A full-f quasi-neutrality solver for gyrokinetic simulations of tokamaks in presence of poloidal asymmetries

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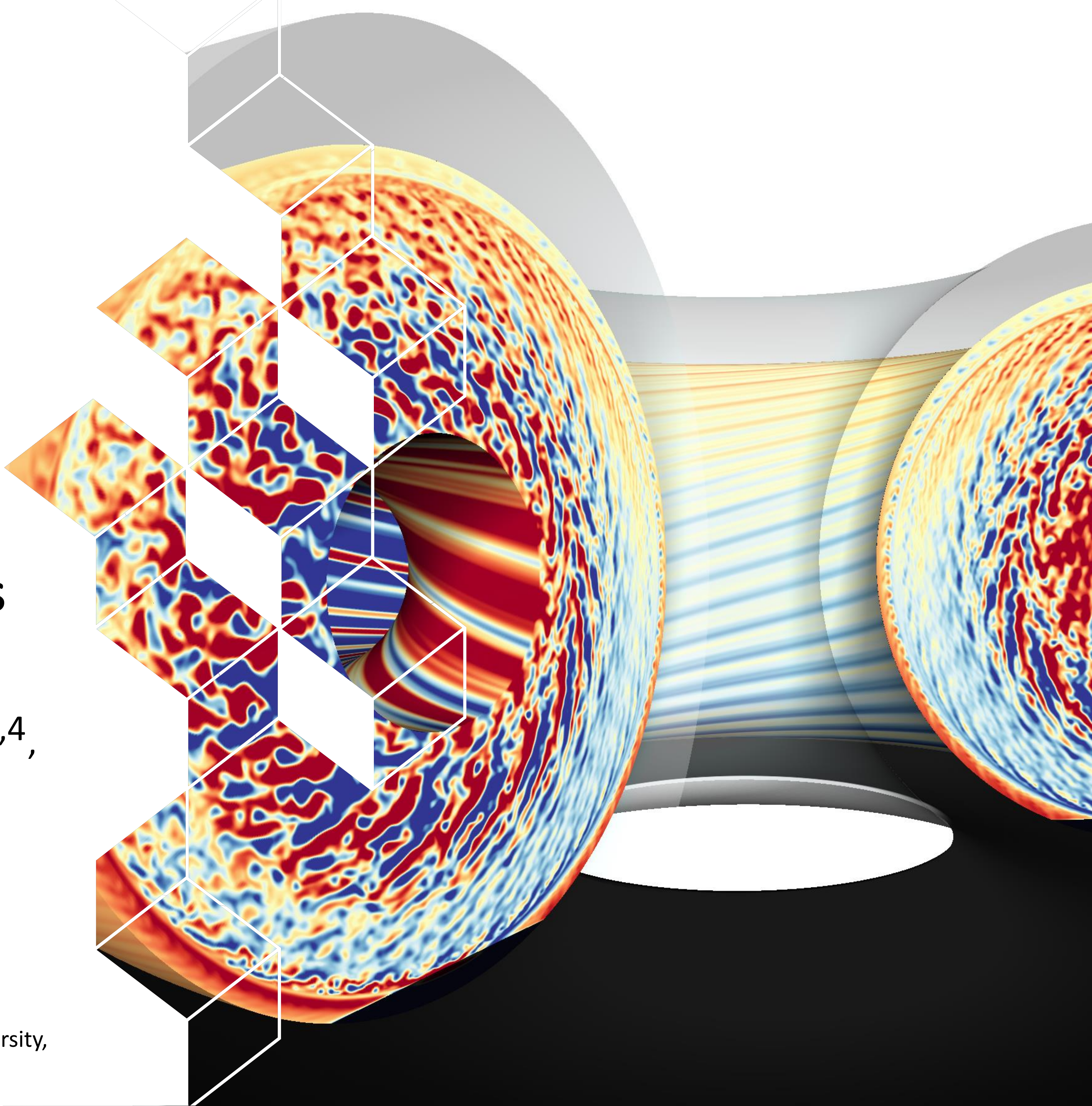


Motivation

- The goal is to simulate the LH transition with a global gyrokinetic code

- The requirement is to simulate consistenlty the edge and the SOL,

- The SOL is caracterized by large poloidal asymmetries induced by the interaction with the wall (limiter or divertor)

- Poloidal asymmetries are also experimentally measured in the edge when there is a transport barrier [Churchill 2015]

➢ **Need for a quasi-neutrality solver able to treat large poloidal asymmetries**

TSVV1 deliverable:
D2.10: Report on the impact of 2D (radial/poloidal) density variations in the quasi-neutrality equation

Outline

1. Motivation
2. Quasi-neutrality equation
3. Test case: GAM damping
4. Illustration on a simple case
5. Numerical consideration
6. Conclusion and outlook

Derivation of the quasi-neutrality equation

Gyrokinetic Poisson equation:

$$-\nabla^2\phi = \frac{1}{\epsilon_0}\sum_s e_s n_s = \frac{1}{\epsilon_0}\sum_s e_s\left(\bar{n}_s + n_{pol,s}\right)$$

$\lambda_D \ll L$

We assume the quasineutrality fullfilled at initial time

$$-\sum_s e_s \delta n_{pol,s} = \sum_s e_s \delta \bar{n}_s$$

Polarisation term (= Finite Larmor Radius effect)

Gyrocenter density

Two approximations for the polarisation term

Long wavelength approximation (LWA): valid for $k_\perp \rho_i \ll 1$

$$e_s \delta n_{pol,s}^{LWA}(\boldsymbol{X},t) = \boldsymbol{\nabla}_\perp \cdot (\alpha_s \boldsymbol{\nabla}_\perp \phi)$$

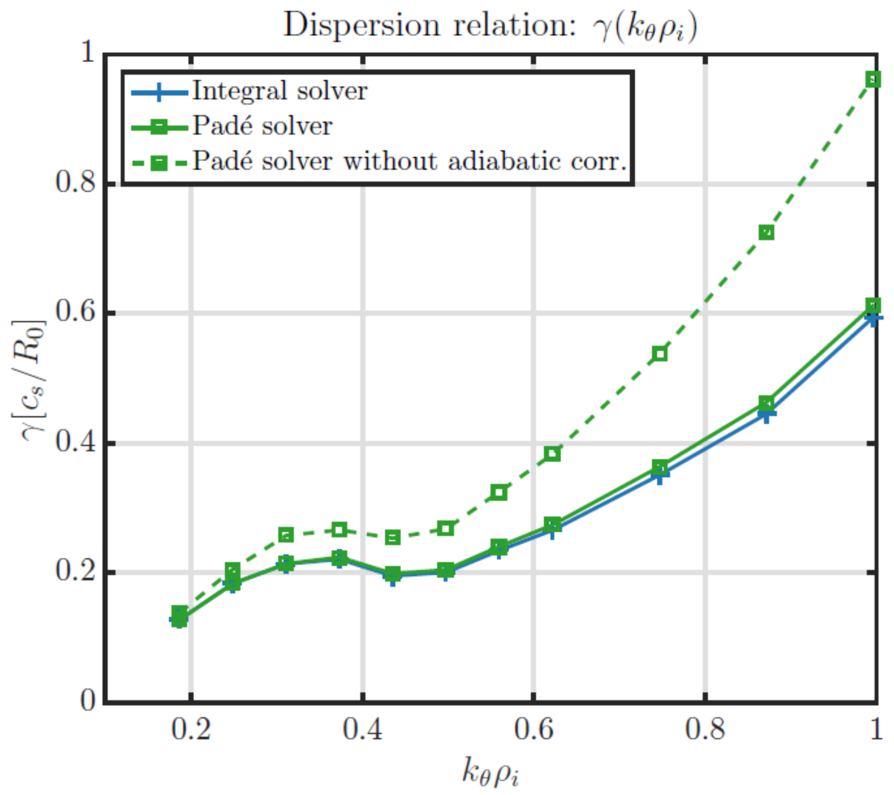
$$\alpha_s = \frac{m_s \langle \bar{n}_s \rangle_\varphi}{\langle B \rangle_\varphi^2}$$

Historically: assumed constant in time and therefore 1D
This work: 2D function + time dependent

Padé approximation : valid for $k_\perp \rho_i \leq 1$

$$e_s \delta n_{pol,s}^{Pade}(\boldsymbol{X},t) = \left[1 - \boldsymbol{\nabla}_\perp \cdot \rho_s^2 \boldsymbol{\nabla}_\perp \cdot\right]^{-1} \left[\boldsymbol{\nabla}_\perp \cdot (\alpha_s \boldsymbol{\nabla}_\perp \phi)\right]$$

[Lanti 2016]



Three electron models

Full Kinetic Electrons (FKE)

$$-\nabla_\perp \cdot \left(\alpha \nabla_\perp \phi^{FKE}\right) = \left[1 - \nabla_\perp \cdot \kappa \nabla_\perp \cdot\right] \left(\sum_i e_i \delta \bar{n}_i\right) - \left[1 - \nabla_\perp \cdot \kappa \nabla_\perp \cdot\right] e \delta \bar{n}_e$$

2D time dependent $\alpha = \sum_i \alpha_i$, $\kappa = \sum_i \rho_i^2$

Adiabatic Electrons (AE)

$$-\nabla_\perp \cdot \left(\alpha \nabla_\perp \phi^{AE}\right) + \left[1 - \nabla_\perp \cdot \kappa \nabla_\perp \cdot\right] \left[\beta^{AE} \left(\phi^{AE} - \langle \phi^{AE} \rangle_{FS}\right)\right] = \left[1 - \nabla_\perp \cdot \kappa \nabla_\perp \cdot\right] \left(\sum_i e_i \delta \bar{n}_i\right)$$

$$\beta^{AE} = e^2 \frac{\langle \bar{n}_e \rangle_{\mathrm{FS}}}{\langle T_e \rangle_{\mathrm{FS}}}$$

Hybrid Kinetic Electrons (HKE)

$$\begin{aligned} -\nabla_\perp \cdot \left(\alpha \nabla_\perp \phi^{TKE}\right) + \left[1 - \nabla_\perp \cdot \kappa \nabla_\perp \cdot\right] \left[\beta^{TKE} \phi^{TKE}\right] &= \left[1 - \nabla_\perp \cdot \kappa \nabla_\perp \cdot\right] \left(\sum_i e_i \delta \bar{n}_i\right) - \left[1 - \nabla_\perp \cdot \kappa \nabla_\perp \cdot\right] e \delta \bar{n}_{e,trap.} \\ &+ \left[1 - \nabla_\perp \cdot \kappa \nabla_\perp \cdot\right] \left[\beta^{TKE} \langle \phi^{FKE} \rangle_{FS}\right] \end{aligned}$$

$$\beta^{TKE} = e^2 \frac{\langle \bar{n}_{e,pas.} \rangle_{\mathrm{FS}}}{\langle T_e \rangle_{\mathrm{FS}}}$$

1D time dependent

$$\phi^{HKE} = \phi^{TKE} - \langle \phi^{TKE} \rangle_{FS} + \langle \phi^{FKE} \rangle_{FS}$$

Numerical solver

The solver uses B-splines & Finite Element [Bourne 2023] to solve the equation of the form

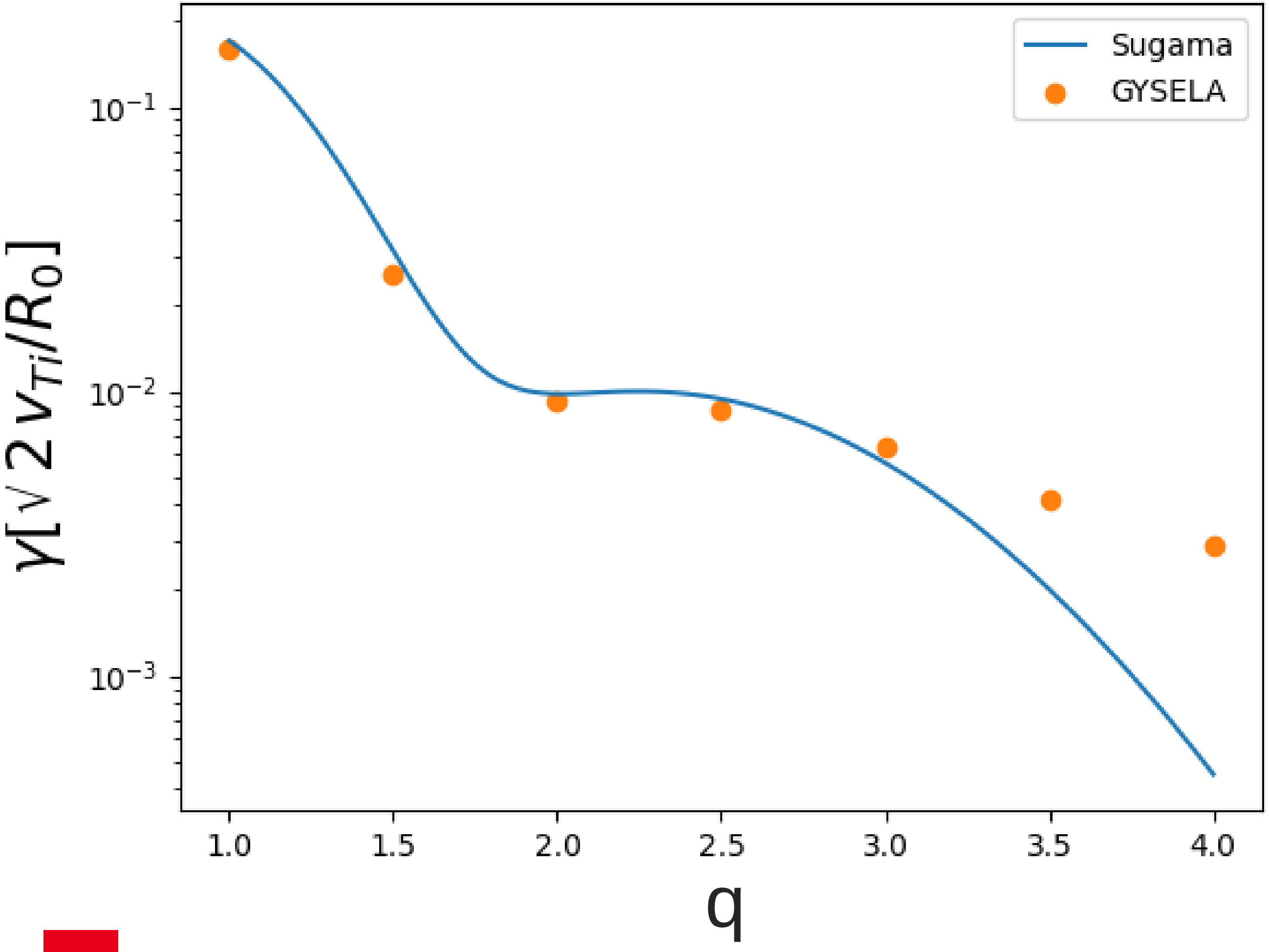
$$-\nabla_\perp \cdot \left[A\left(r,\theta\right)\nabla_\perp\phi\right] + B\left(r,\theta\right)\phi = RHS\left(r,\theta\right)$$

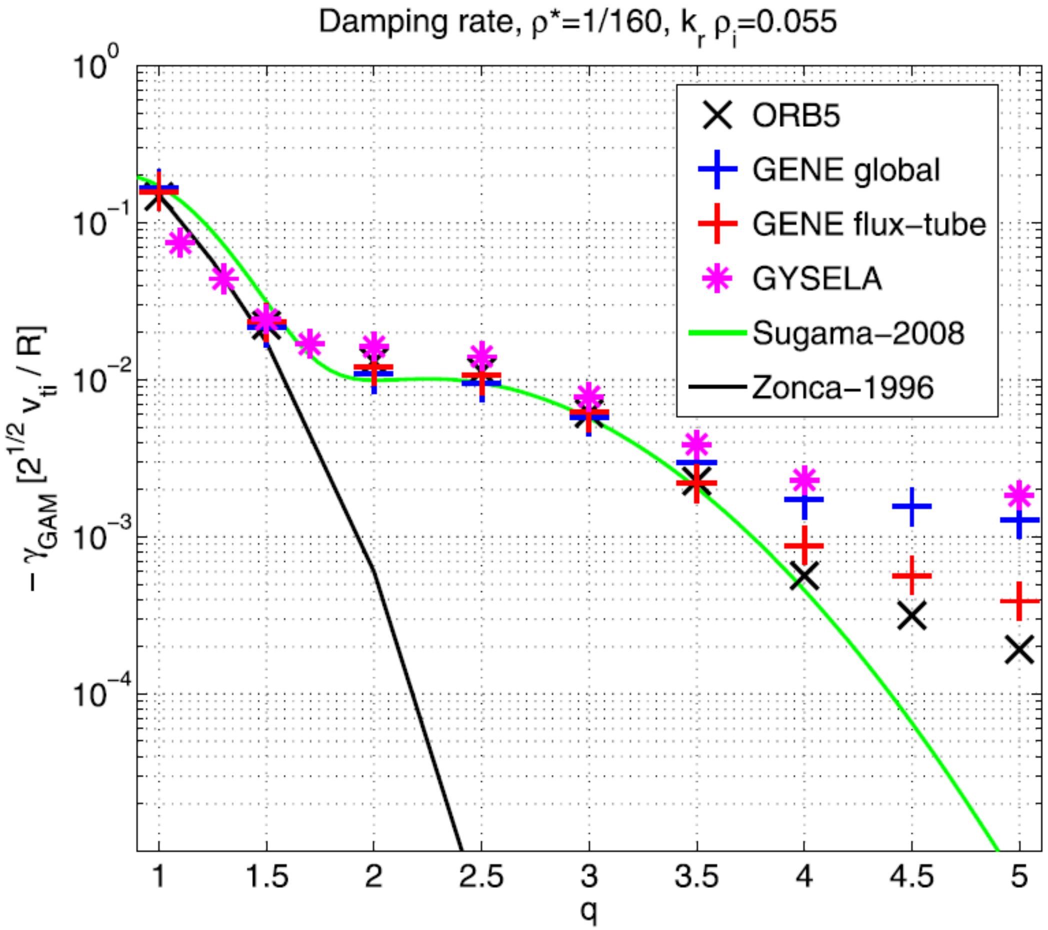
- A priori adiabatic electron response not included in the model. A fixed point method has been developed to compute $\langle\phi^{AE}\rangle_{FS}$. The method has been extensively tested in the circular geometry. Same result obtained with the old solver.
- We assume $\mathbf{e}_{\|} = \mathbf{e}_{\varphi}$ (2D matrix system instead of 3D). Shaped plasmas are included.

$$\nabla_\perp\phi = \frac{\partial\phi}{\partial x^j} g^{jk} e_k = \left(\frac{\partial\phi}{\partial r} g^{rr} + \frac{\partial\phi}{\partial\theta} g^{\theta r}\right) \boldsymbol{e}_r + \left(\frac{\partial\phi}{\partial r} g^{r\theta} + \frac{\partial\phi}{\partial\theta} g^{\theta\theta}\right) \boldsymbol{e}_\theta$$

Test of the solver: Geodesic Acoustic Modes (GAMs) damping retrieved as in the previous version of the solver

ρ_*^{-1}=160, $k_r \rho_i = 0.039$





[Biancalani 2017]

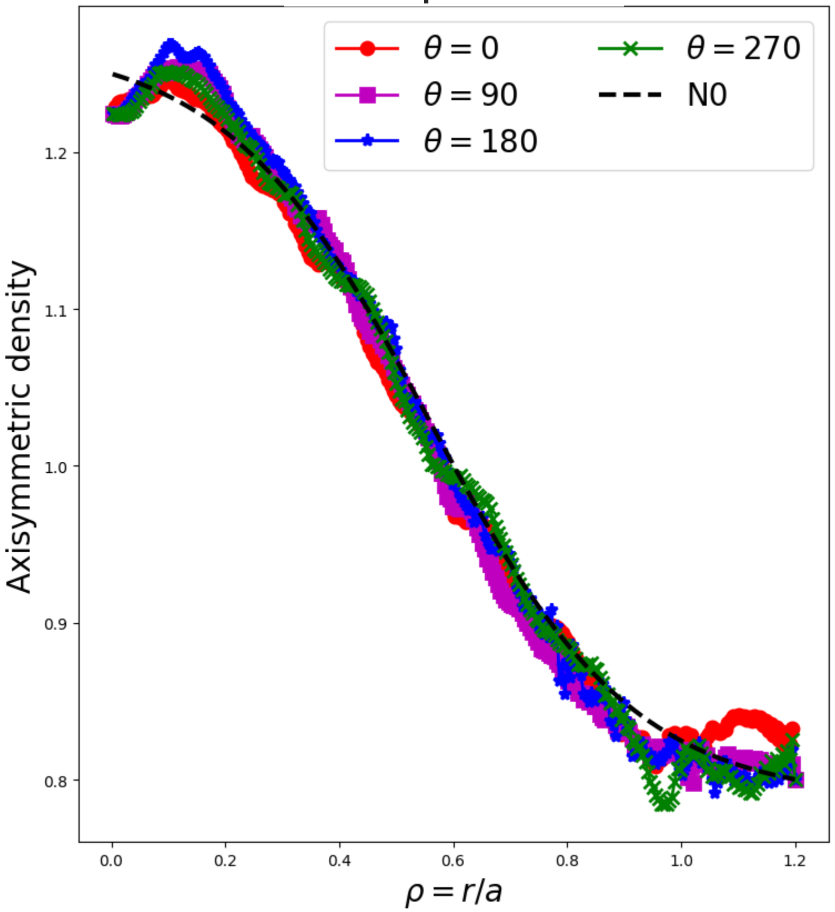
Impact of updating the QN coefficients: minimal test case

- Standard profiles
- Hybrid Kinetic Electrons
- No source
- Poloidally symmetric buffer region in the edge
- **No change in the linear phase as expected**
- Simulation runned in the non linear regime but not really long

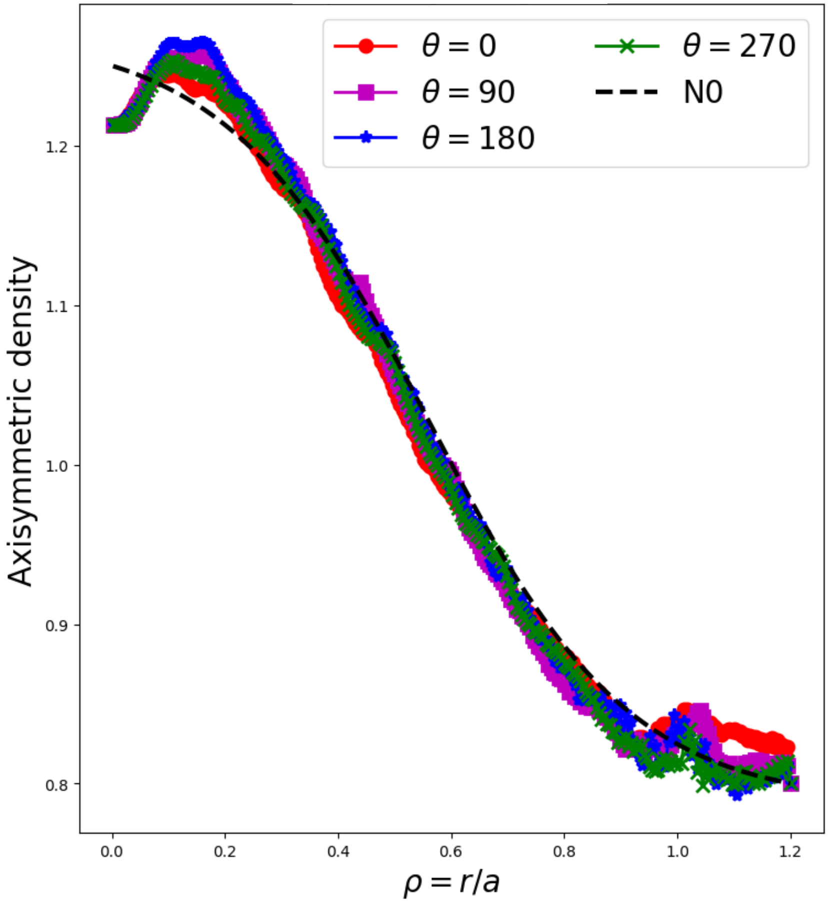
No external source of poloidal asymmetries

Small impact of updating the QN coefficients on the density evolution

No update



Update every 500 Ω_{ci}^{-1}

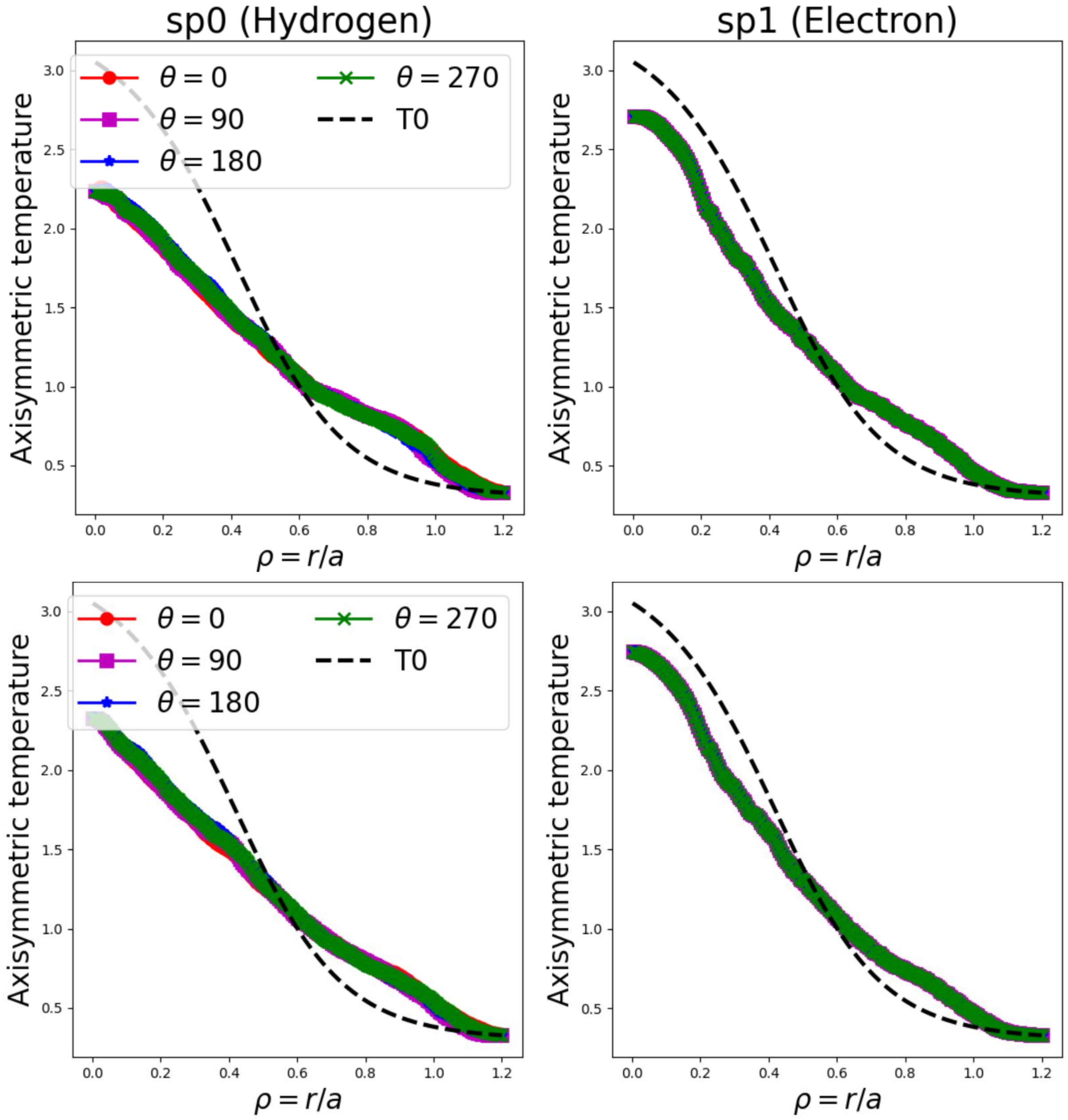


Main difference: less poloidal asymmetries with the update

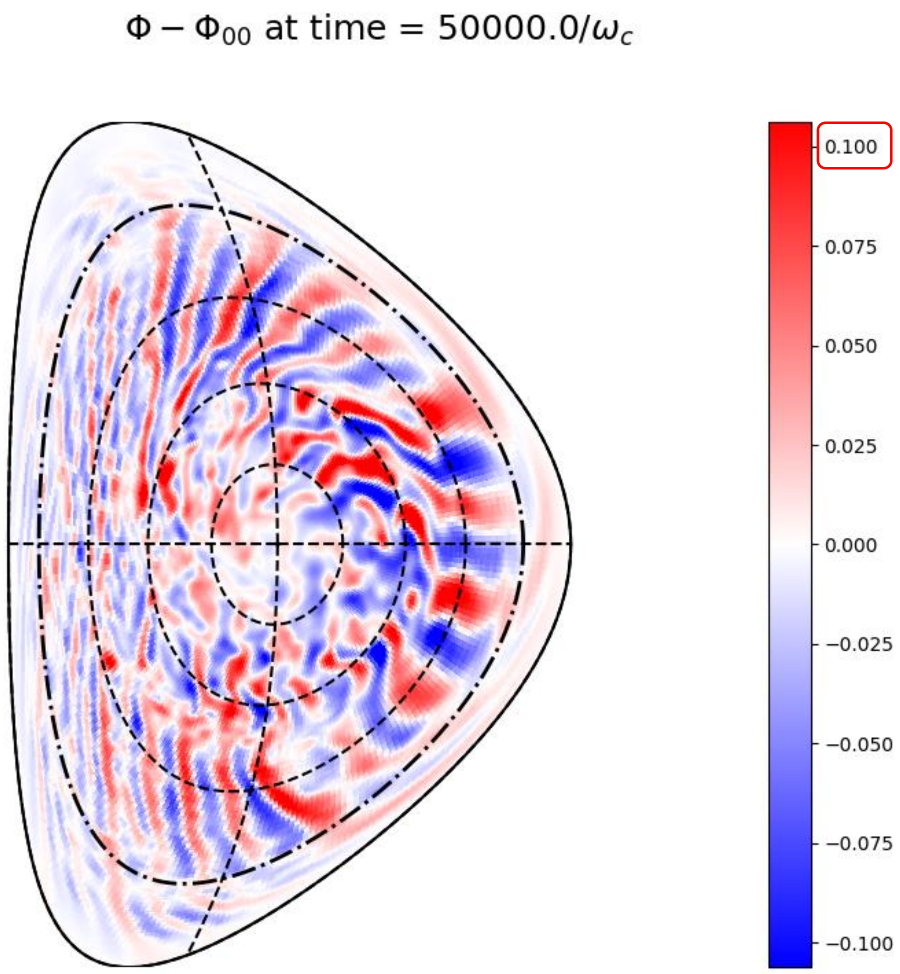
Small impact of updating the QN coefficients on the temperature evolution

No update

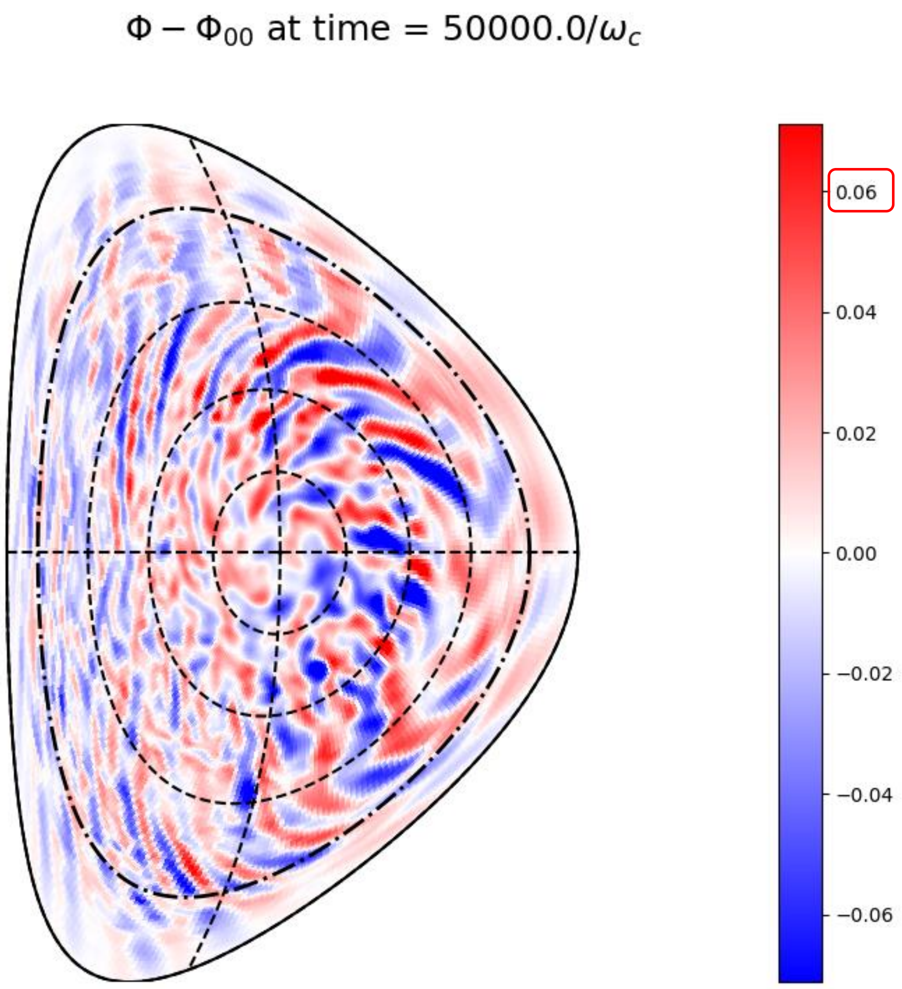
Update every 500 Ω_{ci}^{-1}



Turbulence intensity reduced when updating the QN coefficients

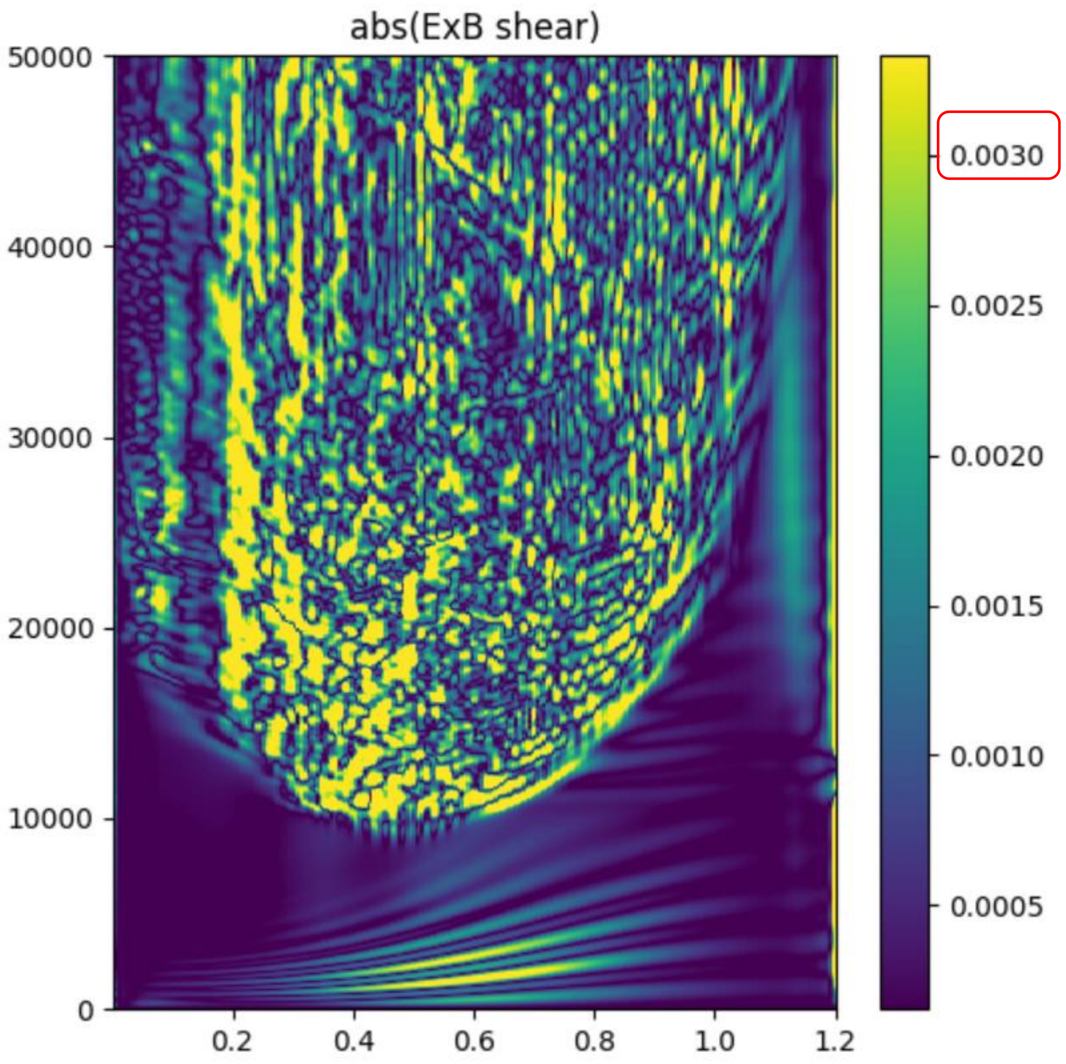


No update

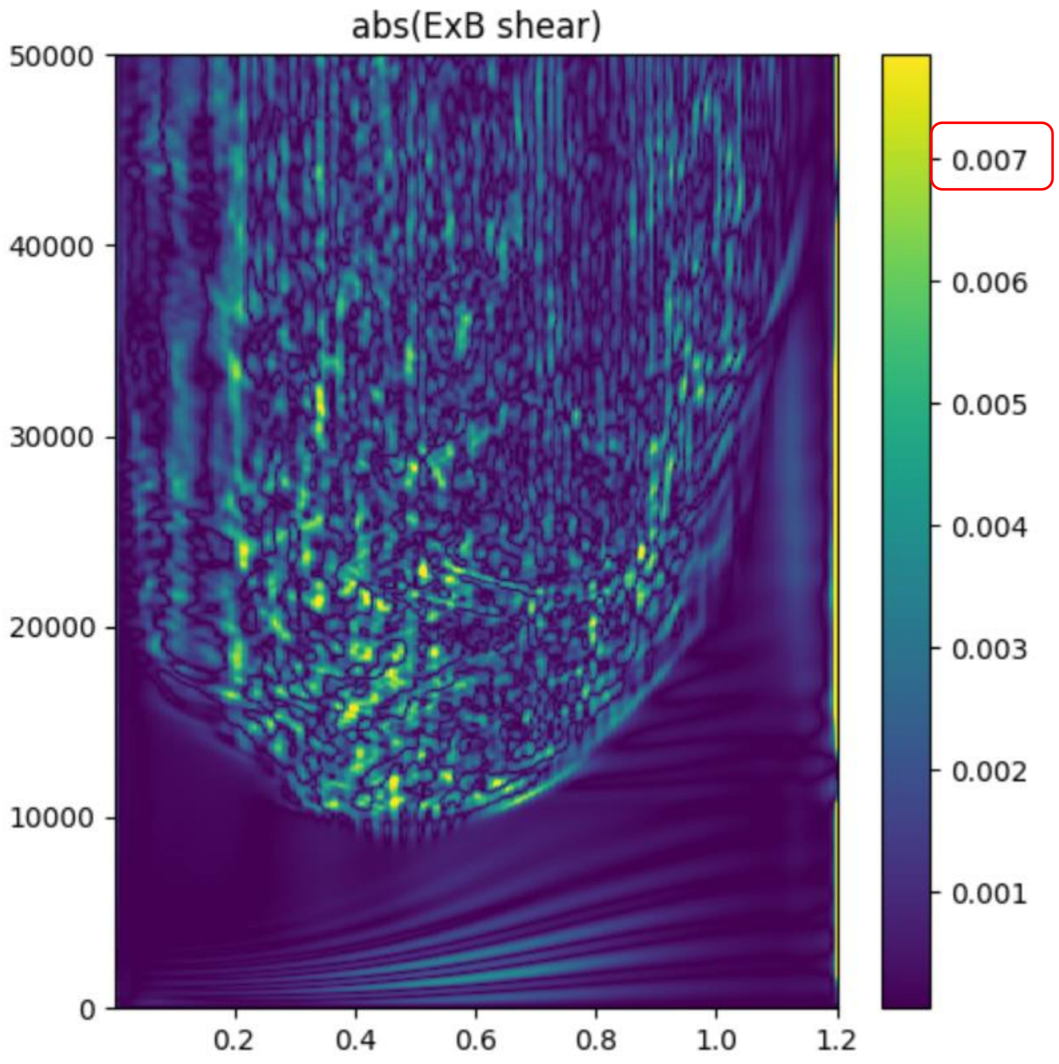


Update every 500 Ω_{ci}^{-1}

Larger Zonal flows when updating the QN coefficients

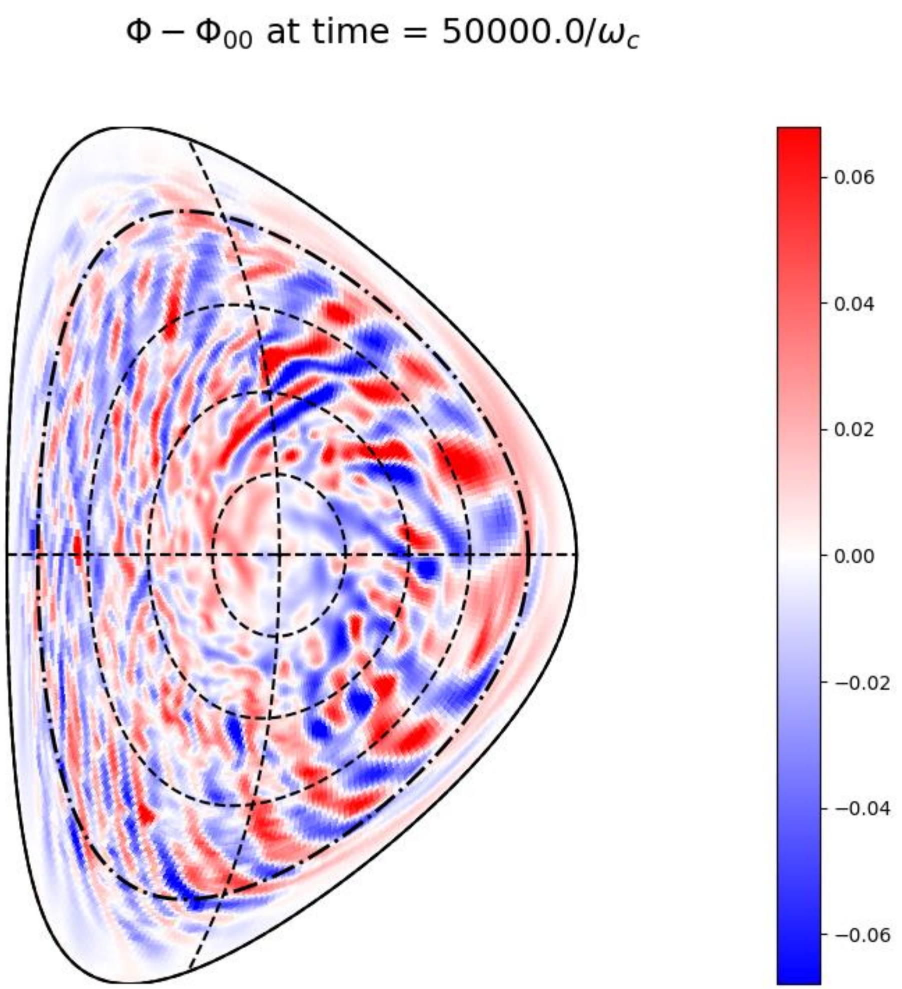


No update



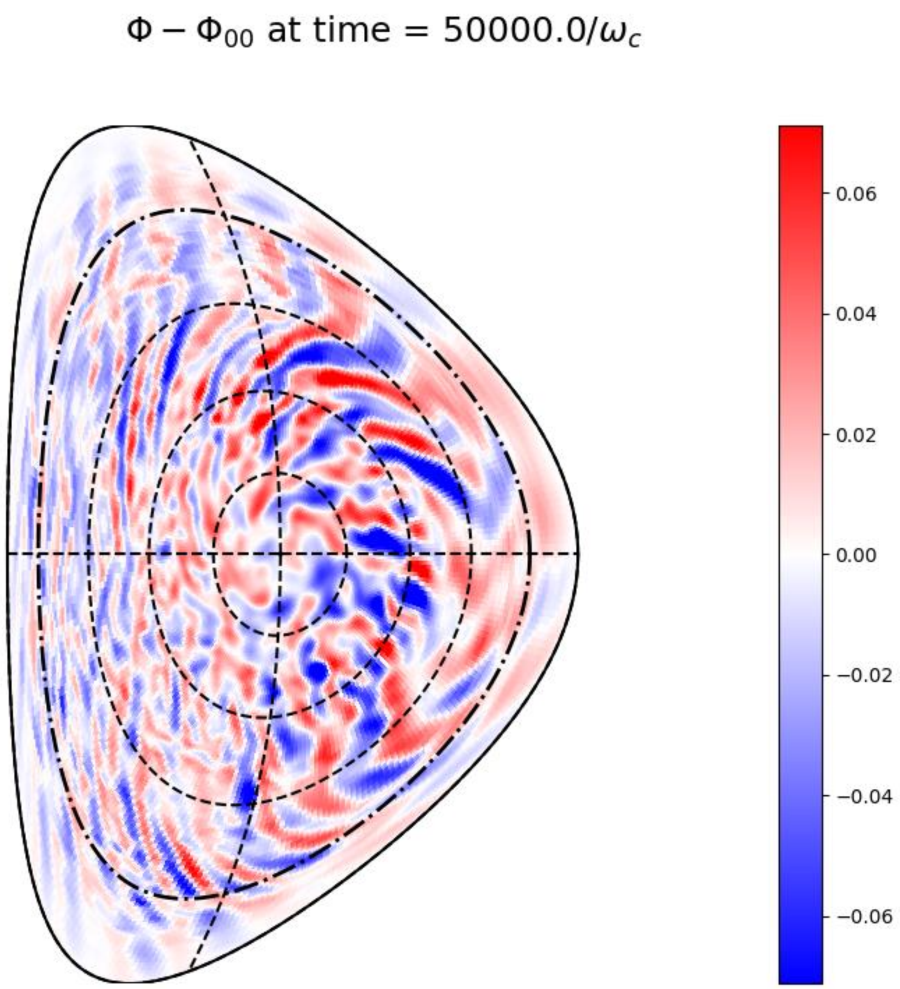
Update every 500 Ω_{ci}^{-1}

Updating the coefficients: compromise between the numerical cost & the impact on the numerical results



Update every 50 Ω_{ci}^{-1}

$\overset{?}{\approx}$



Update every 500 Ω_{ci}^{-1}

For now, updating rate set manually and a priori

➢ Need for a more systematic criterion for choosing when updating the coefficients

Conclusion and outlook

- A new quasi-neutrality solver has been implemented in GYSELA. Its main features are:
  - Allows to simulate shaped plasmas (novelty in GYSELA)
  - LWA & Padé (novelty in GYSELA) versions of the polarization
  - Handle poloidal asymmetries + time evolution (novelty in GYSELA) → requirement for edge and SOL

- Even in the simplest case, updating of the coefficients has a major impact on the turbulence.

- A larger impact is expected when sources/sinks with poloidal asymmetries will be added

- An automatizisation of the updating procedure is still needed: compromise between numerical cost and reliability of results

- The same solver is being developed to be used for solving Ampère → evolution of the magnetic equilibrium?