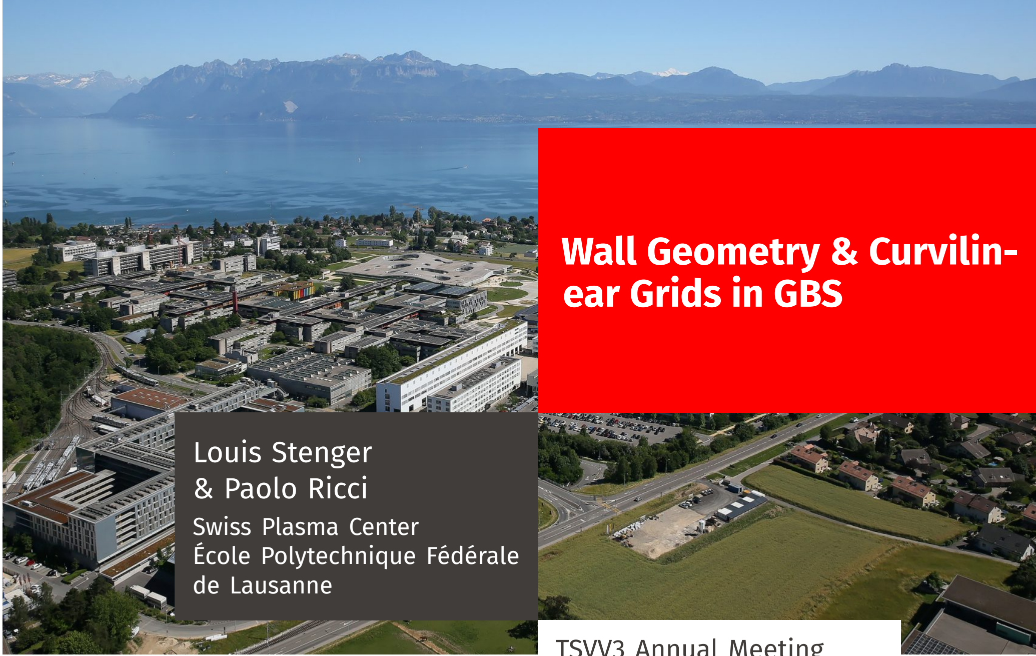
EPFL



Wall Geometry & Curvilinear Grids in GBS

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Swiss Plasma Center
École Polytechnique Fédérale de Lausanne

TSVV3 Annual Meeting

Swiss Plasma Center

Outline

Plasma Turbulence

GBS Evolves the Drift-Reduced Braginskii Equations

- Quasi-neutrality
- Ordering of turbulence $\tau \ll \Omega_{ci}^{-1}, \rho_s \ll L_\perp$
- Large aspect ratio
- Strong toroidal, axisymmetric magnetic field

$$\frac{\partial n}{\partial t} = -\frac{\rho_*^{-1}}{B}\{\phi, n\} + \frac{2}{B}\left[C(p_e) - nC(\phi)\right] - \nabla_\parallel(n v_{\parallel e}) + D_n \nabla_\perp^2 n + S_n + \nu_{iz} n_n - n_i \nu_{rec}$$

Neutral Dynamics

GBS Evolves a Kinetic Neutral System

- Mono-atomic species,
- Short and long neutral mean free paths considered,
- Physical processes: ionization, charge exchange, recombination, recycling, reflection

At each timestep, need to compute and invert the “kernel matrix”,

$$\begin{bmatrix} n_n \\ \Gamma_{\text{out},n} \end{bmatrix} = \begin{bmatrix} v_{\text{cx}} K_{p\to p} & (1-\alpha_{\text{refl}}) K_{b\to p} \\ v_{\text{cx}} K_{p\to b} & (1-\alpha_{\text{refl}}) K_{b\to b} \end{bmatrix} \begin{bmatrix} n_n \\ \Gamma_{\text{out},n} \end{bmatrix} + \begin{bmatrix} n_{n[\text{rec}]} + n_{n[\text{out,i}]} \\ \Gamma_{\text{out},n[\text{rec}]} + \Gamma_{\text{out},n[\text{out,i}]} \end{bmatrix}$$

n_n: neutral density, Γ_{out}: outflowing neutral flux, α_{refl}: reflection coefficient, p: plasma, b: boundary, cx: charge exchange, rec: recombination, i: ionization

Neutral Dynamics

The Kernel Matrix Elements are Path Integrals

- Each element composed of direct and reflected paths, e.g.
  $K_{p\to p} = K^{\mathrm{dir}}_{p\to p} + \alpha_{\mathrm{refl}} K^{\mathrm{refl}}_{p\to p}$
- For each path, compute a path integral, e.g.

$$K^{\mathrm{dir}}_{p\to p}(\mathbf{x},\mathbf{x}') = \int_0^{+\infty} \frac{1}{r_\perp} \Phi_{\perp i}(\mathbf{v}_\perp) \exp\left[-\frac{1}{v_\perp} \int_0^{r_\perp} \nu_{\mathrm{eff}}(\mathbf{x}'')\, \mathrm{d}r''_\perp\right] \mathrm{d}v_\perp .$$

The GBS Code

Plasma:

- GBS evolves
  - 6 fields explicitely in time n, $v_{\parallel e}$, $v_{\parallel i}$, T_e, T_i, ω
  - 2 potentials ϕ (electrostatic), ψ (electromagnetic)
- 4^{th} order spatial finite differences
- Dual φ, Z-staggered Cartesian grid
- Runge-Kutta 4^{th} order in time

Neutrals:

- Low-resolution Cartesian grid,
- Dense matrix inversion

Matrix systems solved with PETSc (GMRES).

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Especially for Neutrals

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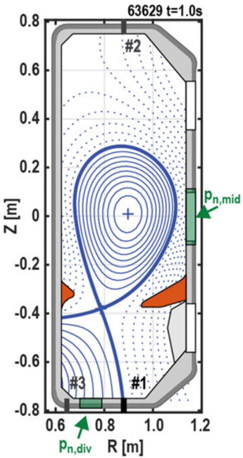

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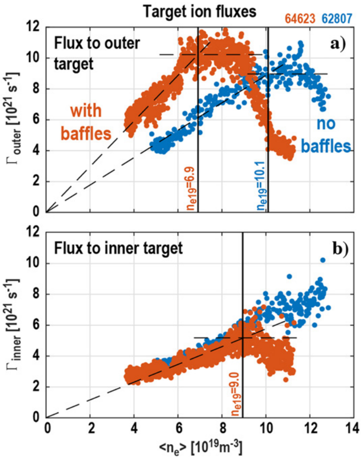

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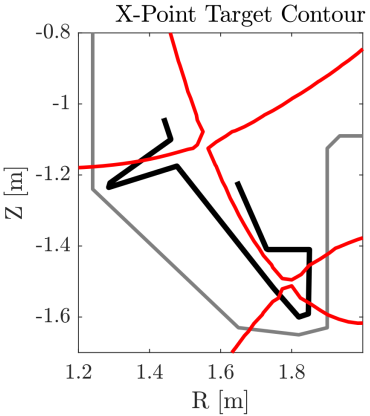

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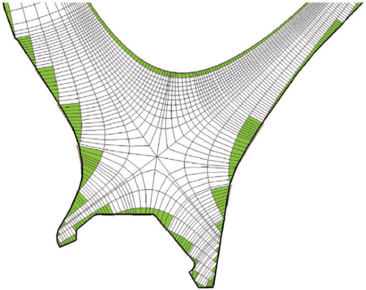

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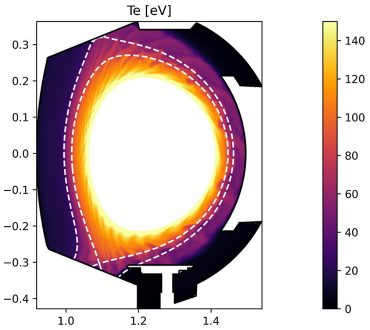

Techniques to Include Wall Geometry Exist

- Finite difference (FD) method on curvilinear grids,
- Finite element method, Galerkin method, Finite volume method or discontinuous Galerkin method,
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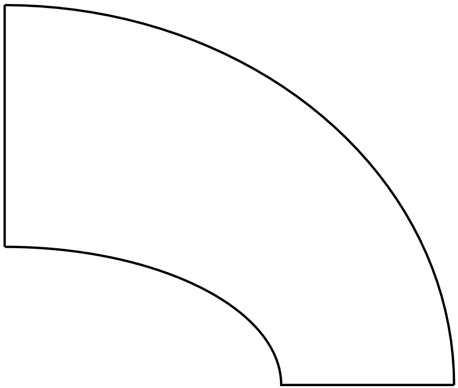

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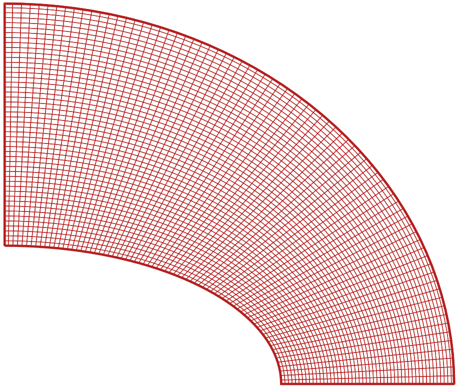

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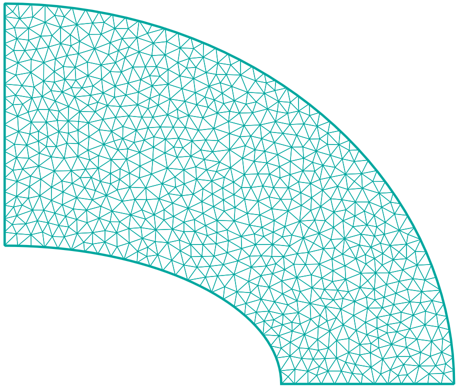

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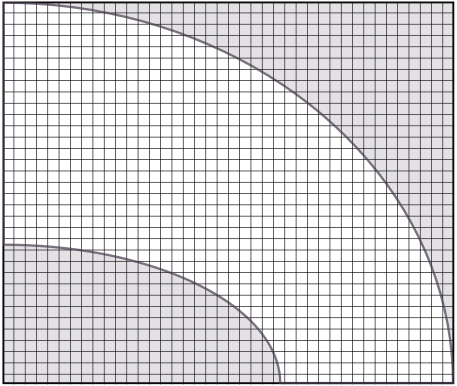


Formalism for Curvilinear Grids

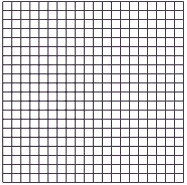
Consider "boundary-fitted" grids

- Coordinate transformation:
  Computational variables $\{\xi^i\}_i \rightarrow (R, \varphi, Z)$

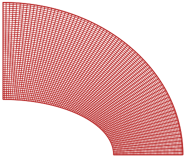
$$\frac{\partial n}{\partial R} = \frac{\partial \xi^i}{\partial R} \frac{\partial n}{\partial \xi^i}$$

- GBS is axisymmetric, transform only a single poloidal plane $\xi^3 = \varphi$
- Retain finite difference convergence
- (Almost) No refactoring needed. User perspective: one new optional input to provide.

Logical grid:

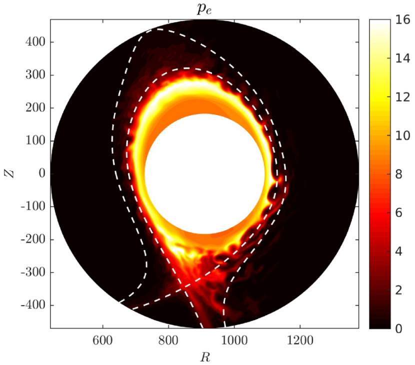


Physical grid:



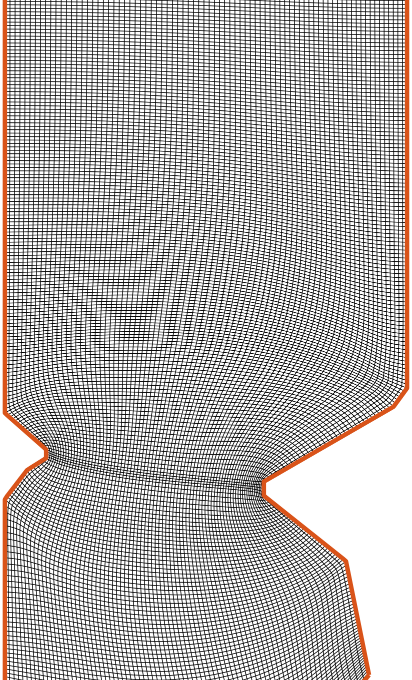
How to Generate a Grid?

- Analytically, e.g. toroidal coordinates
  $(R, Z) = \big((R_0 + r)\cos(\theta), r\sin(\theta)\big)$



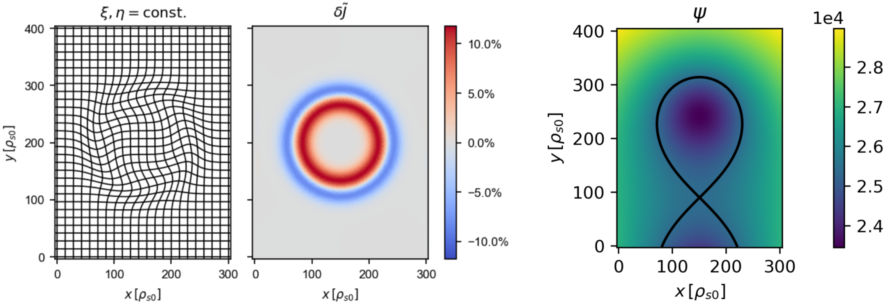
How to Generate a Grid?

- Analytically, e.g. toroidal coordinates
  $(R, Z) = \left((R_0 + r)\cos(\theta), r\sin(\theta)\right)$
- Numerically
  - Transfinite interpolation (TFI)
  - Elliptic methods (EGG)
  - Spline-based EGG (ongoing collaboration MNS, EPFL)

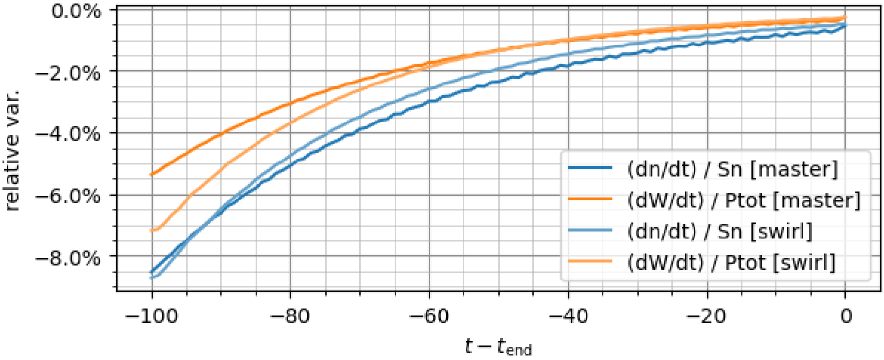


A “Swirl” Case to Test the Inner Domain

Analytical definition. Keep the boundary conditions free from side effects. Activate all metric elements. Direct comparison to main version.



Comparison of Rectilinear and Curvilinear Cases



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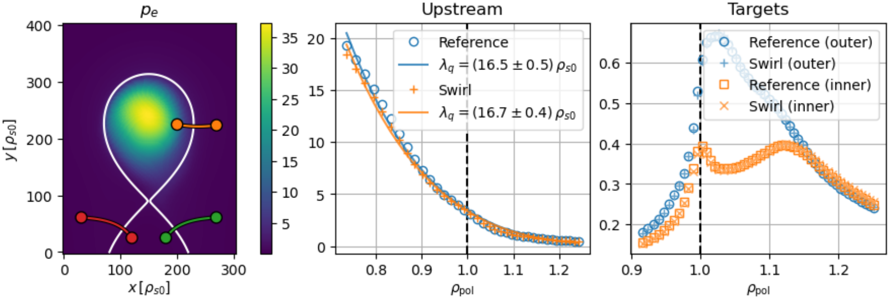
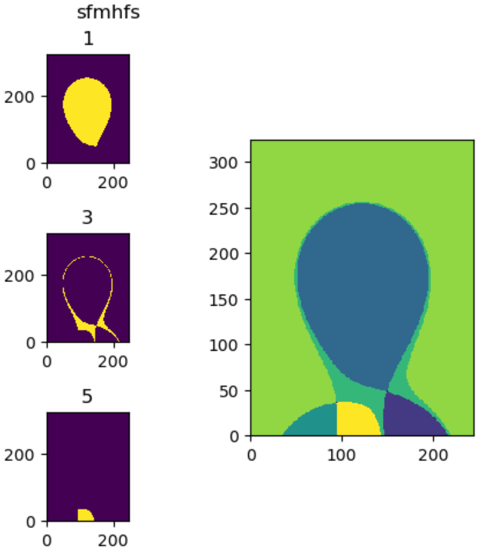


Intermezzo: Generating Field Aligned Grids

- Orthogonal coordinates useful to split B-aligned and cross-field transport.
- $\mathbf{B} = B_\varphi \mathbf{e}_\varphi + \nabla\varphi \times \nabla\psi$
- Flux surfaces: ψ-isolines
- Orthogonal coordinate “χ”

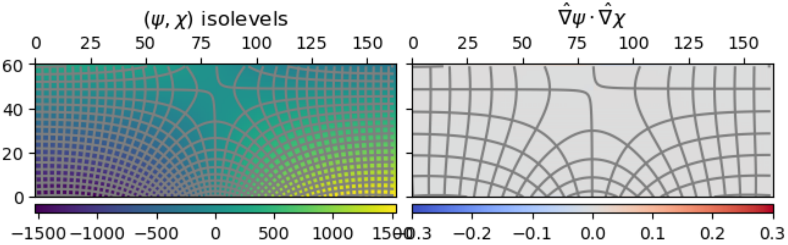


Finding χ given ψ

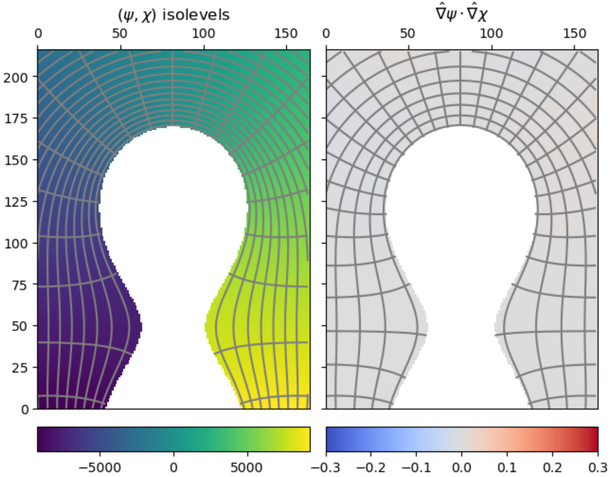
The simplest system, $\nabla\chi = \nabla\psi^T$ is overdetermined,

$$\mathbf{Ax} = \begin{bmatrix} D_x \\ D_y \end{bmatrix}_{2n\times n} [\chi]_n = \begin{bmatrix} D_y \\ -D_x \end{bmatrix}_{2n\times n} [\psi]_n = \mathbf{b}$$

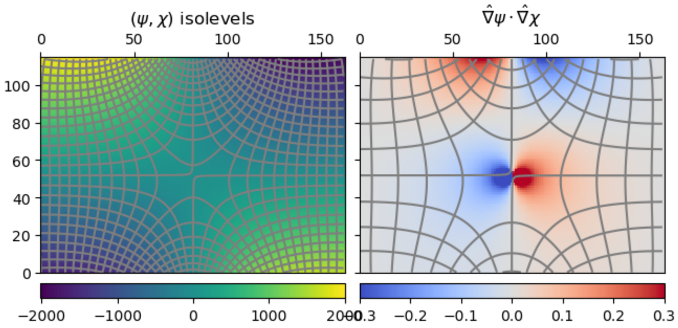
but a LSQR solver (minimize $\|\mathbf{Ax} - \mathbf{b}\|$) converges



… also works for “sparse” domains …



… but fails towards the core



Why did it work so far? We have been solving the Cauchy-Riemann equations. If a solution exists, also imply $\Delta\psi = \Delta\chi = 0$ and a corresponding conformal map is defined $f = \psi + i\chi$. But in our case, $\nabla \times \mathbf{B}_{\text{pol}} = \Delta\psi = J \neq 0$.

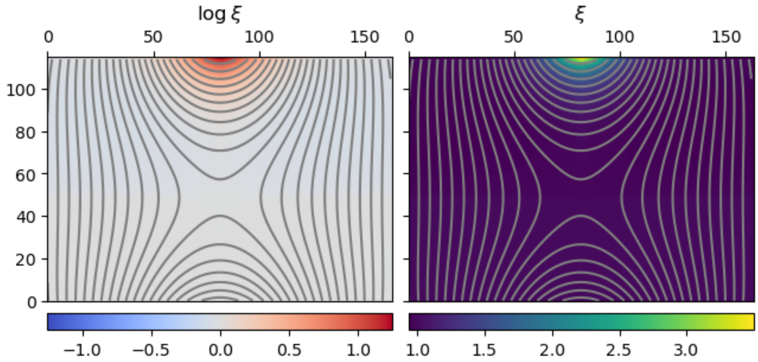
Let’s assume ψ can be rescaled

Let $\psi_h = h(\psi)\psi$,

$$\nabla\psi_h = (h + h'\psi)\nabla\psi = \xi\nabla\psi,$$

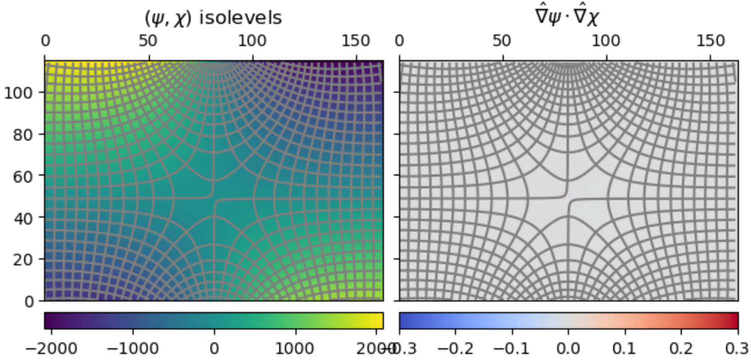
require $\xi > 0$. Require also $\Delta\psi_h = 0 = \xi\Delta\psi + (\nabla\psi \cdot \nabla)\xi$. Solve

$$(\nabla\psi \cdot \nabla)\log\xi = -\Delta\psi$$

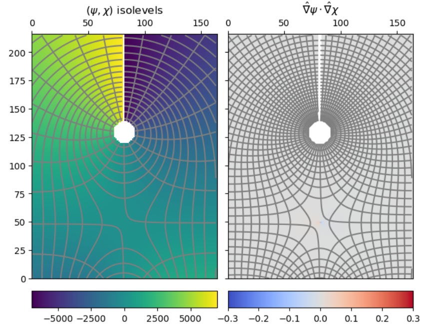


Solve for χ using the rescaled ψ

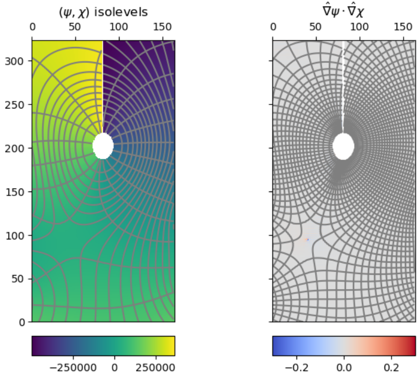
We're solving the over-determined system again, this time with ψ_h in place of ψ, $\nabla\chi = \nabla\psi_h^T$.



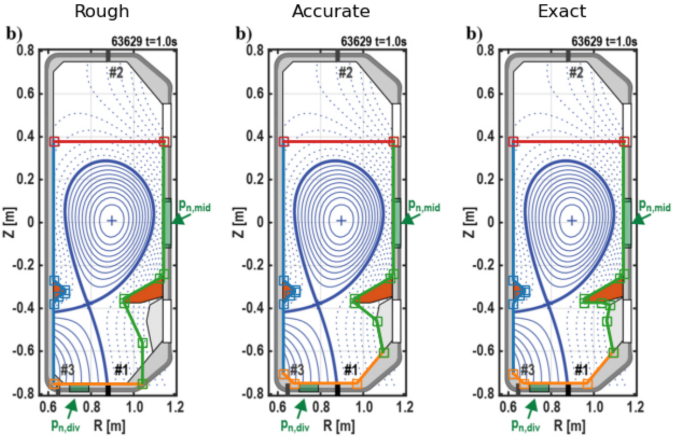
Naive approach works outside closed field line regions



Actually “fixes” X21 defects



Baffled TCV Setup



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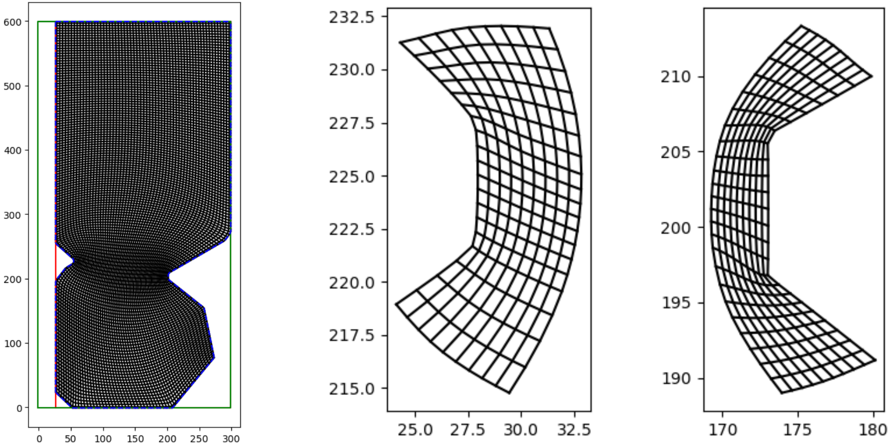

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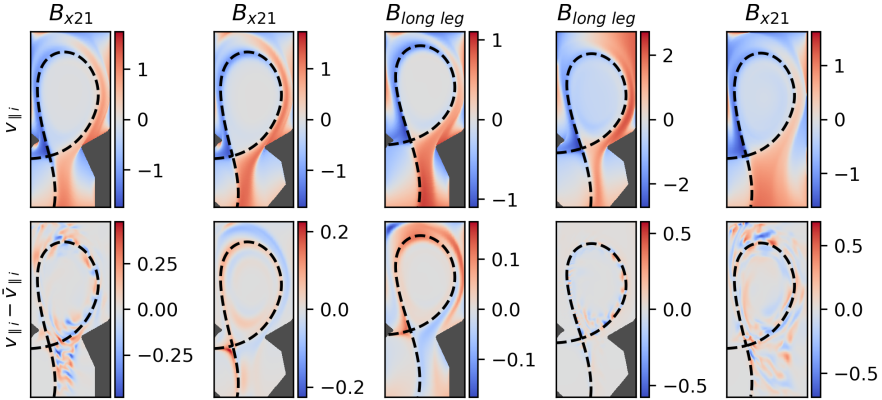
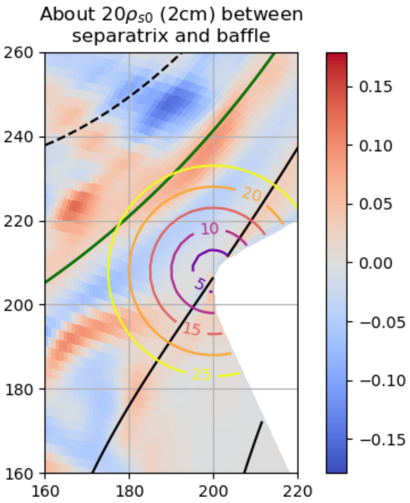


Baffled TCV Results

- Needs additional runtime before convergence
- Qualitative differences observed
  - Baffles enhance SOL parallel flows: very close to plasma (reduced domain, low-res ⅓rd TCV)
  - Electrostatic potential (not shown): Stronger gradients due to baffle proximity, BC $\phi = \lambda T_e$



What's Next?

Updated from last year

- Run with a more realistic, larger, baffled TCV grid
- Refactor, cleanup, and merge back into currently developped GBS version
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What Needs to be Done for the Neutrals?

For the inner domain, modifications *look* simple. The computation as it is done now,

1. Plasma fields are interpolated on the neutral grid
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The hard part: Reflections, and in general, any computation involving wall's orientation.

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