INSTITUTE OF PLASMA PHYSICS
OF THE CZECH ACADEMY OF SCIENCES

Some updates on plasma boundary conditions at divertor and limiter surfaces

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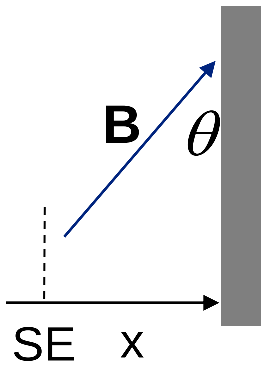
MINISTRY OF EDUCATION, YOUTH AND SPORTS

Boundary conditions at the divertors / limiters



BC for	1D				2D	
	Single ion	Multi-ion	Collisional	Multi-fraction	Single ion	Multi-ion, -fraction / collisional
ϕ	green	yellow	yellow	yellow	yellow	red
$V_{i,\parallel}$	green	yellow	green	yellow	light green	red
$\partial T/\partial x$, or $q_{\parallel}$	green	yellow	red	yellow	red	red
Higher moments, vorticity	red				green	red

[J. Loizu, PoP, 2012]

- green: BC exist and relatively easy to implement
- yellow: BC exist, but hard to implement (contains strong gradients, or code becomes unstable)
- red: BC does not exist
- light green: Newly proposed BC

Multi-fraction ~ time dependent ($\tau_{sheath} << \tau_{other}$)

1D Boundary conditions (classical)

Multi-positive-ion-component plasma sheath

$$\Gamma_i = n_i V_i \,, \quad \Gamma_e = I/e - \sum_{i=1}^{N} Z_i \Gamma_i \,, \quad \Delta\phi = \psi \frac{T_e}{e} \quad \psi = \ln\left[\sqrt{\frac{T_e}{2\pi m_e}} \frac{1}{\sum_{i=1}^{N} s_i Z_i V_i - I/en_e}\right],$$

$$Q_e = (2+\psi)\Gamma_e T_e \,, \quad Q_i = \left(2.5T_i + m_i V_i^2/2\right)\Gamma_i \,, \quad i = 1,\ldots,N$$

.

Single-ion plasma sheath

$$V_{\|,i} = C_{s,i} = \sqrt{(T_i + Z_i T_e)/m_i}$$

$$\psi = 2 \div 5$$

$$Q_e = (2+\psi)\Gamma_e T_e$$

$$Q_i = (3T_i/T_e + Z_i/2)\Gamma_i T_e$$

Magnetic sheath entrance

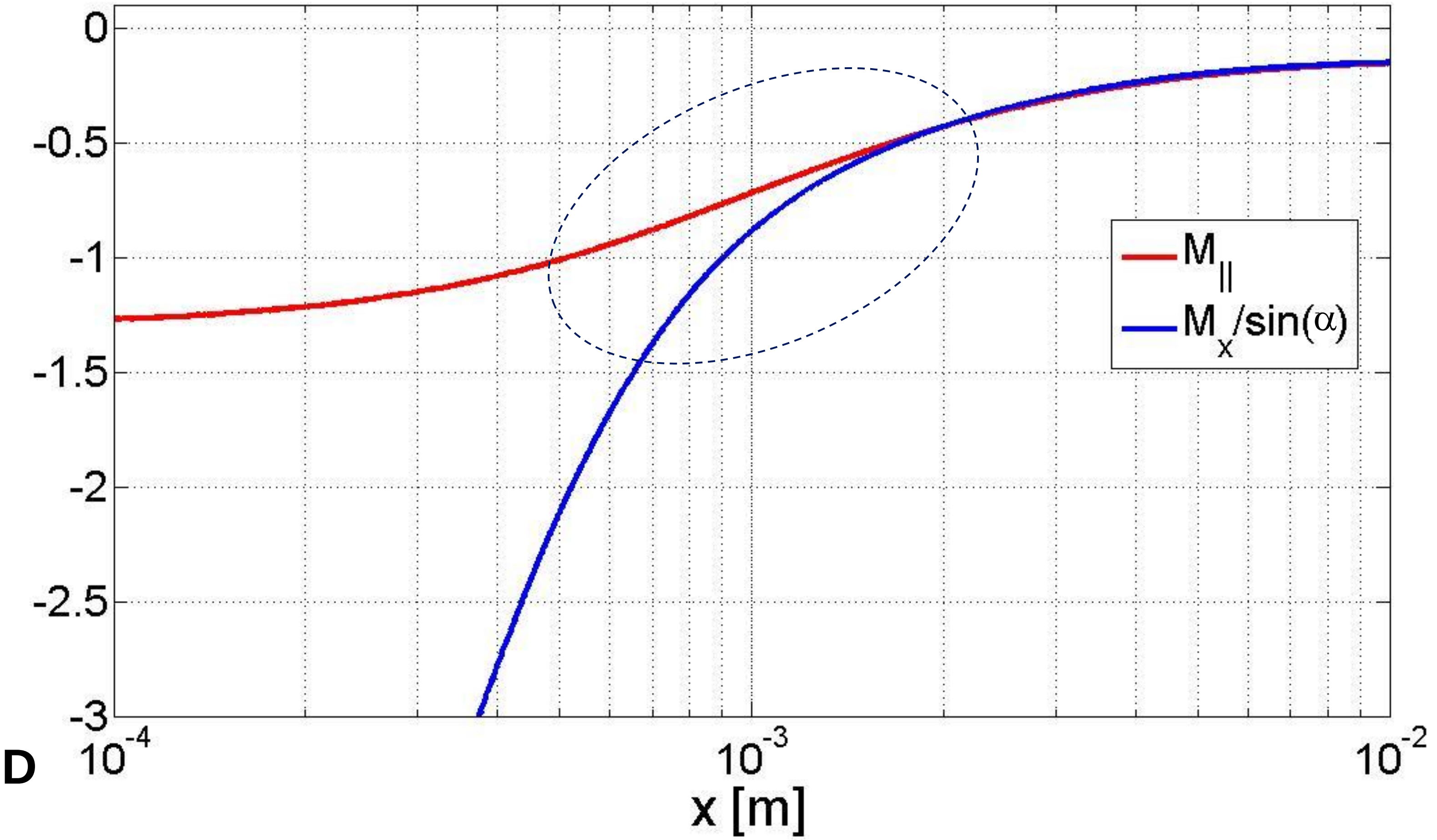
$$1 = T_e \sum_{i=1}^{N} \frac{s_i Z_i^2}{m_i V_i^2 - T_i}$$

$$V_{\|,i} = \sqrt{\left(T_i + Z_i \frac{\partial_x \ln n_e}{\partial_x \ln n_i} T_e\right)/m_i}$$

$$\gamma_i \approx 1$$

[Tskhakaya, JNM 2005]

On definition of the sheath edge



ID

Profiles of parallel and normal Mach numbers at the AUG ID (Ar seeded case)

Sheath edge

$$\left|M_{\parallel}\right| = 1$$

$$M = \frac{V}{c_s}, \quad c_s = \sqrt{\frac{T_e + T_i}{m_i}}$$

Sheath edge is **not fully magnetized**

1D BC for second moments of the VDF

$$q_{\parallel}^{SH} = -\chi_j \frac{\partial}{\partial s} T_j \ , \quad q_{\parallel}^{Sheath} = \gamma_j F_j T_j , \qquad \left| q_{\parallel}^{Convective} \right| << \left| q_{\parallel}^{SH} \right|$$

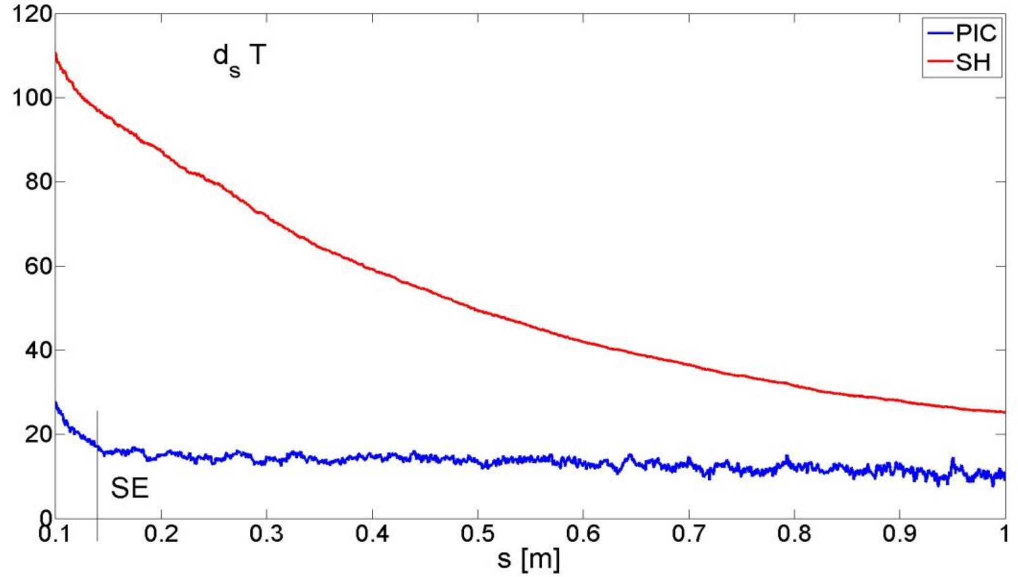
$$\gamma_e \sim 2 + 3 = 5$$

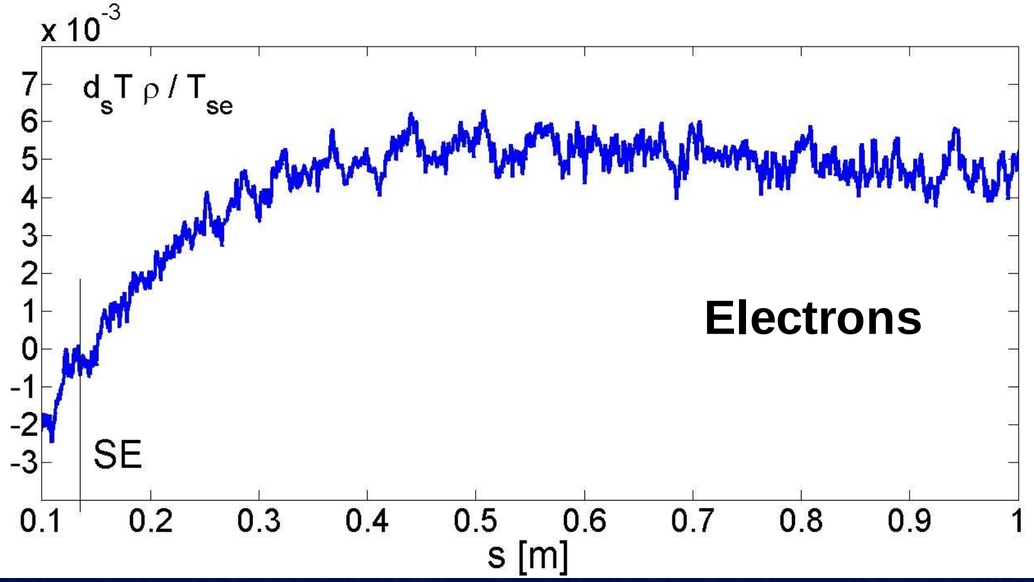
$$\gamma_i \sim 3$$

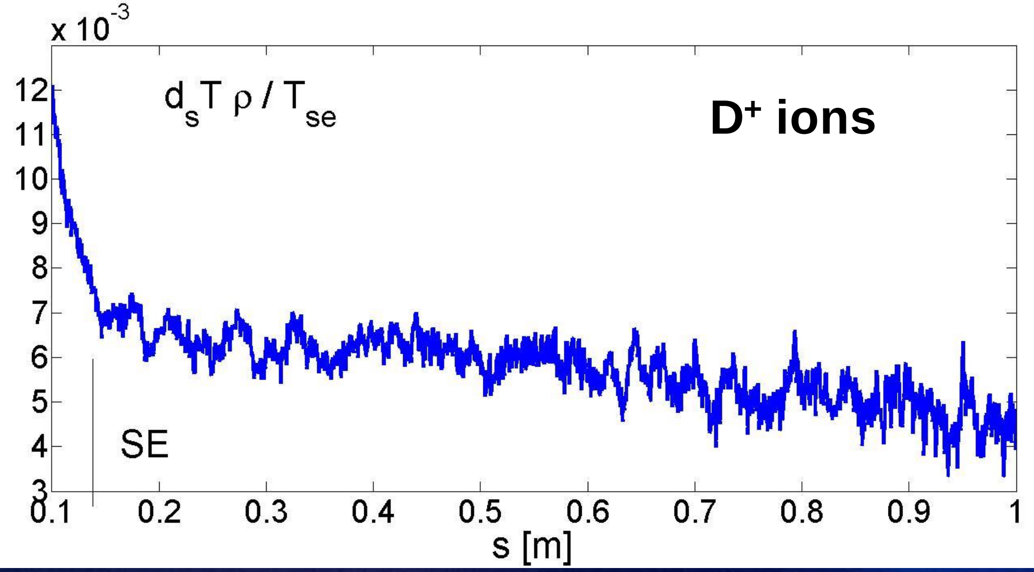
?

$$\frac{1}{T_j}\frac{dT_j}{ds} = -\frac{\gamma_j F_j}{\chi_j}$$

$$\frac{\partial T_j}{\partial s}\frac{\rho_j}{T_j} << 1 \Rightarrow \frac{\partial}{\partial s} T_j \approx 0$$

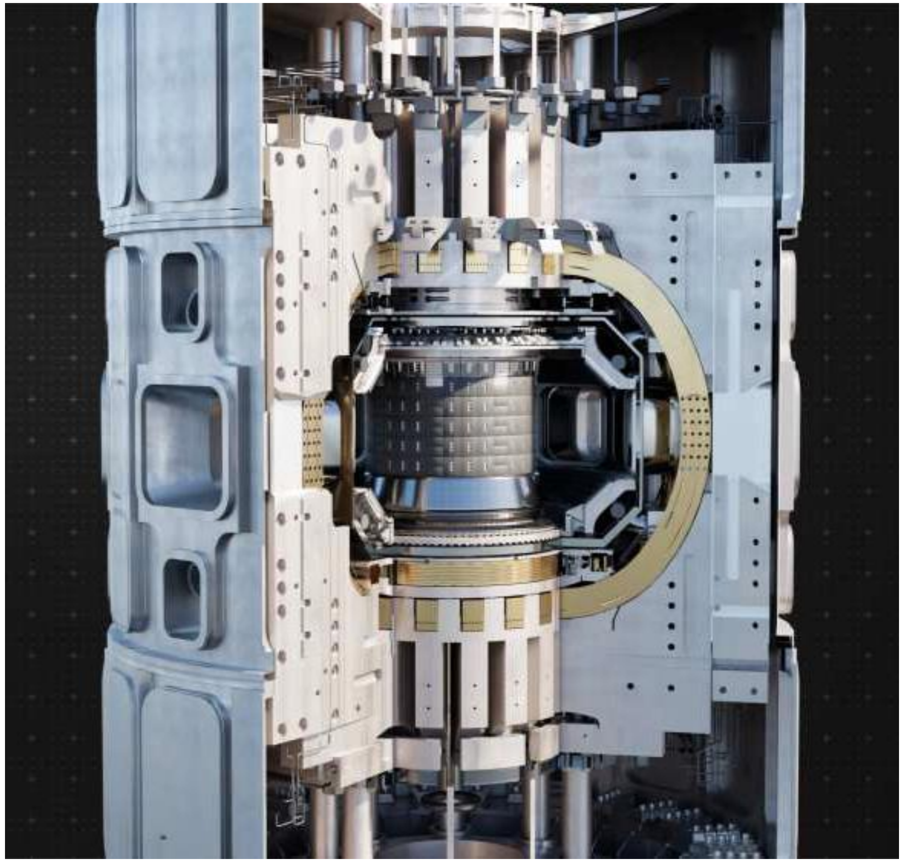






Boundary conditions for the collisional sheath

COMPASS-U



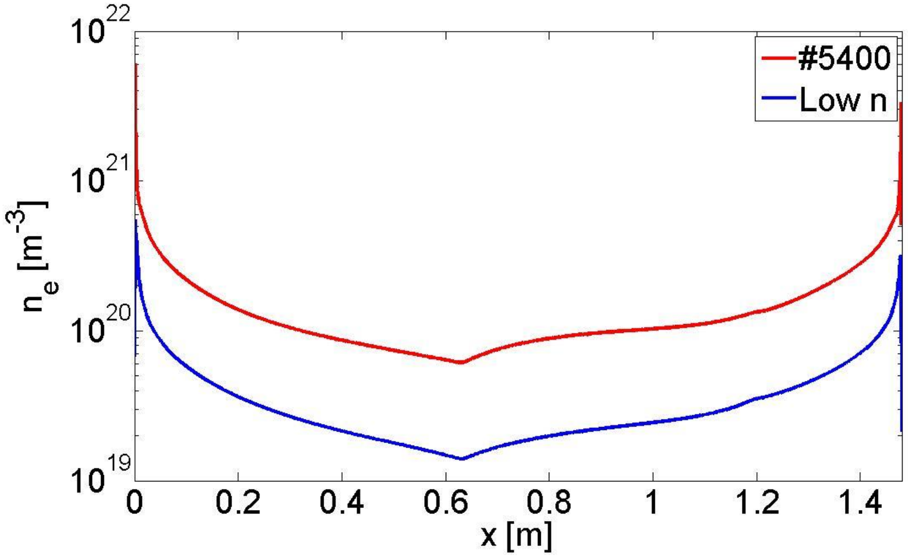
https://www.ipp.cas.cz/Compass_U/

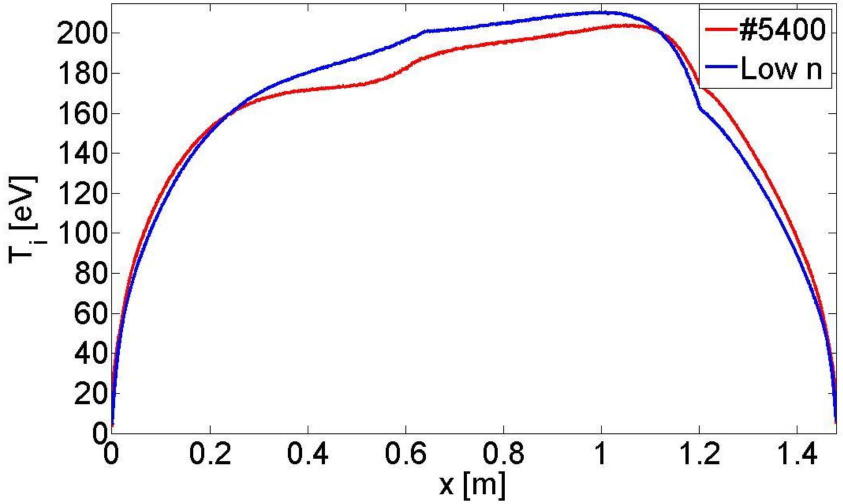
BC for	1D	2D
	Collisional	Collisional
ϕ	🟡	🔴
$V_{i,\parallel}$	🟢	🔴
$\partial T/\partial x$, or $q_{\parallel}$	🔴	🔴
Higher moments, vorticity	🔴	🔴

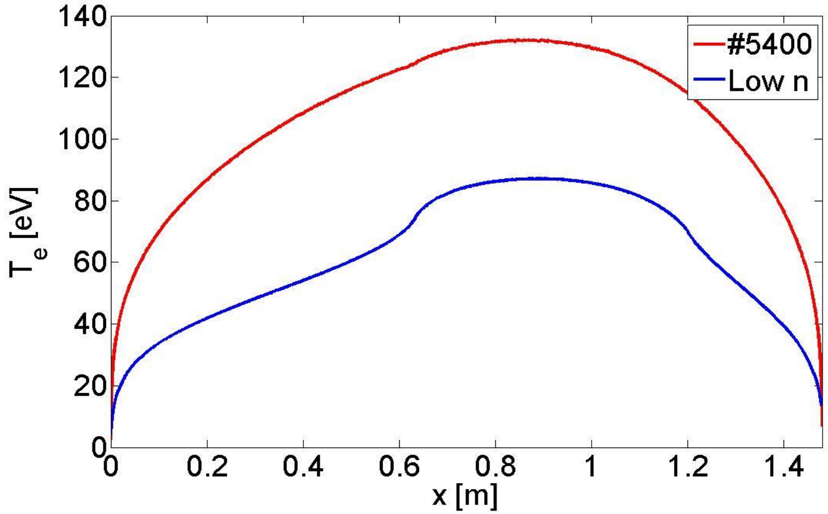
- ✓ R= 0.894 m, a = 0.275 m
- ✓ High magnetic field (BT ≤ 5 T) and high-current (Ip ≤ 2MA)
- ✓ $n_{sep} \sim 10^{20}$ [m^{-3}]
- ✓ High power fluxes in the divertor ($\lambda_q \sim 0.7 - 1$ mm)
- ✓ Metallic first wall and/or liquid metal divertor

Kinetic simulations of the COMPASS-U

	NBI+ECRH [MW]	B [T]	T_{sep} [eV]	n_{sep} [10^{19} m^{-3}]
#5400	4 + 2	5.0	~200	1.0
Low n	?	5.0	~150	0.3

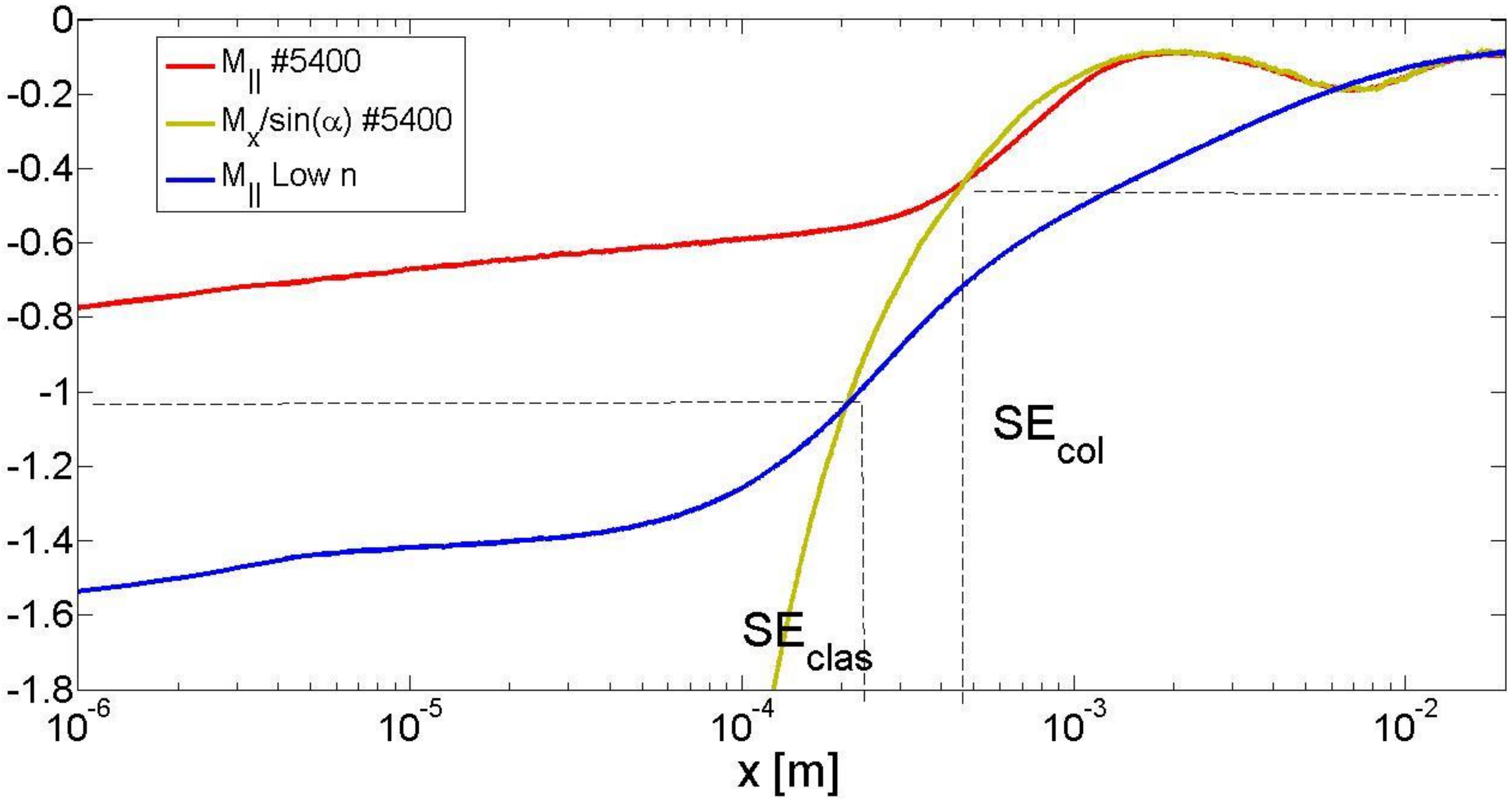






- ✓ No seeded impurity
- ✓ Simulation “price”: ~200 M CPU hours

Kinetic simulation: Mach numbers



Parallel Mach number profiles at the ID

Sheath edge = last point with

$$M_{\parallel} \approx \frac{M_{pol}}{\sin(\alpha)}$$

[D. Tskhakaya, TSVV-4, 22.4.2025]

Kinetic simulations: results

	Sheath	Pot./T_e	$n_{e,div}$ [10^{21} m^{-3}]	$T_{e,div}$ [eV]	$T_{i,div}$ [eV]	SHTF
#5400	col. / col.	**1.8** / 2.9	**6.0 / 3.3**	6.5 / 11.3	5.6 / 7.1	5.4 / 7.4
Low n	clas./clas.	3.0 / **3.4**	0.32 / 0.15	4.8 / 13.7	5.1 / 6.6	21.6 / 14.5

Super-thermal electrons[1]

Classical

$$\varphi \approx 0.5\ln\left(M_i / 4\pi m_e\right) \sim 3$$

$$\left.\frac{E_w^i}{F^i T_i}\right|_{cl} \sim 6.5, \qquad \left.\frac{E_w^{e+i}}{F^i T_i}\right|_{cl} \sim \left.\frac{E_w^i}{F^i T_i}\right|_{cl} + 2 \geq 8.5$$

$$\frac{E_w^i}{F^i T_i} \approx 0.5 M_{\parallel}^2 \left(1+\tau\right) + 2.5 + \varphi\tau, \qquad \tau = T_e / T_i$$

Collisional

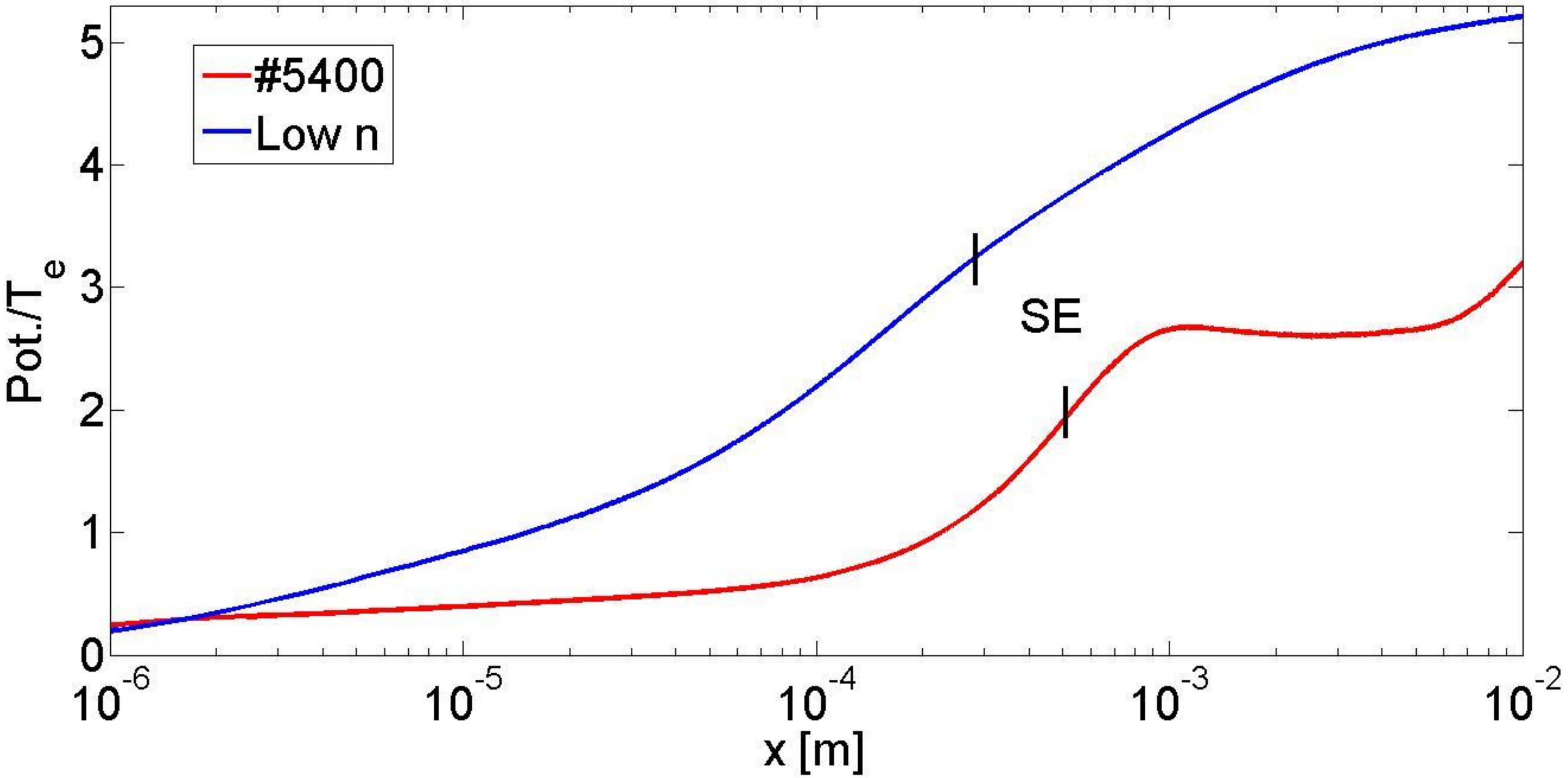
$$1.8 < \varphi < 3$$

$$\left.\frac{E_w^i}{F^i T_i}\right|_{col.} \approx 2.5 + \varphi \sim 4.0 \div 5.5$$

$$\left.\frac{E_w^{e+i}}{F^i T_i}\right|_{col.} \sim 5 \div 8$$

[1] D. Tskhakaya, PPCF 2017

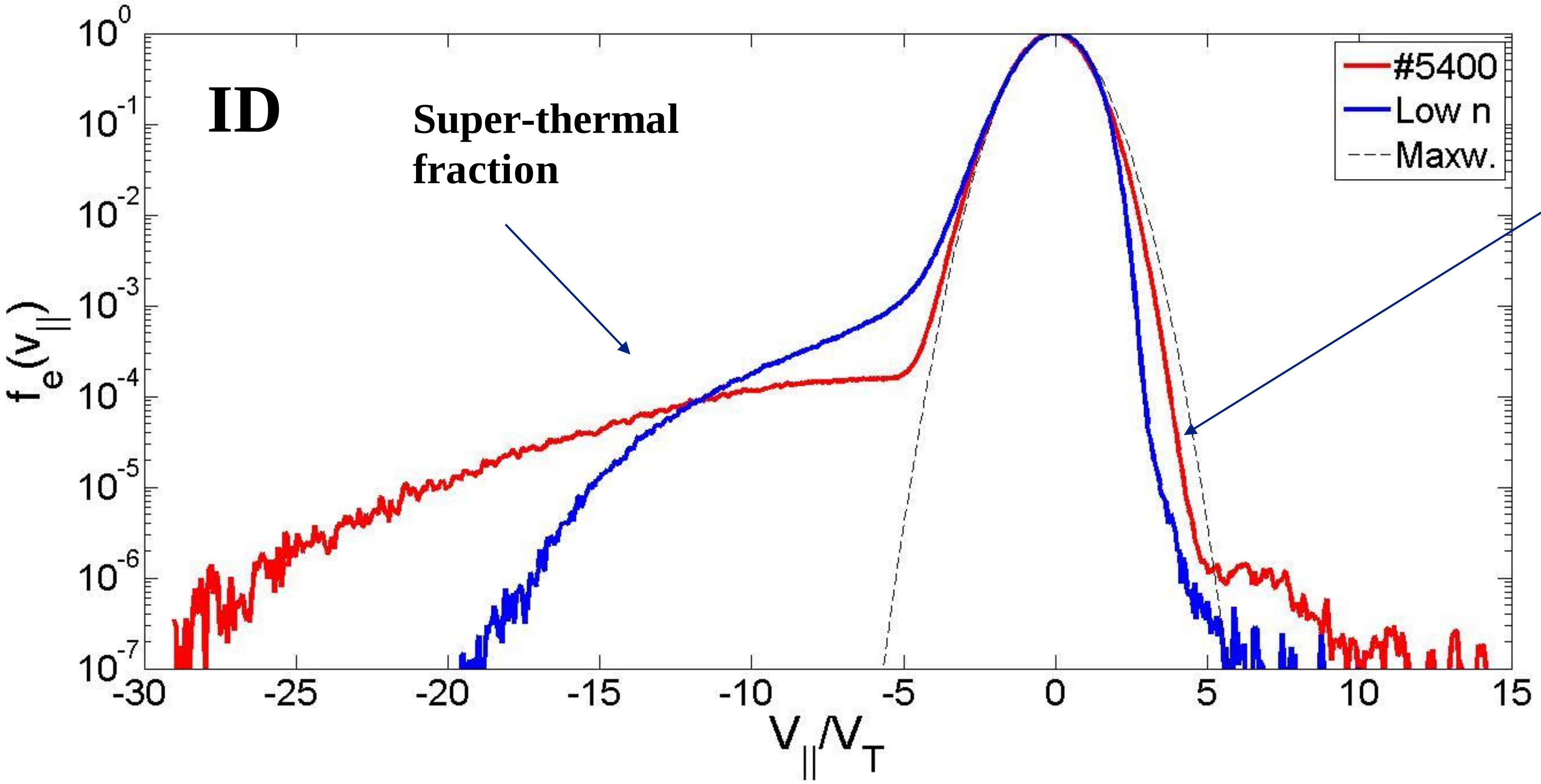
Kinetic simulations: the potential



Profiles of the normalized potential at the ID

[D. Tskhakaya, TSVV-4, 22.4.2025]

Electron VDF at the divertor plate



Collision effect

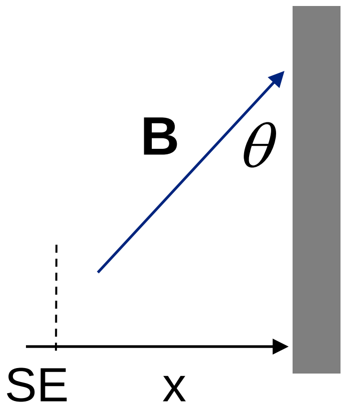
Reduced sheath potential drop

Nonclassical VI characteristics?

[D. Tskhakaya, TSVV-4, 22.4.2025]

1D boundary conditions for the collisional sheath

$$M_{\parallel} = 1 + \chi - \sqrt{\chi^2 + 2\chi}$$

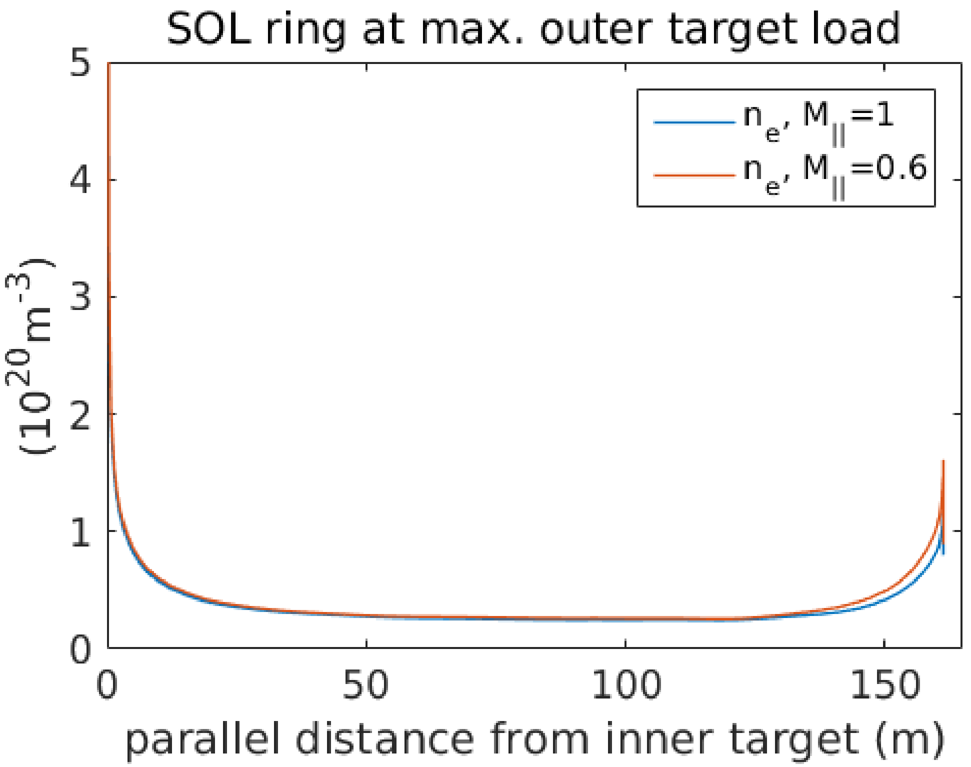


$$\chi = \frac{\left(\upsilon_{mt}(1-\alpha) + \upsilon_{ei}\right) x_0}{2 c_s \sin(\theta)}$$

$$\alpha = V_{\parallel}^{n} / V_{\parallel}$$

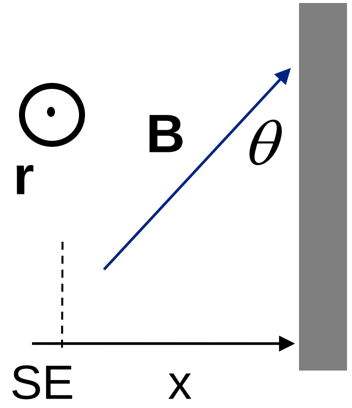
$$M_{\perp}(x_0) = \sin(\theta), \quad x_0 \approx x_{wall} \sim 20\rho_i$$

Implemented in SOLPS-ITER and GBS



[D. Moulton, ISFN DivSOL, 2021]

2D boundary conditions



Intuitive BC

$$V_{x,i} = C_{s,i} \sin\theta$$

$$V_{\|,i} = C_{s,i} + V_{ExB} \cot\theta$$

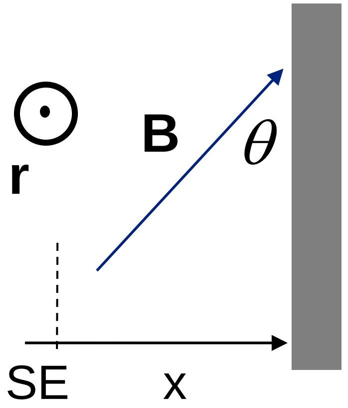
But if $\theta << 1, \ \cot\theta >> 1$ we can have flow reversal at the wall

$$V_{\|,i} < 0$$

2D boundary conditions

[SOLPS]

$$V_{\|,i} = \max\left(C_{s,i},\ 0.1C_{s,i} + V_{ExB}\cot\theta\right)$$



[GBS, J. Loizu, PoP, 2012]

$$V_{\|,i} = C_{s,i}\left(1 + H_n\cot\theta - \frac{1}{2}H_T\cot\theta\right) + V_{ExB}\cot\theta, \qquad T_i \ll T_e$$

[BIT1, D. Tskhakaya, CPP, 2002]

$$V_{\|,i} = C_{s,i}\left(1 \pm \frac{\cot\theta}{1 + T_i/T_e}H_E\right) + V_{ExB}\cot\theta, \qquad \frac{\partial_r T_e}{T_e} \gg \frac{\partial_r nT_i}{nT_i}$$

$$H_n = \frac{\rho_i\partial_r n}{2n}, \quad H_T = \frac{\rho_i\partial_r T_e}{2T_e}, \quad H_E = \left|\frac{\rho_i\partial_r E_x}{2E_x}\right|$$

Directed against ExB

Diamagnetic drift neglected

For simplicity $\quad H_k \ll 1$

$$C_{s,i} = \sqrt{(T_i + Z_i T_e)/m_i}, \quad i = 1,\ldots,N$$

Proposed 2D boundary condition for $V_{\|,i}$

$$H_E = \left|\frac{\rho_i \partial_r E_x}{2E_x}\right| \approx \frac{\rho_i}{2L_r}$$

$$\eta_i = \frac{\rho_i}{2L_r}\frac{\cot\theta}{1+T_i/T_e}$$

$$V_{\|,i} = C_{s,i}\left(\sqrt{1+\eta_i^2} \pm \eta_i\right) + V_{ExB}\cot\theta$$

The sign is „against ExB“

$$L_r = \left|\frac{\partial_r E_x}{E_x}\right| \sim L_{T_e} = \left|\frac{\partial_r T_e}{T_e}\right|$$

$$C_{s,i} = \sqrt{(T_i + Z_i T_e)/m_i}, \quad i = 1,\ldots,N$$

Flow reversal can be avoided!

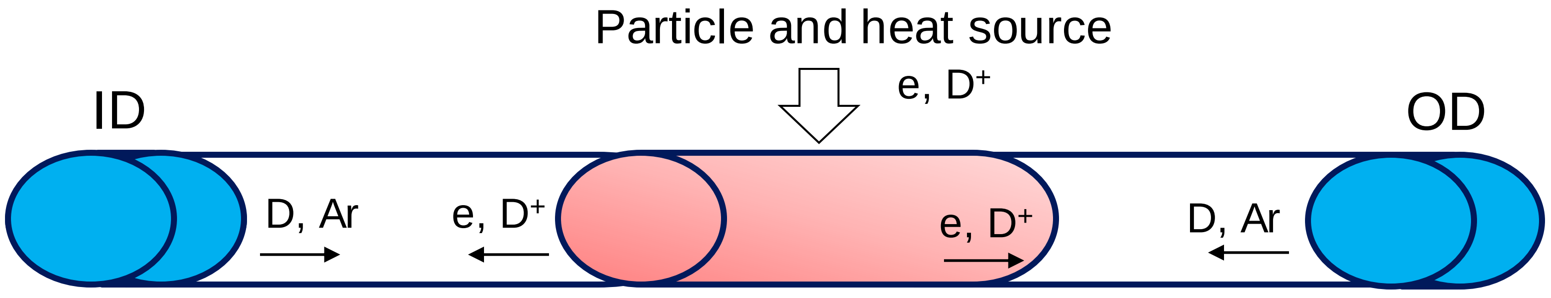
Diamagnetic drift is neglected

$$\frac{\partial_r T_e}{T_e} >> \frac{\partial_r n T_i}{n T_i}$$

Boundary conditions for the multi-fraction sheath

BC for	1D	2D
	Multi-fraction	Multi-fraction
ϕ	🟡	🔴
$V_{i,\parallel}$	🟡	🔴
$\partial T/\partial x$, or $q_{\parallel}$	🟡	🔴
Higher moments, vorticity	🔴	🔴

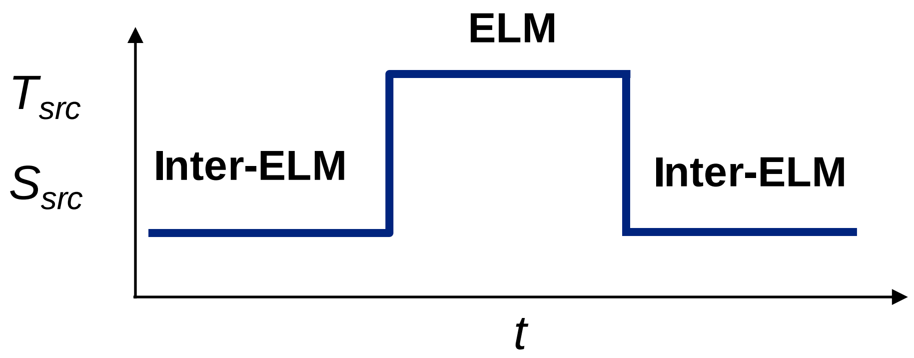
Descriptiion of the model



BIT1 – 1D3V electrostatic PIC + MC

ELM model[1]

Fixed connection length

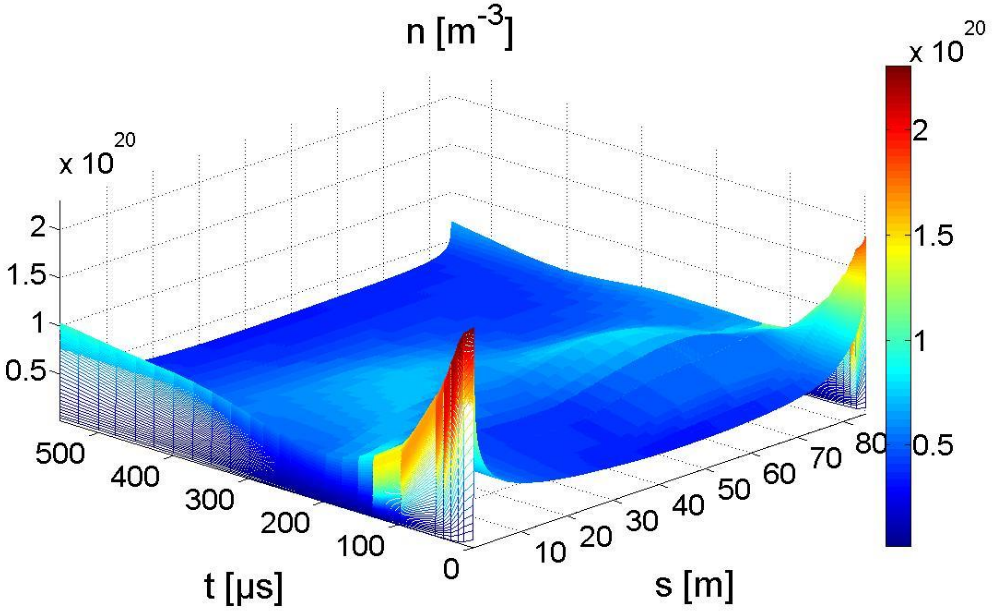


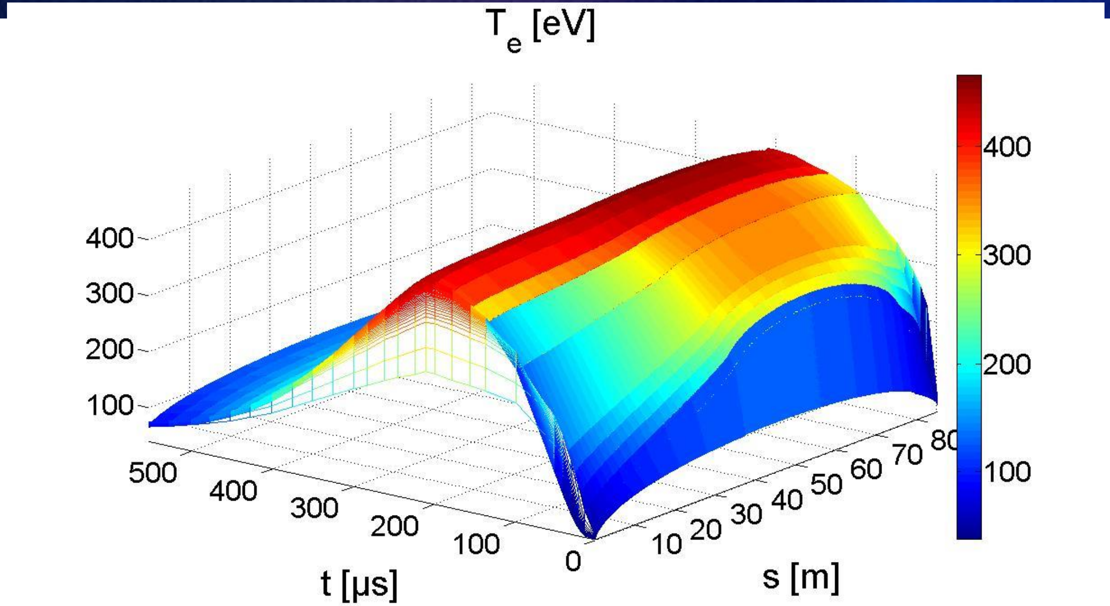
- ✓ Validation of the divertor power loads[2]
- ✓ Plasma sheath parameters[3]
- ✓ W errosion rates[4]
- ✓ Divertor temperatures[5]

[1] D. Tskhakaya, et al., J. Nucl. Mater. (2009)

[2] R.A. Pitts, et al., Nucl. Fus., (2007)
[3] D. Tskhakaya, et al., J. Nucl. Mater., (2011)
[4] J A. Huber, et al., Phys. Scr., (2021)
[5] J. Horacek, et al., Nucl. Fus., (2023)

SOL profiles during the ELM (unseeded)





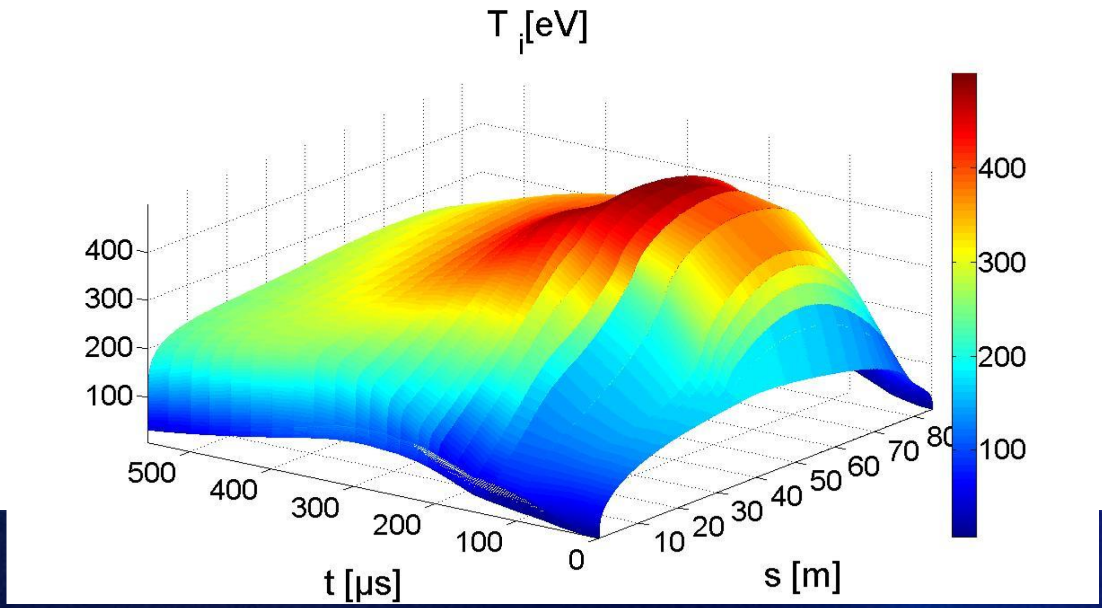
W_{ELM} ~ 60 kJ

$T_{e,\,ELM}$ = 540 eV

$T_{i,ELM}$ = 700 eV

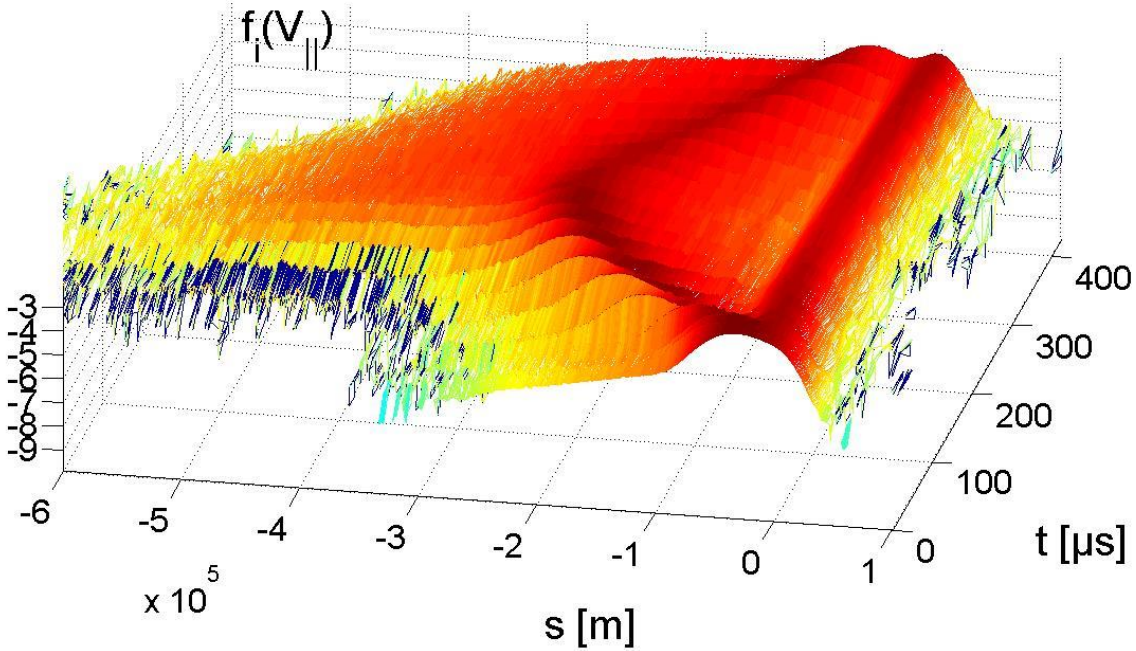
τ_{ELM} = 0.2 ms

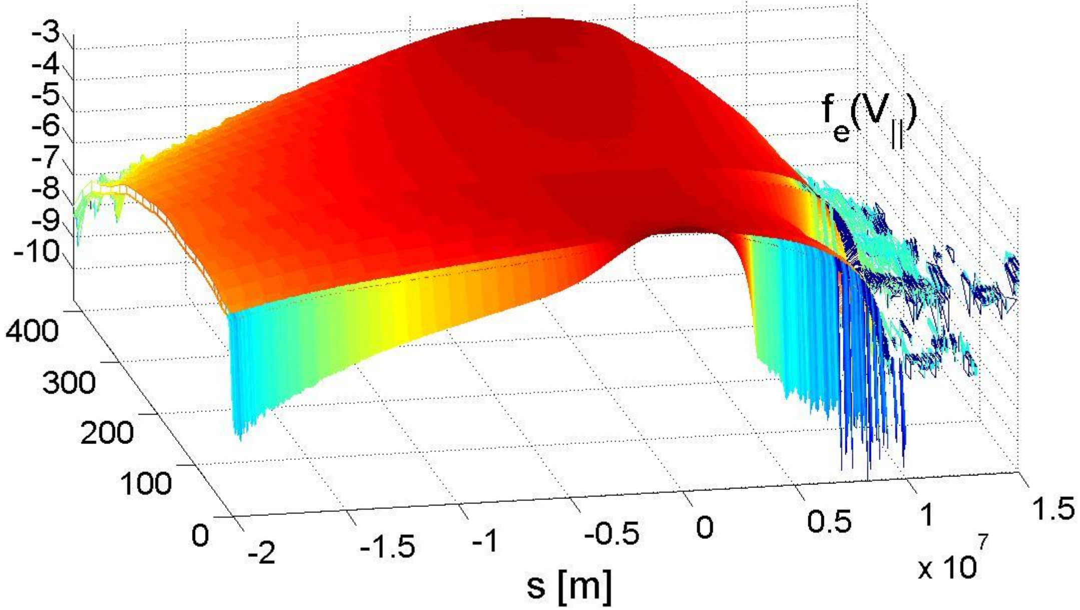
Motivation: ELM mitigation in ASDEX via Ar seeding[1]



[1] M. Komm, et al., Nucl. Fus., (2023)

Plasma VDFs at the ID sheath

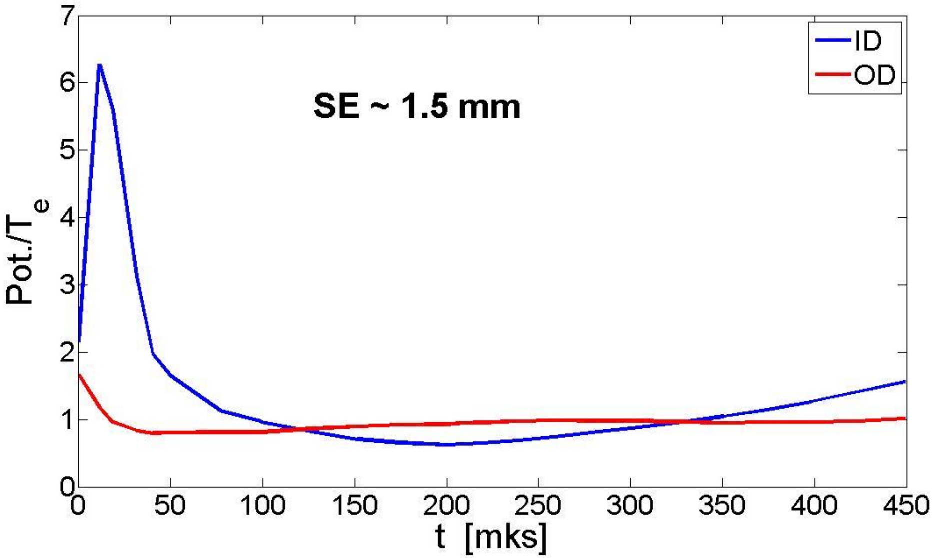




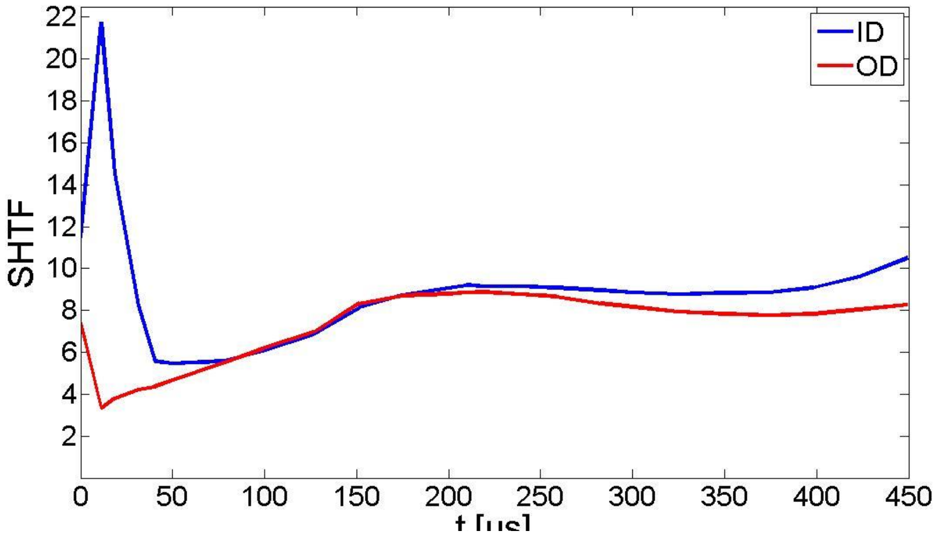
Clear **double Maxwellian** structure of the **ion VDF** corresponding to the ELM and thermal ions

Cut-off Maxwellian electron VDF corresponding to the ELM electrons. Thermal electrons are expelled from the sheath by increased potrntial frop

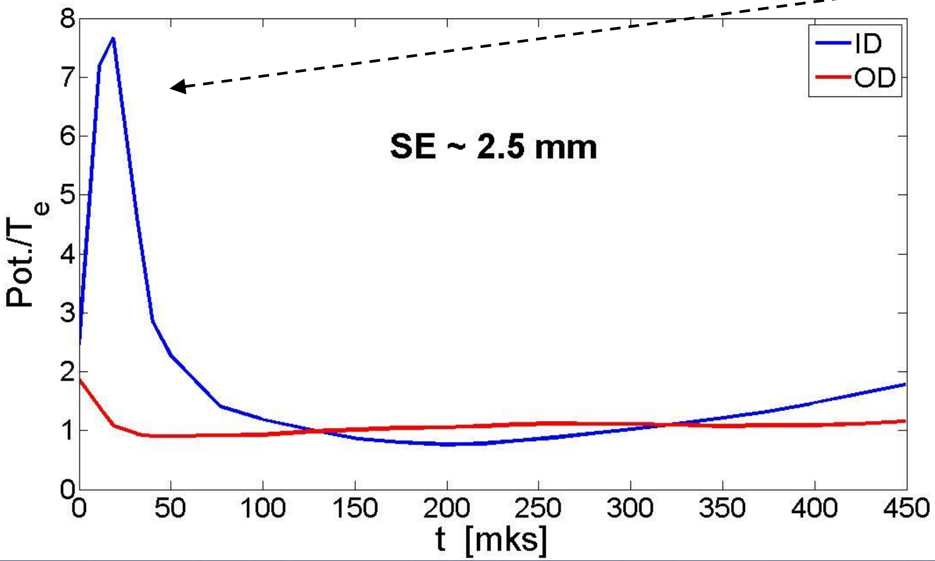
Boundary conditions



BC depend weakly on the position of the SE

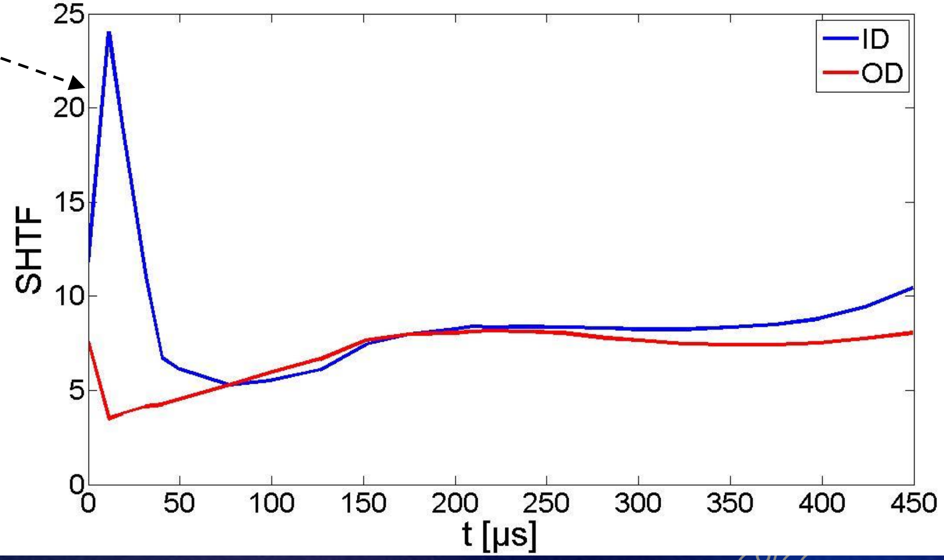


Electrons coming from ionization



$$pot. = \frac{e\varphi}{T_e}$$

$$SHTF = \frac{q_{div}}{F_i T_e}$$



Boundary conditions at the divertors during the ELM

$$K_s(\tau_i) = K_s(0) + \left(\frac{A_s}{(\delta_s\tau_s)^{Z_s}} + B_s\right)\exp\left(-\frac{1}{2(\delta_s\tau_s)^2}\right), \quad s = e,\, i,\, pot$$

$$K_{e,i} \equiv \gamma_{e,i}, \quad K_{pot} = \psi, \quad \tau_s = \frac{t}{t_s}, \quad t_{pot} = t_e, \quad Z_i = 2.5, \quad B_e = B_{pot} = 0,$$

$$A_i = 0.7\frac{T}{T_0}\left(1 + 0.35\ln\frac{n}{n_0}\right) - 0.5, \quad B_i = 2.25 - \gamma_i(0), \quad \delta_i = 0.09\left(\frac{nT}{n_0T_0}\right)^{0.6} + 0.5,$$

$$A_e = 3.9\frac{T}{T_0}\left(1 + 0.9\frac{n}{n_0}\right) - 12.2, \quad \delta_e = 0.5\ln\left(1 + \sqrt{\frac{T_0}{T}}\right) + 0.0035\left(\frac{n}{n_0}\right)^2 + 0.2,$$

$$A_{pot} = 22.0\frac{(T/T_0)^{2.5}}{760(n_0/n)^2 + (T/T_0)^{2.5}} + 0.008\left(\frac{n}{n_0}\right)^3 + 0.5, \quad \delta_{pot} = 0.0008\left(\frac{n}{n_0}\right)^2 + 0.4,$$

$$Z_e = 0.14\frac{T}{T_0} + 0.65, \quad Z_{pot} = 0.028\left(\frac{nT}{n_0T_0}\right) + 0.75,$$

Implemented in EDGE2D

[D. Tskhakaya, F. Subba, X. Bonnin, D.P. Coster, W. Fundamenski, R.A. Pitts, CPP 2008]

[D.M. Harting, et al. JNM 2015]

Summary

- 1D classical BC for the potential and up to the second moments of the VDF exist. Multi-ion case contains density derivatives → hard to use
- 1D BC for the collisional sheath exist and can be implemented (as this was done for SOLPS-ITER and GBS)
- A new (simplified) 2D BC has been proposed
- There is amodel for BC for the large ELMs
- New task: study BCs for

$$\frac{\partial}{\partial s} T_{\|, j} \quad \frac{\partial}{\partial s} T_{\perp, j}$$