Fluid turbulence simulations of stellarators with GRILLIX

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Outline

Technical background:

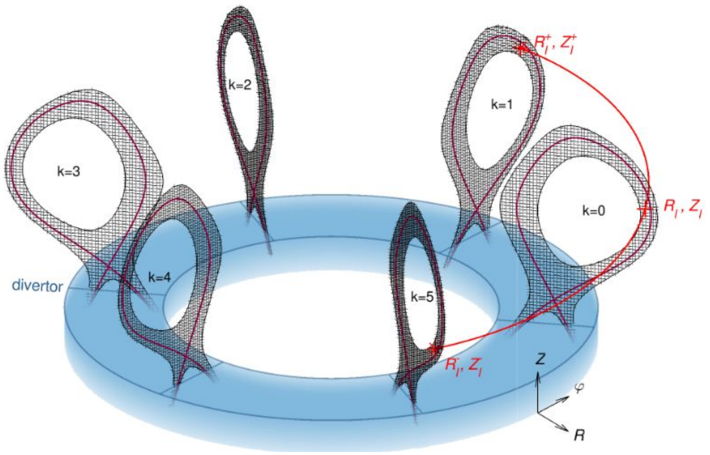
- ❑ Modifications of GRILLIX for 3D geometries
- ❑ Verifications

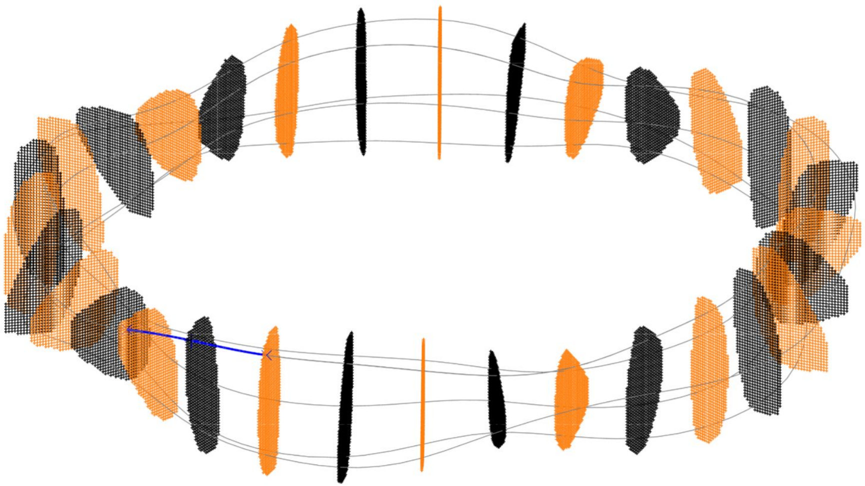
Applications

- ❑ Magnetic island studies
- ❑ Wendelstein 7-AS

Conclusions / Discussion

FCI: Axisymmetric to 3D



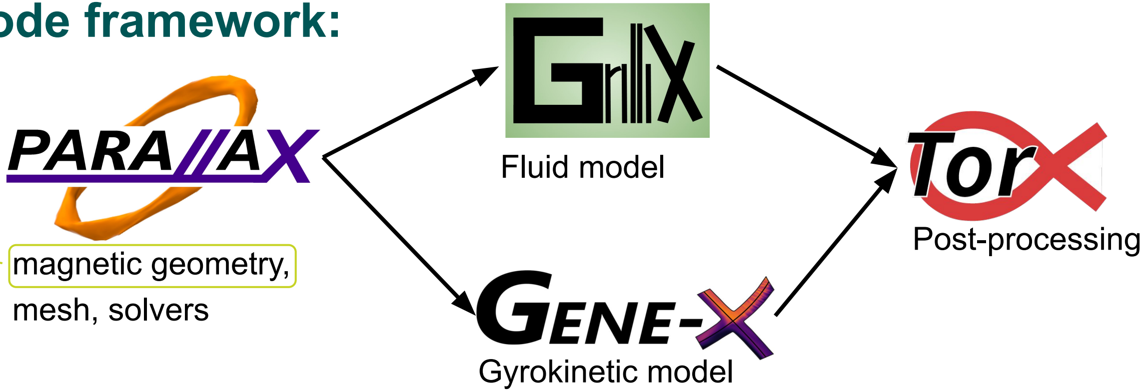


Generalisation of FCI to 3D

- FCI is not limited to axisymmetric geometries ($B_{tor} >> B_{pol}$)
  [B. Shanahan et al. doi:10.48550/arXiv.2403.18220, 2024]
- Efficient for highly anisotropic conditions ($k_{\perp} >> k_{||}$)
- Mostly technical modifications to code are necessary

Technical adaptions

Code framework:



[D. Michels et al., 10.1016/j.cpc.2021.107986 2021]

```
type(mesh_t), target :: mesh_cano
type(mesh_t), pointer :: mesh_stag
...
dphi = 2 * PI / nprocs
phi_cano = rank * dphi
phi_stag = phi_cano + 0.5 * dphi

call mesh_cano%create(phi_cano)

if (is_axisymmetric) then
    mesh_stag => mesh_cano
else
    allocate(mesh_stag)
    call mesh_stag%create(phi_stag)
endif
```

Modifications

- ❑ Extend interfaces to 3d: (R, Z) → (R, phi, Z)
- ❑ Implementation of Dommaschk configurations
  [W. Dommaschk, 10.1016/0010-4655(86)90109-8, 1986]
- ❑ Each MPI process holds separate mesh and staggered mesh

→ No special code branch for 3D geometries, almost all features readily available for 3D

MMS verification

Method of manufactures solutions for 3D geometry

- ❑ In rotating (close to) elliptical geometry (Dommaschk)
- ❑ Full model (Plasma + Neutrals)
- ❑ Export of source expressions via Mathematica

→ **Expected Convergence order achieved**

	$O(16 \rightarrow 32)$	$O(32 \rightarrow 64)$	$O(64 \rightarrow 128)$
n	2.0191	2.0730	1.9560
ϕ	2.2233	1.9749	1.9541
T_e	2.3163	2.2726	1.9956
T_i	2.0075	2.0183	1.9710

Can we resolve the magnetic geometry ?

Analysis of numerical diffusion

- ❑ Mimetic finite difference method drastically reduce numerical diffusion.

  [A. Stegmeir, 10.1016/j.cpc.2015.09.016, 2016]

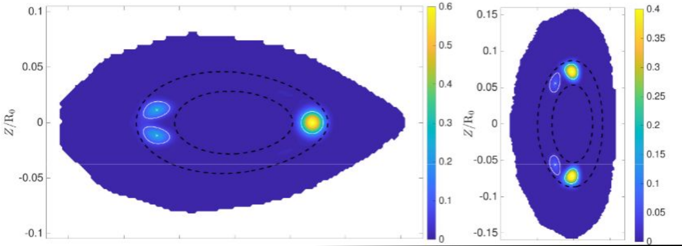
  Does this also hold true for 3D geometries?
- ❑ Consider anisotropic parallel heat equation *($\epsilon_\perp$<<1)*

$$\partial_t u = \nabla \cdot (\mathbf{b}_0 \nabla_{\|,0} u) + \epsilon_\perp \nabla_\perp^2 u$$

→If not sufficiently suppressed, numerical diffusion leads to spurious perpendicular broadening of structures.

Mimetic finite differences:

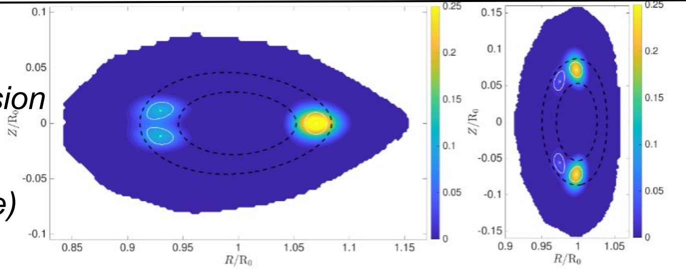
Blob footprints coincide with traced field lines



Direct discretisation:

Numerical perpendicular diffusion visible

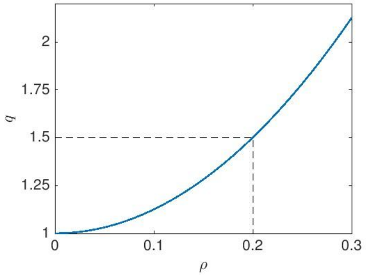
(also note different colourscale)

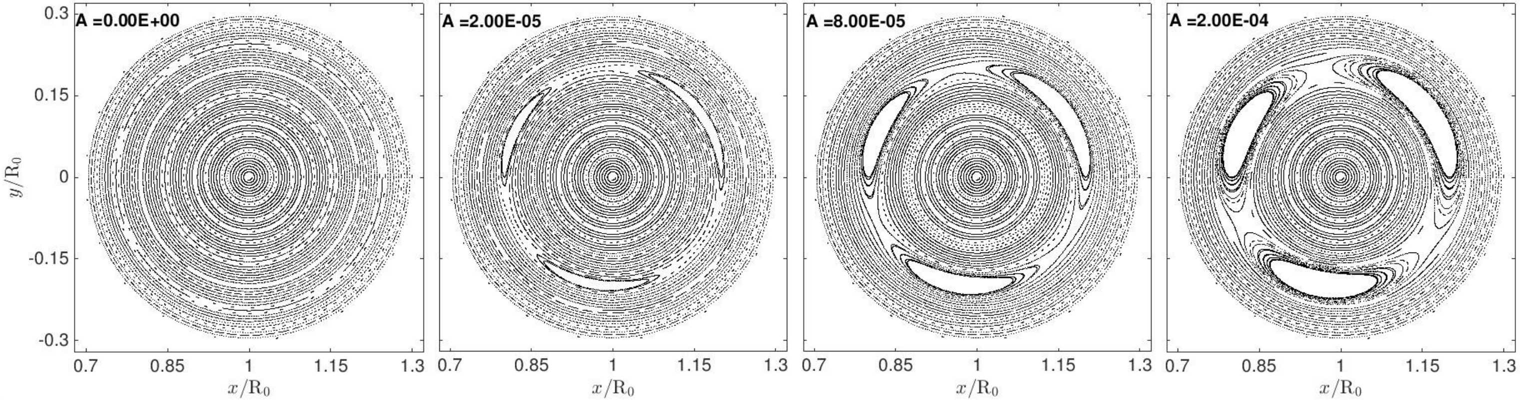


Application 1: Magnetic islands

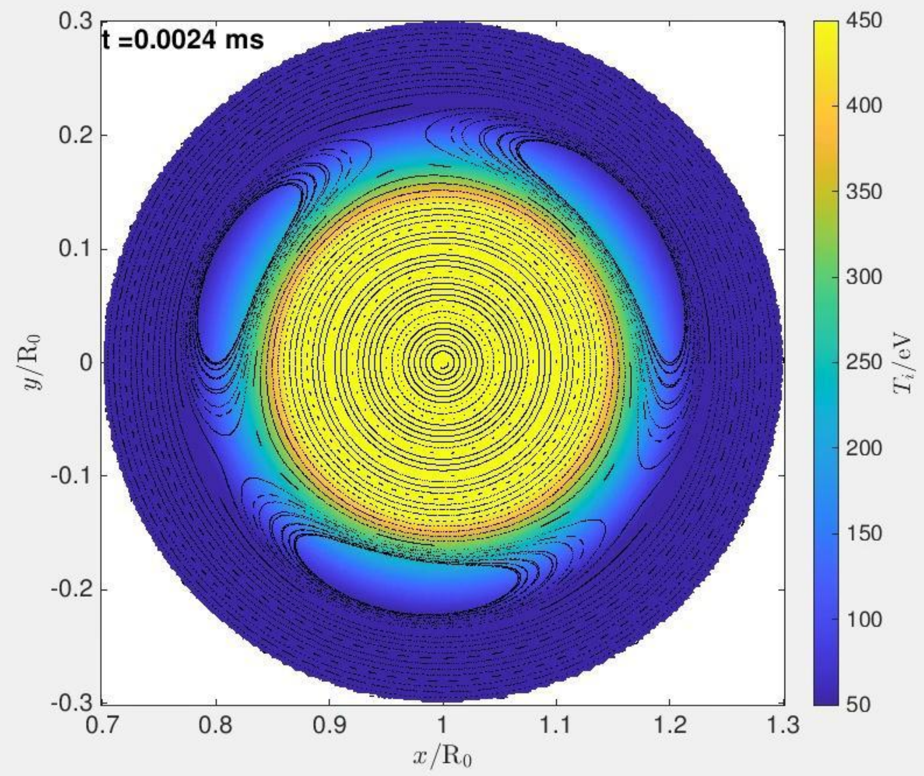
Setup

- Impose magnetic island to circular toroidal flux surfaces: $\psi = \psi_\circ + f(\rho)\sin(m\eta - n\varphi)$
- No target boundaries, fix density/temperatures at outermost flux surface, maintain profiles near magnetic axis
- R_0=1.65m, a=0.5m, B_0=2.5T, $n_0=2\cdot10^{19}m^{-3}$, $T_{e0}= T_{i0}$=750eV, $m_i/ m_p = 2$

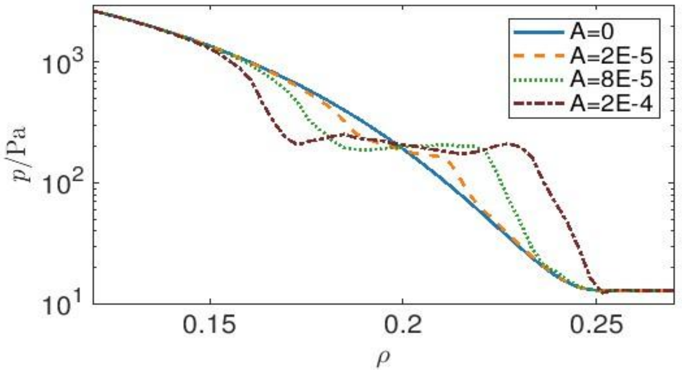




Application 1: Magnetic islands



Pressure profile through magnetic island for various island sizes



- ❑ Time-scales for flattening consistent with theoretical predictions t ∝ $(m/m_e)^{1/2}$ / w *[R. Fitzpatrick, doi.org/10.1063/1.87143, 1995]*
- ❑ Enhanced turbulence around edges of islands due to strong gradients

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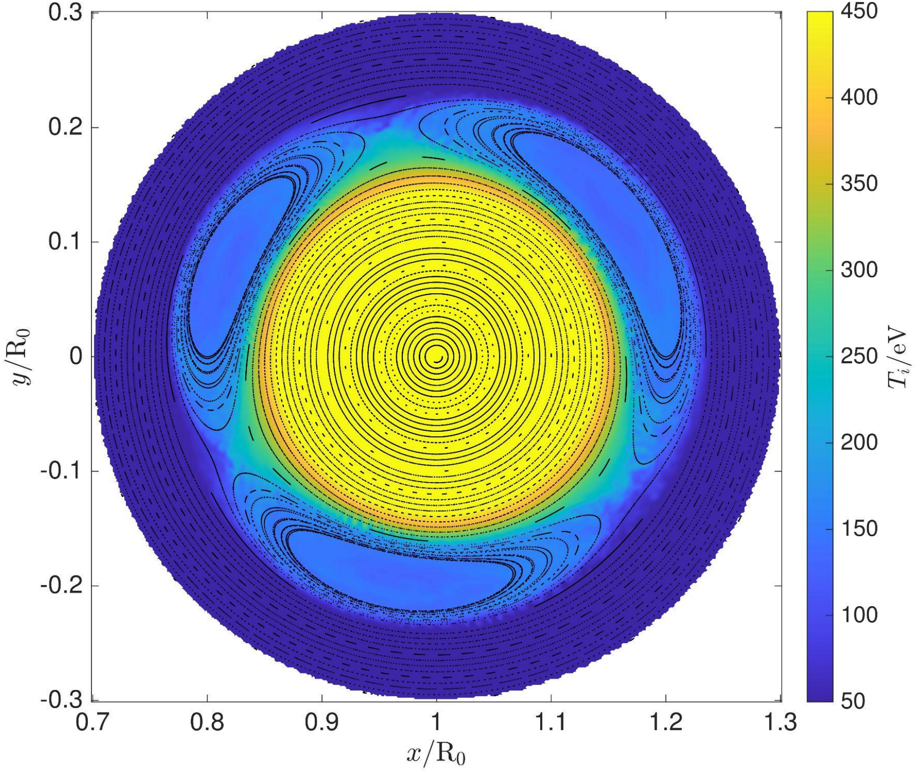

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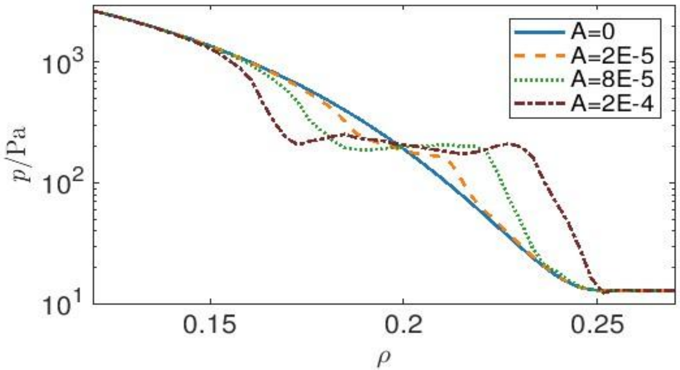

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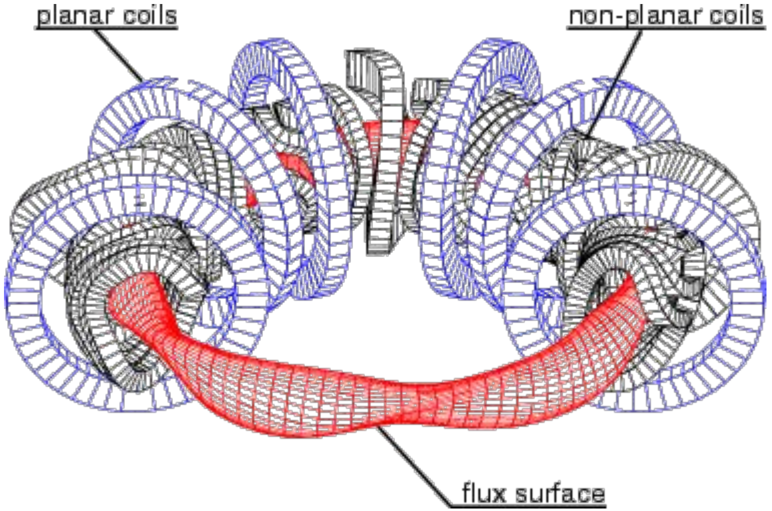
Application 2: Wendelstein 7-AS, Overview



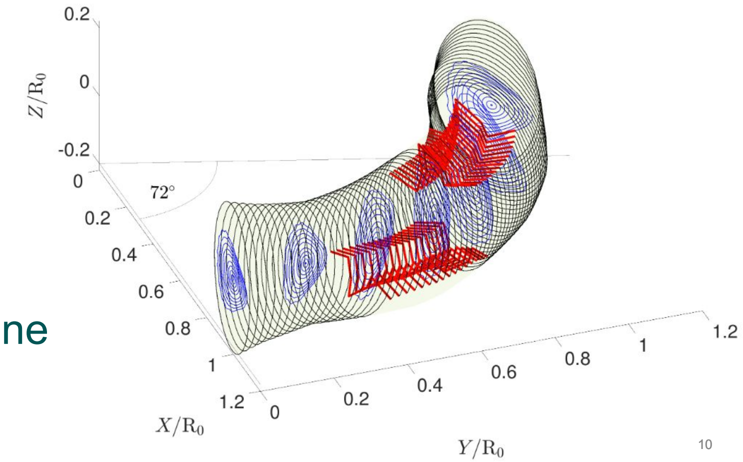
Wendelstein 7-AS:

[M. Hirsch et al., 10.1088/0741-3335/50/5/053001, 2008]

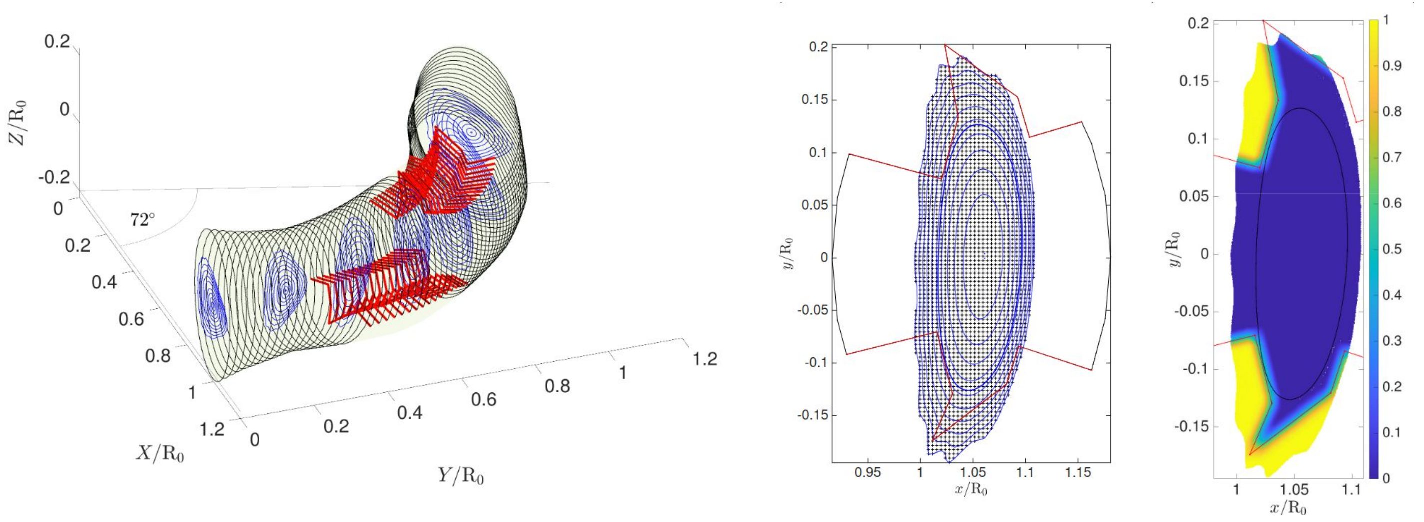
- Operated between 1988 - 2002
- Stellarator with 5 fold periodicity
- Various limiter concepts and tests of first island divertor

Modelling in GRILLIX:

- Fit suitable Dommaschk coefficients
- Target plates prescribed via polygons on each plane



Wendelstein 7-AS: Boundaries



Immersed boundaries for segmented targets:

- Conceptually same approach as for tokamaks: $\partial_t f = (1-\chi)\,F + \frac{\chi}{\epsilon}\,(f_{bnd} - f)$
- Smoothing done in ad-hoc fashion
- Sufficient number of planes are required to resolve boundaries

Wendelstein 7-AS: Simulation details

Further setup

- ❑ Still simple geometry (no major island divertors etc.)
- ❑ Realistic parameters and full torus

  R_0=2m, B_0=2.5T, n_0=5·$10^{19}$$m^{-3}$, T_{e0}= T_{i0}=750eV, m_i/ m_p = 2
- ❑ Full blown model:

  full-f, electromagnetic, no Boussinesq approximation, trans-collisional extensions,....

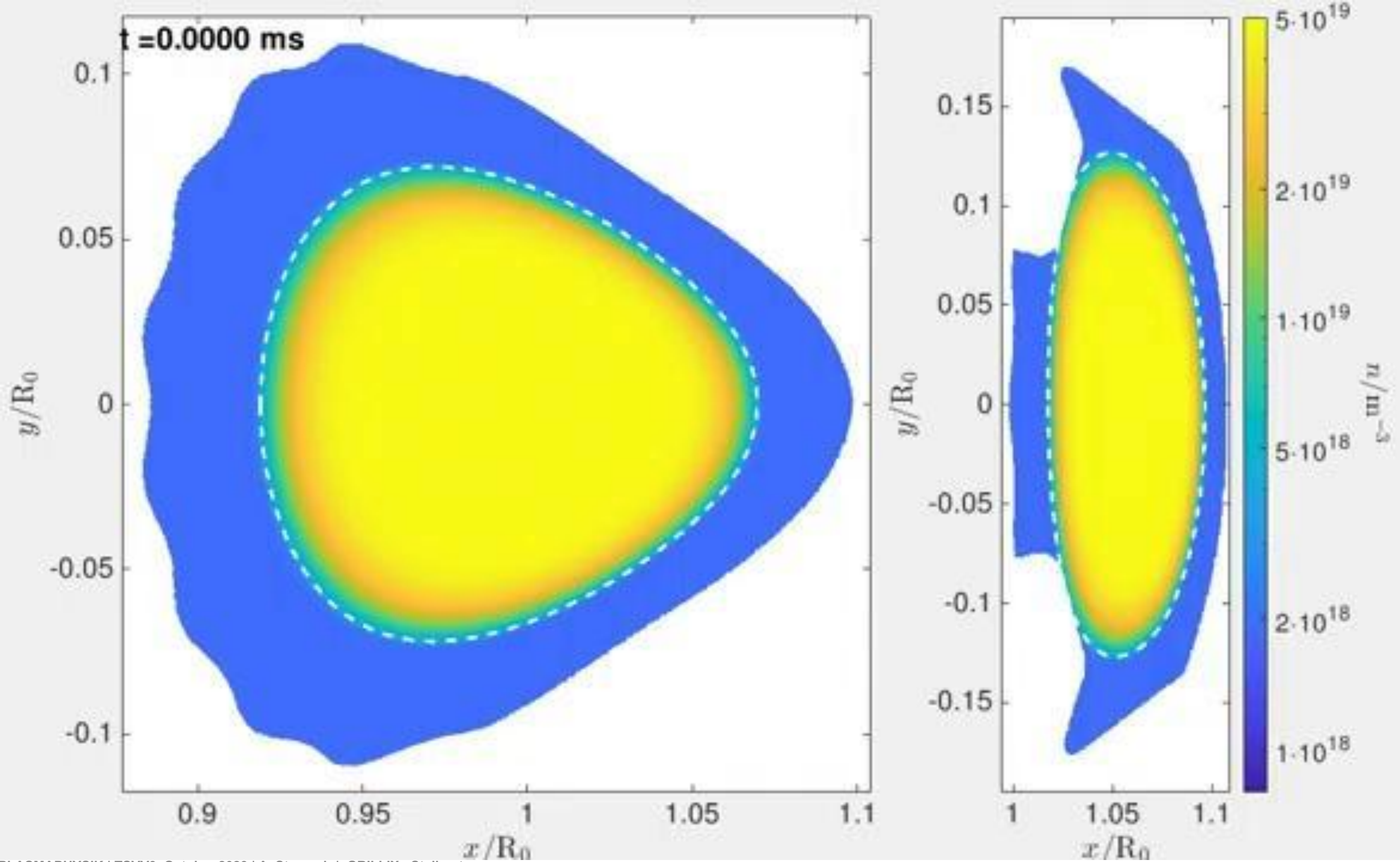
  [W. Zholobenko, 10.1088/1741-4326/ad7611, 2024]
- ❑ Running with fluid neutral model (N=const. at boundaries)
- ❑ No external power sources → decaying state

Computational background

- ❑ Mesh points ~250.000 x 128
- ❑ Marconi SKL (only CPUs)

	8 nodes (386 cores)	32 nodes (1544 cores)
time / timestep	3 s	1 s
time / millisec	60 days	20 days
cpuh / millisec	0.5E6	0.75E6

Wendelstein 7-AS: Simulation results



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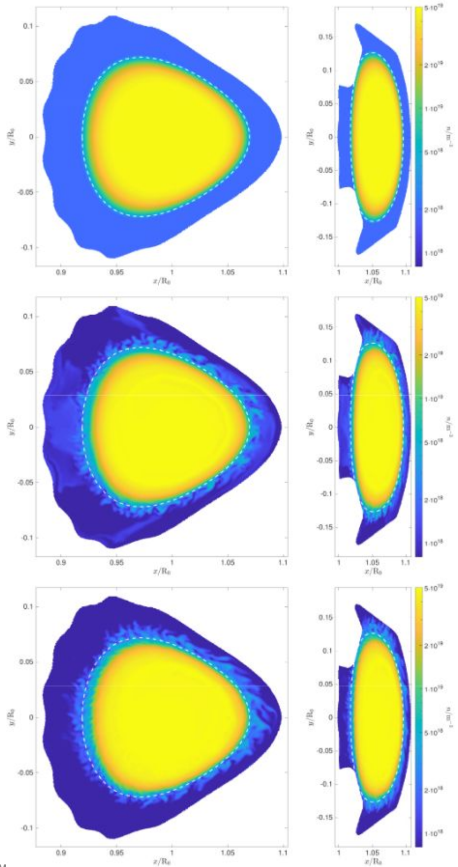


Observations

- Very initial phase: Plasma column slightly shifts outwards
  Shafranov Shift:

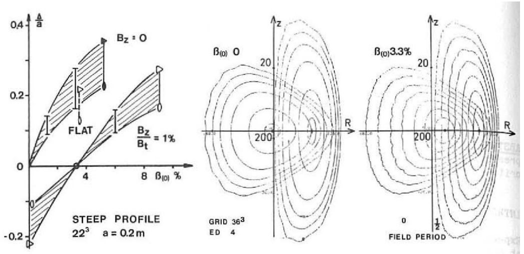
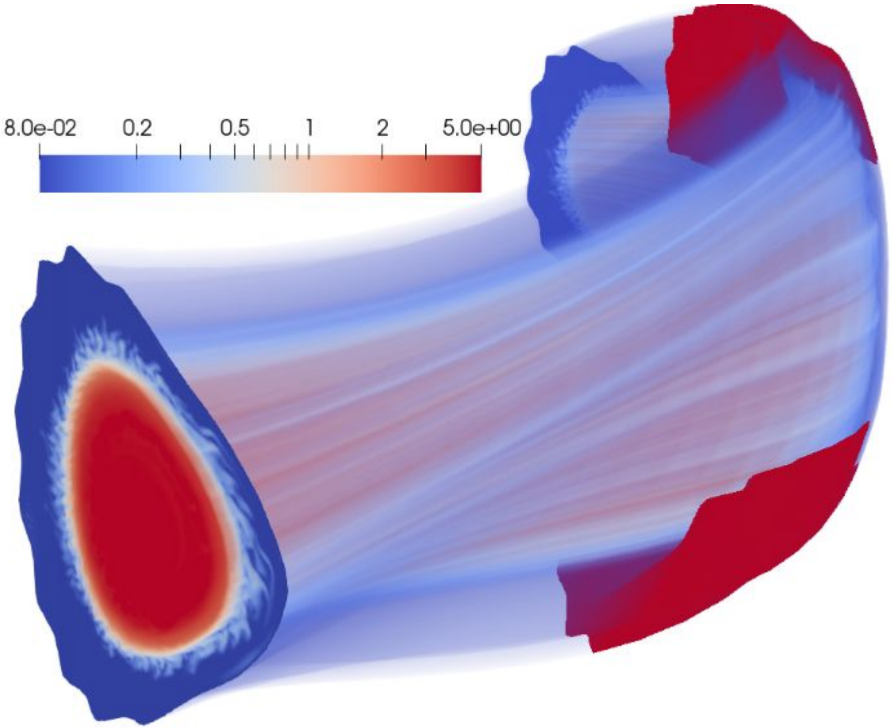


Fig.1 SHAFRANOV shift in W VII-AS for the "standard configuration" as well as for $B_z / B_t = 1$ % (grid 22), and magnetic surfaces at $\beta(o) = 0$ and 3.3 % (grid 36).

[R. Chodura, 11th EPS, A12, 1983]

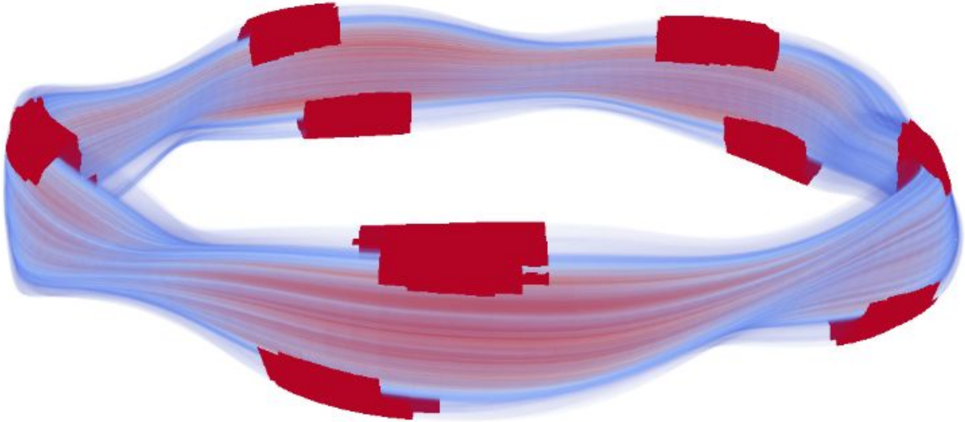
- Plasma is flushed out in open field line region
- Small scale turbulence and blobs are observed, no strong asymmetry between inboard and outboard
- No large scale mode so far, as observed by GBS
  [A.J. Coelho, 10.1088/1741-4326/ad4ef5, 2024]
- Rotates counter-clockwise

Wendelstein 7-AS: Simulation results



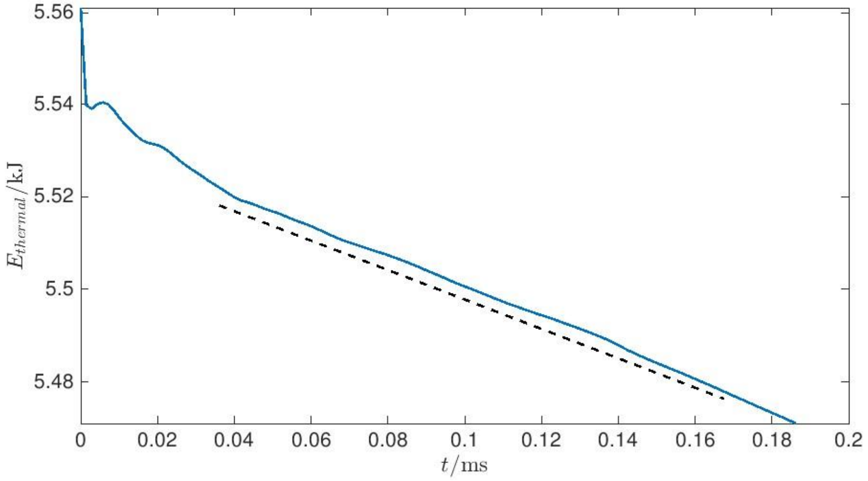
...

- Strongly elongated along magnetic field lines



Wendelstein 7-AS: Confinement time

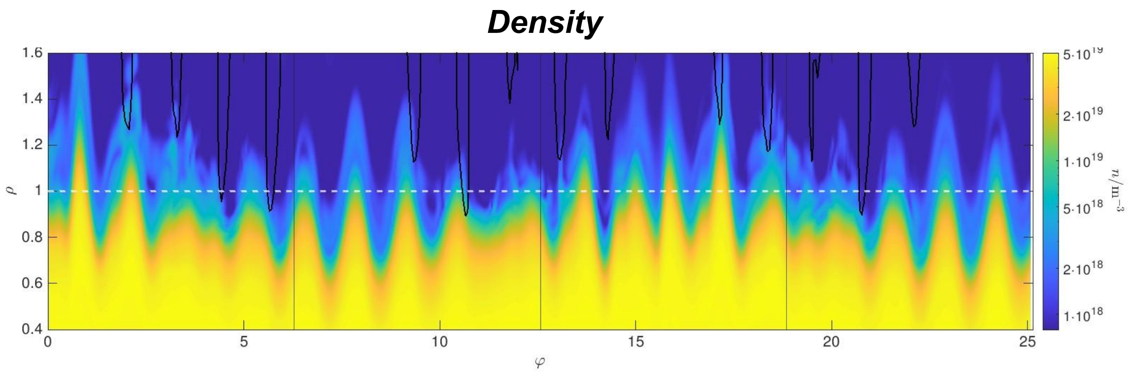
No power sources → Stored plasma energy decays

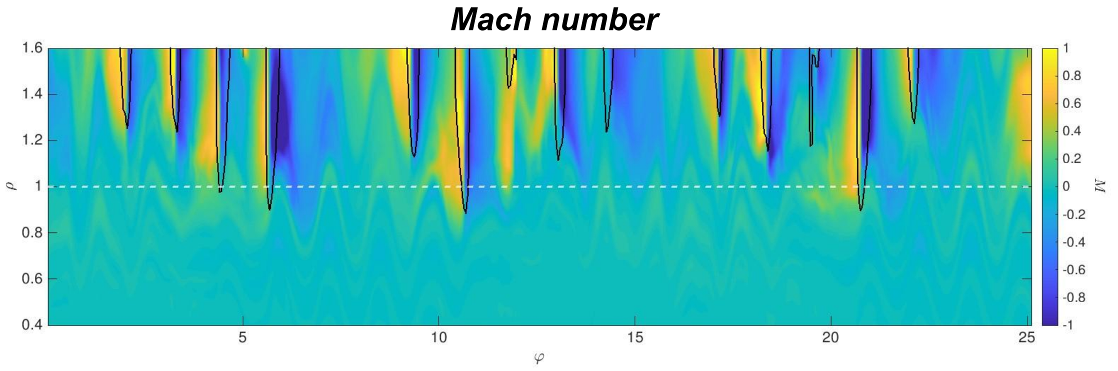


Estimate for confinement time: t_c = 17.4 ms

Wendelstein 7-AS: View along field lines

View along field aligned ribbon





- ❑ Large scale bulges due to Shafranov shift
- ❑ Mode with parallel mode number of 5 matching discrete toroidal symmetry of W7-AS
- ❑ Mach numbers approach ±1 in penalisation region, Secondary SOLs are apparent
- ❑ Penalisation smears into closed field line region

GRILLIX: A unified code for tokamaks and stellarators

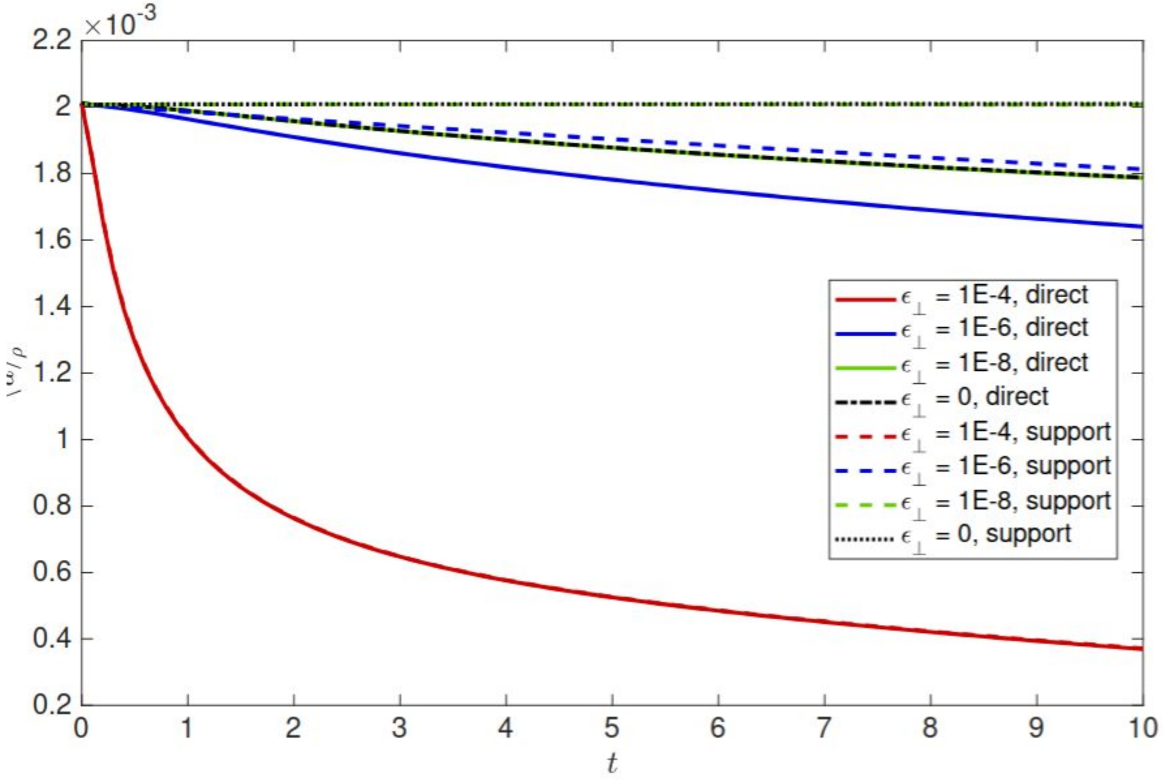
Conclusions

- ❑ Unified extension of GRILLIX and related framework to 3D
- ❑ Numerical tests and verification demonstrate code's capability to resolve 3D geometries
- ❑ Full turbulence model applied to magnetic island and W7AS configuration
  - ❑ Shafranov shift observed, as only vacuum field is prescribed background
  - ❑ No large scale mode so far, as observed by GBS *[A.J. Coelho, 10.1088/1741-4326/ad4ef5, 2024]*
  - ❑ Small scale field-aligned turbulent structures
  - ❑ Parallel mode observed, representing discrete symmetry of W7-AS

Future directions

- ❑ Sources and diagnostics for 3D
- ❑ Further numerical geometries (FLARE, mgrid files) *[H. Frerichs, 10.1088/1741-4326/ad7303, 2024]*
- ❑ Handling of high beta cases (true 3D equilibrium field required)
- ❑ Boundary treatment → Leg value fill approach *[P. Hill et al., 10.1016/j.cpc.2016.11.004, 2017]*

Backup slide: Quantification of numerical diffusion



L1 norm integrated over the volumes enclosed by the flux surfaces indicated by the dashed black lines in Fig on slide 6 for various values of $\varepsilon_\perp$. The solid coloured lines show the result for the direct scheme and the dashed coloured lines for the support schemes for values of $\varepsilon_\perp > 0$. The red dashed line ($\varepsilon\perp = 1 \cdot 10{-}4$, support) is not visible as it is overlapped by the solid red line $\varepsilon_\perp = 1 \cdot 10{-}4$, direct)