MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Electromagnetic turbulence with GRILLIX

ASDEX Upgrade

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EUROfusion

Courtesy to Bruce Scott

We cannot stress enough: our work builds heavily on past work by Bruce Scott, see

B. Scott. Turbulence and Instabilities in Magnetised Plasmas, Volume 1: Fluid drift turbulence. IOP Publishing (2021).

B. Scott. Turbulence and Instabilities in Magnetised Plasmas, Volume 2. IOP Publishing (2021)

B. Scott. Three-dimensional computation of drift Alfvén turbulence. Plasma Physics and Controlled Fusion, 39 (10), 1635 (1997).

B. Scott. Low frequency fluid drift turbulence in magnetised plasmas (2001). IPP Report 5/92 (Garching: Max-Planck-Institut für Plasmaphysik)

This talk is based on 2 papers:

Kaiyu Zhang et al 2024 Nucl. Fusion 64 036016, https://doi.org/10.1088/1741-4326/ad1b93

W. Zholobenko et al 2024 Submitted to Nucl. Fusion, https://doi.org/10.48550/arXiv.2403.10113

Prolog: Gyrokinetic background

$$\frac{\partial f_\sigma}{\partial t} + \dot{\boldsymbol{R}} \cdot \nabla f_\sigma + \dot{v}_{||} \frac{\partial f_\sigma}{\partial v_{||}} = 0$$

$$\boldsymbol{\nabla} \times A_{1||}\boldsymbol{b} \approx \boldsymbol{\nabla} A_{1||} \times \boldsymbol{b}$$

Flutter

$$\dot{\boldsymbol{R}} = v_{||}\frac{\boldsymbol{B}^*}{B^*_{||}} + \frac{c}{q_\sigma B^*_{||}} \boldsymbol{b} \times (\mu \boldsymbol{\nabla} B + q_\sigma \boldsymbol{\nabla} \phi_1),$$

$$\dot{v}_{||} = -\frac{\boldsymbol{B}^*}{m_\sigma B^*_{||}} \cdot (\mu \boldsymbol{\nabla} B + q_\sigma \boldsymbol{\nabla} \phi_1) - \frac{q_\sigma}{m_\sigma c}\frac{\partial A_{1||}}{\partial t}$$

$$\boldsymbol{B}^* := \boldsymbol{B} + m_\sigma v_{||}\frac{c}{q_\sigma} \boldsymbol{\nabla} \times \boldsymbol{b} + \boldsymbol{\nabla} A_{1||} \times \boldsymbol{b},$$

$$B^*_{||} := \boldsymbol{b} \cdot \boldsymbol{B}^* = B + m_\sigma v_{||} \frac{c}{q_\sigma} \boldsymbol{b} \cdot \boldsymbol{\nabla} \times \boldsymbol{b}.$$

Induction: $E_\parallel = -\nabla_\parallel \phi - \partial_t A_\parallel$

D. Michels, Phys. Plasmas 29, 032307 (2022)

Very briefly on magnetic induction

- $\partial_t A_\parallel$ **in Ohm's law requires a Helmholtz solver, if electron inertia is kept**
- **However, this introduces shear Alfvén waves, which is good!**
  - THE physical phenomenon which determines the parallel dynamics.
  - Alfvén time is usually still much longer than current diffusion at any realistic collisionality!
- **This is known for a long time, see e.g.**

B. Scott. Three-dimensional computation of drift Alfvén turbulence. Plasma Physics and Controlled Fusion, 39 (10), 1635 (1997)

Dannert T. and Jenko F. 2004 Vlasov simulation of kinetic shear Alfvén waves Comput. Phys. Commun. 163 67–78

Dudson B.D., Newton S.L., Omotani J.T. and Birch J. 2021 On Ohm's law in reduced plasma fluid models Plasma Phys. Control. Fusion 63 125008

Magnetic flutter in drift-fluid models is a mess

- Flutter terms approximated to Poisson bracket
- Discretized by Arakawa scheme.

$$\nabla_1 f = -\frac{\mathbf{B}_0}{B_0^2}\nabla A_1 \times \nabla f \approx [A_1, f]/B_0$$

$$\nabla_1 \rightarrow \nabla_\parallel$$

- continuity
$$\frac{\mathrm{d}}{\mathrm{d}t} n = nC(\phi) - C(p_e) + \nabla_\parallel j_\parallel - \nabla_\parallel (n u_\parallel) + \mathcal{D}_n(n) + S_n$$

- vorticity
$$\nabla \cdot \left[\frac{n}{B^2}\left(\frac{\mathrm{d}}{\mathrm{d}t} + u_\parallel \nabla_\parallel\right)\left(\nabla_\perp \phi + \zeta \frac{\nabla_\perp p_i}{n}\right)\right] = -C(p_e + \zeta p_i) + \nabla_\parallel j_\parallel - \frac{\zeta}{6} C(G) + \mathcal{D}_\Omega(\Omega)$$

- parallel momentum
$$\left(\frac{\mathrm{d}}{\mathrm{d}t} + u_\parallel \nabla_\parallel\right) u_\parallel = -\frac{\nabla_\parallel p_e}{n} - \frac{\nabla_\parallel (\zeta p_i)}{n} + \zeta T_i C(u_\parallel) - \frac{2}{3}\zeta \frac{B^{3/2}}{n} \nabla_\parallel \frac{G}{B^{3/2}} + \mathcal{D}_u(u_\parallel)$$

- Ohm's law
$$\beta_0 \frac{\partial}{\partial t} A_\parallel + \frac{M_e}{M_i}\left(\frac{\mathrm{d}}{\mathrm{d}t} + v_\parallel \nabla_\parallel\right)\frac{j_\parallel}{n} = -\left(\frac{\eta_{\parallel 0}}{T_e^{3/2}}\right) j_\parallel - \nabla_\parallel \phi + \frac{\nabla_\parallel p_e}{n} + 0.71 \nabla_\parallel T_e + \mathcal{D}_\Psi(\Psi_m)$$

- electron temperature
$$\frac{3}{2}\left(\frac{\mathrm{d}}{\mathrm{d}t} + v_\parallel \nabla_\parallel\right) T_e = T_e C(\phi) - \frac{T_e}{n} C(p_e) - \frac{5}{2} T_e C(T_e) - T_e \nabla_\parallel v_\parallel + 0.71 \frac{T_e}{n} \nabla_\parallel j_\parallel$$
$$+ \frac{1}{n}\nabla_\parallel \left[\mathcal{L}_{\nabla_\parallel}(\nabla_\parallel T_e)\right] - 2\nu_{e0}\mu \left(\frac{n}{T_e^{3/2}}\right)(T_e - \zeta T_i) + \left(\frac{\eta_{\parallel 0}}{T_e^{3/2}}\right)\frac{j_\parallel^2}{n} + \frac{3}{2}\left(\mathcal{D}_{T_e}(T_e) + S_{T_e}\right)$$

- ion temperature
$$\frac{3}{2}\left(\frac{\mathrm{d}}{\mathrm{d}t} + u_\parallel \nabla_\parallel\right) T_i = T_i C(\phi) - \frac{T_i}{n} C(p_e) + \frac{5}{2}\zeta T_i C(T_i) - T_i \nabla_\parallel u_\parallel + \frac{T_i}{n}\nabla_\parallel j_\parallel$$
$$+ \frac{1}{n}\nabla_\parallel \left[\mathcal{L}_{\nabla_\parallel}(\nabla_\parallel T_i)\right] + 2\nu_{e0}\mu \left(\frac{n}{T_e^{3/2}}\right)\left(\frac{1}{\zeta} T_e - T_i\right) + \frac{2 w_{GTi}}{9\eta_{i0}} \frac{G^2}{n T_i^{5/2}} + \frac{3}{2}\left(\mathcal{D}_{T_i}(T_i) + S_{T_i}\right)$$

- Ampere's law
$$\nabla_\perp^2 A_\parallel = -j_\parallel.$$

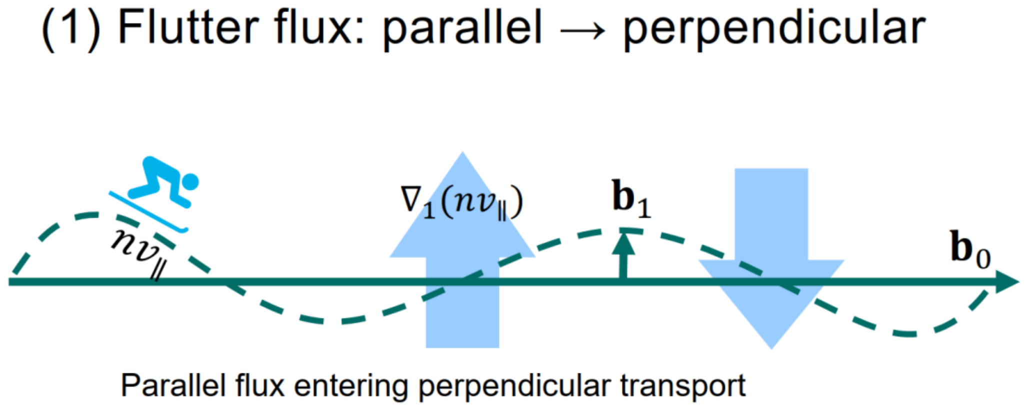
- Viscous stress
$$G = -\eta_0^i \left[\frac{2}{B^{3/2}} \nabla_\parallel \left(u_\parallel B^{3/2}\right) - \frac{1}{2}\left(C(\varphi) + \frac{1}{en} C(p_i)\right)\right]$$

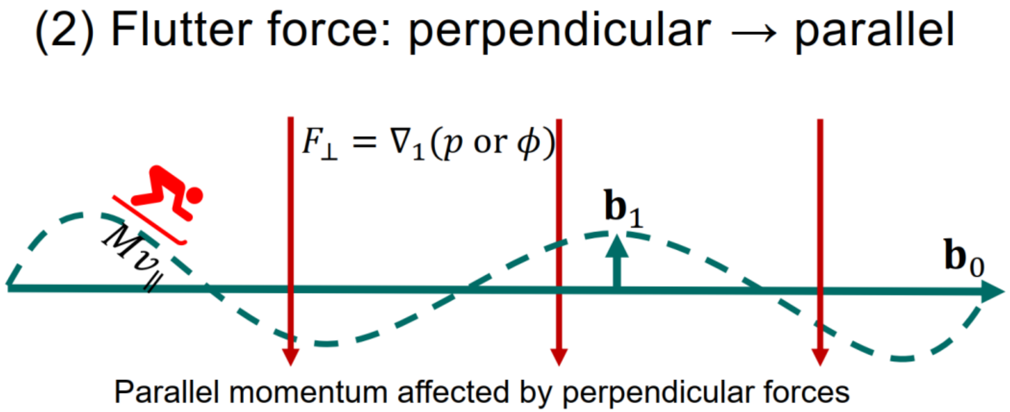
- 25 explicit terms in timestepping
- 6 implicit terms in Helmholtz solvers

Good news: most flutter terms are unimportant

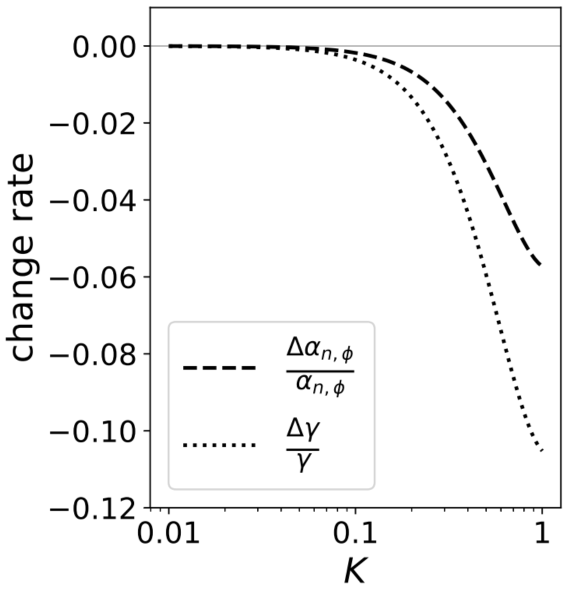
There are essentially 2 effects:

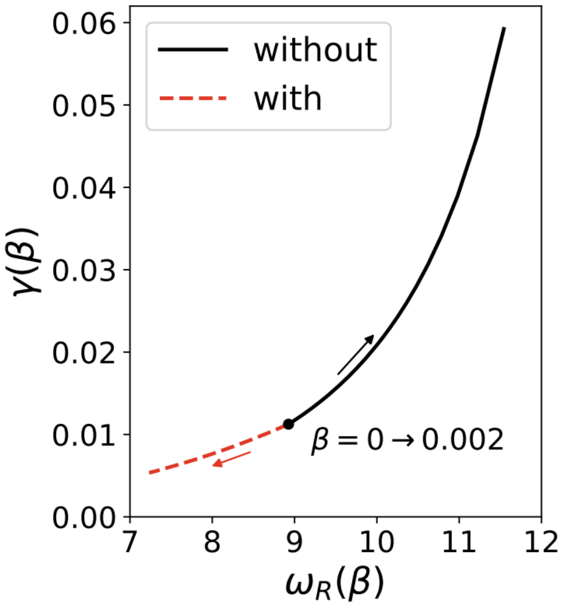
Only 1 linear term: $\nabla_{\|} p_e$ in Ohm's law!





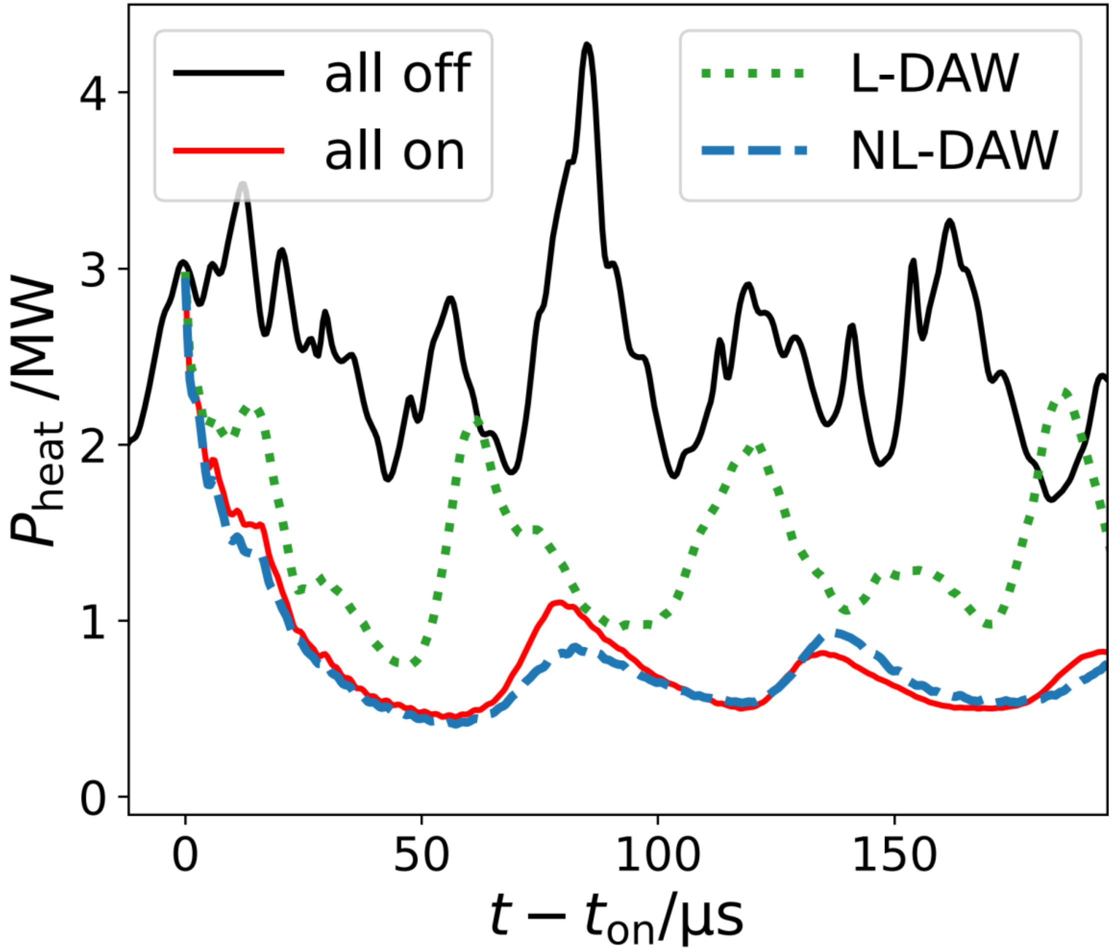
- The critical mechanism is the **flutter force in Ohm's law**, coupled to state variables via $\nabla \cdot j_{\|}\mathbf{b}$, which leads to a **stabilization of drift-wave turbulence** – until the ballooning limit.
- **Flutter balances destabilization by induction**
- See again Bruces references above for linear and non-linear theory.
- In Kaiyu Zhang et al 2024 Nucl. Fusion **64** 036016, the theory is extended for the GRILLIX model (with temperature dynamics) – density and temperature (also Ti!) are closely coupled!



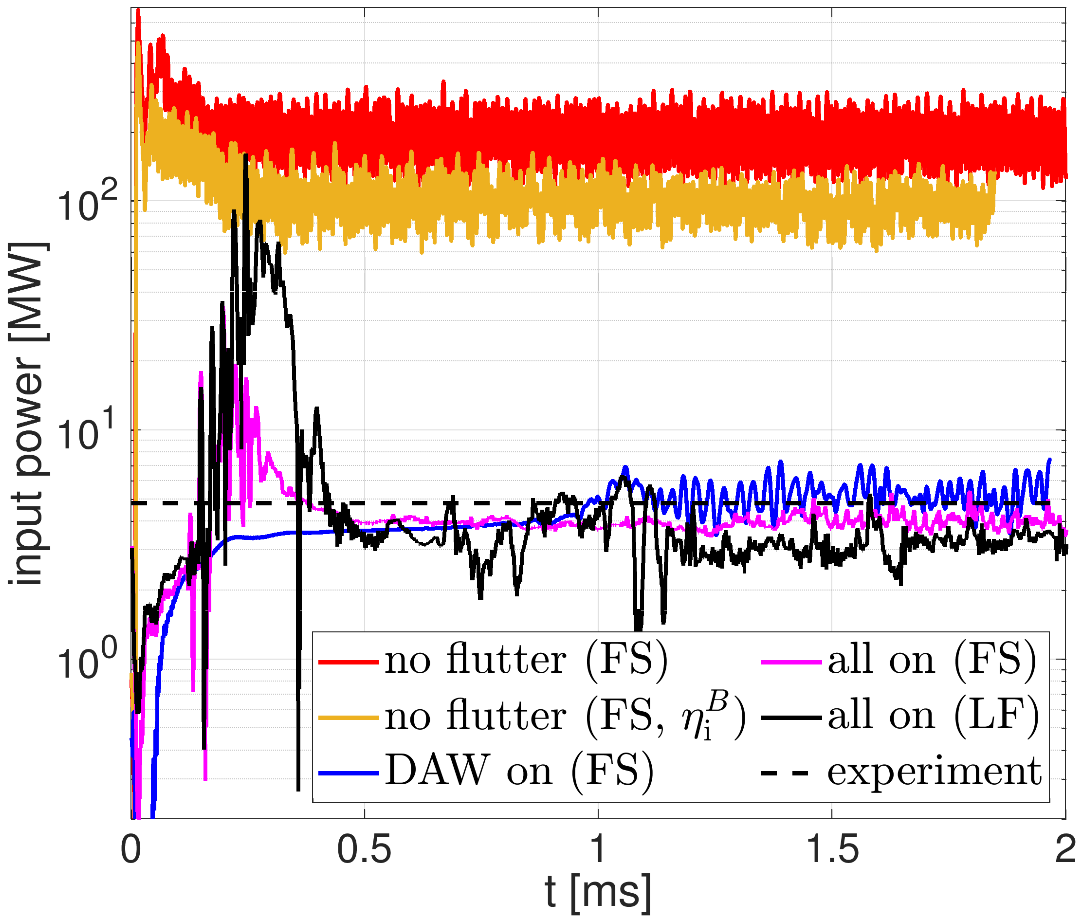


Nonlinear stabilization, L- and H-mode

- Flutter in non-linear drift-wave dynamics even more stabilizing than linear!
- In L-mode, stabilization factors of 2-5.

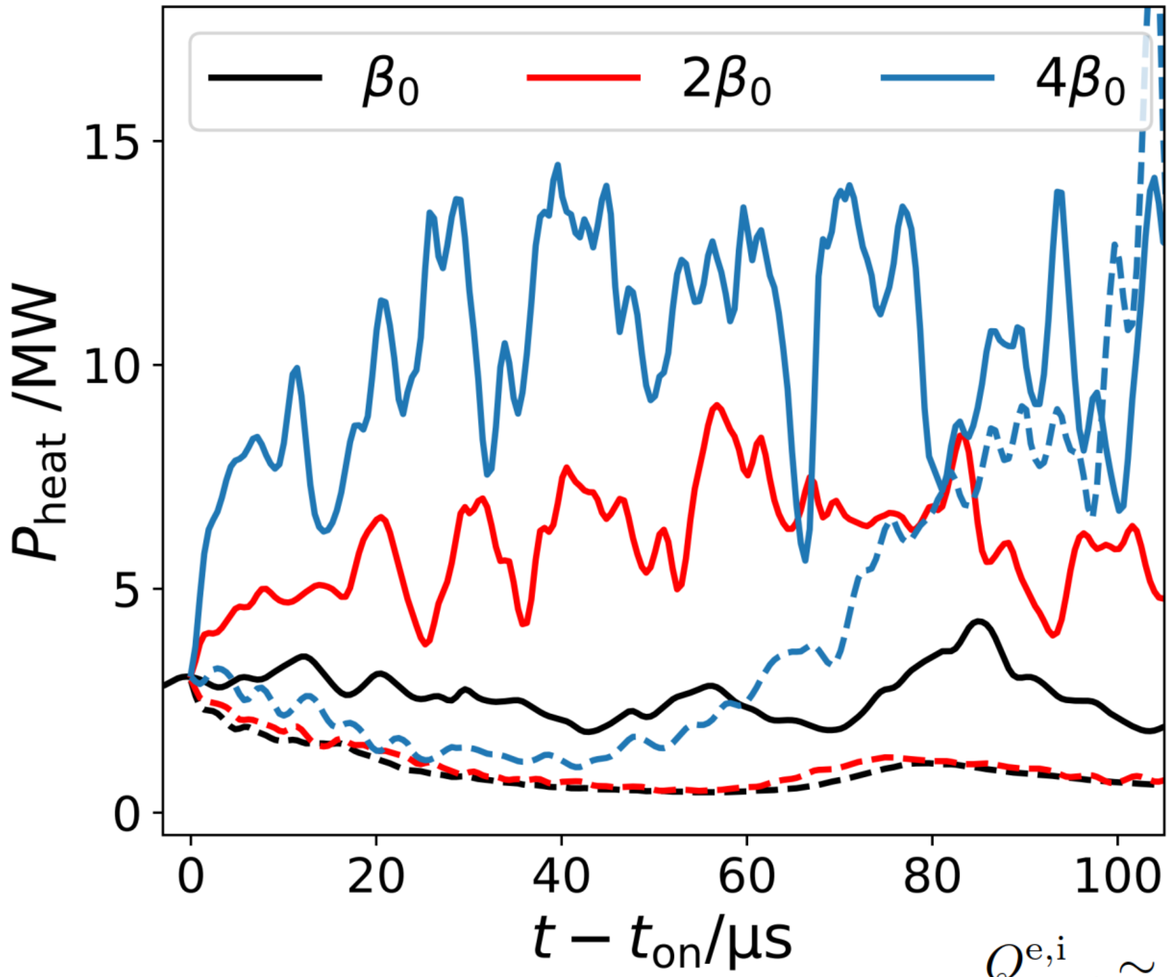


- **<u>In H-mode, magnetic flutter changes fluxes over 2 orders of magnitude!</u>**
- Cannot be compensated by fluid closure (ion viscosity, heat flux).

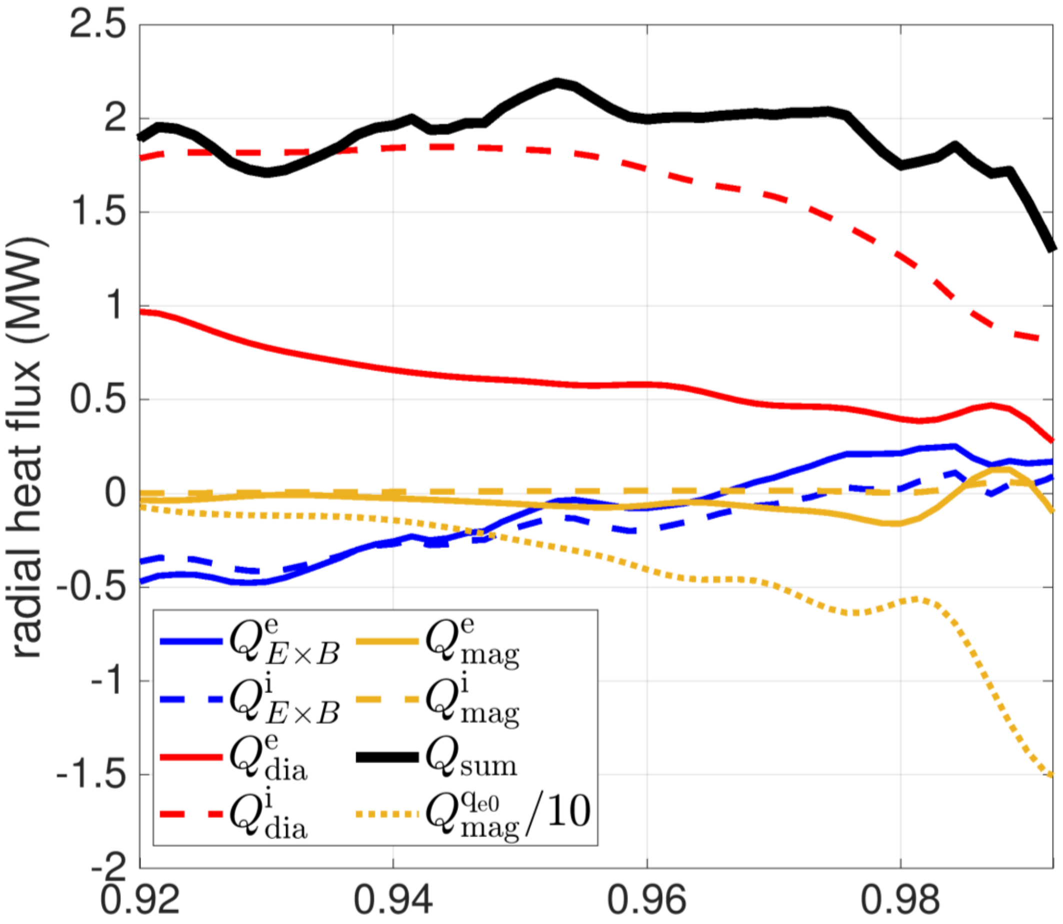


Beta threshold of MHD-like instabilities

- Simply increasing beta, at some point triggers electromagnetic transport.



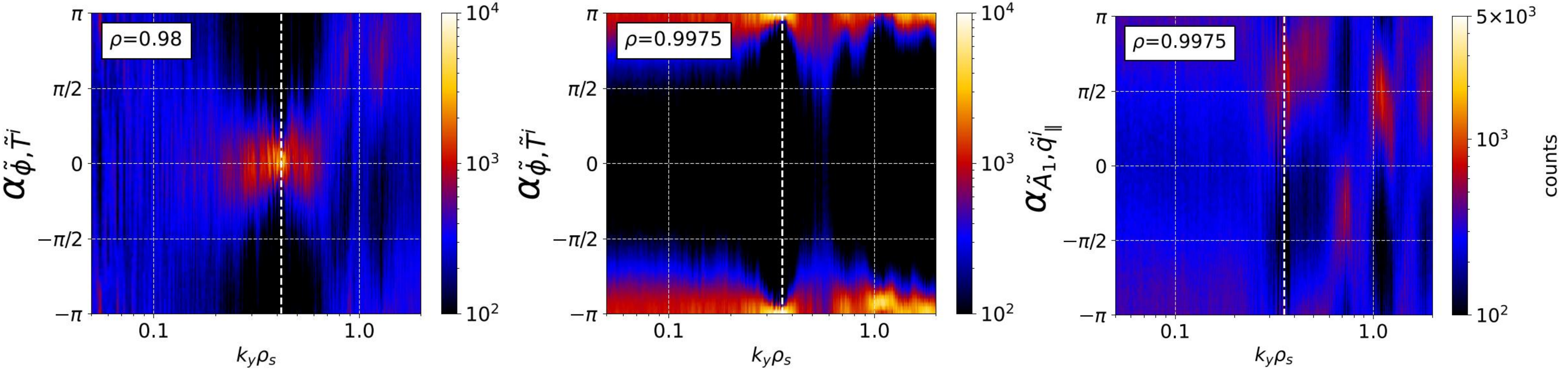
- In our H-mode simulation, we see EM > ES transport near the separatrix.



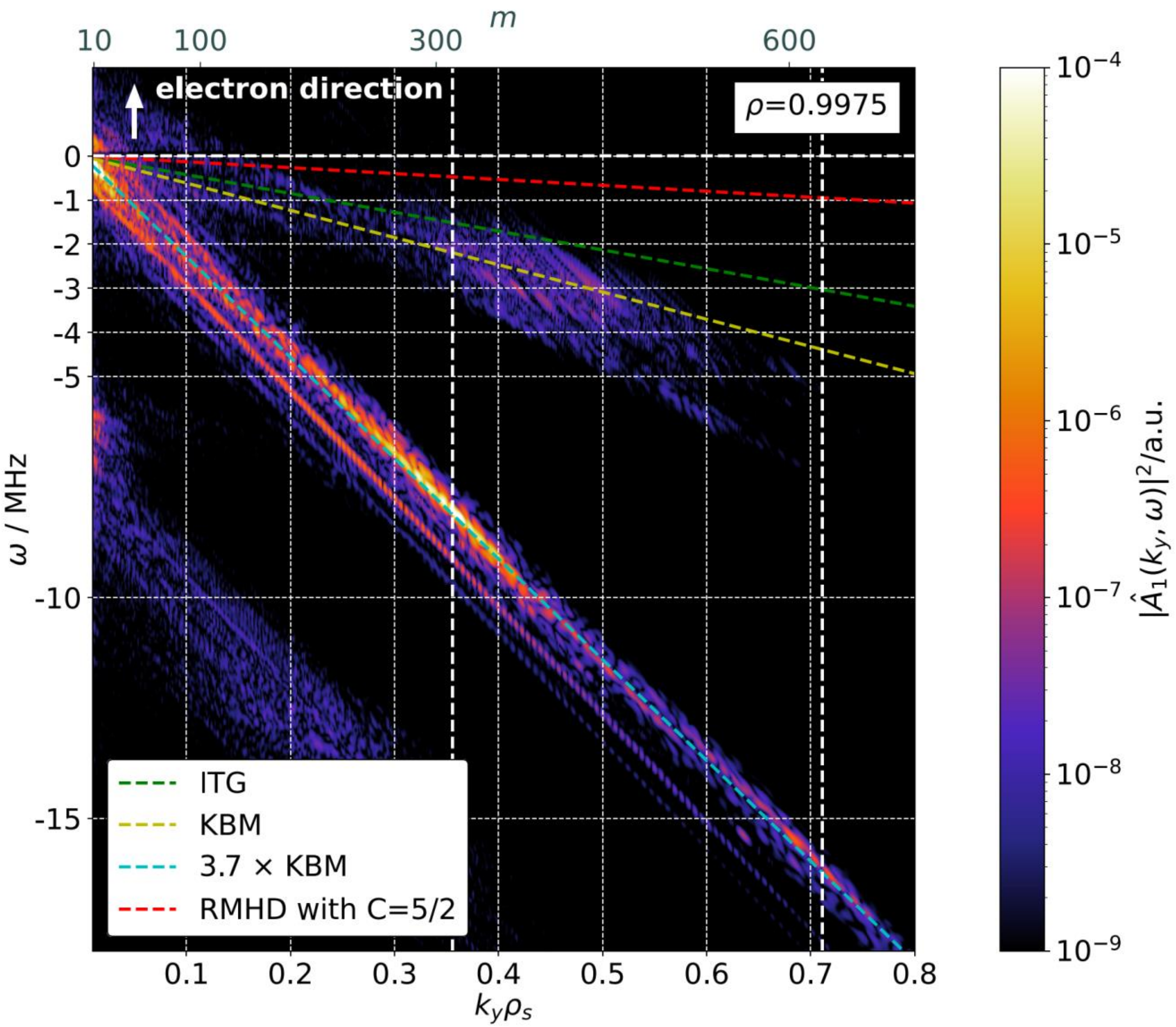
$$Q^{e,i}_{mag} \sim q_{\|e,i}\mathbf{b}_1 \cdot \mathbf{e}_r = -\frac{q_{\|e,i}}{B}\partial_y A_1 \sim -k|\tilde{q}_{\|e,i}||\tilde{A}_1|\sin(\alpha_{\tilde{A}_1,\tilde{q}_{\|e,i}})$$

Kinetic ballooning turbulence mode structure

- Normal drift-wave mode structure at $\rho_{\mathrm{pol}} < 0.99$
- Kinetic ballooning mode structure near the separatrix:
    - $\pm\pi$ for density and temperature, $\pm\pi/2$ for parallel heat flux



KBM dispersion relation



- Strongly electromagnetic, with $E_{\parallel} \approx 0$
- Mode propagates in ion diamagnetic direction (excludes MTM)
- Frequency is 4 x flux-tube GK ideal (collisionless) KBM: deviation due to geometry or resistivity?

⇒ Quite clearly a kinetic ballooning mode

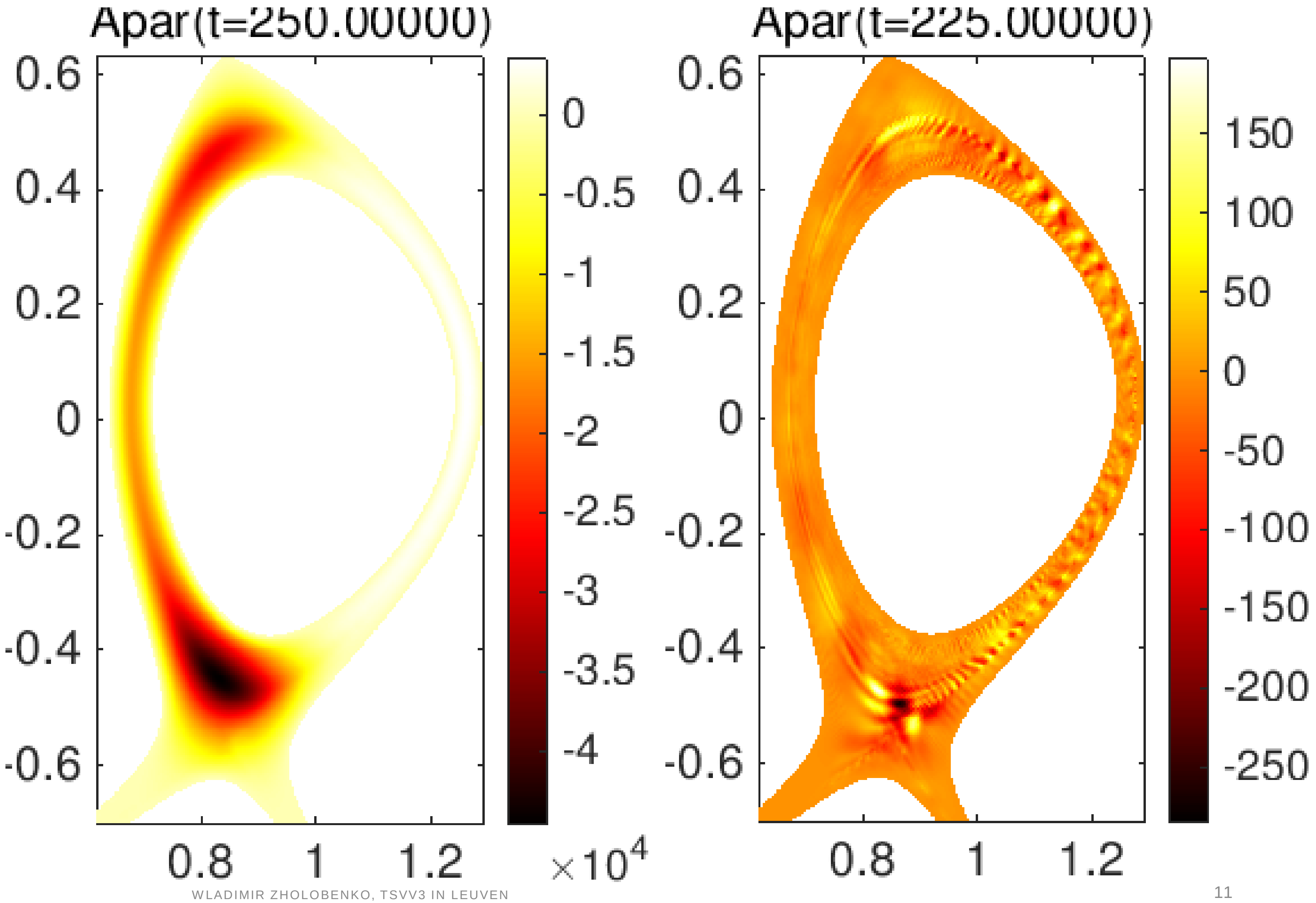
Numerical treatment of the Shafranov shift

- Locally field-aligned numerics in GRILLIX (and GENE-X) require strong background field B_0, and allow only small perturbations

$$B_1/B_0 < h/\Delta s \sim \frac{\mathrm{n_{pol}}\rho_\mathrm{i}}{\mathrm{R_0}} \sim 5\times 10^{-3}$$

- Global full-*f* models evolve the full plasma pressure, which leads to Pfirsch-Schlüter (and bootstrap) currents and a Shafranov shift
- **The Shafranov shift must be removed from parallel operators, it is included in the equilibrium**

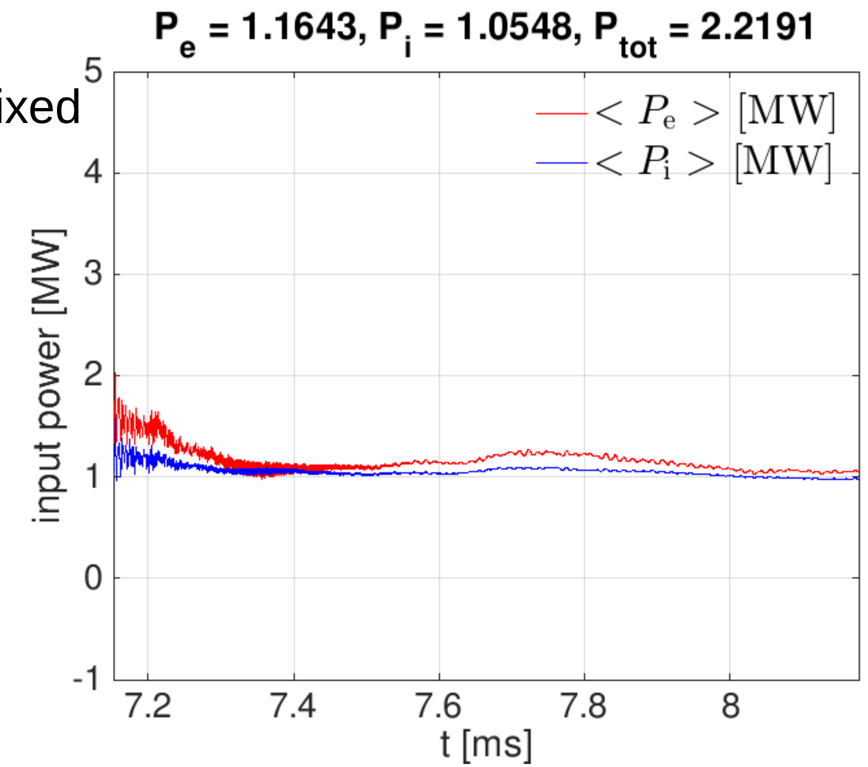
$B_1/B_0 \sim 10^{-2}$ $B_1/B_0 \sim 10^{-3}$



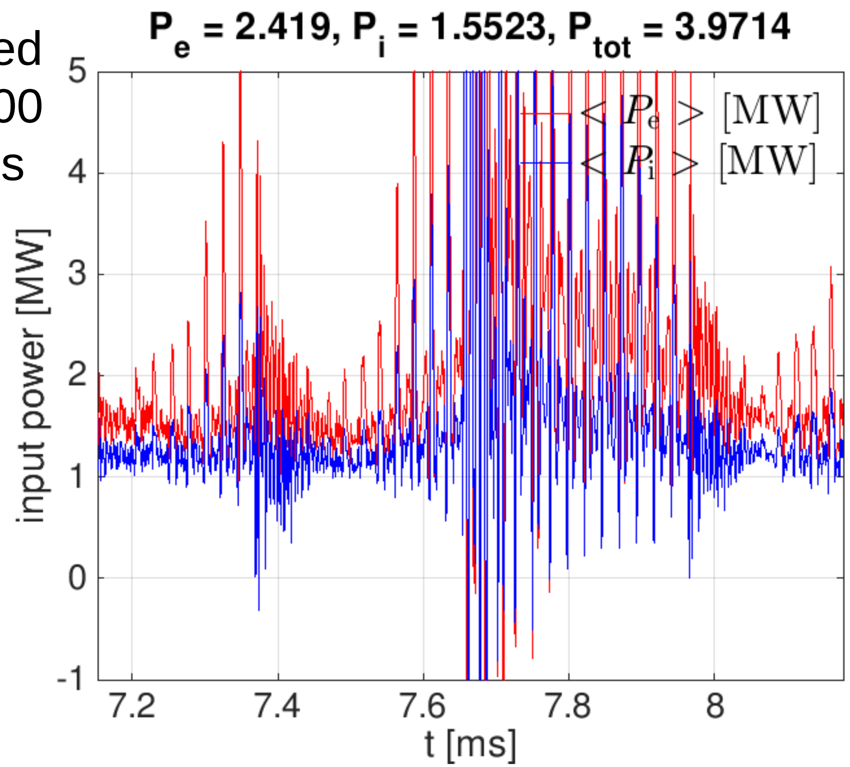
Removing the Shafranov shift correctly is not trivial

- We want to extract $A_1 = A_\parallel - A_s$, where A_s is the mean (averaged!) Shafranov shift
- The trivial method [1-3], $A_s = \langle A_\parallel \rangle_{\text{tor}}$ **at every time step**, does not work! Introduces (subtle) artefacts even in L-mode, and really bad ones in H-mode, **because it destroys the correct drift-Alfvén wave response,**
- Our current solution: use $A_s = \langle A_\parallel \rangle_{\text{t,tor}}$ and update periodically. But also this was found to cause problems!
- Fix A_s in saturated state. For the future, we work on a smooth A_s adaptation scheme.

A_s fixed



A_s updated every 1000 time steps



[1] B. Scott. Contributions to Plasma Physics, 46 (7-9), 714 (2006)
[2] R. Hager et al.. Physics of Plasmas, 27 (6), 062301 (2020)
[3] M. Giacomin et al.. Journal of Computational Physics, 463, 111294 (2022)

Conclusions

- ✓ Role of magnetic induction and flutter in drift-wave turbulence is very well understood
  - Until the ballooning threshold, induction is de- and flutter is stabilizing
  - Flutter is most important in the non-adiabaticity force in Ohm's law
  - Still need to put it everywhere for energy conservation (and div(B)=0)
- ✓ At higher beta (in H-mode), drift-wave ($E \times B$) turbulence is suppressed (with flutter, and E_r)
- ✓ New modes appear, like the KBM, which cause EM heat transport (by flutter)

- ➢ Care has to be taken when magnetic fluctuations are so large that they destroy the field-alignment of the code
- ➢ Especially the Shafranov shift background has to be treated carefully in global codes

Thank you for your attention!