Estimation of Turbulent Transport Coefficients by the Conditional Variance Method

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Outline

- **Problem: Measurements of turbulent transport**
- **Theoretical background of the transport estimation model**
- **Two applications; Simulations (GEMR) & Experimental data (MPM)**
- **Summary**

Problem: Measurements of turbulent transport

- **Turbulence**
  - Important for plasma confinement capabilities of fusion devices
  - Affects all areas; from the core[1] to the edge[2] and plasma-facing materials[3-5]
- **Measurements**
  - Probes are measuring density and electrostatic potential fluctuations which are used to calculate turbulent transport[6,7]
  - Invasive diagnostic: subject to heat loads ➡ limited operational space
  - Potential measurements are the limiting factor to calculate the turbulent transport, while density measurements are widely available
- **Goal**
  - Find a model to estimate turbulent transport
  - Utilize single-point density measurements
  - Circumvent the need for potential measurements

Theoretical background

$$n = \langle n \rangle + \tilde{n}$$
$$\boldsymbol{u} = \langle \boldsymbol{u} \rangle + \tilde{\boldsymbol{u}}$$

Time evolution of density variance

$$\frac{\partial \langle \tilde{n}^2 \rangle}{\partial t} + \nabla \mathrm{F}_{\tilde{n}} = \mathrm{P}_{\tilde{n}} - \epsilon_{\tilde{n}}$$

[Turbulent Flows, Stephen B. Pope, 2000]

- Flux of variance: $F_{\tilde{n}} = \langle \tilde{\boldsymbol{u}} \tilde{n}^2 \rangle - \kappa \nabla \langle \tilde{n}^2 \rangle$
- Dissipation: $\epsilon_{\tilde{n}} = 2\kappa \langle \nabla \tilde{n} \cdot \nabla \tilde{n} \rangle$
- Production: $P_{\tilde{n}} = -2 \langle \tilde{\boldsymbol{u}} \tilde{n} \rangle \cdot \nabla \langle n \rangle$

Thought experiment:

- Reset fluid into state of negligibly small level of density fluctuations but fully developed velocity field
- Flux of variance and dissipation are expected to be small
- Variance growth is ought to grow linearly at a rate given by $P_{\tilde{n}}$

$$\frac{1}{2(\nabla \langle n \rangle)^2} \frac{\partial \langle \tilde{n}^2 \rangle}{\partial t} = -\frac{\langle \tilde{\boldsymbol{u}} \tilde{n} \rangle}{\nabla \langle n \rangle} = D_T$$

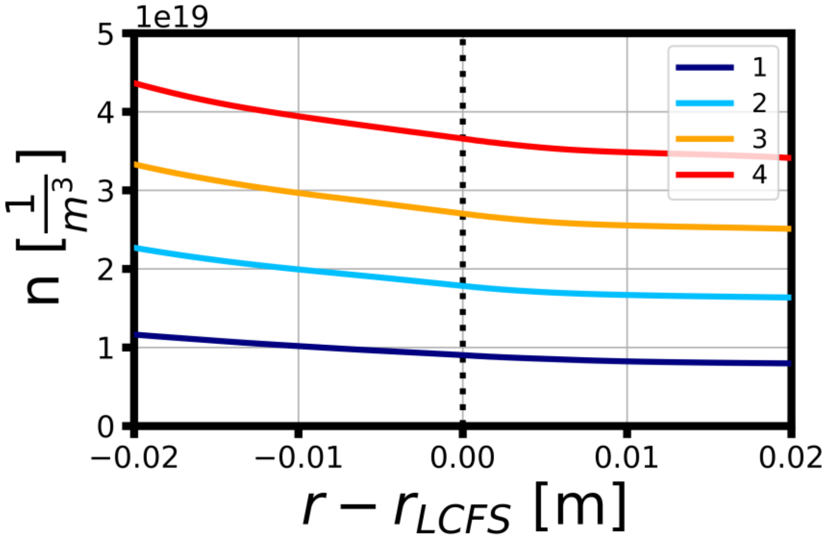
Theoretical background

- Radial transport across magnetic flux surfaces can be quantified by the turbulent diffusivity $D_T = -\frac{\langle \tilde{u}_r \tilde{n} \rangle}{\nabla_r \langle n \rangle}$
- Deduce time-averaged flux $\Gamma_r = \langle \tilde{u}_r \tilde{n} \rangle$ from density fluctuation statistics at one point
- Variance growth is the key quantity
- Following the thought experiment; calculate the variance growth when fluctuations are small.
  - Step 1: $R_n(t) = \frac{\tilde{n}(t)}{\nabla_r \langle n \rangle}$ time-dependent mixing length
  - Step 2: $D_{CV} = \lim_{\tau \to 0} \frac{1}{2\tau} \langle [R_n(t+\tau) - R_n(t)]^2 | R_n(t) = 0 \rangle \stackrel{?}{=} D_T$ GOAL

Time derivative — Squared increments between timesteps — Conditioning

Application: Simulations (GEMR)

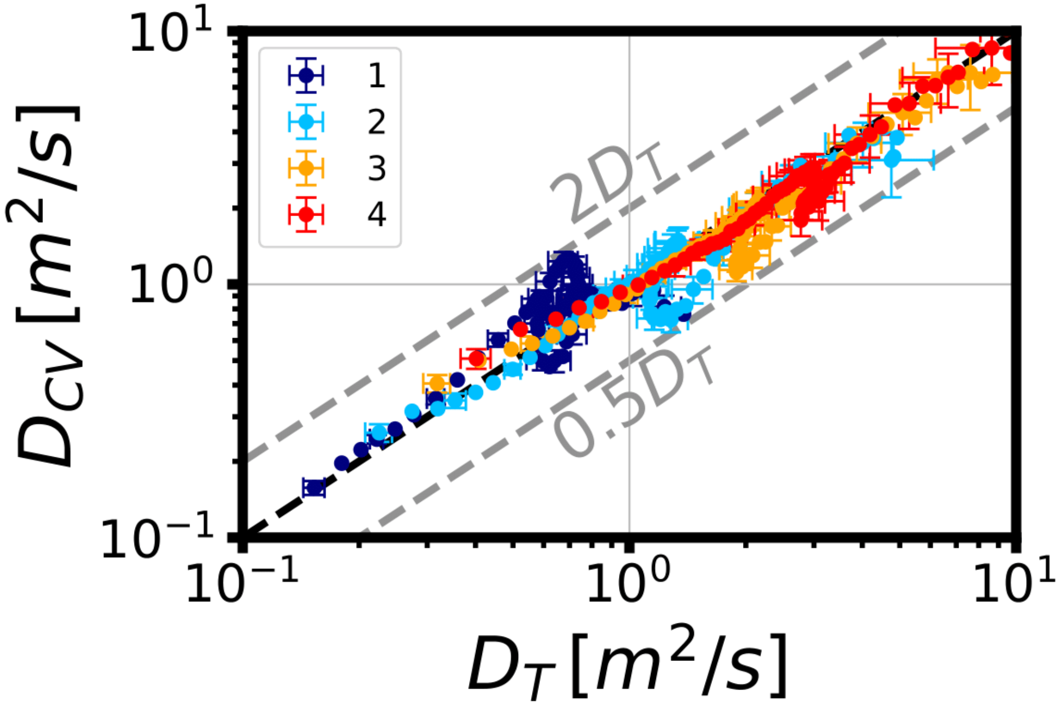
- 3D electromagnetic delta-f gyrofluid code for tokamaks (here for AUG)
- 2D drift planes (outer midplane); ~2 cm around the LCFS
  - Plasma edge, near-SOL & partly far-SOL (far-SOL discarded for transport analysis)
- 4 different density runs
- Calculate D_T using v_{ExB}; electrostatic transport
  - $\Gamma_r = \frac{\langle \tilde{n} \tilde{E}_y \rangle}{B}$
  - $D_T = -\Gamma_r / \nabla_r \langle n \rangle$



Results: GEMR

$$D_{CV} \stackrel{?}{=} D_T$$

Particle transport

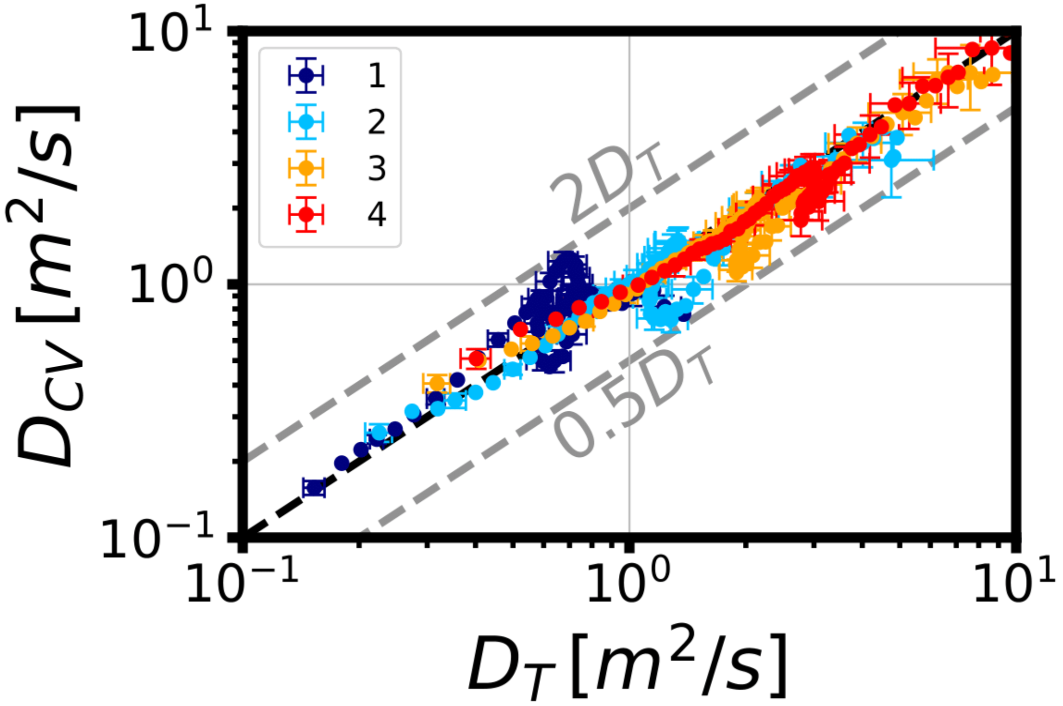


- **Colors indicate density runs**
  - Blue lowest; red highest
- **80 radial positions per run for comparison**
- **Grey lines show factor 2 deviations**

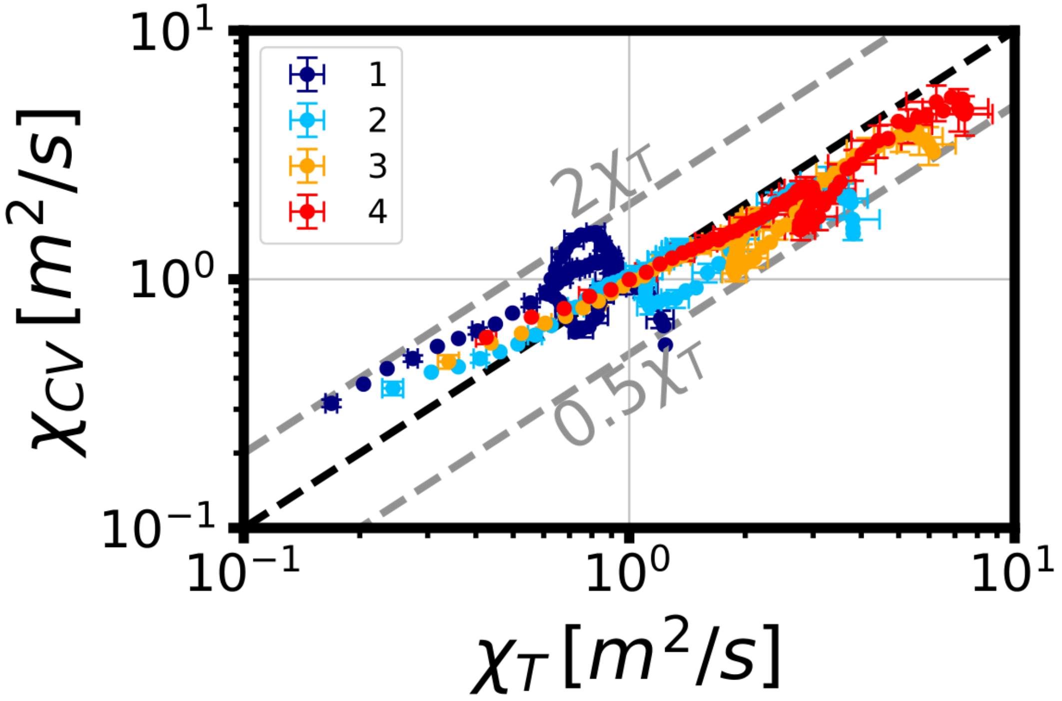
Results: GEMR

$$D_{CV} \stackrel{?}{=} D_T$$

Particle transport



Heat transport



$$X_T = -\frac{2}{3}\frac{\langle \tilde{u}_r \tilde{T}_e \rangle}{\nabla_r \langle T_e \rangle}$$

Application: Experimental data (MPM)

- **Method‘s favorable area of application is the core**
  - Transport measurements are not available
- **Difficult to validate methods without a precise counterpart**
  - GEMR showed decent applicability in the near-SOL

↘ *Use MPM for first exp. heuristic validation*

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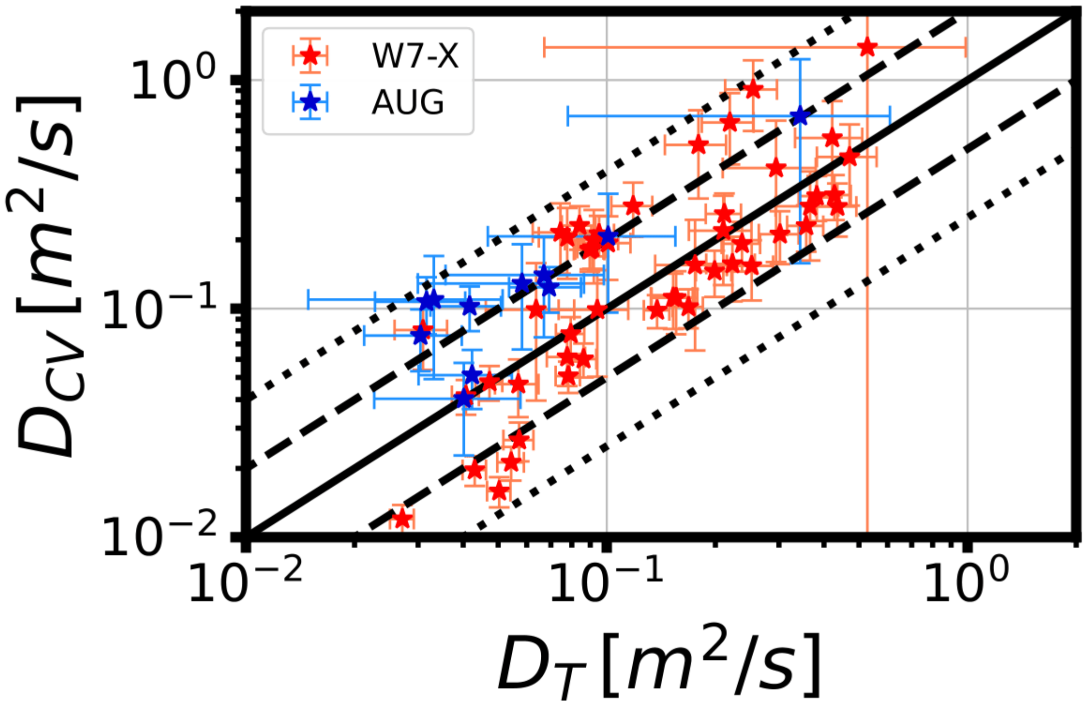
- **MPM provides similar data as used for GEMR**
  - Ion saturation current: density measurement
  - Floating potential: plasma potential
- **MPM data from ASDEX Upgrade (11 timeseries) and W7-X (51 timeseries)**
  - Different heating powers, magnetic configurations

Results: MPM

$$D_{CV} \stackrel{?}{=} D_T$$



- $\Gamma_r = \frac{\langle \tilde{n}\tilde{E}_\theta \rangle}{B}$ $\qquad D_T = -\Gamma_r / \nabla_r \langle n \rangle$
- Deviation lines
  - Dashed: factor 2
  - Dotted: factor 4
- **Deviations not larger than 4 for AUG & W7-X**
- AUG
  - Overestimation by factor 2 (?)
  - Larger uncertainties due to profiles
- W7-X
  - Reproduction of overall trends
  - Mostly withing a factor of 2

Summary

- **Estimation of diffusivity calculating growth of density fluctuation variance conditioned to small pertubations**
- **Two applications**
  - Simulations (GEMR)
  - Experimental data (MPM from AUG and W7-X)
- **Deviations not larger than 4 for most of the data**
- **Overall reproduction of trends**

Bibliography

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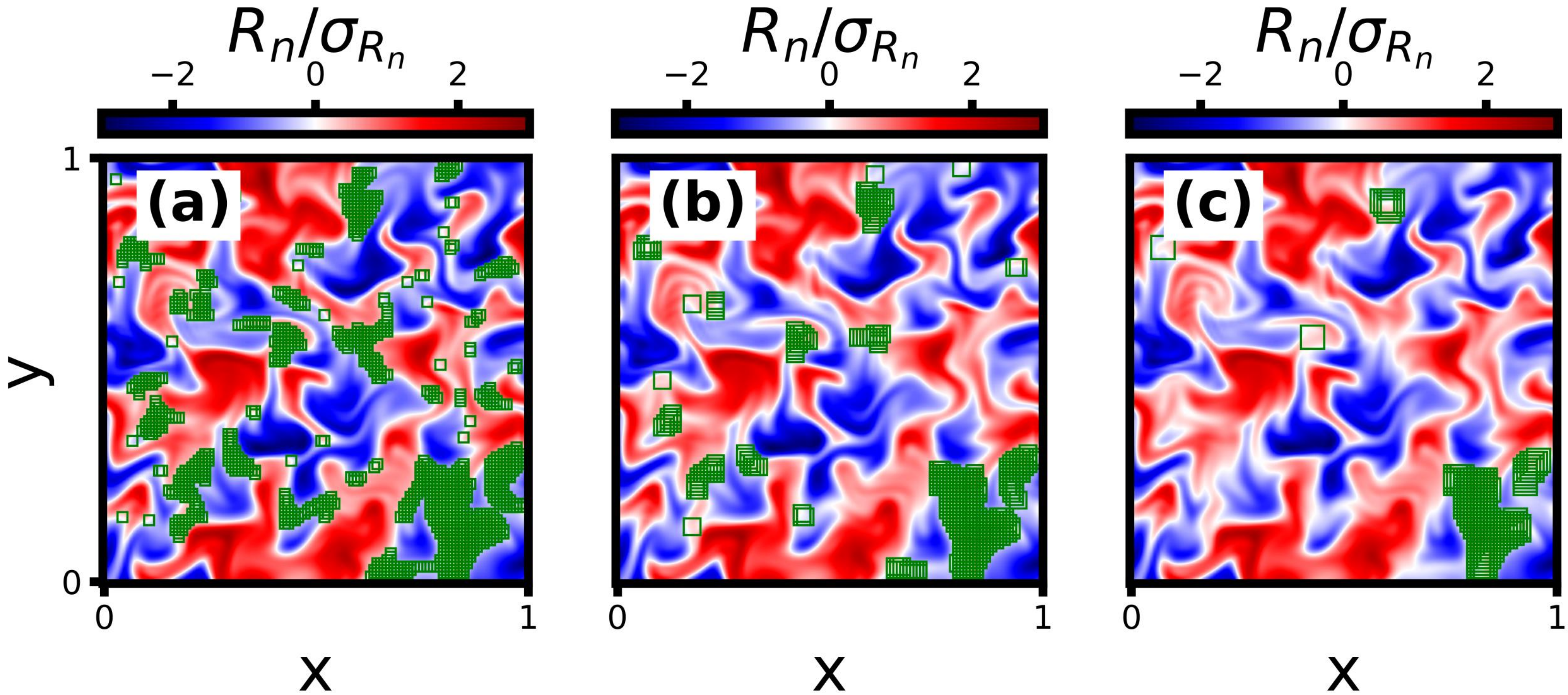
BACKUP - Multiconditioning



BACKUP – Local vs. Non-local

$$\frac{\partial}{\partial t}\tilde{n} + \tilde{\mathbf{u}}\nabla\langle n\rangle + \nabla(\tilde{\mathbf{u}}\tilde{n}) = \kappa\nabla^2\tilde{n}$$

Multiply by $\tilde{n}$ and averaging yields

$$\frac{1}{2}\frac{\partial}{\partial t}\langle\tilde{n}^2\rangle + \langle\tilde{\mathbf{u}}\tilde{n}\rangle\nabla\langle n\rangle + \langle\tilde{n}\tilde{\mathbf{u}}\nabla\tilde{n}\rangle = \kappa\langle\tilde{n}\nabla^2\tilde{n}\rangle$$