Beat-driven and spontaneous excitations of zonal flows & Updates on low-frequency AE

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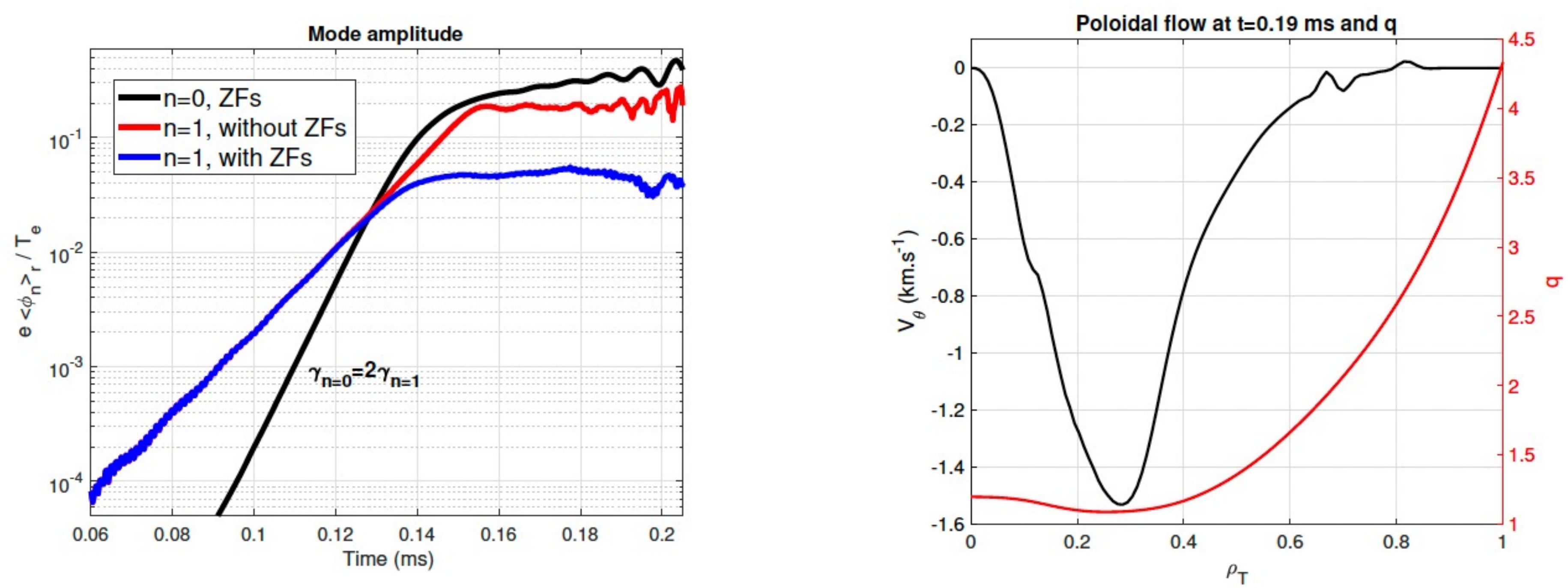
Acknowledgment: Ningfei Chen

Beat-driven and spontaneous

- Beat-driven spontaneous ZF excitations are observed in simulations [G. Brochard et al, submitted to NF]



(a) (b)

Figure 7: (a) Time evolution of volume-averaged perturbed electrostatic potential $e\langle\phi\rangle/T_e$ (n=0,1).(b) Zonal poloidal flow V_θ (km.s^{-1}) after saturation at t=0.19ms. Figure (a) is reproduced from [56]

- Discussed at the recent TSVV10 meeting on Feb. 19°
  ➔highlights here

Recent theories

- M. V. Falessi et al: self-consistent evolution of zonal e.m. fields and corresponding phase space zonal structures as zonal states, describing nonlinear plasma equilibria in the presence of finite fluctuation spectrum and sources/sinks, collisions ➔ New Journal of Physics (2023) **25** 123035

- N. Chen et al: Drift wave soliton formation via forced-driven zonal flow and implication on plasma confinement.
➔ Phys. Plasmas (2024) **31** 042307

- L. Chen et al: On beat-driven and spontaneous excitations of zonal flows by drift waves.
➔ Phys. Plasmas (2024) **31** 040701

Recent theories

- Follow L. Chen et al POP 2024 and develop theoretical paradigm based on e-DW to illuminate the respective roles of beat-driven and spontaneous components of ZF and possible synergies.
- Quasineutrality condition (single-n e-DW + ZF in slab; extension to general geometry and fluctuation spectrum Falessi et al NJP 2023)

$$\frac{N_0 e^2}{T_e}\left(1+\frac{T_e}{T_i}\right)\delta\phi_k = \sum_{j=e,i}\langle eJ_k\delta g_k\rangle_j ,$$

$$\delta f_{kj} = -\left(\frac{e}{T}\right)_j F_{Mj}\delta\phi_k + e^{-i\boldsymbol{\rho}\cdot\mathbf{k}_\perp}\delta g_{kj}$$

$$i(k_\parallel v_\parallel - \omega_k)\delta g^{(1)}_{kj} = -i(\omega-\omega_{*j})_k (e/T)_j F_{Mj} J_k \delta\phi_k ,$$

$$i(k_\parallel v_\parallel - \omega_k)\delta g^{(2)}_{kj} = -(c/B_0)\Lambda^{k'}_{k''}\left(J_{k'}\delta\phi_{k'}\delta g_{k''}\right)_j .$$

$$\Lambda^{k'}_{k''} = \mathbf{b}\cdot\mathbf{k}''\times\mathbf{k}' \text{ satisfying } \mathbf{k}=\mathbf{k}'+\mathbf{k}''$$

$$\delta\phi_k = \delta\phi_d + \delta\phi_Z ,$$

$$\delta\phi_d = \phi_d(x,t)\exp(i(k_y y + k_\parallel z - \omega_{dr} t)) + c.c.,$$

$$\delta\phi_Z = \phi_Z(x,t) + c.c..$$

Recent theories

- Solve kinetic equations by small amplitude expansion, noting
  $|k_\parallel v_{te}| \gg |\omega_k| \gg |k_\parallel v_{ti}|$ and $|k_\perp \rho_e| \ll 1$; $\omega_k = \omega_{kr} + i\partial_t$.

- Linear responses

$$\delta g_{ki}^{(1)} \simeq \left(1 - \frac{\omega_{*i}}{\omega}\right)_k \frac{e}{T_i} J_k F_{Mi} \delta\phi_k, \qquad \delta g_{ke}^{(1)} \simeq O(\omega / k_\parallel v_{te}) \ll 1.$$

- Zonal component

$$\delta g_{Zi}^{(1)} = \frac{e}{T_i} F_{Mi} J_Z \delta\phi_Z, \qquad \delta g_{Ze}^{(1)} = -\frac{e}{T_e} F_{Me} \delta\phi_Z.$$

Recent theories

- Nonlinear responses: obtained by solving kinetic equations based on the small amplitude expansion

- Zonal component

$$\frac{\partial}{\partial t}\delta g^{(2)}_{Zi} = -\frac{c}{B_0}\Lambda^{k'}_{k''}\left(J_{k'}\delta\phi_{k'}\delta g_{k''}\right)_i . \qquad \delta g^{(2)}_{Ze} \simeq 0;$$

- Two mechanisms for zonal component excitations

- First: the beat-driven process due to the ponderomotive force produced by the self beating of the eDWs $|k'_x| = |k''_x|$

- Second: the spontaneous excitation produced by the nonlinear interactions between the radial sidebands $|k'_x| \neq |k''_x|$

Recent theories

$$\delta g_{Zi}^{(2)} = \delta g_{Zi,A}^{(2)} + \delta g_{Zi,B}^{(2)},$$

- A: beat-driven (forced-driven) process

$$\delta g_{Zi,A}^{(2)} = \frac{c}{B_0} k_y J_k^2 \left(\frac{\omega_{*i}}{\omega_r^2} \right)_d \frac{e}{T_i} F_{Mi} \frac{\partial}{\partial x} |\phi_d|^2 .$$

- B: spontaneous process

$$\frac{\partial}{\partial t} J_Z \delta g_{Zi,B}^{(2)} = i \frac{c}{B_0} k_y \frac{\partial}{\partial x} [(J_Z J_{k'} - J_{k''} + J_{k''})_i \delta g_{k''i} \delta \phi_{k'} - (J_Z J_{k''} - J_{k'} + J_{k'})_i \delta g_{k'i} \delta \phi_{k''}] .$$

Recent theories

$$\frac{N_0 e^2}{T_i}\left(1-\Gamma_Z\right)\delta\phi_Z = \left\langle eJ_Z\left(\delta g^{(2)}_{Zi,A} + \delta g^{(2)}_{Zi,B}\right)\right\rangle .$$

$$\phi_Z = \phi_{Zb} + \phi_{Zs}$$

- A: beat-driven (forced-driven) process

$$\frac{\partial^2}{\partial x^2}\phi_{Zb} = -\frac{c}{B_0}k_y\frac{\omega_{*in}}{\omega_{dr}^2\rho_i^2}\frac{\partial}{\partial x}\left|\phi_d\right|^2 ,$$

- B: spontaneous process

$$\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2}\phi_{Zs}\right) = -\frac{T_i}{N_0 e^2\rho_i^2}\frac{\partial}{\partial t}\left\langle eJ_Z\delta g^{(2)}_{Zi,B}\right\rangle$$

$$\simeq i\frac{c}{B_0}k_y\alpha_i\frac{\partial^2}{\partial x^2}\left(\phi_d\frac{\partial}{\partial x}\phi_d^* - \phi_d^*\frac{\partial}{\partial x}\phi_d\right) .$$

Recent theories

- Explore the interactions of the two processes on the modulational instability

$$\epsilon_d \phi_d = \frac{c}{B_0} \frac{k_y}{\omega_{dr}} \phi_d \frac{\partial}{\partial x} \left(\phi_{Zb} + \phi_{Zs} \right),$$

$$\epsilon_d \simeq 1 - \alpha_i \rho_s^2 \nabla_\perp^2 - \frac{\omega_{*en}}{\omega_{dr}} + i \frac{\omega_{*en}}{\omega_{dr}^2} \frac{\partial}{\partial t}$$

$$\bar{\alpha}_i = (1 - \omega_{*pi}/\omega_{dr}) \simeq 1 + (T_i/T_e)(1 + \eta_i)$$

- Modulation interaction with sidebands

$$\phi_d = A_0 + A_+ \exp(\gamma_Z t + i k_Z x) + A_-^* \exp(\gamma_Z t - i k_Z x)$$

$$\phi_Z = A_Z \exp(\gamma_Z t + i k_Z x)$$

$$A_{Zb} = -i \frac{c k_y \omega_{*en}}{B_0 k_Z \rho_s^2 \omega_{0r}^2} \left(A_0 A_- + A_0^* A_+ \right), \qquad \gamma_Z A_{Zs} = -\frac{c}{B_0} k_y k_Z \alpha_i \left(A_0 A_- - A_0^* A_+ \right).$$

Recent theories

- QN equation for sidebands

$$\epsilon_{d\pm} A_{\pm} = i \frac{c k_y k_Z}{B_0 \omega_{0r}} (A_{Zb} + A_{Zs}) \begin{pmatrix} A_0 \\ A_0^* \end{pmatrix}, \qquad \epsilon_{d\pm} = \alpha_i b_{Zs} \pm i \gamma_Z (\omega_{*en}/\omega_{0r}^2).$$

- Dispersion relation for modulationall instability is found by substitution into nonlinear expressions for the ZF components

$$A_{Zb} = -i \frac{c k_y \omega_{*en}}{B_0 k_Z \rho_s^2 \omega_{0r}^2} (A_0 A_- + A_0^* A_+), \quad \gamma_Z A_{Zs} = -\frac{c}{B_0} k_y k_Z \alpha_i (A_0 A_- - A_0^* A_+).$$

$$\left(\frac{\gamma_Z \omega_{*en}}{\omega_{0r}^2} \right)^2 = \alpha_i b_{Zs} \left[(\alpha_b + \alpha_s) \left| \overline{A}_0 \right|^2 - \alpha_i b_{Zs} \right],$$

where $\overline{A}_0 = eA_0/T_e$, $b_{Zs} = k_Z^2 \rho_s^2$, and

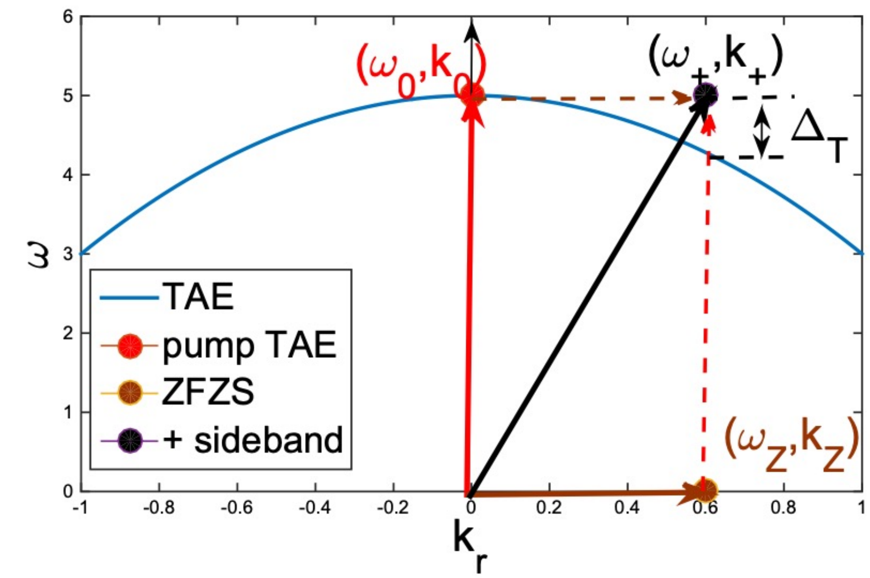
$$\alpha_b = \alpha_s = 2 b_{ys} (\omega_{*en}/\omega_{0r}) (\Omega_{ci}/\omega_{0r})^2.$$

Main results

- Instability threshold

$$|\overline{A}_0|^2_{th} = \alpha_i b Z_s/(\alpha_b + \alpha_s)$$

- α_b and α_s correspond, respectively, to beat-driven (forced-driven) and spontaneous ZF components contributions and $\alpha_b = \alpha_s$
- This demonstrates that the beat-driven component provides an O(1) contribution to the spontaneous excitation and, thus, reduces the instability threshold
- Physically, this is understood because the beat-driven component gives rise to a nonlinear frequency shift that reduces the frequency mismatch



Z. Qiu et al, RMPP 2023

Conclusions I

- The theoretical framework for describing excitations of ZF by beat-driven or modulational instability is fully developed. Supported by numerical simulation results. (qualitatively, verification needed)

- It shows that both processes are important in determining ZF levels. Recent analyses [N. Chen et al POP 2024] suggest that this has impact on the turbulence spreading by soliton formation

- Theoretical framework is demonstrated by simple eDW slab paradigm, but it is fully deployed for general geometry and analysis of NL dynamics in phase space by action-angle approach [Falessi et al NJP 2023]

Update on low-frequency AE

- Y. Li (reported earlier): general expression for enhanced inertia can be written explicitly in action angle coordinates (small orbit) *arXiv:2401.04600v1 [physics.plasm-ph]*

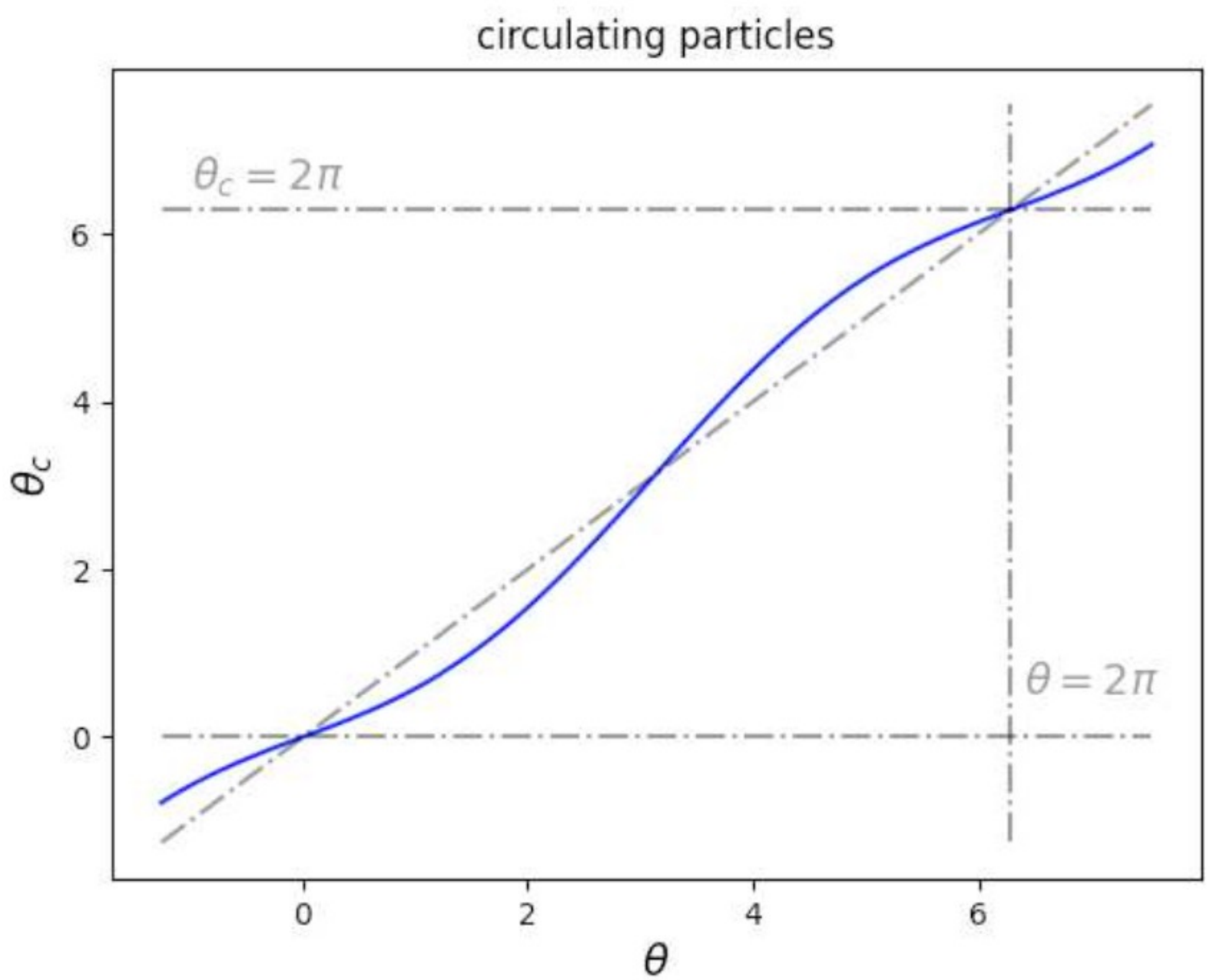
- Recent updates: general approach to action angle coordinates for arbitrary orbits (M.V. Falessi). General mapping and lifting operators (this presentation). Implemented in EQUIPE

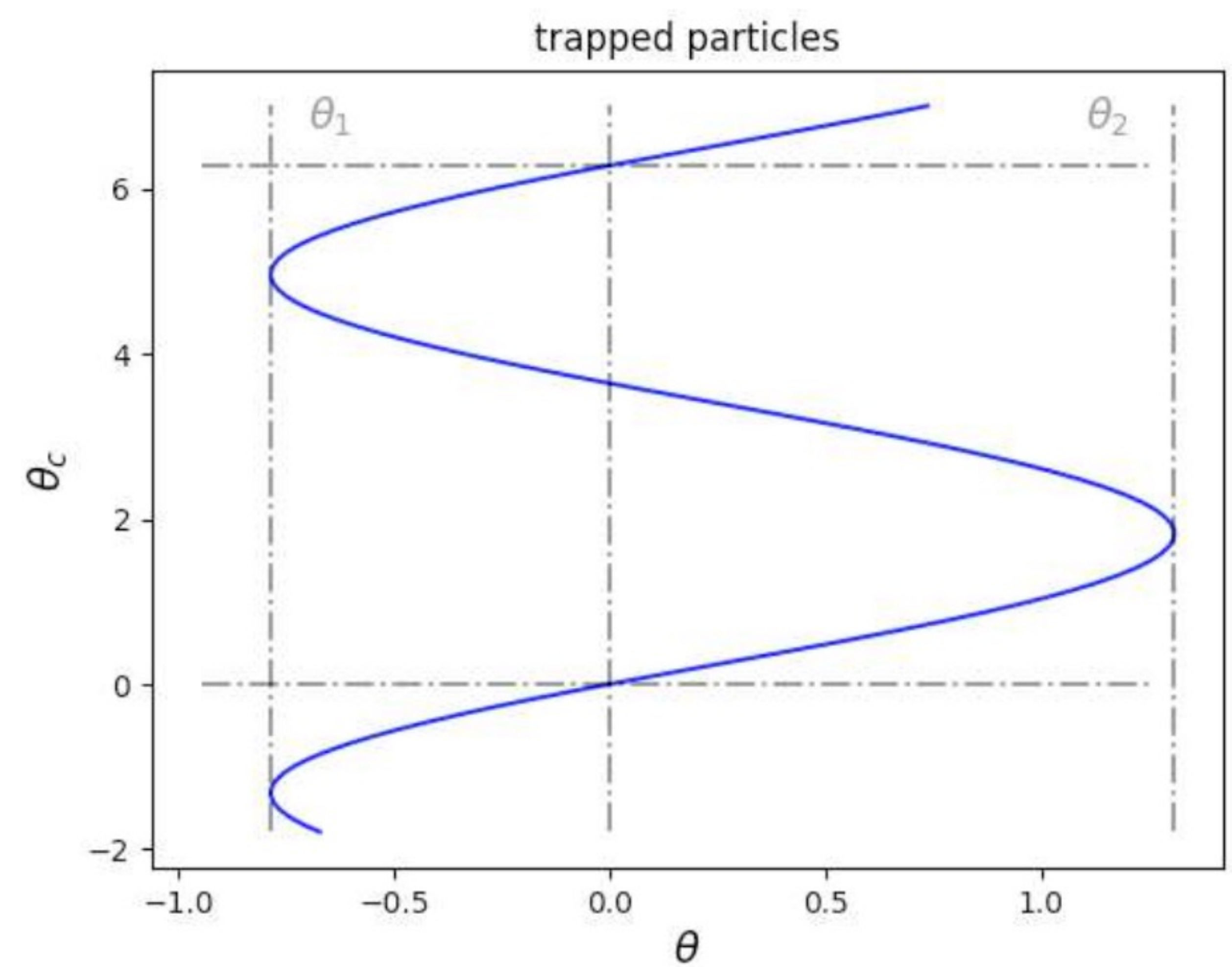
- General result on the low frequency continuum, including KAW

- Extension to finite (arbitrary) frequency

General action-angle

$$r = r_c + \tilde{\rho}_c(\theta_c) \ , \qquad \theta = \bar{\theta}_c + \tilde{\Theta}_c(\theta_c) \ , \qquad \zeta = \zeta_{c0} + \bar{\omega}_d \tau + \bar{q}\sigma\theta_c + \tilde{\Xi}_c(\theta_c) \ .$$





Implemented in EQUIPE

Lifting to phase space

the map $h:(r,\theta,\zeta)\mapsto(r_c,\theta_c,\zeta_c)$ is a *lifting* of f to the phase space

$$f(r,\theta,\zeta) = e^{in\zeta}\sum_{m\in\mathbb{Z}} e^{-im\theta} f_{m,n}(r)$$

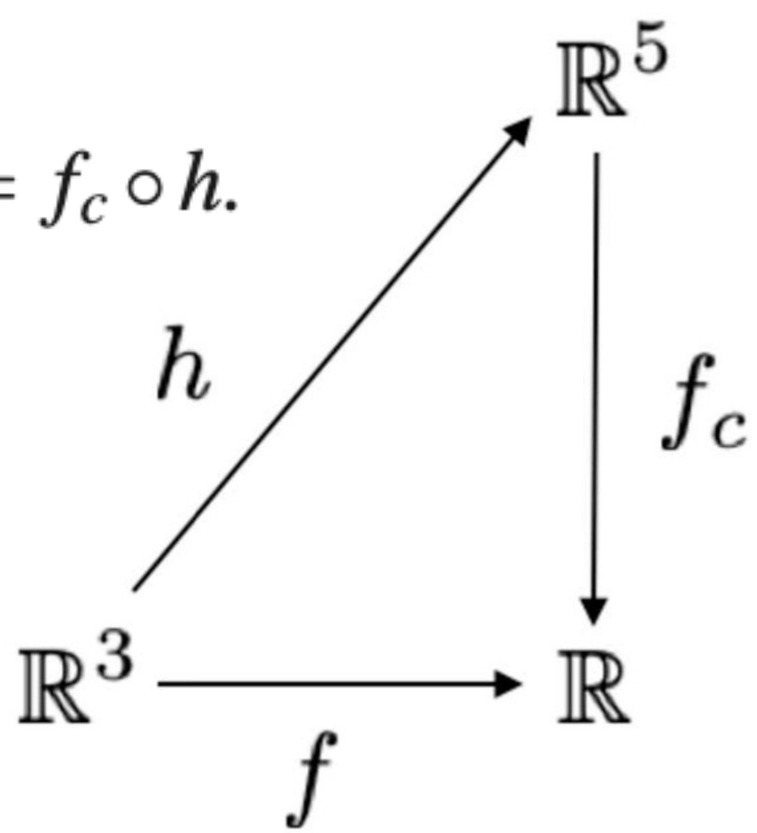
$$\mapsto f_c(r_c,\theta_c,\zeta_c) = e^{in(\zeta_c-\bar{q}\bar{\theta}_c)}\sum_{m,\ell\in\mathbb{Z}} \lambda_{m,n}(r_c,\theta_c)e^{i\ell\theta_c}\mathscr{F}_{m,n,\ell}(r_c) \ ,$$

← push-forward

$$\lambda_{m,n}(r_c,\theta_c) = \exp\left[i\,(n\bar{q}-m)\,\bar{\theta}_c\right]$$

$$\mathscr{F}_{m,n,\ell}(r_c) = \frac{1}{2\pi}\oint \exp\left\{in\tilde{\Xi}_c(\theta_c') - im\tilde{\Theta}_c(\theta_c')\right\} \times f_{m,n}(r_c+\tilde{\rho}_c(\theta_c'))e^{-i\ell\theta_c'}d\theta_c' \ .$$

$$f = f_c\circ h.$$



pull back →

$$f(r,\theta,\zeta) = e^{-iQ_B} f_c\left(r,\theta_c(\theta),\zeta\right)$$

$$= f_c\left(r-\tilde{\rho}_c(\theta_c),\theta_c(\theta),\zeta-\tilde{\Xi}_c(\theta_c)\right)$$

$$f(r,\theta,\zeta) = e^{-iQ_B}\sum_{\ell\in\mathbb{Z}} e^{i\ell\theta_c}\overline{e^{-i\ell\theta_c}e^{iQ_B}f\left(r,\theta(\theta_c),\zeta\right)} \ .$$

…to be implemented in EQUIPE

Generalized inertia

$$\Lambda^2 = \overline{\left[\frac{\omega^2 \mathscr{J}^2 B_0^2}{v_A^2}\left(1-\frac{\omega_{*pi}}{\omega}\right)\left(1+\delta\bar{\Phi}_{\parallel 0}/\delta\bar{\Psi}_0^{(0)}\right)\right]_{1e}} + \sum\int \mathscr{E} d\mathscr{E} d\lambda$$
$$\times \sum_\sigma \tau_b \overline{\left[\frac{4\pi\omega \mathscr{J} B_0/\bar{B}_0}{k_\vartheta c}\kappa_g \mathrm{sgn}(s\vartheta_1)\frac{m(\mu B_0 + v_\parallel^2)e^{-i\omega t}e^{-i\hat{Q}_B}\delta\hat{K}_{Bn,\omega}}{|\nabla r|\delta\bar{\Psi}_0^{(0)}}\right]};$$

$$\Lambda^2 = \overline{\left[\frac{\omega^2 \mathscr{J}^2 B_0^2}{v_A^2}\left(1-\frac{\omega_{*pi}}{\omega}\right)\left(1+\delta\bar{\Phi}_{\parallel 0}/\delta\bar{\Psi}_0^{(0)}\right)\right]_{\vartheta_0}}$$
$$+\sum\int \mathscr{E} d\mathscr{E} d\lambda \sum_\sigma \tau_b \overline{\left[\frac{4\pi\omega}{k_\vartheta c}\frac{F(\psi)}{\bar{B}_0 d\psi/dr}\mathrm{sgn}(s\vartheta_1) m v_\parallel^2 \partial_{\vartheta_0}\left(\frac{\delta\hat{K}}{|\nabla r|^2\delta\bar{\Psi}_0^{(0)}}\right)\right]}.$$

➔ $\Lambda^2 \simeq \omega(\omega-\omega_{*pi})(1+\Delta)/\omega_A^2$ $\qquad$ $\Delta = \left(1.6(R_0/r_s)^{1/2}+0.5\right)q^2$

Particle response & QN

➔ $$\delta\hat{K} = e^{-i\hat{Q}_B} e^{-i\omega t} \delta\hat{K}_{Bn,\omega}$$

$$e^{-i\omega t}\delta\hat{K}_{Bn,\omega,\ell} = \sum_l \frac{e}{m} \overline{\frac{e^{i\hat{Q}_B} Q\bar{F}_0 (J_0|\nabla r|\delta\bar{\Phi}_{\|0}/\hat{\kappa}_\perp) e^{-il\theta_c}}{n\bar{\omega}_d + l\omega_b + \Lambda\dot{\bar{\theta}}_c \mathrm{sgn}(\vartheta) - \omega}}\Bigg|_{n,\ell} e^{il\theta_c}$$
$$+\sum_l \frac{e}{m} \overline{\frac{e^{i\hat{Q}_B} Q\bar{F}_0 (\omega_d/\omega)(J_0|\nabla r|\delta\bar{\Psi}_0^{(0)}/\hat{\kappa}_\perp) e^{-il\theta_c}}{n\bar{\omega}_d + l\omega_b + \Lambda\dot{\bar{\theta}}_c \mathrm{sgn}(\vartheta) - \omega}}\Bigg|_{n,\ell} e^{il\theta_c} .$$

$$A_{l,l'}\left(|\nabla r|\delta\bar{\Phi}_{\|0}/\hat{\kappa}_\perp\right)_{l'} = C_l\left((|\nabla r|\delta\bar{\Psi}_0^{(0)}/\hat{\kappa}_\perp\right) ,$$

$$A_{l,l'} = -\sum \int \mathscr{E} d\mathscr{E} d\lambda \sum_\sigma \tau_b \sum_{l''} \frac{T_{0i}}{n_0 m} \overline{(\mathscr{J}\bar{B}_0)^{-1} e^{-i\hat{Q}_B} e^{il''\theta_c} e^{-il\vartheta_0}}$$
$$\times \overline{\frac{e^{i\hat{Q}_B} Q\bar{F}_0 e^{il'\vartheta_0} e^{-il''\theta_c}}{n\bar{\omega}_d + l''\omega_b + \Lambda\dot{\bar{\theta}}_c \mathrm{sgn}(\vartheta) - \omega}}\Bigg|_{n,\ell} + \left(1+\frac{1}{\tau}\right)\delta_{l,l'} ,$$

$$C_l = \sum \int \mathscr{E} d\mathscr{E} d\lambda \sum_\sigma \tau_b \sum_{l''} \frac{T_{0i}}{n_0 m} \overline{(\mathscr{J}\bar{B}_0)^{-1} e^{-i\hat{Q}_B} e^{il''\theta_c} e^{-il\vartheta_0}}$$
$$\times \overline{\frac{e^{i\hat{Q}_B} Q\bar{F}_0 (\omega_d/\omega) e^{-il''\theta_c}}{n\bar{\omega}_d + l''\omega_b + \Lambda\dot{\bar{\theta}}_c \mathrm{sgn}(\vartheta) - \omega}}\Bigg|_{n,\ell} .$$

🡐 Recover and extend previous results by Y. Li

🡐 Reproduce earlier results by I. Chavdarovski (2009)

Conclusions II

- The theoretical framework for describing low-frequency AE is fully deployed and allows calculating kinetic continuous spectrum and mode structures for general geometry

- Previous results are recovered in the proper limit. This allows various kinds of simplifications in terms of geometry and representations of particle orbits

- Numerical implementation in underway adopting EQUIPE

- Results will be ready for benchmark against LIGKA and/or other codes

THANK YOU!

Your questions are welcome