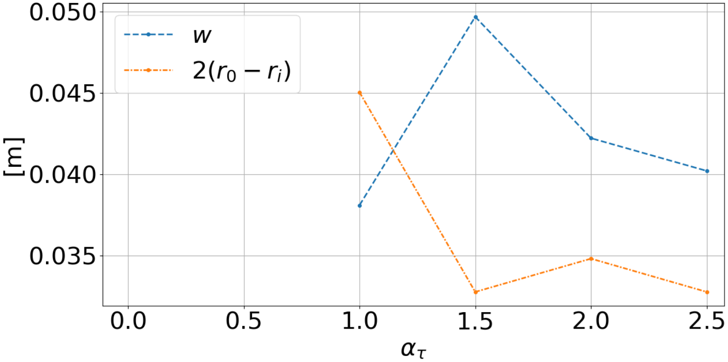
τ scaling of transition region length



$$T_i = T_{\exp,i}, \quad n = n_{\exp}, \quad T_e = \alpha_\tau \left(T_{\exp,e} - T_{\exp,i} \right) + T_{\exp,i}. \quad (9)$$

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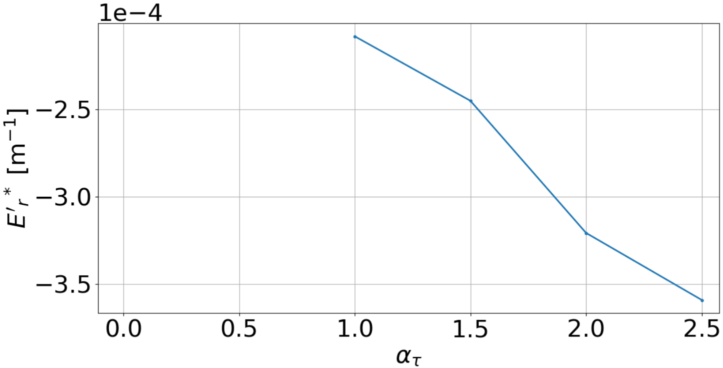
τ scaling of maximum value of the electric field derivative



$$T_i = T_{\exp,i}, \quad n = n_{\exp}, \quad T_e = \alpha_\tau \left(T_{\exp,e} - T_{\exp,i} \right) + T_{\exp,i}. \quad (10)$$

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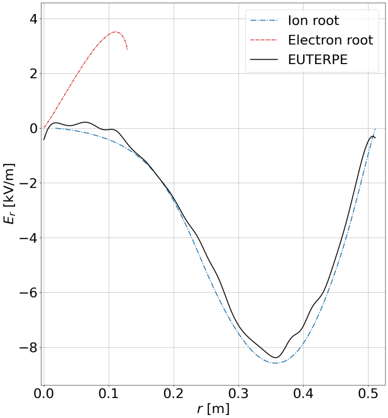
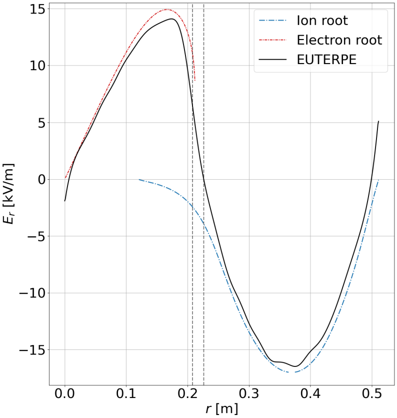
Phenomena unexplained by the local theory



(a) Transition does not appear



(b) Transition outside of the overlap

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Summary

1. EUTERPE is now capable of calculating the global, radial electric field self-consistently.
2. Scaling behaviour of the transition location is understood qualitatively via the derived criterion (r_*).
3. The code is a viable tool in understanding phenomena outside of the scope of the local neoclassical transport theory.

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Outlook

1. Global, non-linear turbulence simulations with the self-consistent, global, neoclassical electric field.
2. Combining neoclassical and turbulent simulations in one.
3. Simulations of the 3D structure of the electric field with magnetic islands.

ITG simulations without E_r

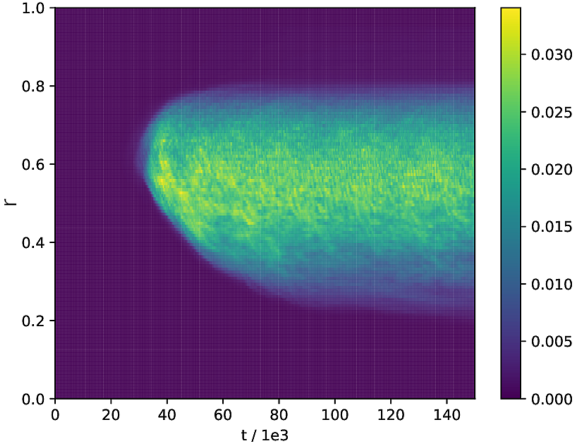

Figure: 1.2 gyro-Bohm units

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ITG simulations with E_r

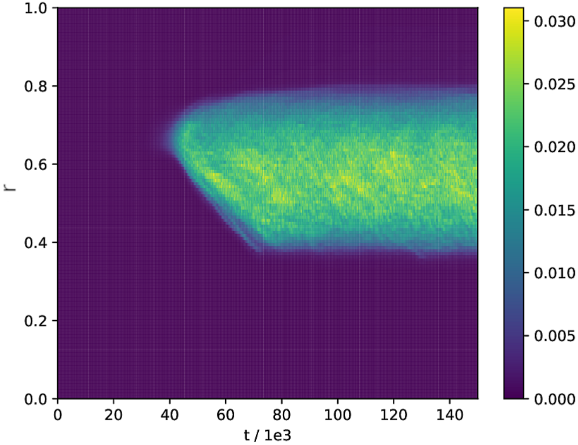

Figure: 1.09 gyro-Bohm units

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