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Beat-driven and spontaneous excitations of zonal flows

Liu Chen[1,2], Zhiyong Qiu[3,2] and Fulvio Zonca[2,1]

[1]IFTS, School of Physics, Zhejiang University, Hangzhou, 310027 P.R. China
[2]Center for Nonlinear Plasma Science and ENEA C.R. Frascati, 00044 Frascati, Italy
[3]Institute of Plasma Physics, Chinese Academy of Sciences, Hefei, 230031 P.R. China

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浙江大學聚变理論與模擬中心

Institute for Fusion Theory and Simulation, Zhejiang University

The role of zonal flows

Z. Lin et al, Science 1998

Turbulent Transport Reduction by Zonal Flows: Massively Parallel Simulations

Z. Lin,* T. S. Hahm, W. W. Lee, W. M. Tang, R. B. White

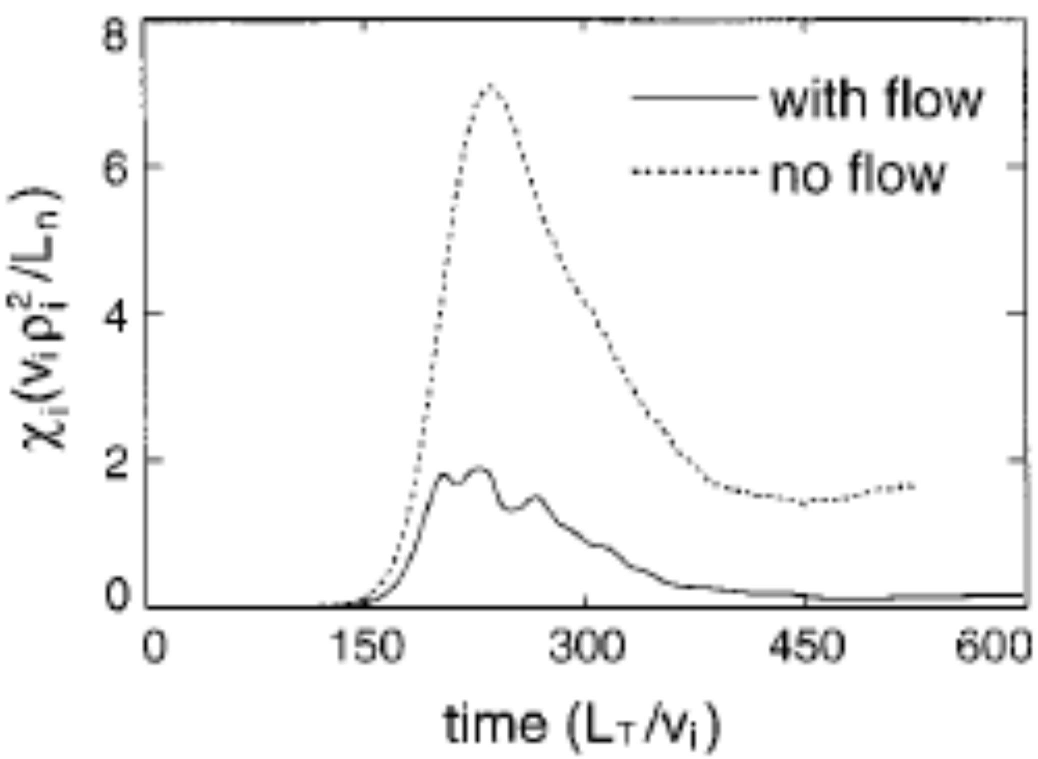


Fig. 1. Time history of ion heat conductivities with (solid) and without (dotted) **E** × **B** flows in global simulations with realistic plasma parameters.

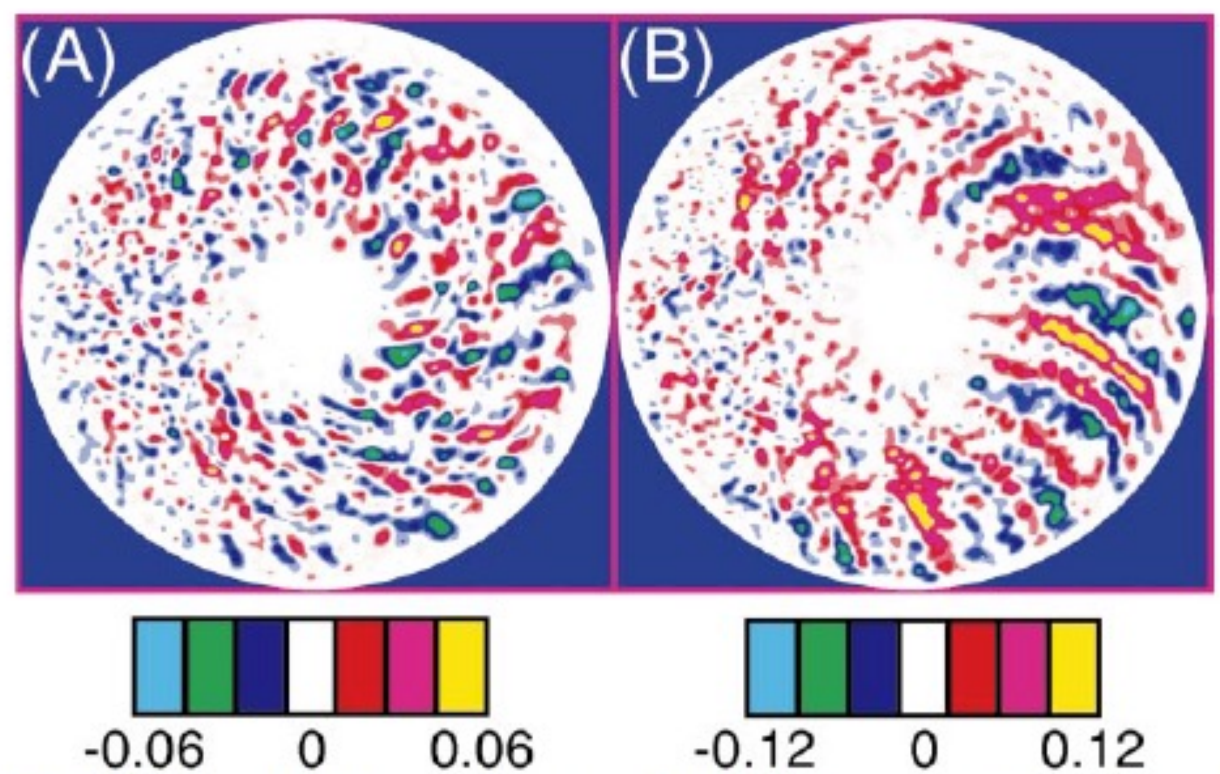


Fig. 2. Poloidal contour plots of fluctuation potential ($e\Phi/T_i$) in the steady state of nonlinear global simulation with **E** × **B** flows included (**A**) and with the flows suppressed (**B**). The dominant poloidal spectrum $k_\theta = 0$ mode is filtered out to highlight the differences in the turbulent eddy size.

Shearing vs. scattering

- The important effect of zonal flows on fluctuations can be understood in term of two fundamental processes: shearing of turbulent eddies and scattering of fluctuations in k-space

- Shearing: impact on transport due to enhanced wave-particle decorrelation [Hasegawa et al. 79]

- Scattering: coupling to the stable part of k-spectrum and regulation of fluctuation level [Diamond et al 98, Chen et al 00]

- Relative role of shearing vs. scatteting can be measured within NL GK simulations

Spontaneous excitation of ZF

- Since the early work on ZF interaction with drift-waves, it was proposed that the mechanism underlying ZF generation might be modulational instability [Diamond et al 98, Chen et al 00]

- Modulational instability: was originally investigated for surface waves in deep water [Benjamin & Feir 1967] and consists of the reinforcement by non-linearity of the deviation from wave periodic behavior, which may lead to spectral sidebands and possibly to breaking of the periodic fluctuation into modulated pulses

- The importance of investigating the process for drift-Alfvén waves was emphasized by [Chen et al 01]

- Generation ZF and its importance for long wavelength Alfvén waves (TAEs) was noted in simulations by [Spong et al 94; Todo et al 2010]

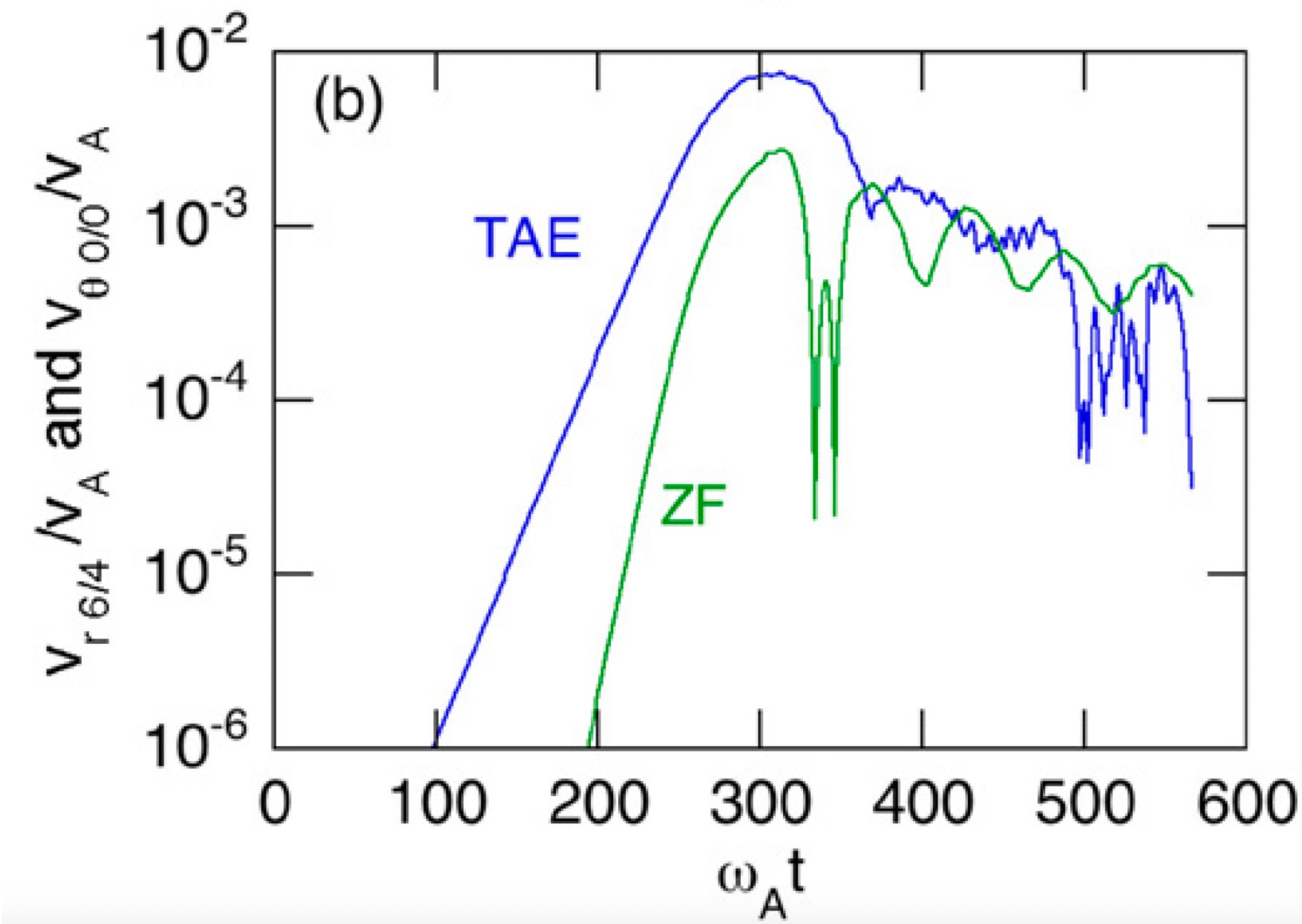
Forced-(beat-)driven excitation of ZF

Nucl. Fusion **50** (2010) 084016 (9pp) doi:10.1088/0029-5515/50/8/084016

Nonlinear magnetohydrodynamic effects on Alfvén eigenmode evolution and zonal flow generation

Y. Todo[1,2], H.L. Berk[3] and B.N. Breizman[3]

ZF grows at twice the TAE growth rate



Additional twists

- Theoretical analyses of ZF and zonal currents generated by DAW and AEs was carried out by [C&Z 2012,Z&C2015] pointing out that ZF and zonal currents need to be considered self-consistently

- The crucial role of geometry was shown by [Qiu et al 2017], since finite radial scales connected with realistic equilibrium enhance nonlinear couplings

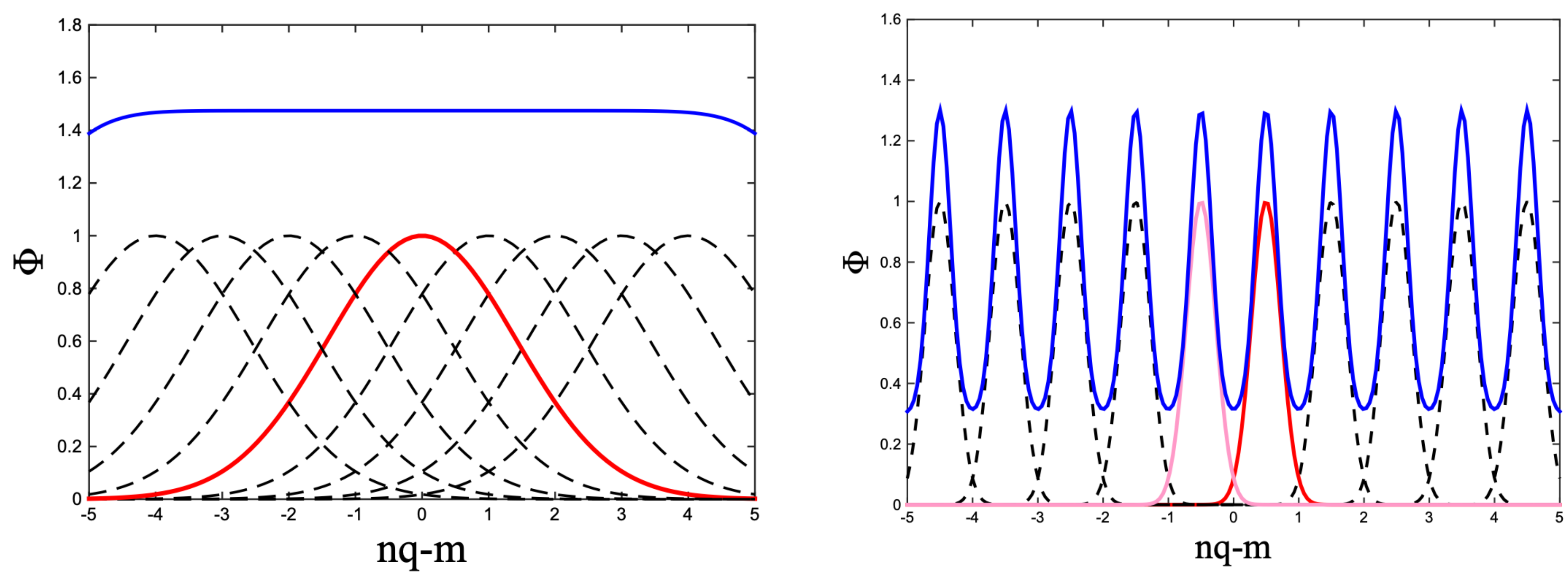


Fig. 3 Cartoon for ZFS excitation by strongly ballooning DWs (left panel) v.s. weakly ballooning SAW instabilities (right panel). Here, the dashed curves correspond to the parallel mode structure $\Phi_0(nq - m)$ for DWs (left panel) and SAW instabilities (right panel), respectively; while the solid blue curves in both panels correspond to $\sum_m |\Phi_0|^2$. Thus, for DWs with $\sum_m |\Phi_0|^2$ being almost independent of r (Zonca et al. 2004), radial envelope modulation leads to meso-scale ZF excitation; while for SAW instabilities, fine-scale structure ZFS is excited

Additional twists

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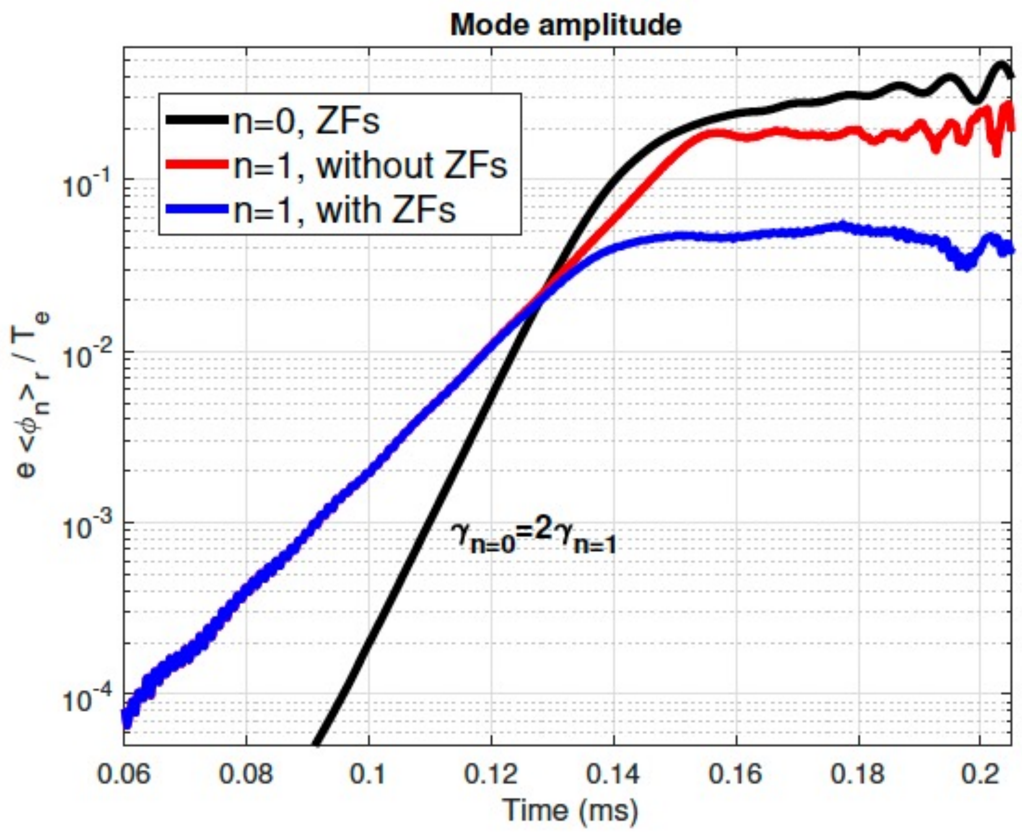
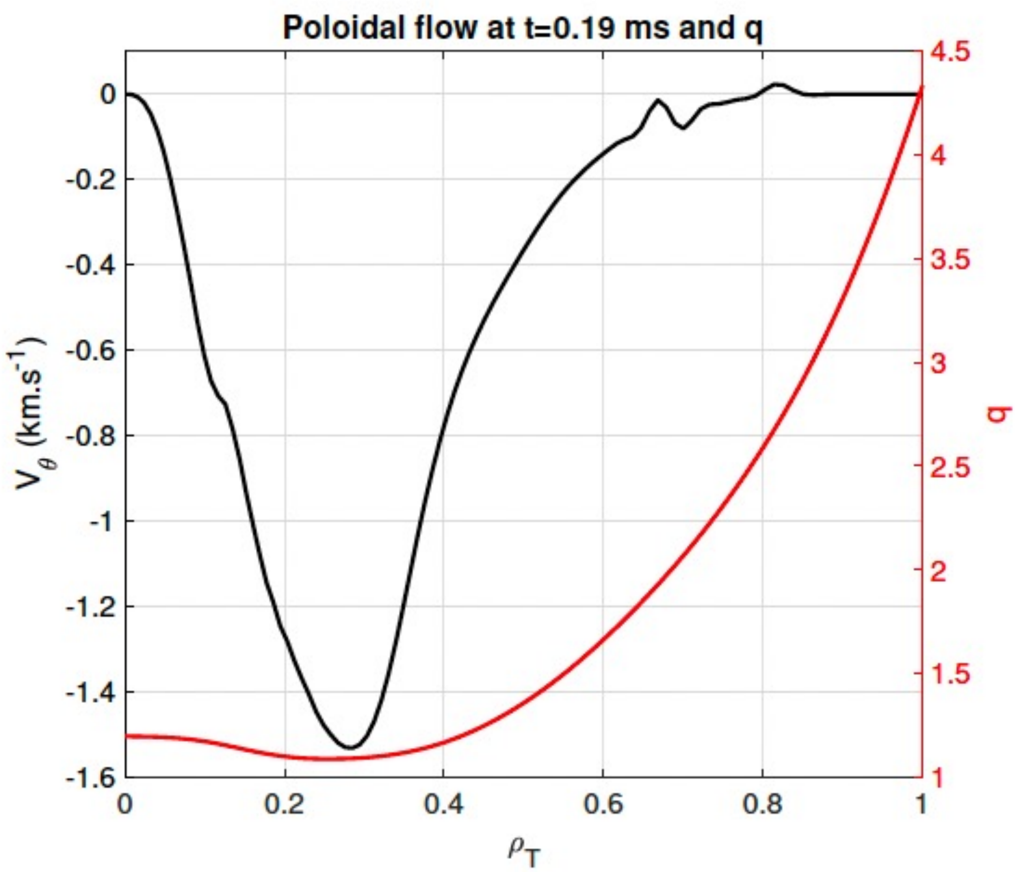
- The crucial role of geometry was shown by [Qiu et al 2017], since finite radial scales connected with realistic equilibrium enhance nonlinear couplings
- The crucial role of resonant EPs was emphasized by [Qiu et al 2016], showing that they render the ZF excitation into a forced-driven process (beat-driven)
- Recent review by Z. Qiu et al, Reviews of Modern Plasma Physics (2023) 7:28

Beat-driven and spontaneous

- Beat-driven spontaneous ZF excitations are observed in simulations [G. Brochard et al, submitted to NF]



(a)



(b)

Figure 7: (a) Time evolution of volume-averaged perturbed electrostatic potential $e\langle\phi\rangle/T_e$ (n=0,1).(b) Zonal poloidal flow V_θ (km.s^{-1}) after saturation at t=0.19ms. Figure (a) is reproduced from [56]

Recent theories

- M. V. Falessi et al: self-consistent evolution of zonal e.m. fields and corresponding phase space zonal structures as zonal states, describing nonlinear plasma equilibria in the presence of finite fluctuation spectrum and sources/sinks, collisions ➔ New Journal of Physics (2023) **25** 123035

- N. Chen et al: Drift wave soliton formation via forced-driven zonal flow and implication on plasma confinement. ➔ Submitted to POP https://users.euro-fusion.org/repository/pinboard/EFDA-JET/journal/111424_1674587_0_unknown_upload_24079272_s810cg_sc.pdf

- L. Chen et al: On beat-driven and spontaneous excitations of zonal flows by drift waves. ➔ Submitted to POP https://users.euro-fusion.org/repository/pinboard/EFDA-JET/journal/111811_1685700_0_unknown_upload_24111954_s8kny3_sc.pdf

Recent theories

- Follow L. Chen et al POP 2024 and develop theoretical paradigm based on e-DW to illuminate the respective roles of beat-driven and spontaneous components of ZF and possible synergies.
- Quasineutrality condition (single-n e-DW + ZF in slab; extension to general geometry and fluctuation spectrum Falessi et al NJP 2023)

$$\frac{N_0 e^2}{T_e}\left(1+\frac{T_e}{T_i}\right)\delta\phi_k = \sum_{j=e,i}\langle eJ_k\delta g_k\rangle_j,$$

$$\delta f_{kj} = -\left(\frac{e}{T}\right)_j F_{Mj}\delta\phi_k + e^{-i\boldsymbol{\rho}\cdot\mathbf{k}_\perp}\delta g_{kj}$$

$$i(k_\parallel v_\parallel - \omega_k)\delta g^{(1)}_{kj} = -i(\omega-\omega_{*j})_k(e/T)_j F_{Mj}J_k\delta\phi_k,$$

$$i(k_\parallel v_\parallel - \omega_k)\delta g^{(2)}_{kj} = -(c/B_0)\Lambda^{k'}_{k''}\left(J_{k'}\delta\phi_{k'}\delta g_{k''}\right)_j.$$

$$\Lambda^{k'}_{k''} = \mathbf{b}\cdot\mathbf{k}''\times\mathbf{k}' \text{ satisfying } \mathbf{k}=\mathbf{k}'+\mathbf{k}''$$

$$\delta\phi_k = \delta\phi_d + \delta\phi_Z,$$

$$\delta\phi_d = \phi_d(x,t)\exp(i(k_y y + k_\parallel z - \omega_{dr}t)) + c.c.,$$

$$\delta\phi_Z = \phi_Z(x,t) + c.c..$$

Recent theories

- Solve kinetic equations by small amplitude expansion, noting
  $|k_\parallel v_{te}| \gg |\omega_k| \gg |k_\parallel v_{ti}|$ and $|k_\perp \rho_e| \ll 1$, $\omega_k = \omega_{kr} + i\partial_t$.

- Linear responses

$$\delta g_{ki}^{(1)} \simeq \left(1 - \frac{\omega_{*i}}{\omega}\right)_k \frac{e}{T_i} J_k F_{Mi} \delta\phi_k, \qquad \delta g_{ke}^{(1)} \simeq O(\omega / k_\parallel v_{te}) \ll 1.$$

- Zonal component

$$\delta g_{Zi}^{(1)} = \frac{e}{T_i} F_{Mi} J_Z \delta\phi_Z, \qquad \delta g_{Ze}^{(1)} = -\frac{e}{T_e} F_{Me} \delta\phi_Z.$$

Recent theories

- Nonlinear responses: obtained by solving kinetic equations based on the small amplitude expansion

- Zonal component

$$\frac{\partial}{\partial t}\delta g^{(2)}_{Zi} = -\frac{c}{B_0}\Lambda^{k'}_{k''}\left(J_{k'}\delta\phi_{k'}\delta g_{k''}\right)_i . \qquad \delta g^{(2)}_{Ze} \simeq 0;$$

- Two mechanisms for zonal component excitations

- First: the beat-driven process due to the ponderomotive force produced by the self beating of the eDWs $|k'_x| = |k''_x|$

- Second: the spontaneous excitation produced by the nonlinear interactions between the radial sidebands $|k'_x| \neq |k''_x|$

Recent theories

$$\delta g^{(2)}_{Zi} = \delta g^{(2)}_{Zi,A} + \delta g^{(2)}_{Zi,B},$$

- A: beat-driven (forced-driven) process

$$\delta g^{(2)}_{Zi,A} = \frac{c}{B_0} k_y J_k^2 \left(\frac{\omega_{*i}}{\omega_r^2}\right)_d \frac{e}{T_i} F_{Mi} \frac{\partial}{\partial x} |\phi_d|^2 .$$

- B: spontaneous process

$$\frac{\partial}{\partial t} J_Z \delta g^{(2)}_{Zi,B} = i\frac{c}{B_0} k_y \frac{\partial}{\partial x} [(J_Z J_{k'} - J_{k''} + J_{k''})_i \delta g_{k''i} \delta \phi_{k'} \\ -(J_Z J_{k''} - J_{k'} + J_{k'})_i \delta g_{k'i} \delta \phi_{k''}] .$$

Recent theories

$$\frac{N_0 e^2}{T_i}\left(1-\Gamma_Z\right)\delta\phi_Z = \left\langle eJ_Z\left(\delta g^{(2)}_{Zi,A} + \delta g^{(2)}_{Zi,B}\right)\right\rangle .$$

$$\phi_Z = \phi_{Zb} + \phi_{Zs}$$

- A: beat-driven (forced-driven) process

$$\frac{\partial^2}{\partial x^2}\phi_{Zb} = -\frac{c}{B_0}k_y\frac{\omega_{*in}}{\omega^2_{dr}\rho_i^2}\frac{\partial}{\partial x}\left|\phi_d\right|^2 ,$$

- B: spontaneous process

$$\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2}\phi_{Zs}\right) = -\frac{T_i}{N_0 e^2\rho_i^2}\frac{\partial}{\partial t}\left\langle eJ_Z\delta g^{(2)}_{Zi,B}\right\rangle$$

$$\simeq i\frac{c}{B_0}k_y\alpha_i\frac{\partial^2}{\partial x^2}\left(\phi_d\frac{\partial}{\partial x}\phi_d^* - \phi_d^*\frac{\partial}{\partial x}\phi_d\right).$$

Recent theories

- Explore the interactions of the two processes on the modulational instability

$$\epsilon_d \phi_d = \frac{c}{B_0} \frac{k_y}{\omega_{dr}} \phi_d \frac{\partial}{\partial x} \left(\phi_{Zb} + \phi_{Zs} \right),$$

$$\epsilon_d \simeq 1 - \alpha_i \rho_s^2 \nabla_\perp^2 - \frac{\omega_{*en}}{\omega_{dr}} + i \frac{\omega_{*en}}{\omega_{dr}^2} \frac{\partial}{\partial t}$$

$$\alpha_i = (1 - \omega_{*pi}/\omega_{dr}) \simeq 1 + (T_i/T_e)(1 + \eta_i)$$

- Modulation interaction with sidebands

$$\phi_d = A_0 + A_+ \exp(\gamma_Z t + i k_Z x) + A_-^* \exp(\gamma_Z t - i k_Z x)$$

$$\phi_Z = A_Z \exp(\gamma_Z t + i k_Z x)$$

$$A_{Zb} = -i \frac{c k_y \omega_{*en}}{B_0 k_Z \rho_s^2 \omega_{0r}^2} \left(A_0 A_- + A_0^* A_+ \right), \qquad \gamma_Z A_{Zs} = -\frac{c}{B_0} k_y k_Z \alpha_i \left(A_0 A_- - A_0^* A_+ \right).$$

Recent theories

- QN equation for sidebands

$$\epsilon_{d\pm} A_\pm = i\frac{ck_y k_Z}{B_0\omega_{0r}}(A_{Zb} + A_{Zs})\begin{pmatrix} A_0 \\ A_0^* \end{pmatrix}, \qquad \epsilon_{d\pm} = \alpha_i b_{Zs} \pm i\gamma_Z(\omega_{*en}/\omega_{0r}^2).$$

- Dispersion relation for modulationall instability is found by substitution into nonlinear expressions for the ZF components

$$A_{Zb} = -i\frac{ck_y\omega_{*en}}{B_0 k_Z \rho_s^2 \omega_{0r}^2}(A_0 A_- + A_0^* A_+), \quad \gamma_Z A_{Zs} = -\frac{c}{B_0}k_y k_Z \alpha_i (A_0 A_- - A_0^* A_+).$$

$$\left(\frac{\gamma_Z \omega_{*en}}{\omega_{0r}^2}\right)^2 = \alpha_i b_{Zs}\left[(\alpha_b + \alpha_s)\left|\overline{A}_0\right|^2 - \alpha_i b_{Zs}\right],$$

where $\overline{A}_0 = eA_0/T_e$, $b_{Zs} = k_Z^2\rho_s^2$, and

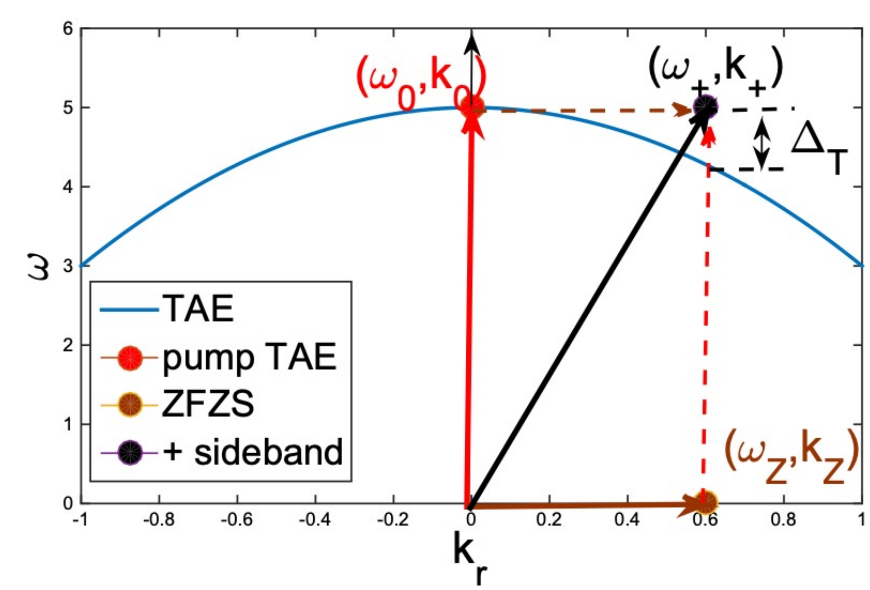
$$\alpha_b = \alpha_s = 2b_{ys}(\omega_{*en}/\omega_{0r})(\Omega_{ci}/\omega_{0r})^2.$$

Main results

- Instability threshold

$$|\overline{A}_0|^2_{th} = \alpha_i b Z_s/(\alpha_b + \alpha_s)$$

- α_b and α_s correspond, respectively, to beat-driven (forced-driven) and spontaneous ZF components contributions and $\alpha_b = \alpha_s$
- This demonstrates that the beat-driven component provides an O(1) contribution to the spontaneous excitation and, thus, reduces the instability threshold
- Physically, this is understood because the beat-driven component gives rise to a nonlinear frequency shift that reduces the frequency mismatch



Z. Qiu et al, RMPP 2023

Conclusions

- The theoretical framework for describing excitations of ZF by beat-driven or modulational instability is fully developed. Supported by numerical simulation results. (qualitatively, verification needed)

- It shows that both processes are important in determining ZF levels. Recent analyses [N. Chen et al POP 2024] suggest that this has impact on the turbulence spreading by soliton formation

- Theoretical framework is demonstrated by simple eDW slab paradigm, but it is fully deployed for general geometry and analysis of NL dynamics in phase space by action-angle approach [Falessi et al NJP 2023]

THANK YOU!

Your questions are welcome