MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

Paper Rehearsal:

Modelling resistive-inductive evolution of currents in Wendelstein 7-X

Nucl. Fusion

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This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.

Abstract

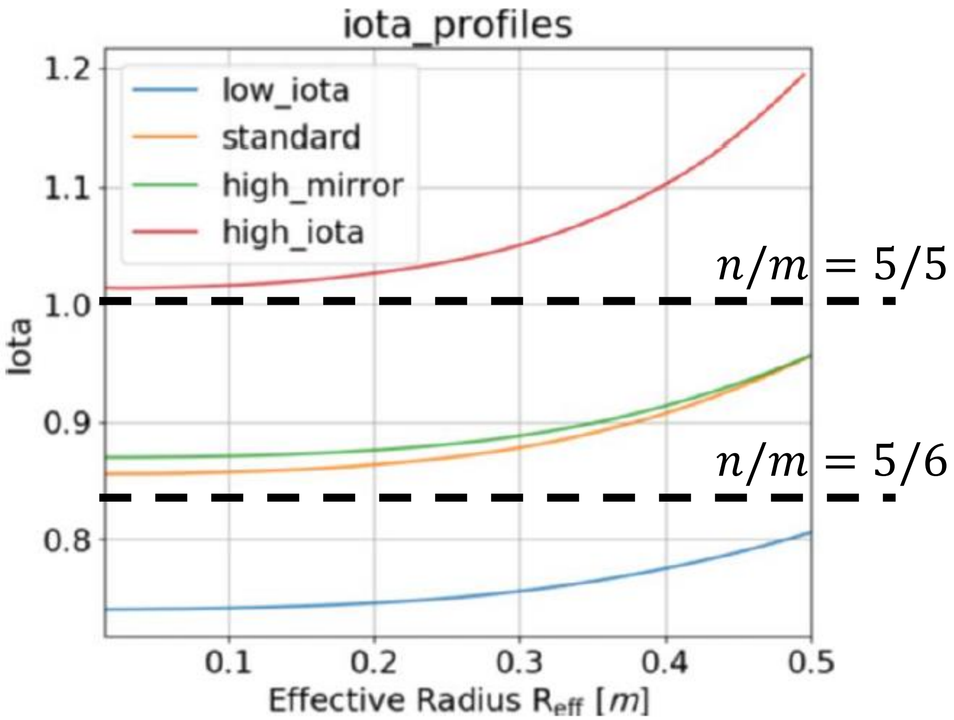
- The THRIFT code, capable of modelling toroidal current evolution in fusion plasmas, has been modernised and integrated into STELLOPT
- Several W7-X programs have been recreated using THRIFT and BOOTSJ, showing:
  - THRIFT characteristic timescales **agree** with experimental observations
  - BOOTSJ bootstrap current predictions **disagree** with experimental observations
  - Numerical issues still remain in THRIFT regarding edge plasma temperature
- ECCD and heating step simulations show THRIFT is capable of responding to changes in current sources

Rotational transform and the island divertor in W7-X

Currents affect the rotational transform: [Strand, P.I. & Houlberg, W.A. 2001 *Phys. Plasmas* **8** 2782]

$$\iota = \frac{\mu_0}{S_{11}\Phi'} I - \frac{S_{12}}{S_{11}}$$

μ_0 – vacuum permeability
I – toroidal current
Φ –toroidal magnetic flux
S_{ij} –susceptance matrix elements



Pedersen, T. et al. *Plasma Phys. Control Fusion* **61** (2019)

Effect on island divertor: [Yu Gao et al 2019 *Nucl. Fusion* **59** 106015]

- Shifting of strike-lines ($\sim 9\mathrm{mm/kA}$, standard)
- Entering of limited configuration

Current sources:

- Bootstrap current
- Externally driven currents (ECCD, NBCD)

➢ Need for accurate source current models

Currents evolve on inductive-resistive timescales

Total current evolves with $\tau_{L/R}$ (~30s in W7-X)→ >100s before saturation is reached

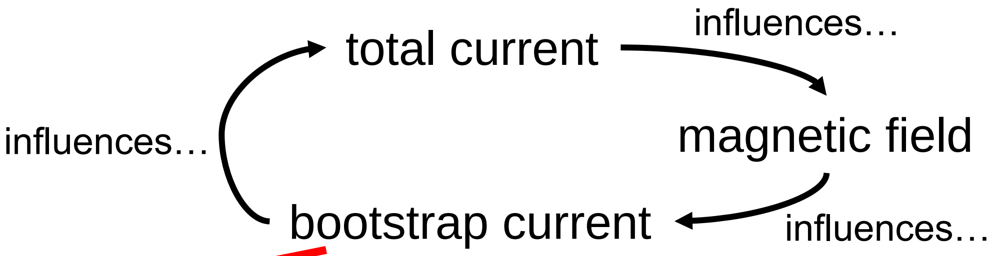
Mechanism:

1. $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ toroidal current changes poloidal $\mathbf{B}$-component
2. $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ parallel electric field is induced
3. $J_{\text{shield}} = \sigma_{\|} E_{\|}$ "shielding" current opposes total current

Modelled with simple fit: $I(t) = I_\infty \left(1 - \exp\left(-\frac{t}{\tau_{L/R}}\right)\right)$

Downsides:

- $I_\infty, \tau_{L/R}$ not necessarily time-independent
- Only gives total current, not the distribution
- Doesn't include effect of magnetic field



➢ **THRIFT self-consistently models this process**

The THRIFT code (1): System of equations

Diffusion equation for ι: [Strand, P.I. & Houlberg, W.A. 2001 *Phys. Plasmas* **8** 2782]

$$\frac{\mathrm{d}\iota}{\mathrm{d}t} = \frac{\mathrm{d}\iota}{\mathrm{d}\Phi}\frac{\mathrm{d}\Phi}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}\Phi}\left[\frac{\eta_\parallel}{\mu_0}\Phi'(S_{11}\iota + S_{12})^2\frac{\mathrm{d}}{\mathrm{d}\rho}\left(\frac{S_{11}\iota + S_{12}}{S_{21}\iota + S_{22}}\right) - \eta_\parallel\langle J_s B\rangle\frac{\mathrm{d}V}{\mathrm{d}\Phi}\right]$$

ρ – radial variable
J_s – net source current
V – plasma volume

$$u = S_{11}\iota + S_{12} = \frac{\mu_0}{\Phi_a}I$$

[Schmitt, J.C. PhD thesis (2011)]

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{S_{11}}{\Phi_a^2}\frac{\mathrm{d}}{\mathrm{d}s}\left[\eta_\parallel V'\left(\frac{\langle B^2\rangle}{\mu_0}u' + p'u - \langle J_s B\rangle\right)\right]$$

$s = \Phi/\Phi_a$
p – pressure

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \alpha_1 + \alpha_2 u + \alpha_3 u' + \alpha_4 u''$$

$$\alpha_1 = -\frac{S_{11}}{\Phi_a^2}\frac{\mathrm{d}}{\mathrm{d}s}[\eta_\parallel V'\langle J_s B\rangle]$$
$$\alpha_2 = \frac{S_{11}}{\Phi_a^2}\frac{\mathrm{d}}{\mathrm{d}s}[\eta_\parallel V' p']$$
$$\alpha_3 = \frac{S_{11}}{\Phi_a^2}\left(\frac{\mathrm{d}}{\mathrm{d}s}\left[\eta_\parallel V'\frac{\langle B^2\rangle}{\mu_0}\right] + \eta_\parallel V' p\right)$$
$$\alpha_4 = \frac{S_{11}}{\Phi_a^2}\eta_\parallel V'\frac{\langle B^2\rangle}{\mu_0}$$

❖ Discretise radial $s = \Phi/\Phi_a$-grid (ns points) & apply Backwards Time, Centred Space (BTCS) scheme

Boundary conditions:

- Magnetic axis $(s = 0)$: No current enclosed → $u = 0$
- Plasma edge $(s = 1)$: Voltage in inductor → $E_\parallel = -\frac{L_{ext}}{2\pi R}\frac{\mathrm{d}I}{\mathrm{d}t}$

L_{ext} – external inductance
R – major radius

Initial condition: No currents anywhere → $u = 0$

$T \to 0 \Rightarrow \eta_\parallel \to \infty$
***T* clamped to > 14eV**

The THRIFT code (2): Solving system of equations

Write system of equations in TDM form:

$$
\begin{bmatrix}
b_1 & c_1 & & & & 0 \\
a_2 & b_2 & c_2 & & & \\
 & a_3 & b_3 & \ddots & & \\
 & & \ddots & \ddots & c_{\mathrm{ns}-2} & \\
 & & & a_{\mathrm{ns}-1} & b_{\mathrm{ns}-1} & c_{\mathrm{ns}-1} \\
0 & & & & a_{\mathrm{ns}} & b_{\mathrm{ns}}
\end{bmatrix}
\begin{bmatrix}
u_1^n \\ u_2^n \\ u_3^n \\ \vdots \\ u_{\mathrm{ns}-2}^n \\ u_{\mathrm{ns}-1}^n \\ u_{\mathrm{ns}}^n
\end{bmatrix}
=
\begin{bmatrix}
y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_{\mathrm{ns}-2} \\ y_{\mathrm{ns}-1} \\ y_{\mathrm{ns}}
\end{bmatrix}
$$

→ $u^n = u(t^n)$ → I^n → VMEC → (back to coefficients)

Coefficients depend on:

- Densities and temperatures
- Resistivity $\eta_{\parallel}$ → Sauter resistivity [Sauter, O. et al, *Phys. Plasmas* **6**, 2834-2839 (1999)]
- Magnetic geometry → VMEC [Hirshman, S.P. & Whitson, J.C. *Phys. Fluids* **26**, 3553 (1983)]

Solved by DGTSV (Linear Algebra PACKage) using Gaussian elimination with partial pivoting

Verifying the THRIFT implementation

Test case:

1. Large aspect ratio, circular cross-section tokamak
2. Spatially uniform resistivity
3. Time-independent source currents

F_n - Bessel function
f_n - amplitude
τ_n- decay time

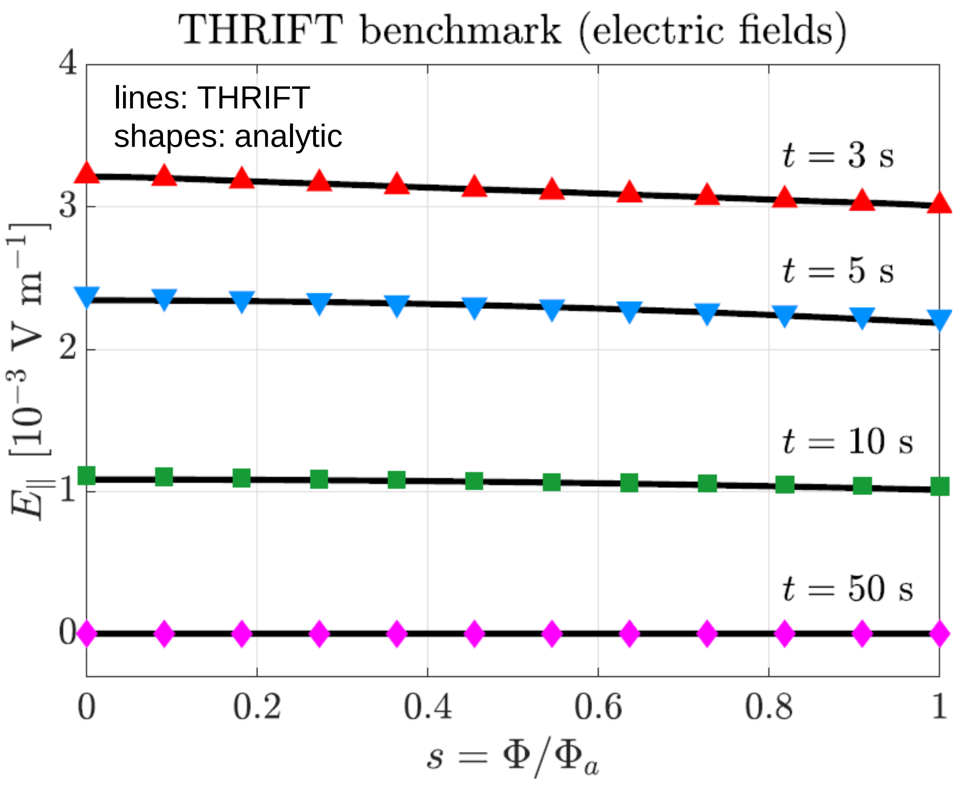
$$\mu_0 \sigma_\parallel \frac{\partial E_\parallel}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial E_\parallel}{\partial r}\right) \Rightarrow E^{FB}(x,t) = \sum_{n=1}^{\infty} f_n F_n(x) \exp\left(-\frac{t}{\tau_n}\right)$$

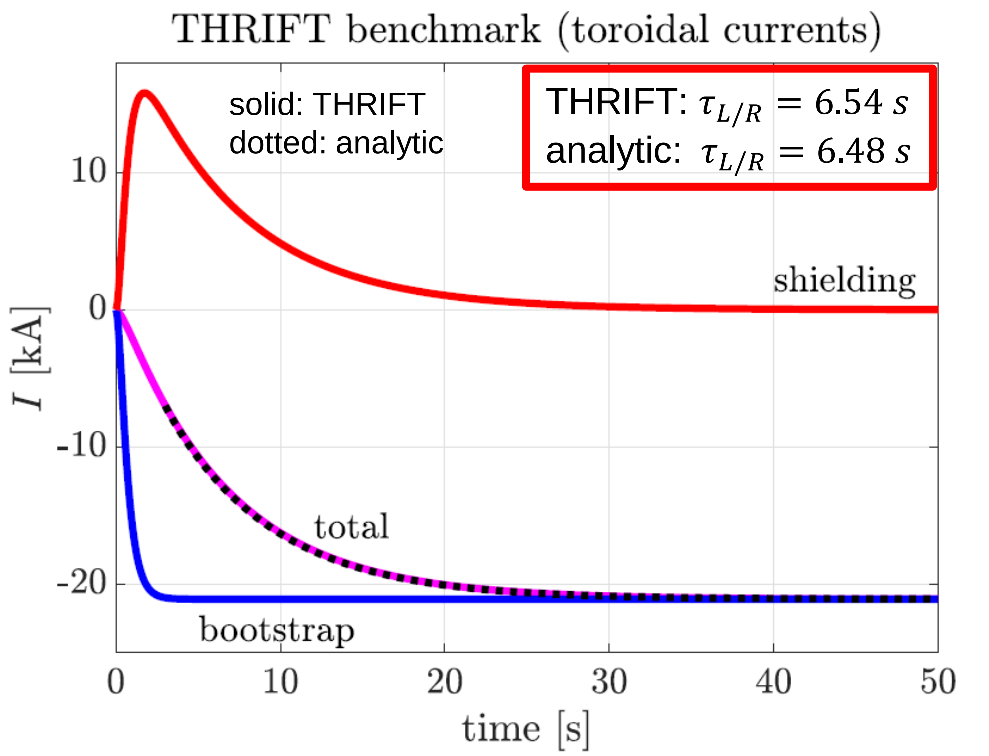
THRIFT case:

1. Generate VMEC equilibrium circular cross section, $R/a = 1000$ tokamak
2. η_{Sauter} independent of T_i → Generate spatially uniform T_e, n_e, n_i profiles
3. Generate spatially varying T_i → ∇T_i-driven bootstrap current $J_{bsc}(s) = \frac{\sqrt{\epsilon}R}{\Phi_a} p's$

- $I(s, t = 0) = 0$ → VMEC cannot sustain pressure gradient
  - Evolve n, T profiles to steady state values over ~3 seconds
  - Determine f_n, τ_n at $t = 3$s from THRIFT data, then compare at later times

Verification results

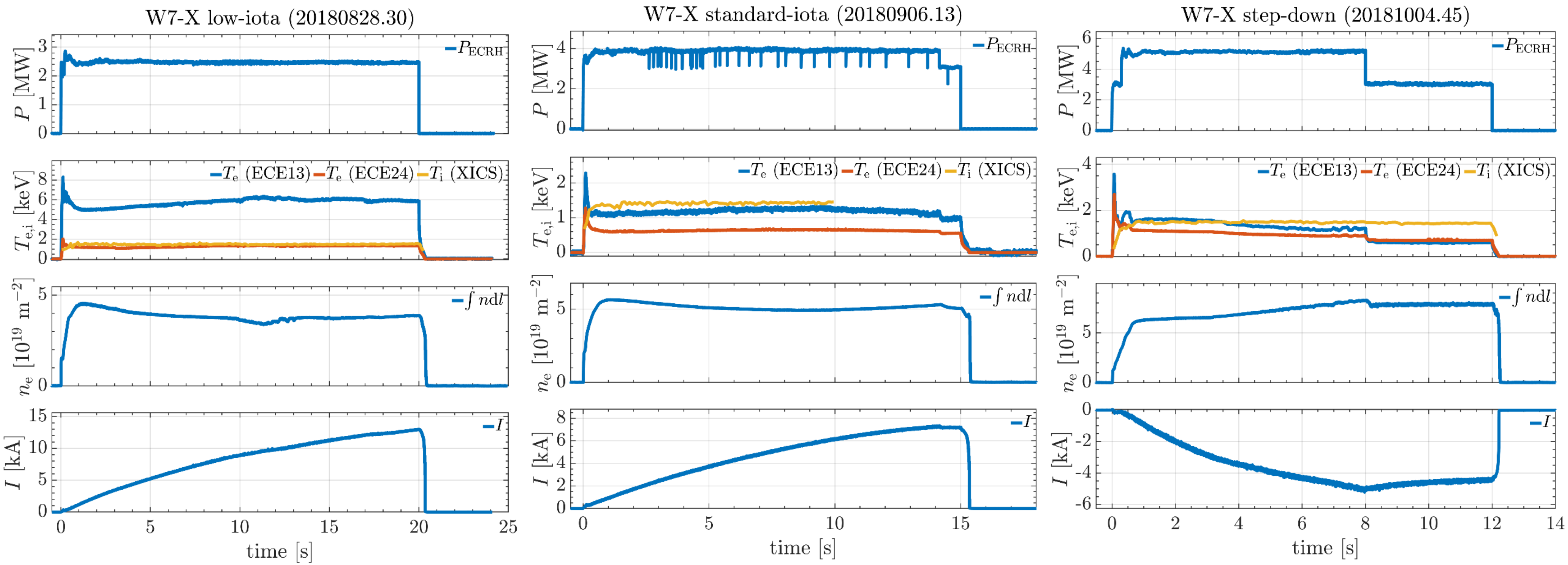




- THRIFT electric field closely follows analytical field for $t > 3$ s
- Decay times for THRIFT and analytical currents agree

Implementation successful

Current evolution simulations for W7-X



fixed heating, low-iota

fixed heating, standard

heating step-down,
low-iota, reverse field

Set-up in THRIFT:

- n, T profiles produced by WAPID_FIT
- $\eta_{\parallel}$ calculated using Sauter model
- Bootstrap current calculated by BOOTSJ

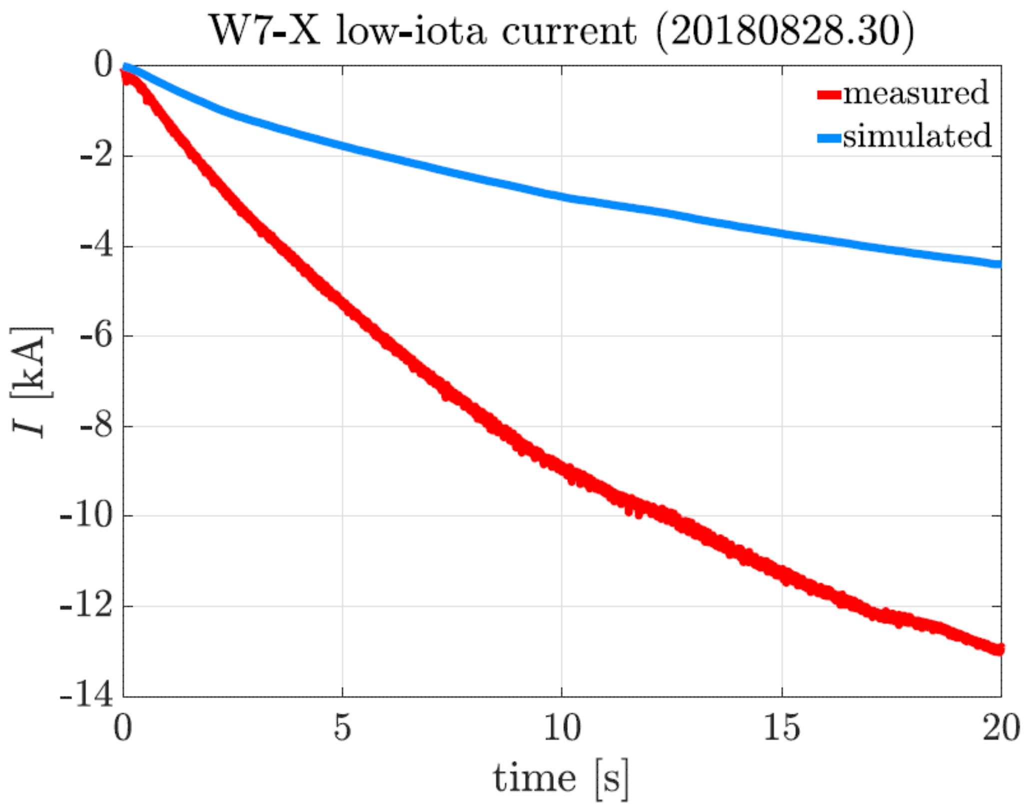
Results for low-iota, fixed heating case (I)

Curve fits:

$$I(t) = I_\infty\left(1 - \exp\left(-t/\tau_{L/R}\right)\right)$$

	$\tau_{L/R}$ [s]	I_∞ [kA]	I_{BS} [kA]
experiment	12.76 ± 0.36	16.32 ± 0.24	–
simulation	13.22 ± 0.33	5.53 ± 0.03	8.65

experimental uncertainties from fitting;
simulation uncertainties from profile uncertainties



- Quantitative agreement in $\tau_{L/R}$ (~4%)
- Disagreement in currents:
  - Bootstrap current ~half the experimental I_∞
  - Noticeable difference (few kA) between simulation I_∞ and I_{BS}

➢ BOOTSJ **underestimates** the bootstrap current

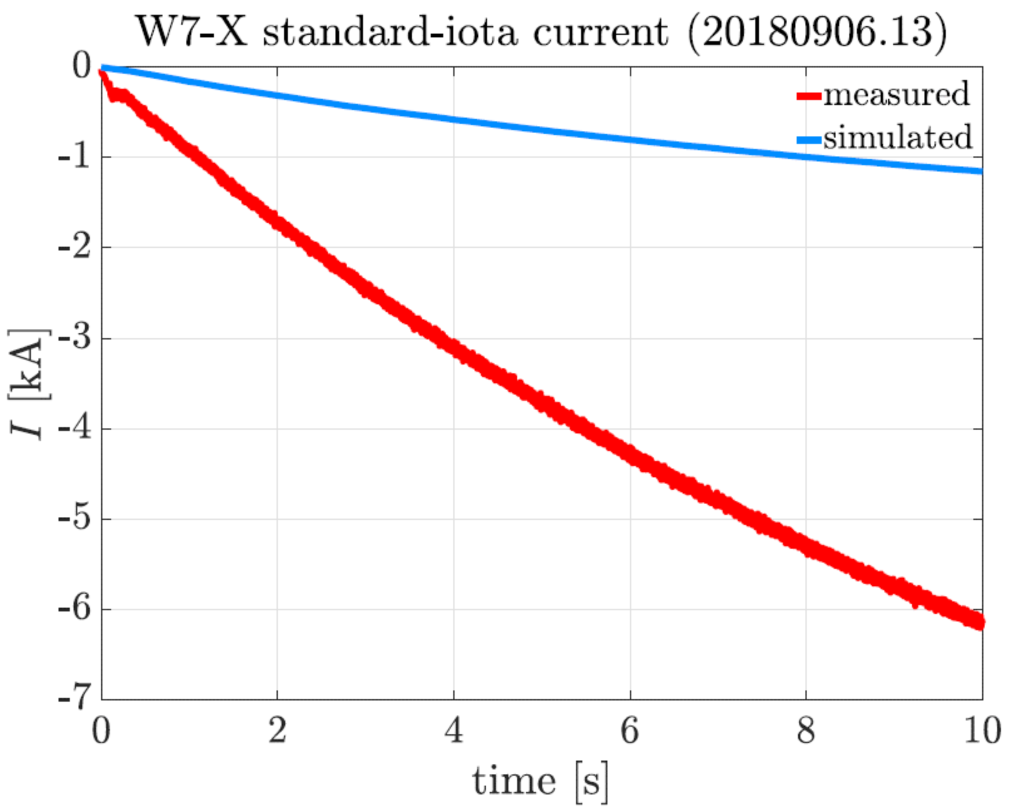
Results for standard-iota, fixed heating case (II)

Curve fits:

$$I(t) = I_\infty\left(1 - \exp\left(-t/\tau_{L/R}\right)\right)$$

	$\tau_{L/R}$ [s]	I_∞ [kA]	I_{BS} [kA]
experiment	12.16 ± 0.07	5.53 ± 0.03	–
simulation	11.09 ± 0.003	1.91 ± 0.001	3.31

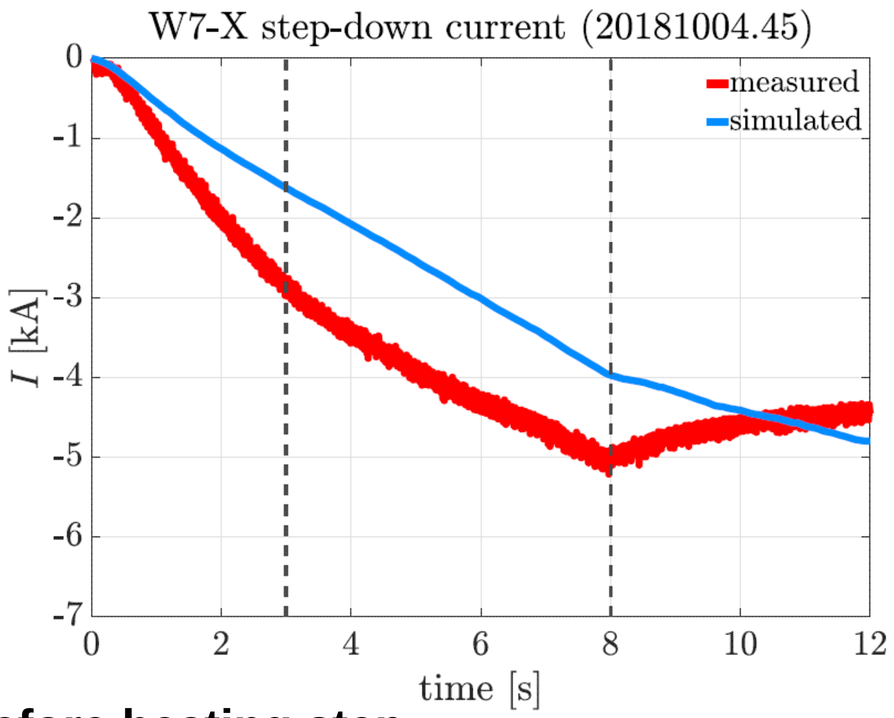
uncertainties from fitting

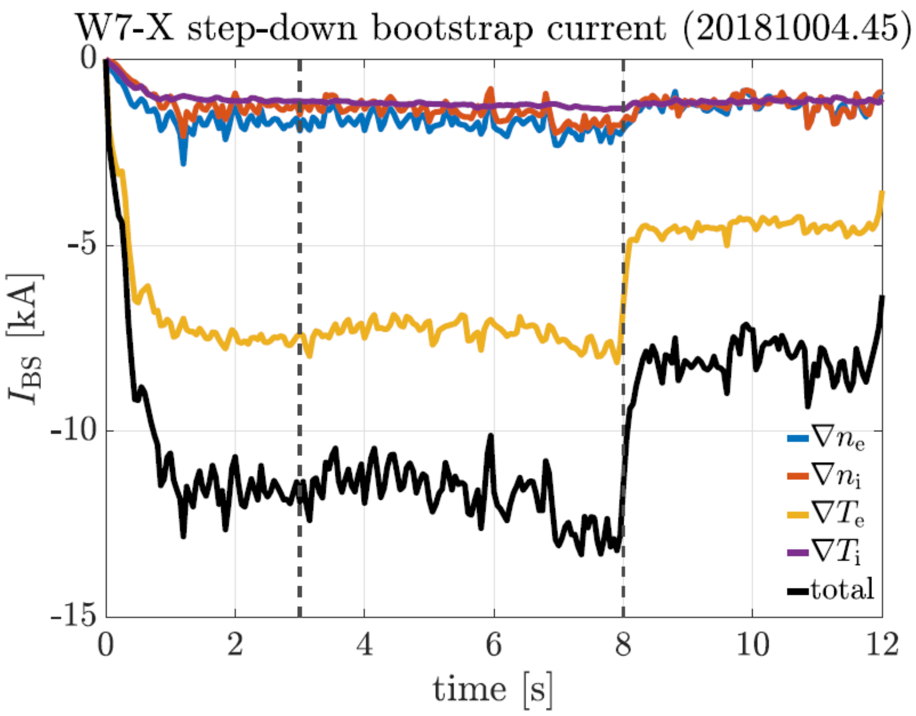


- Quantitative agreement in $\tau_{L/R}$ (~10%)
- Disagreement in currents:
  - Bootstrap current $\sim 0.6 \times$ experimental I_∞
  - Small difference between simulation I_∞ and I_{BS}

➢ BOOTSJ **underestimates** the bootstrap current

Results for reverse-field, low-iota, heating step down case (III)





Before heating step:
- Disagreement in currents

After heating step:
- Disagreement in currents:
  - Experimental current decays
  - Simulated current continues growing

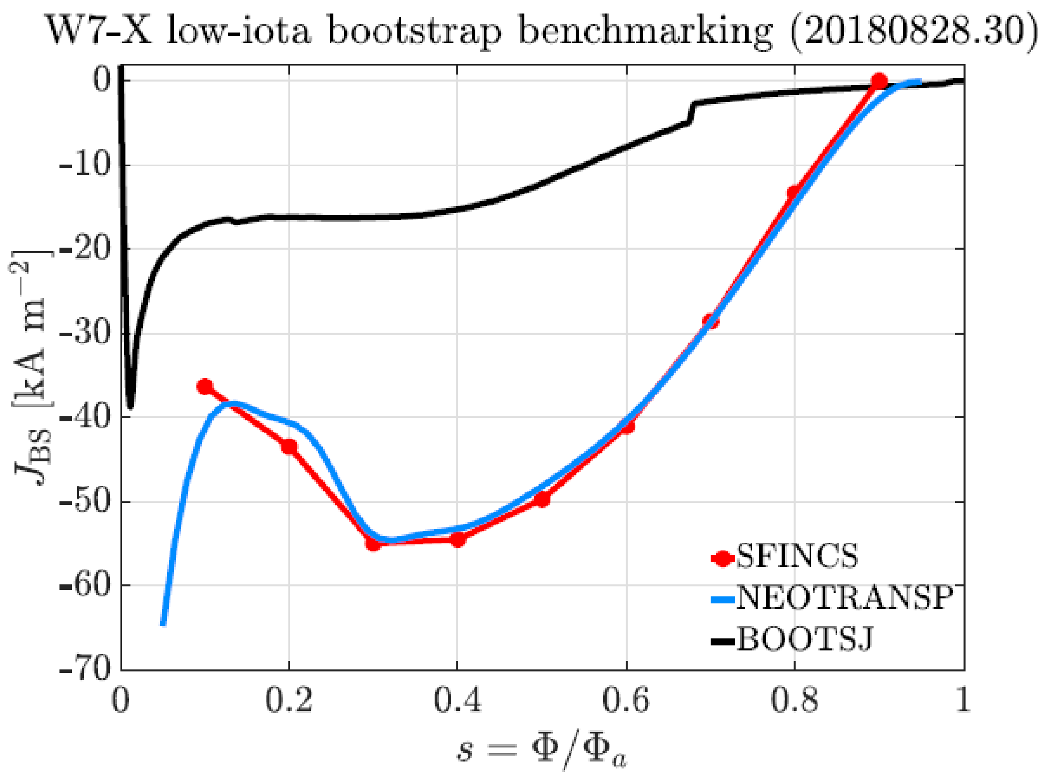
BOOTSJ bootstrap before step: ~12 kA
BOOTSJ bootstrap after step: ~8 kA

➢ BOOTSJ **overestimates** the bootstrap current

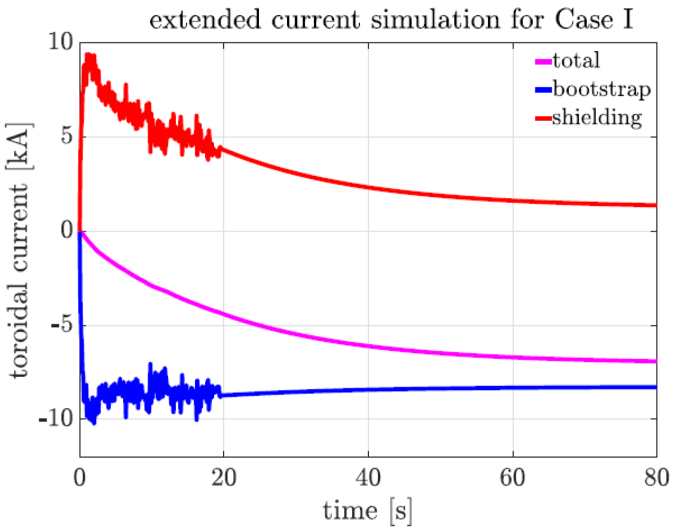
(I) Bootstrap current predictions by BOOTSJ

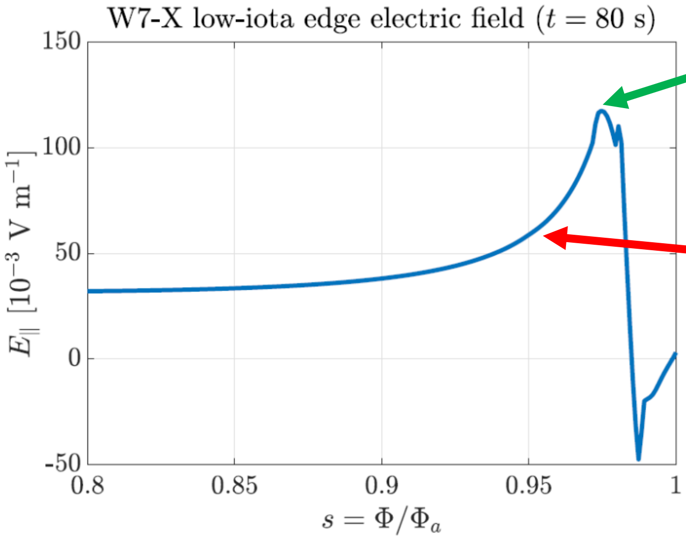
- BOOTSJ model is derived in the asymptotic collisionless limit
- For this case, compare estimate of bootstrap current from BOOTSJ with:
  - SFINCS
  - NEOTRANSP

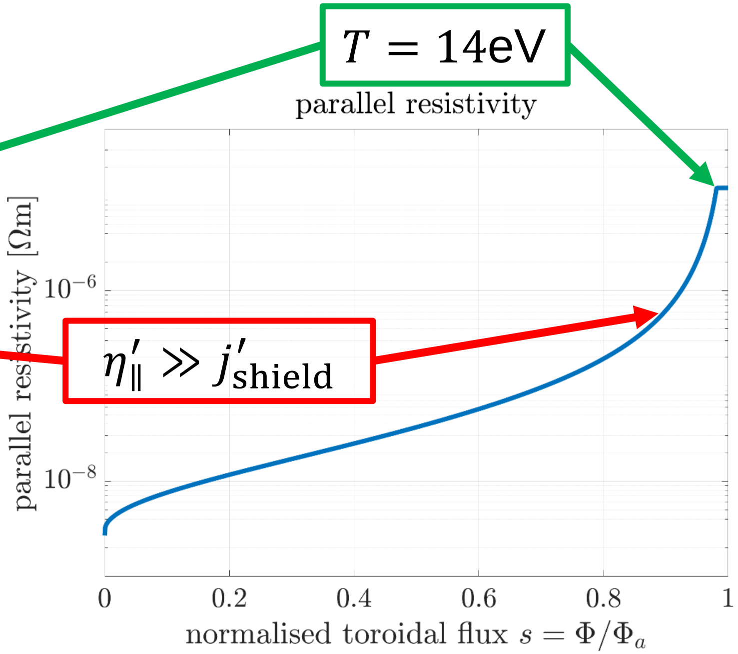
➢ BOOTSJ inaccurate at finite collisionalities



(I) Shielding current does not vanish in steady-state







- Shielding current $\not\to 0$ in simulation

- Edge BC: $E_\parallel = -\frac{L_{ext}}{2\pi R}\frac{\mathrm{d}I}{\mathrm{d}t} \to 0$
- Core electric field becomes **“frozen-in”** due to edge structure

$$E_\parallel = \eta_\parallel j_{\mathrm{shield}}$$

- $\eta_\parallel$ flattens when T clamped
- Small residual shielding current in the plasma edge

…but $\eta_\parallel$ **increases** faster than j_{shield} **decreases**.

➢ Different edge temperature model/clamping (~ 100eV) or a different resistivity model necessary

Current acceleration with ECCD

- Some divertor scenarios require certain $I = I_{\mathrm{BS}} = I_{\mathrm{target}}$

$$I(t) = I_\infty\left(1 - \exp\left(-t/\tau_{L/R}\right)\right)$$
$$\rightarrow \dot{I}(t) \propto I_\infty = I_{\mathrm{BS}} + I_{\mathrm{ext}}$$

- Temporarily drive current I_{ext} to reach $I = I_{\mathrm{target}}$

Set-up in THRIFT:

- Add ECCD to low-iota, fixed heating case:

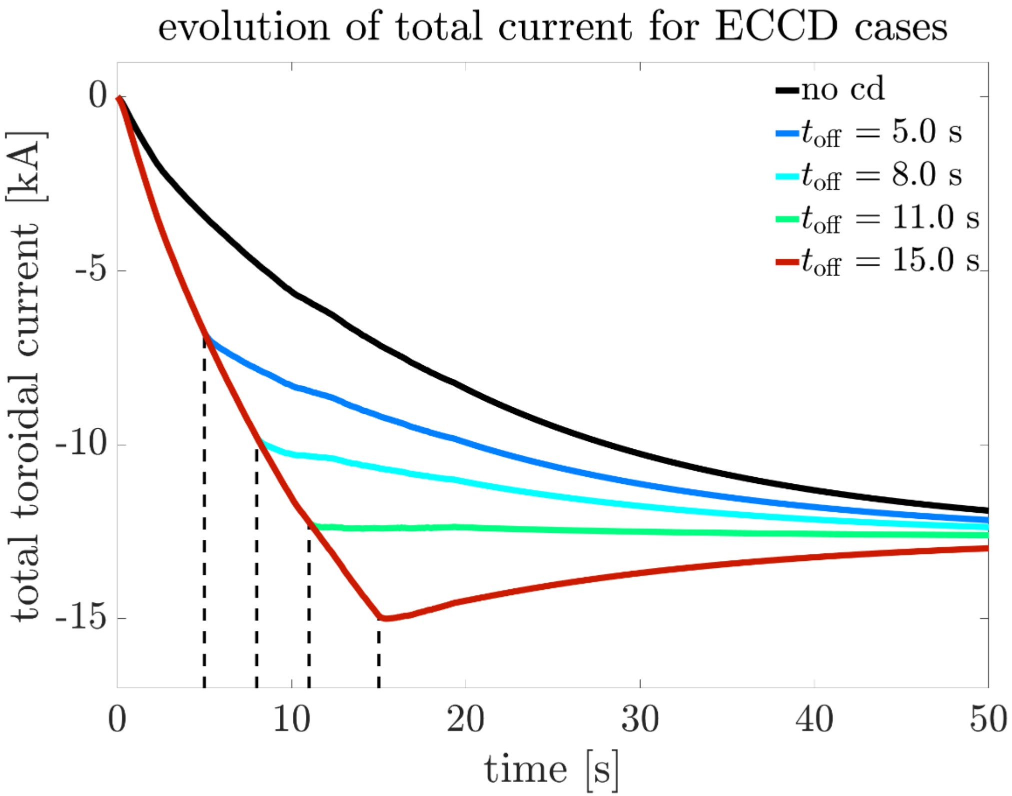
$$J_{\mathrm{ECCD}}(\rho) = I_{\mathrm{norm}} \exp\left(-\frac{(\rho - r_c)^2}{w^2}\right)$$

$\rho = \sqrt{s} = \sqrt{\Phi/\Phi_a}$
$r_c = 0.3$ – deposition location
$w = 0.175$ – deposition width
[Yu Turkin et al, *Fusion Sci. Technol.* **50**, 387-394 (2006)]

- Apply $I_{\mathrm{ECCD}} = 20$ kA for durations $t_{\mathrm{ECCD}} = 5$ s, 8 s, 11 s, 15 s

Current acceleration with THRIFT

- No ECCD: I_∞ in > 50 s
- ECCD with $t_{\text{off}} = 5$ s $\rightarrow$ undershoot I_∞
- ECCD with $t_{\text{off}} = 8$ s $\rightarrow$ undershoot I_∞
- ECCD with $t_{\text{off}} = 11$ s $\rightarrow I_\infty$
- ECCD with $t_{\text{off}} = 15$ s $\rightarrow$ overshoot I_∞



- ➢ THRIFT captures changes in source currents
- ➢ CD scenario investigation possible with THRIFT:
  - Amount of CD necessary to reach target currents
  - Effect of CD on iota profiles

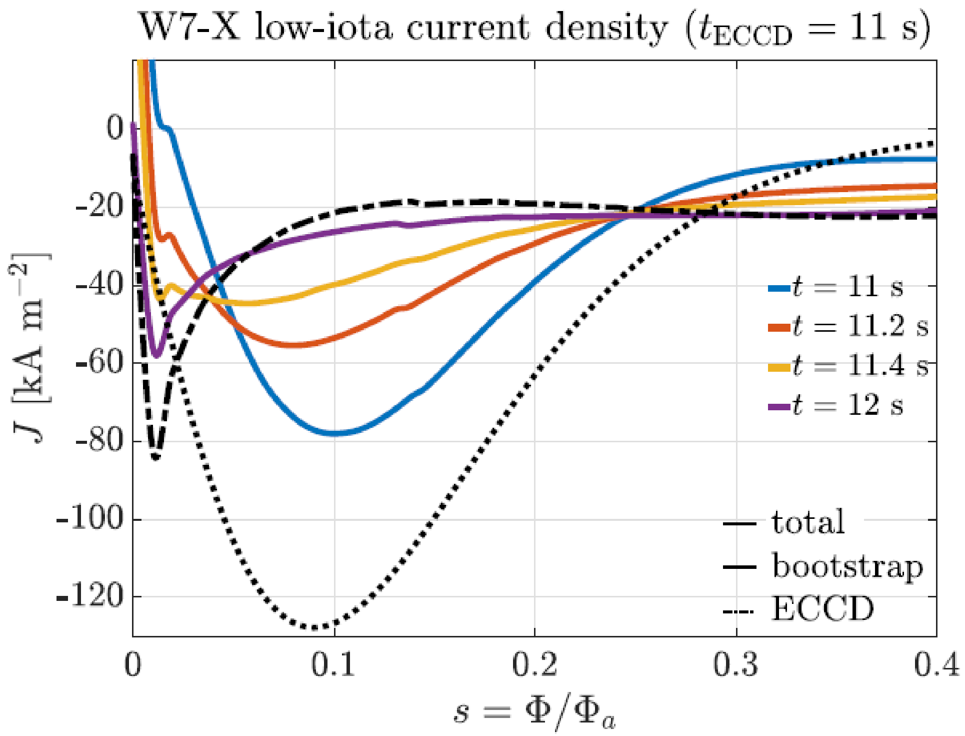
Conclusion

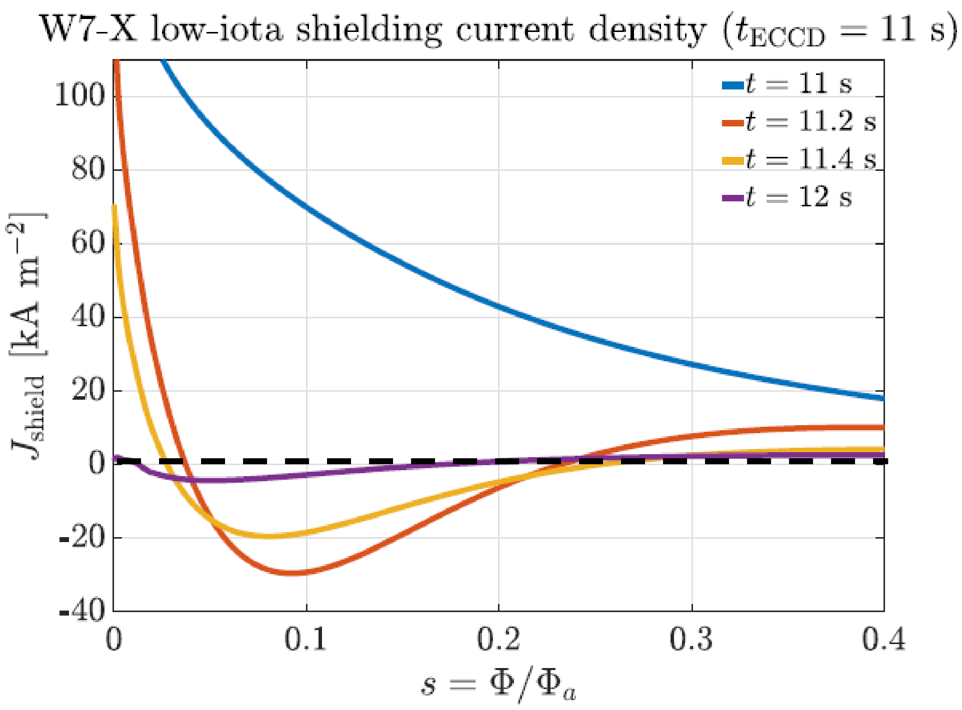
- Reimplemented and verified the THRIFT code
- Decay times of plasma currents in W7-X are recreated by THRIFT
- BOOTSJ predictions inaccurate → Plasma collisionality
- THRIFT shielding current does not vanish → Different treatment of edge T necessary
- Changes in source currents captured by THRIFT

Future potential uses of THRIFT:

- Validating/benchmarking other source current codes
- Investigating current drive scenarios

Current density for $t_{\mathrm{ECCD}} = 11$ s

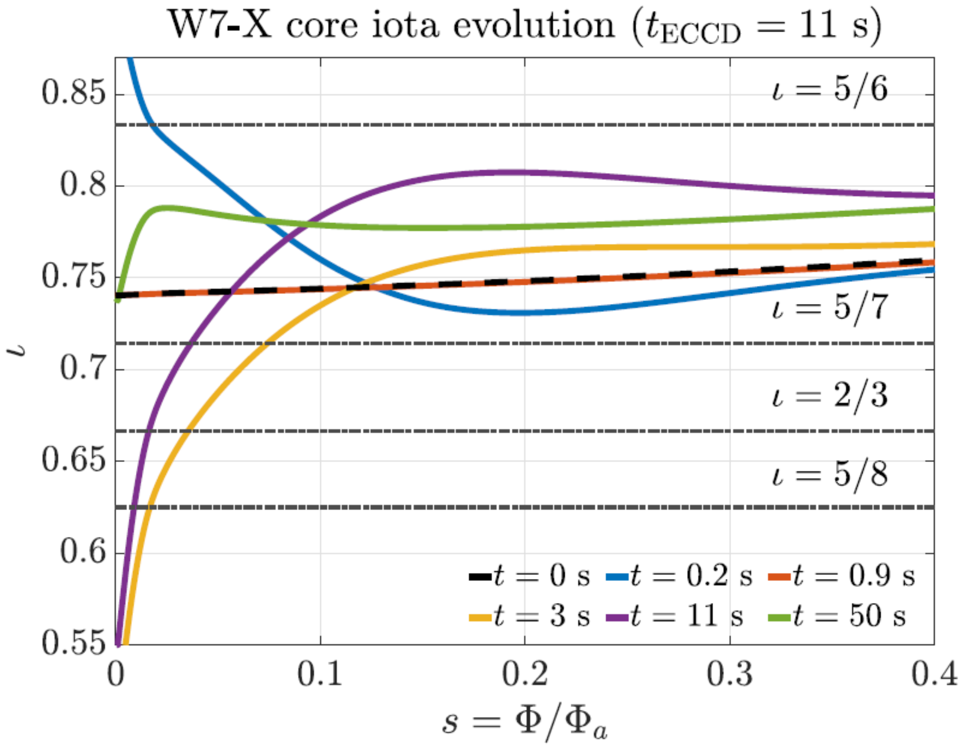


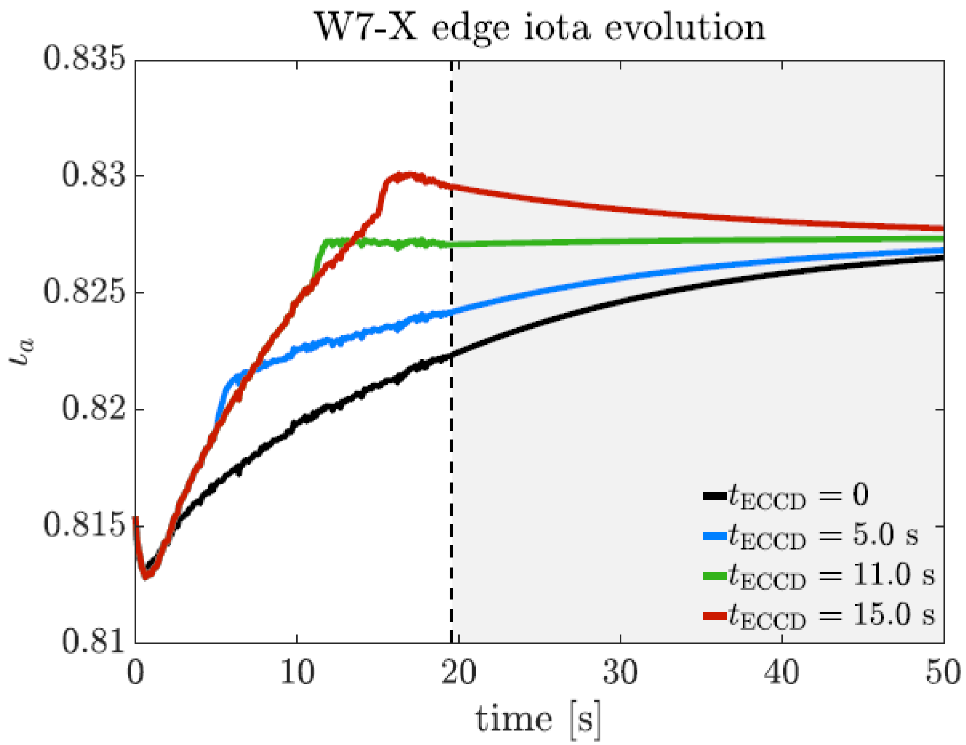


- Total current remains unchanged
- Total current density redistributes to match bootstrap current
- Redistribution occurs on timescale of ~1s

- Shielding current ∝ $E_{\|}$
- $E_{\|}$ becomes locally negative (in direction of total current) → rapid diffusion from “negative” to “positive” locations
- $\int E_{\|} \mathrm{d}s = 0$

Iota evolution with ECCD

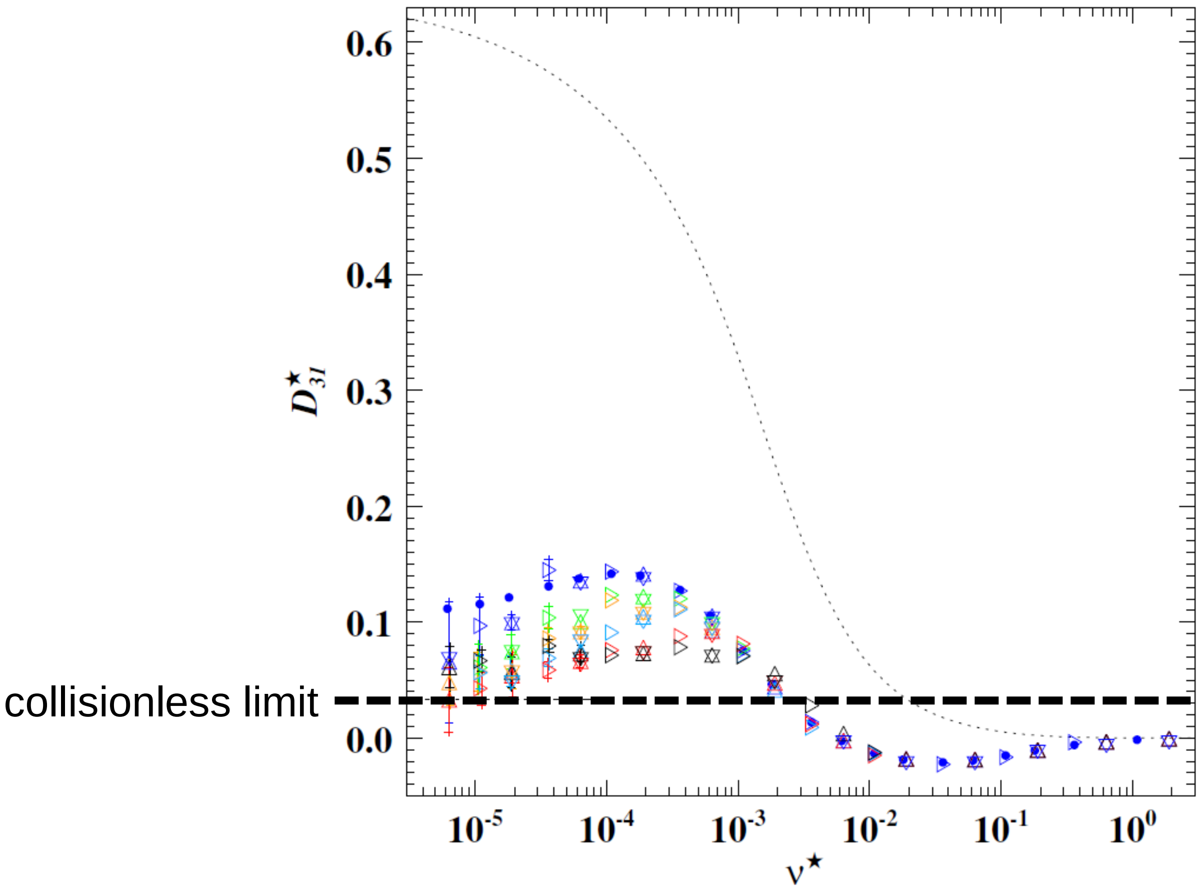




Mono-energetic bootstrap current coefficient for W7-X

BOOTSJ underpredicts at small (finite) collisionalities, overpredicts at large collisionalities

mono-energetic bootstrap current coefficient (standard-iota)



Beidler C.D. et al. *Nucl. Fusion* ***51*** *(2011) 076001*