3-fluid burning plasma and hybrid kinetic-fluid models

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Outline

- 3-fluid burning plasma model
- Hybrid kinetic-fluid models

3-fluid burning plasma model

Dynamics of a burning fusion plasma

$$D + T \longrightarrow \ {}^4\text{He}(3.5 \text{ MeV}) + \text{n}(14.1 \text{ MeV})$$

In a fusion reactor the neutrons leaving the plasma is expected in part to breed tritium, by reacting with lithium, and in part produce electricity.

The alpha particles is expected to be confined by the magnetic field and through collisions to heat the plasma species.

If this alpha heating rate is equal to the plasma energy loss rate then the plasma will ignite, and the plasma burning process will be self-sustaining.

MHD is inappropriate to describe a burning plasma.
Development of multi-fluid burning plasma models on the basis of a self consistent treatment of time-dependent continuity, momentum and energy equations for each fluid species.

3-fluid burning plasma model I

- Ions fluid
  - D-D
  - D-T

  The various species and the relative ratio of ions species is taken into account in the elementary density and charge of the fluid.
- Electrons fluid
- α particle fluid

3-fluid burning plasma model II

The equations for a burning 3-fluid burning plasma consists of the following equations:

- Mass continuity
- Momentum conservation
- Energy conservation
- Maxwell's Equations

3-fluid burning plasma model: 0D case

$$\left.\begin{aligned}\frac{dn_\ell}{dt} &= -\frac{n_\ell}{\tau_n} + S_{n_\ell} \\ \frac{d\epsilon_\ell}{dt} &= -\frac{\epsilon_\ell}{\tau_\epsilon} + S_{\epsilon_\ell}\end{aligned}\right., \ \ell = i, e, \alpha$$

where:

$$S_{n_e} = S_{e_{fuelling}} \qquad S_{n_i} = -S_r + S_{i_{fuelling}} \qquad S_{n_\alpha} = S_r$$

$$S_{\epsilon_e} = Q_{e_{fuelling}} + Q_{e_p} \qquad S_{\epsilon_i} = -Q_{i_r} + Q_{i_{fuelling}} + Q_{i_p} \qquad S_{\epsilon_\alpha} = Q_{\alpha_r} + Q_{\alpha_p}$$

n_e, n_i, n_α: electron-, ion- and α-fluid densities

τ_n, τ_ϵ: particle and energy confinement time

$\epsilon_e, \epsilon_i, \epsilon_\alpha$: electron-, ions- and α-fluid internal energies

S_r: reaction rate

S_f: fueling rate

Q_{i_r} energy depletion terms due to $D-T$ reactions

$Q_{\alpha_r} = S_r\times$ 3.5 MeV: alpha fluid energy term

$Q_{e_p}, Q_{i_p}, Q_{\alpha_p}$: temperature equilibration terms due to Coulomb collisions

Reaction Source

The α particles are produced with a reaction rate

$$S_r = \frac{1}{4} n_i^2 < \sigma v >,$$

where $< \sigma v >$ (cm^3/s) is the fusion reactivity given by:

$$< \sigma v >= C_1 \theta \sqrt{\xi/(m_r c^2 T^3)} e^{-3\xi},$$

with

$$\theta = T/\left[1 - \frac{T(C_2 + T(C_4 + TC_6))}{1 + T(C_3 + T(C_5 + TC_7))}\right], T = \text{temperature}$$

and

$$\xi = (B_G^2/(4\theta))^{1/3}$$

The B_G is the Gamov constant that takes the value $34.3827\sqrt{keV}$, $m_r c^2 = 1124656 keV$ and the parameters C_m are obtained from the Bosch and Halle, Nucl. Fus. **32** , 611 (1992):

C_1	C_2	C_3	C_4	C_5	C_6	C_7
1.17302×10^{-9}	1.51361×10^{-2}	7.51886×10^{-2}	4.60643×10^{-3}	1.35×10^{-2}	-1.0675×10^{-4}	1.366×10^{-5}

Results of 0D Code I

- Results of the 0D code have been presented in:
  1. "A burning plasma model using two-fluid approximations", P. Lalousis, S. Moustaizis, G. Poulipoulis, G. Throumoulopoulos, 15th EFTC, 2013.
  2. "Alpha heating in magnetic and inertial confinement fusion", P. Lalousis, G.N. Throumoulopoulos, G. Poulipoulis, 43rd EPS Conference on Plasma Physics, 2016.
- Collision frequencies based on: J. D. Callen, Fundamentals of Plasma Physics.

Input data:

Start time: 0.0 (s)

End time: 200.0 (s)

Ions fluid species and ratio: D(50%)-T(50%)

Initial ions density: 1.0×10^{19} (m^{-3})

Initial ions temperature: 10.0 (keV)

Initial electrons temperature: 1.0 (keV)

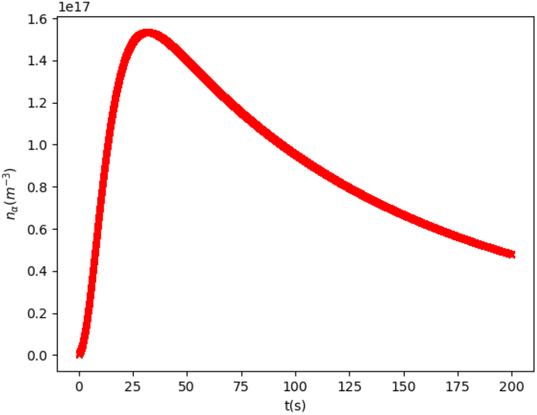
Initial α-particles density: 1.0×10^{5} (m^{-3})

Initial magnetic field: 5.3 (T)

no fuelling, no heating

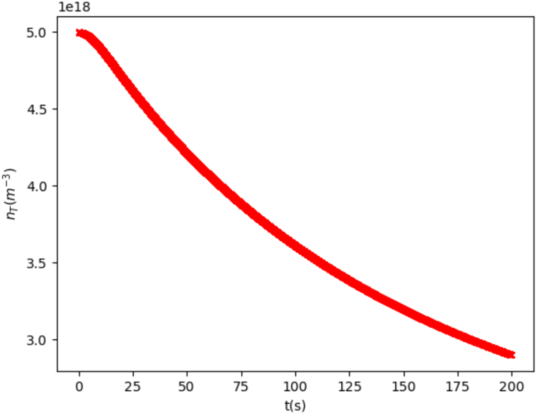
Results of 0D-Code II

α-particles density



Results of 0D-Code III

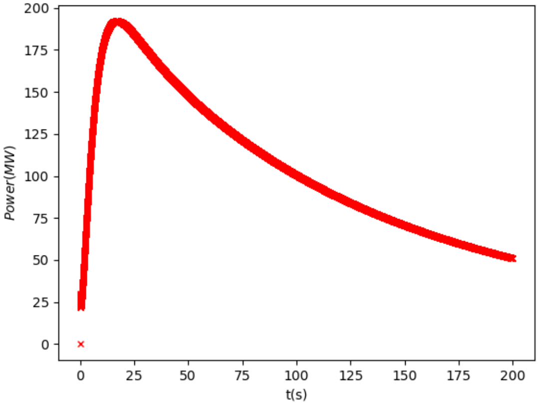
T density



Results of 0D-Code IV

Power output

$$P = V \times S_r \times (14.1 MeV)$$



3-fluid burning plasma model in cylindrical geometry I

- Mass continuity
- Momentum conservation
- Energy conservation
- Maxwell's Equations

Vector form:

$$\frac{\partial \mathbf{A}(r,z,t)}{\partial t} + \frac{C_1}{r}\frac{\partial \mathbf{F}(A,r,z,t)}{\partial r} + C_2\frac{\partial \mathbf{G}(A,r,z,t)}{\partial z} = \mathbf{H}(A,r,z,t) \quad (1)$$

3-fluid burning plasma model in cylindrical geometry II

Contribution to the right-hand source term $\mathbf{H}(A,r,z,t)$:

- Term due to the reactions
- Term due to fuelling
- Term due to heating
- Term due to transport
- Term due to collisions
- Geometric scale factor term
- Field-related term

3-fluid burning plasma model in cylindrical geometry III

Defining $s_\ell = q_\ell / m_\ell$, the vectors **A**, **F**, **G** and **H** are:

Equation	$A(r,z,t)$	C_1	$F(A,r,z,t)$	C_2	$G(A,r,z,t)$	$H(A,r,z,t)$
Mass continuity	ρ_ℓ	1	p_{ℓ_r}	1	p_{ℓ_z}	$Smass_\ell - \frac{p_{\ell_r}}{r}$
Momentum conservation						
r-component	p_{ℓ_r}	1	$\frac{p_{\ell_r}^2}{\rho_\ell} + \bar{P}_\ell$	1	$\frac{p_{\ell_r} p_{\ell_z}}{\rho_\ell}$	$s_\ell \rho_\ell E_r + s_\ell p_{\ell_\phi} B_z - s_\ell p_{\ell_z} B_\phi + \frac{p_{\ell_\phi}^2}{r\rho_\ell} - \frac{p_{\ell_r}^2}{r\rho_\ell} + Smom_{\ell_r}$
ϕ-component	p_{ℓ_ϕ}	1	$p_{\ell_\phi} \frac{p_{\ell_r}}{\rho_\ell}$	1	$p_{\ell_\phi} \frac{p_{\ell_z}}{\rho_\ell}$	$s_\ell \rho_\ell E_\phi + s_\ell p_{\ell_r} B_z - \frac{2 p_{\ell_r} p_{\ell_\phi}}{\rho_\ell r} + Smom_{\ell_\phi}$
z-component	p_{ℓ_z}	1	$p_{\ell_z} \frac{p_{\ell_r}}{\rho_\ell}$	1	$\frac{p_{\ell_z}^2}{\rho_\ell} + \rho_\ell \bar{P}_\ell$	$s_\ell \rho_\ell E_z + s_\ell p_{\ell_z} B_\phi - \frac{p_{\ell_z} p_{\ell_z}}{\rho_\ell r} + Smom_{\ell_z}$
Energy conservation	ϵ_ℓ	1	$[\frac{p_{\ell_r}}{\rho_\ell}(\epsilon_\ell + \bar{P}_\ell)]$	1	$[\frac{p_{\ell_z}}{\rho_\ell}(\epsilon_\ell + \bar{P}_\ell)]$	$s_\ell \sum_{j=r,\phi,z} p_{\ell_j} E_j - \frac{p_{\ell_r}(\epsilon_\ell + \bar{P}_\ell)}{\rho_\ell r} + Seng_\ell$
Ampere's law						
r-component	E_r	0	0	c^2	B_ϕ	$-c^2 \mu_0 \sum_\ell s_\ell p_{\ell_r}$
ϕ-component	E_ϕ	c^2	B_z	$-c^2$	B_r	$-c^2 \mu_0 \sum_\ell s_\ell p_{\ell_\phi}$
z-component	E_z	c^2	B_ϕ	0	0	$-c^2 \mu_0 \sum_\ell s_\ell p_{\ell_z} - c^2 \frac{B_\phi}{r}$
Faraday's law						
r-component	B_r	0	0	1	E_ϕ	0
ϕ-component	B_ϕ	1	E_z	-1	E_r	0
z-component	B_z	1	E_r	0	0	$-\frac{E_r}{r}$

No transport, no fuelling, no heating, no collisions

Reaction Source terms

We assume no collisions and further that the α-particles are produced with zero fluid velocity, so the fluid energy is their internal one $\epsilon_\alpha = 3.5MeV$. Thus, the source terms for each specie and each equation are:

electrons	$Smass_e$	$Smom_{e_r}$	$Smom_{e_\phi}$	$Smom_{e_z}$	$Seng_e$
	0	0	0	0	0
ions	$Smass_i$	$Smom_{i_r}$	$Smom_{i_\phi}$	$Smom_{i_z}$	$Seng_i$
	$-5m_pS_r$	$-S_rm_i\upsilon_{i_r}$	$-S_rm_i\upsilon_{i_\phi}$	$-S_rm_i\upsilon_{i_z}$	$-S_rm_i\epsilon_i$
α	$Smass_\alpha$	$Smom_{\alpha_r}$	$Smom_{\alpha_\phi}$	$Smom_{\alpha_z}$	$Seng_\alpha$
	$4m_pS_r$	0	0	0	$S_rm_\alpha\epsilon_k$

The ion fluid is a D-T one.

Current Status and future plans

Short-term objectives:

- Code in 2D cylindrical geometry (under development).
- Extend to include:
  1. term due to transport.
  2. fuelling term.
  3. term due to binary collisions.

Long-term objectives:

- Additional heating.
- Recycling.
- Integrate into a workflow with equilibrium and transport codes.

Hybrid kinetic-fluid models

Energetic particles

Small fraction of the particles with energies higher than those of the thermal energies of the main population.

- α-particles in a burning plasma (in D-T reaction the α-particles have energy 3.5MeV)
- Additional heating such as ICRH or NBI.
- Also present in astrophysical plasmas.

$\Rightarrow$ Need special treatment

- Fluid models insufficient.
- Fully kinetic computations "expensive"

$\Rightarrow$

Hybrid models

Hamiltonian structure

- Sum of the particle Poisson bracket appearing in Vlasov theory and the Hall MHD bracket expressed in terms of fluid and particle canonical momenta, $\bar{\boldsymbol{M}}$ and $\boldsymbol{\pi}_p$

$$\{F, G\} = \int d^3x \Big\{ \bar{\boldsymbol{M}} \cdot (G_{\bar{\boldsymbol{M}}} \cdot \nabla F_{\bar{\boldsymbol{M}}} - F_{\bar{\boldsymbol{M}}} \cdot \nabla G_{\bar{\boldsymbol{M}}}) + \rho\,(G_{\bar{\boldsymbol{M}}} \cdot \nabla F_\rho - F_{\bar{\boldsymbol{M}}} \cdot \nabla G_\rho) - e^{-1}\,(G_{\boldsymbol{A}} \cdot \nabla F_{n_e} - F_{\boldsymbol{A}} \cdot \nabla G_{n_e}) - \frac{1}{e n_e}(\nabla \times \boldsymbol{A}) \cdot (F_{\boldsymbol{A}} \times G_{\boldsymbol{A}}) + \sum_p \int d^3\boldsymbol{\pi}\, \bar{f}_p [F_{\bar{f}_p}, G_{\bar{f}_p}]_{\boldsymbol{\pi}} \Big\}, \qquad (2)$$

where $[g, h]_{\boldsymbol{\pi}} = \nabla g \cdot \nabla_{\boldsymbol{\pi}} h - \nabla h \cdot \nabla_{\boldsymbol{\pi}} g$, $\bar{f}_p = \bar{f}_p(\boldsymbol{x}, \boldsymbol{\pi}_p, t)$, $\bar{\boldsymbol{M}} := m n_i \boldsymbol{u}_i + e\boldsymbol{A}$ and $\boldsymbol{\pi}_p = m_p \boldsymbol{v} + e_p \boldsymbol{A}$.

- Appropriate sum of the Hamiltonians

$$\mathcal{H} = \int d^3x \left[\rho \frac{|\boldsymbol{u}_i|^2}{2} + \rho U(\rho) + n_e \mathcal{U}_e + \frac{|\boldsymbol{B}|^2}{2\mu_0} \right] + \sum_p \int\int d^3x\, d^3v\, m_p \frac{v^2}{2} f_p \,. \qquad (3)$$

Current Coupling Scheme

- A Hamiltonian model in the current coupling scheme (CCS) follows from the transformation:

$$\bar{\boldsymbol{M}} \to \boldsymbol{u}_i\,, \quad \bar{f}_p(\boldsymbol{x}, \boldsymbol{\pi}_p, t) \to f_p(\boldsymbol{x}, \boldsymbol{v}, t)\,, \tag{4}$$

- Hamilton's equations: $\partial_t \xi = \{\xi, \mathcal{H}\}$

$$\partial_t \rho + \nabla \cdot (\rho \boldsymbol{u}) = 0\,, \quad \partial_t n_e + \nabla \cdot (n_i \boldsymbol{u}) + e^{-1} \nabla \cdot \boldsymbol{J}_k = 0 \tag{5}$$

$$\partial_t \boldsymbol{u} = \boldsymbol{u} \times \nabla \times \boldsymbol{u} - \nabla \left(h + \frac{v^2}{2} \right) + \frac{e}{m} \left(1 - \frac{n_i}{n_e} \right) \boldsymbol{u} \times \boldsymbol{B}$$
$$+ \frac{1}{m n_e} \left(\mu_0^{-1} \nabla \times \boldsymbol{B} - \boldsymbol{J}_k \right) \times \boldsymbol{B} - \frac{\nabla \cdot \mathbb{P}_e}{m n_e}\,, \tag{6}$$

$$\partial_t \boldsymbol{B} = \nabla \times \left[\frac{n_i}{n_e} \boldsymbol{u} \times \boldsymbol{B} - \frac{1}{e n_e} \left(\mu_0^{-1} \nabla \times \boldsymbol{B} - \boldsymbol{J}_k \right) \times \boldsymbol{B} + \frac{\nabla \cdot \mathbb{P}_e}{e n_e} \right]\,, \tag{7}$$

$$\partial_t f_p = -\boldsymbol{v} \cdot \nabla f_p - \frac{e_p}{m_p} \Bigg[\left(\boldsymbol{v} - \frac{n_i}{n_e} \boldsymbol{u} \right) \times \boldsymbol{B}$$
$$+ \frac{1}{e n_e} \left(\mu_0^{-1} \nabla \times \boldsymbol{B} - \boldsymbol{J}_k \right) \times \boldsymbol{B} - \frac{\nabla \cdot \mathbb{P}_e}{e n_e} \Bigg] \cdot \nabla_{\boldsymbol{v}} f_p\,. \tag{8}$$

Pressure Coupling Scheme

- For the derivation of the PCS we replace the ion kinetic energy density by the center-of-mass kinetic energy density in the Hamiltonian.
- **PCS dynamical equations**

$$\partial_t n_i = -\nabla \cdot (n_i \boldsymbol{u}) \,, \tag{9}$$

$$\partial_t n_e = -\nabla \cdot (n_e \boldsymbol{u}) - \frac{1}{e} \nabla \cdot \boldsymbol{J}_k \,, \tag{10}$$

$$\partial_t \boldsymbol{B} = \nabla \times \left[\boldsymbol{u} \times \boldsymbol{B} - \frac{1}{e n_e} \left(\mu_0^{-1} \nabla \times \boldsymbol{B} - \boldsymbol{J}_k \right) \times \boldsymbol{B} + \frac{1}{e n_e} \nabla \cdot \mathbb{P}_e \right] , \tag{11}$$

$$\begin{aligned} \partial_t(\rho \boldsymbol{u}) + \nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) &= -\rho \nabla h_i - \nabla \cdot \mathbb{P}_e \\ &\quad - \sum_p \nabla \cdot \tilde{\mathbb{P}}_p + \mu_0^{-1} (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} \,, \end{aligned} \tag{12}$$

$$\begin{aligned} \partial_t f_p &= -(\boldsymbol{v} + \boldsymbol{u}) \cdot \nabla f_p - \frac{e_p}{m_p} \Big[\boldsymbol{v} \times \boldsymbol{B} + \frac{1}{e n_e} (\mu_0^{-1} \nabla \times \boldsymbol{B} - \boldsymbol{J}_k) \times \boldsymbol{B} \\ &\quad - \frac{1}{e n_e} \nabla \cdot \mathbb{P}_e \Big] \cdot \nabla_v f_p + \nabla_v f_p \cdot \nabla \boldsymbol{u} \cdot \boldsymbol{v} \,. \end{aligned} \tag{13}$$

Hybrid model with kinetic ions and fluid electrons

- Neglect thermal ions
- Model equations:

$$\boldsymbol{v} \cdot \nabla f + d_i^{-2} \left(\mathbf{E} + \boldsymbol{v} \times \mathbf{B} \right) \cdot \nabla_v f = 0 \,, \tag{14}$$

$$-\nabla \Phi = \frac{1}{n_e} \left[\left(\nabla \times \mathbf{B} - \mathbf{J}_k \right) - d_i^2 \nabla P_e \right] \,, \tag{15}$$

$$\mathbf{E} = -\nabla \Phi \,, \quad \nabla \times \mathbf{B} = \mathbf{J} \,, \tag{16}$$

$$\nabla \cdot \mathbf{B} = 0 \,, \quad d_i^2 \beta_A^2 \nabla \cdot E = \left(n - n_e \right) , \tag{17}$$

$$P_e = \kappa n_e \,, \tag{18}$$

where

$$\kappa := \frac{k_B T_{e0}}{d_i^2 m v_A^2}, \quad \beta_A^2 = v_A^2 / c^2 .$$

Cold electrons: $\kappa = 0$, Thermal electrons: $\kappa = 1$

- In the limit $\beta_A^2 \to 0$ we obtain the quasineutrality condition $n_e = n$

Axisymmetric equilibria

- Cylindrical coordinate system (r, ϕ, z) and Axisymmetry: $\forall A \;\; \partial A/\partial\phi = 0$

$$\text{Magnetic field}: \quad \boldsymbol{B} = I\nabla\phi + \nabla\psi(r,z)\times\nabla\phi\,, \tag{19}$$

$$\text{Current density}: \quad \boldsymbol{J} = \nabla\times\boldsymbol{B} = -\Delta^*\psi\nabla\phi + \nabla I\times\nabla\phi\,, \tag{20}$$

- Jeans' theorem: Distribution functions of the form $f = f(C_1, C_2, ...)$, where C_i are particle constants of motion, are solutions to the Vlasov equation.

$$\text{Energy}: \quad \tilde{H} = \frac{v^2}{2} + d_i^{-2}\Phi, \quad \tilde{H} = \frac{H}{d_i^2 m v_A^2} \tag{21}$$

$$\text{Toroidal angular momentum}: \quad \tilde{p}_\phi = rv_\phi + rA_\phi = rv_\phi + d_i^{-2}\psi \tag{22}$$

$$A_\phi: \quad \phi\text{-component of the vector potential}$$

$$f = f(\tilde{H}, \tilde{p}_\phi) = e^{-\tilde{H}} g(\tilde{p}_\phi) \;:\; \text{choice}$$

Generalized Grad-Shafranov equation (GGS)

- $\nabla\psi$ projection of Ohm's law (15):

$$\Delta^*\psi + II'(\psi) + r^2\mathcal{Z}(r,\psi) = 0\,, \tag{23}$$

where

$$\mathcal{Z}(r,\psi) := \frac{1}{r}\overbrace{\int d^3v\, v_\phi f}^{J_{k\phi}} - G'(\psi)\overbrace{\int d^3v\, f}^{n}\,. \tag{24}$$

- $\mathcal{Z}(r,\psi)$ can be derived by a "pseudopotential" as $\mathcal{Z} = \partial V(r,\psi)/\partial\psi\,$,

$$V(\psi, r) = d_i^2(\kappa+1)\left[\frac{2\pi e^{-G(\psi)/d_i^2}}{r}\int_{-\infty}^{+\infty} dp_\phi\, e^{-\frac{(p_\phi-\psi/d_i^2)^2}{2r^2}} g(p_\phi)\right]^{\frac{1}{\kappa+1}}. \tag{25}$$

Consequently, the GGS equation can be written in the form

$$\Delta^*\psi + II'(\psi) + r^2\frac{\partial V}{\partial\psi} = 0\,. \tag{26}$$

- Knowing V enables the solution of the partial differential equation (26) to determine ψ and of the integral equation (25) to determine $g(p_\phi)$.

Procedure for determining f

- Hermite-polynomial expansion of the function g, employing an appropriate multiplication theorem and invoking the orthogonality of Hermite polynomials:

$$\left[\frac{V}{d_i^2(\kappa+1)}\right]^{\kappa+1} e^{G(\psi)/d_i^2} = \sum_m \sum_{\ell=0}^{\lfloor m/2 \rfloor} 2^{m-2\ell+1}\pi^{3/2} \tag{27}$$

$$c_m \frac{m!}{\ell!(m-2\ell)!}(r^2-1)^\ell \left(\frac{\psi}{d_i^2\sqrt{2}}\right)^{m-2\ell} . \tag{28}$$

- Expanding the function $\left\{V/\left[d_i^2(\kappa+1)\right]\right\}^{k+1} \exp\left[G(\psi)/d_i^2\right]$ with respect to ψ

$$\left[\frac{V}{d_i^2(\kappa+1)}\right]^{\kappa+1} e^{G(\psi)/d_i^2} = \sum_m V_m(r)\psi^m . \tag{29}$$

- Consider the equation resulting from equating the RHDs of (28) and (29) to find the coefficients c_m and $V_m(r)$.

Tokamak equilibrium

- Ansatz for the free functions:

$$I(\psi) = (I_0 + I_1\psi + I_2\psi^2)e^{-(\psi-\psi_a)^2/\eta}\,, \quad (30)$$

$$G(\psi) = \alpha(\psi-\psi_a)^2\,. \quad (31)$$

ψ_a : Value of ψ on the magnetic axis.

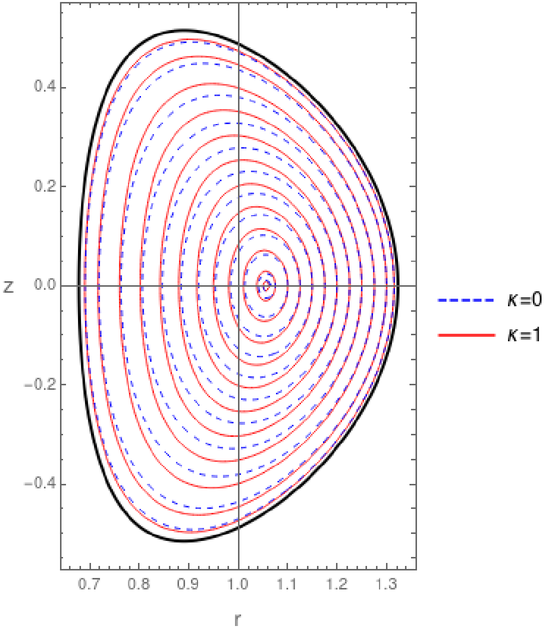
- Expanding the function $\left\{V/\left[d_i^2(\kappa+1)\right]\right\}^{k+1}\exp\left[G(\psi)/d_i^2\right]$ up to second-order in ψ and following the procedure described previously:

$$V = (k+1)d_i^2\left\{e^{-G(\psi)/d_i^2}\left[V_0(r)+V_1\psi+V_2\psi^2\right]\right\}^{1/(k+1)}\,, \quad (32)$$

$$f(H,p_\phi) = \left[c_0+\sqrt{2}c_1p_\phi+c_2(2p_\phi^2-2)\right]e^{-H}\,. \quad (33)$$

- The GSS equation is solved numerically for a fixed D-shaped boundary cross-section, on which $\psi|_{\partial\mathcal{D}}=0$, using ITER parametric values: aspect ratio $\epsilon=0.32$, triangularity $\delta=0.34$, elongation $k=1.6$ major radius $R_0=6.2\,m$, central magnetic field $B_0=5\,T$ and $n_0=2.1\times10^{19}$.

Equilibrium configuration



References

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- "Axisymmetric hybrid Vlasov equilibria with applications to tokamak plasmas", D. A. Kaltsas *et al*, Plasma Phys. Control. Fusion **66** 065016, 2024.

Future plans

Short-term objectives:

- Use of extended MHD models with finite electron inertia for describing the bulk plasma.
- Construction of hybrid equilibria by incorporating realistic electron temperature distribution and fluid ion components.

Long-term objectives:

- Derivation of sufficient stability criteria.
- Examine whether the equilibrium model can be derived through a Hamiltonian energy-Casimir variational principle or/and alternative Hamiltonian variational method upon utilizing dynamically accessible variations.
- Extend these studies to the more general class of helically symmetric equilibria in connection with stellarator optimization.