Status update
Implementation of 3 fields edge plasma model in the EBC HDG code

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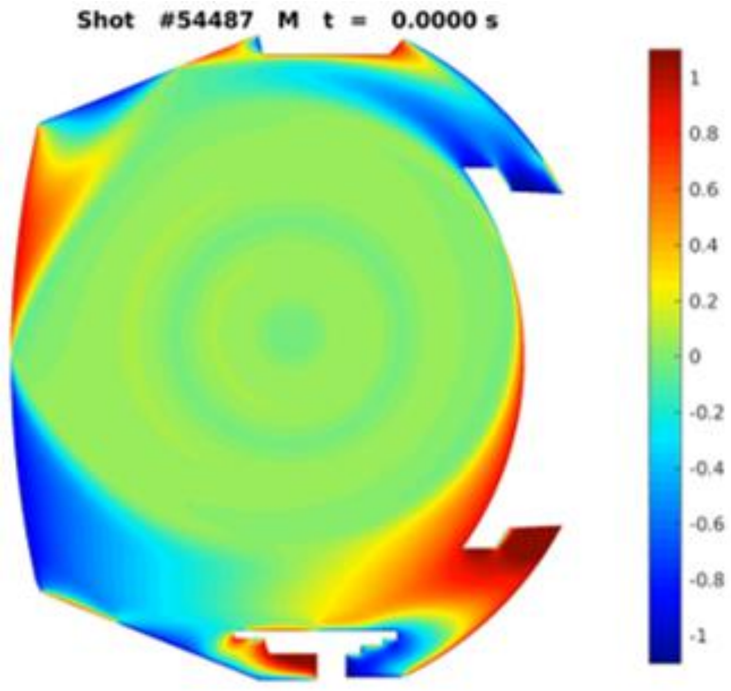
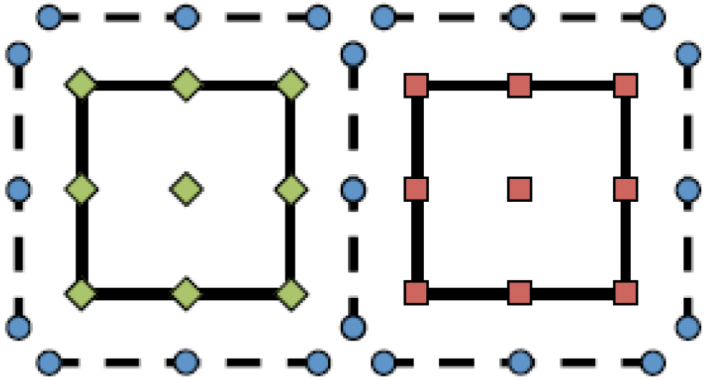
Motivation

- Why HDG-approach?
  - Allows detailed geometrical description of the wall
  - Grid flexibility
  - High order, p-adaptivity
  - Highly parallelizable
  - Cheaper than standard implicit FEM

- Drawbacks HDG-approach?
  - High order $\Rightarrow$ badly conditioned matrices





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The 3-field mean-field model

- Continuity, plasma

$$\partial_t n + \nabla \cdot \left(\left(\frac{\Gamma_{||}}{n} \boldsymbol{b} \right) n - D \nabla_\perp n \right) = S_n$$

- Parallel momentum, plasma

$$\partial_t(\Gamma_{||}) + \nabla \cdot \left(\left(\frac{\Gamma_{||}}{n} \boldsymbol{b} - \frac{D}{n} \nabla_\perp n \right) \Gamma_{||} + (\boldsymbol{b}) n c_s^2 - \frac{1}{mn} \mu_{||} \boldsymbol{b} \nabla_{||}(\Gamma_{||}) - \frac{1}{mn} \mu_\perp \nabla_\perp(\Gamma_{||}) + \frac{\Gamma_{||}}{mn^2} \mu_{||} \boldsymbol{b} \nabla_{||}(n) + \frac{\Gamma_{||}}{mn^2} \mu_\perp \nabla_\perp(n) \right)$$
$$= \frac{1}{m} S_{u_{||}} + \frac{1}{m} p \nabla \cdot \boldsymbol{b}$$

- Continuity, neutrals (AFN)

$$\partial_t n_\mathrm{n} + \nabla \cdot \left(\left(\frac{n_\mathrm{n,eq} \Gamma_{||}}{n n_\mathrm{n}} \boldsymbol{b} \right) n_\mathrm{n} - D_\mathrm{n}^p T \, \nabla(n_\mathrm{n}) - D_\mathrm{n}^p n_n \, \nabla(T) \right) = -S_n$$

$$n_\mathrm{n,eq} = \frac{n^2 K_\mathrm{r} + n_\mathrm{n} n K_\mathrm{cx,m}}{n K_\mathrm{i} + n K_\mathrm{cx,m}}$$

$$D_\mathrm{n}^p = \frac{1}{m(n K_\mathrm{i} + n K_\mathrm{cx,m})}$$

- Sources: ionization, recombination, charge-exchange

The 3-field mean-field model, normalized

$$\partial_t n + \nabla \cdot \left(\left(\frac{\Gamma_{||}}{n} \boldsymbol{b} \right) n - D \nabla_\perp n \right) = S_n$$

$$\partial_t(\Gamma_{||}) + \nabla \cdot \left(\left(\frac{\Gamma_{||}}{n} \boldsymbol{b} - \frac{D}{n} \nabla_\perp n \right) \Gamma_{||} + M_0(\boldsymbol{b}) n (T_i + T_e) - \frac{1}{n} \mu_{||} \boldsymbol{b} \nabla_{||}(\Gamma_{||}) - \frac{1}{n} \mu_\perp \nabla_\perp (\Gamma_{||}) + \frac{\Gamma_{||}}{n^2} \mu_{||} \boldsymbol{b} \nabla_{||}(n) + \frac{\Gamma_{||}}{n^2} \mu_\perp \nabla_\perp (n) \right)$$

$$= S_{u_{||}} + M_0 n (T_i + T_e) \nabla \cdot \boldsymbol{b}$$

$$\partial_t n_{\mathrm{n}} + \nabla \cdot \left(\left(\frac{n_{\mathrm{n,eq}} \Gamma_{||}}{n n_{\mathrm{n}}} \boldsymbol{b} \right) n_{\mathrm{n}} - D_{\mathrm{n}}^p T \, \nabla(n_{\mathrm{n}}) - D_{\mathrm{n}}^p n_{\mathrm{n}} \, \nabla(T) \right) = -S_n$$

$\Rightarrow$ coupled advection-diffusion-reaction equations for $\boldsymbol{U} = \{\boldsymbol{U}_1, \boldsymbol{U}_2, \boldsymbol{U}_3\}^T = \{n, \Gamma_{||}, n_{\mathrm{n}}\}^T$

$$\partial_t \boldsymbol{U} + \boldsymbol{\nabla} \cdot \left(\boldsymbol{C}(\boldsymbol{U}) + \boldsymbol{C}_p(\boldsymbol{U}) + \boldsymbol{K}(\boldsymbol{U}) \boldsymbol{q} \right) = \boldsymbol{S}(\boldsymbol{U}), \qquad \boldsymbol{q} + \nabla_\perp \boldsymbol{U} = 0$$

Solution with HDG approach

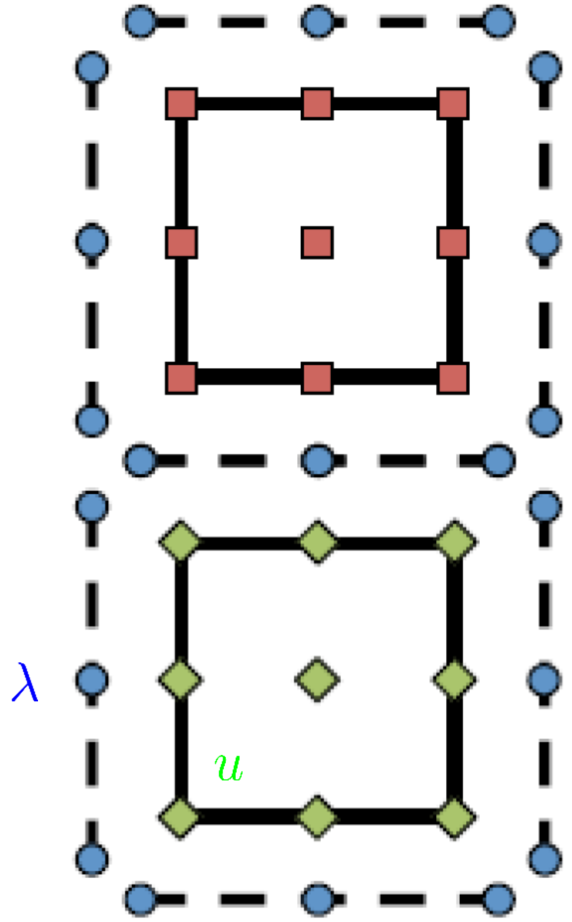
- Implementation: models & operators
  - Model: build matrix contributions at the element level
  - Operator: construct contributions from individual terms in the model
  - Flexibility to implement different models and discretization options

$$\begin{bmatrix} A_{uu} & A_{uq} & A_{ul} \\ A_{qu} & A_{qq} & A_{ql} \\ A_{lu} & A_{lq} & A_{ll} \end{bmatrix} \begin{bmatrix} u \\ q \\ \lambda \end{bmatrix} = \begin{bmatrix} B_u \\ B_q \\ B_l \end{bmatrix}$$

model eq.

$$q + \nabla u = 0$$

trace eq.



Breakdown into operators

- Advection operator: explicit, upwind

$$\boldsymbol{C}(\boldsymbol{U}) = \begin{bmatrix} \boldsymbol{U}_1 \mathbf{v}_1 \\ \dots \\ \boldsymbol{U}_{\mathrm{nEq}} \mathbf{v}_{\mathrm{nEq}} \end{bmatrix}$$

- Diffusion operator: implicit

$$\boldsymbol{K}(\boldsymbol{U})\boldsymbol{q} = \begin{bmatrix} \kappa_{1,1}(\boldsymbol{U}) & \kappa_{1,2}(\boldsymbol{U}) & \dots & \kappa_{1,\mathrm{nEq}}(\boldsymbol{U}) \\ \dots & \dots & \dots & \dots \\ \kappa_{\mathrm{nEq},1}(\boldsymbol{U}) & \kappa_{\mathrm{nEq},2}(\boldsymbol{U}) & \dots & \kappa_{\mathrm{nEq},\mathrm{nEq}}(\boldsymbol{U}) \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_1 \\ \dots \\ \boldsymbol{q}_{\mathrm{nEq}} \end{bmatrix}$$

- Time lagging for the advection velocities & diffusion matrix: evaluate with solution at time step *n*

Breakdown into operators

- Reaction (implicit, time-lagging) and source (explicit) operators

$$\boldsymbol{S}(\boldsymbol{U}) = \begin{bmatrix} R_{1,1}(\boldsymbol{U}) & R_{1,2}(\boldsymbol{U}) & \dots & R_{1,\mathrm{nEq}}(\boldsymbol{U}) \\ \dots & \dots & \dots & \dots \\ R_{\mathrm{nEq},1}(\boldsymbol{U}) & R_{\mathrm{nEq},2}(\boldsymbol{U}) & \dots & R_{\mathrm{nEq},\mathrm{nEq}}(\boldsymbol{U}) \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_1 \\ \dots \\ \boldsymbol{U}_{\mathrm{nEq}} \end{bmatrix}$$

$$\boldsymbol{S}(\boldsymbol{U}) = \begin{bmatrix} f_1(\boldsymbol{U}) \\ \dots \\ f_{\mathrm{nEq}}(\boldsymbol{U}) \end{bmatrix}$$

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Simple advection-diffusion-reaction model

- Single advection-diffusion-reaction equation with linear analytical solution $\bar{u} = 0.1x + 0.2y + 0.3$

$$\partial_t u + \boldsymbol{\nabla}\cdot(u\boldsymbol{v} - D\nabla u) = S - Ru \quad \text{on} \quad \Omega$$
$$u = \bar{u}|_{\partial\Omega} \quad \text{on} \quad \partial\Omega$$

Solenoidal velocity field $\boldsymbol{v} = \{-0.2, 0.1\}^T$; constant, isotropic diffusion tensor D

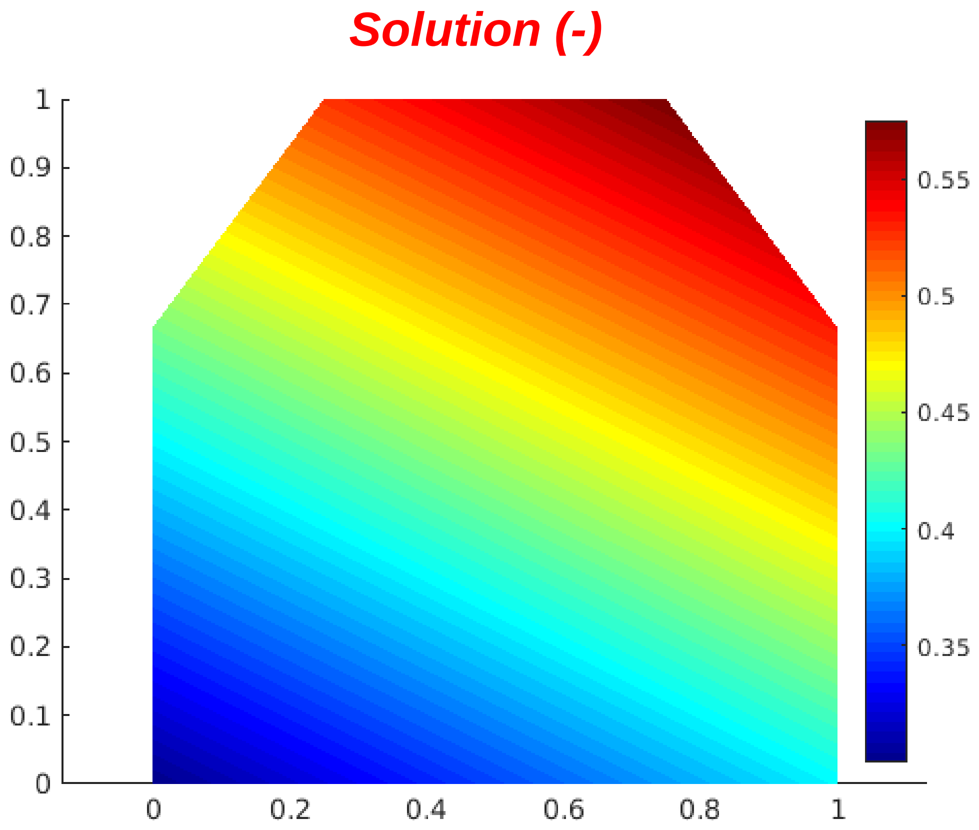
Source chosen to have constant R and $S = R\bar{u}$

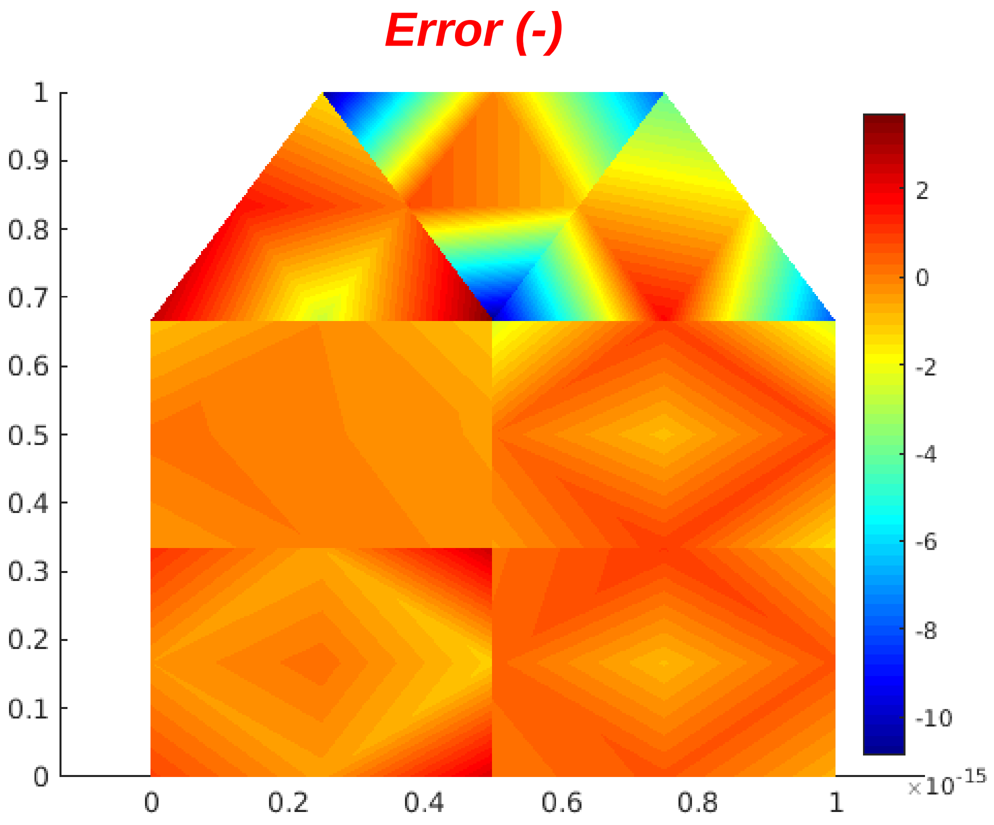
- *N* coupled equations, linear analytical solutions $\bar{u}_i = i\bar{u}$

$$\partial_t u_i + \boldsymbol{\nabla}\cdot\left(u_i\boldsymbol{v} - \sum_{j=1,\mathrm{N}} D_{i,j}\nabla u_j\right) = S_i - \sum_{j=1,\mathrm{N}} R_{i,j}u_j \quad \text{on} \quad \Omega$$
$$u_i = \bar{u}_i|_{\partial\Omega} \quad \text{on} \quad \partial\Omega$$

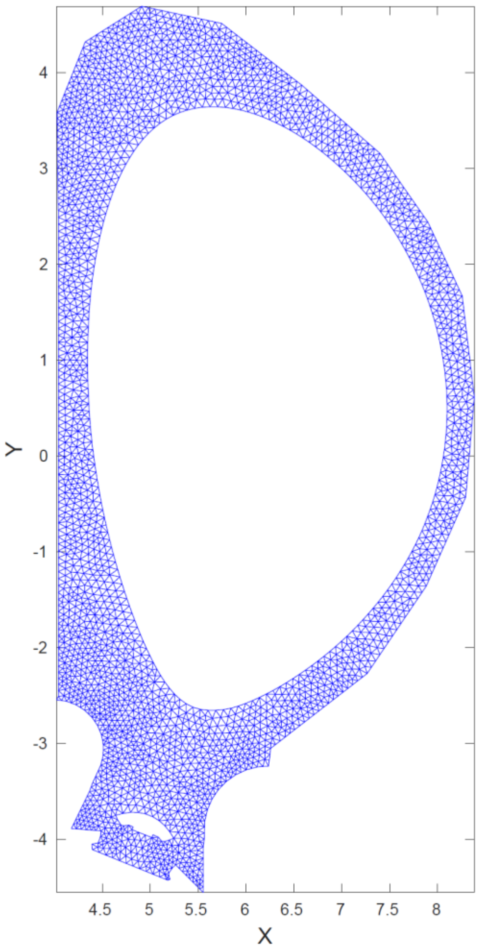
Same velocity field; isotropic diffusion tensor with $D_{i,j} = \frac{i}{jN}D$; source rate $R_{i,j} = \frac{i}{jN}R$

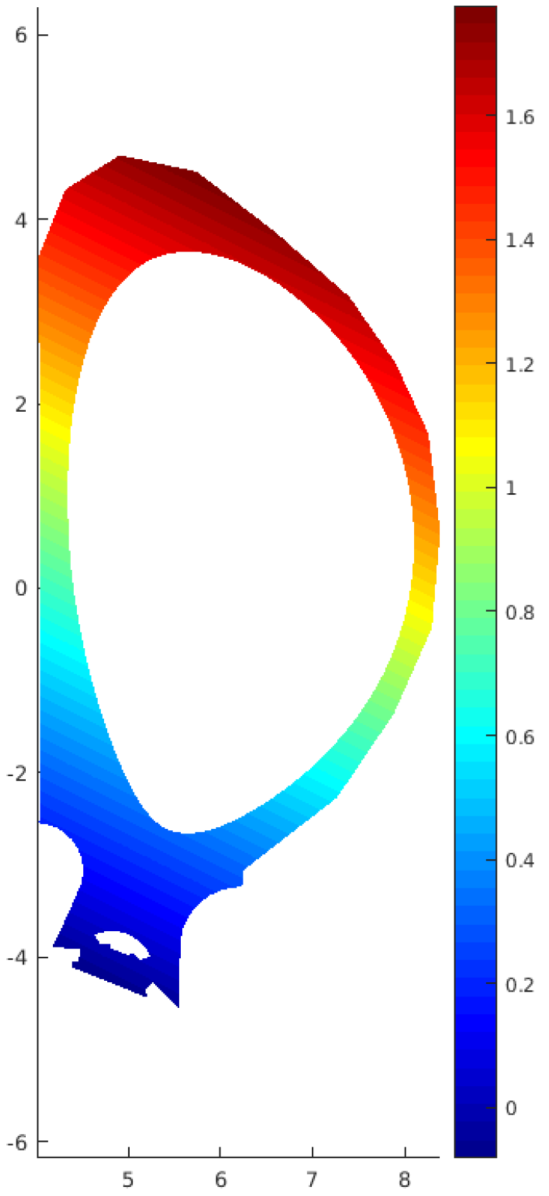
Simple advection-diffusion-reaction model





Solution in tokamak geometry





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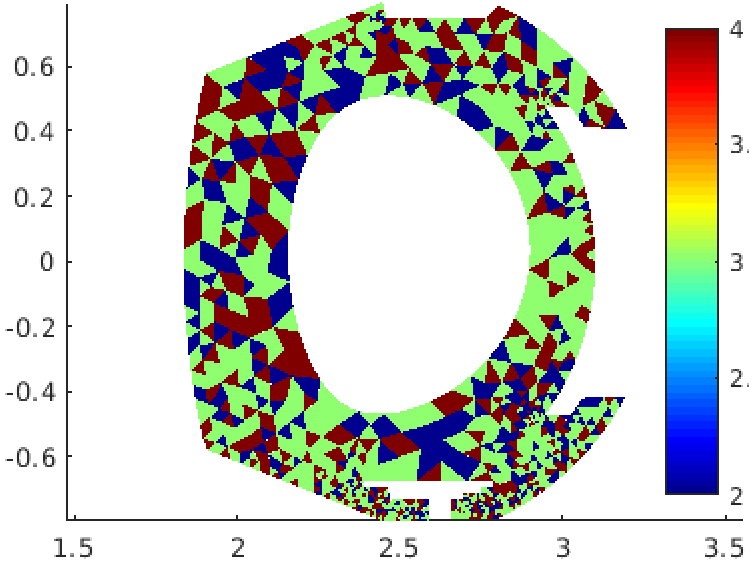
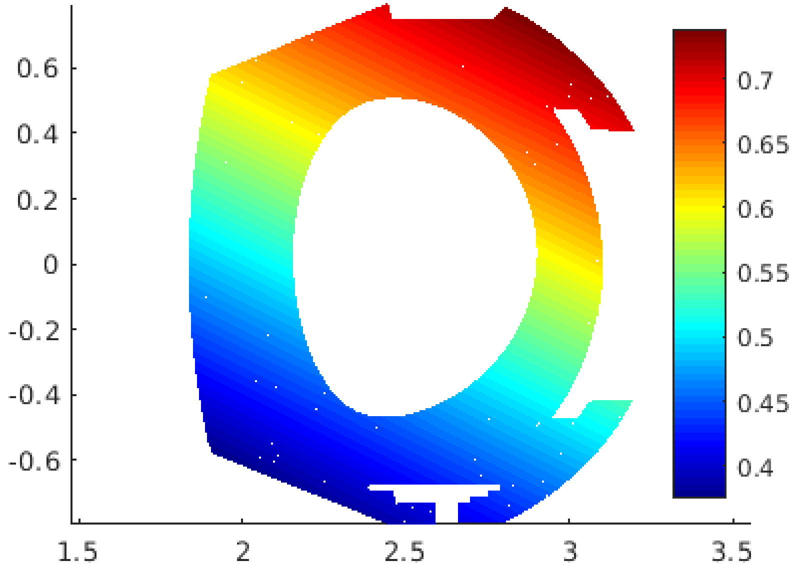
Diffusion equation, arbitrary element order

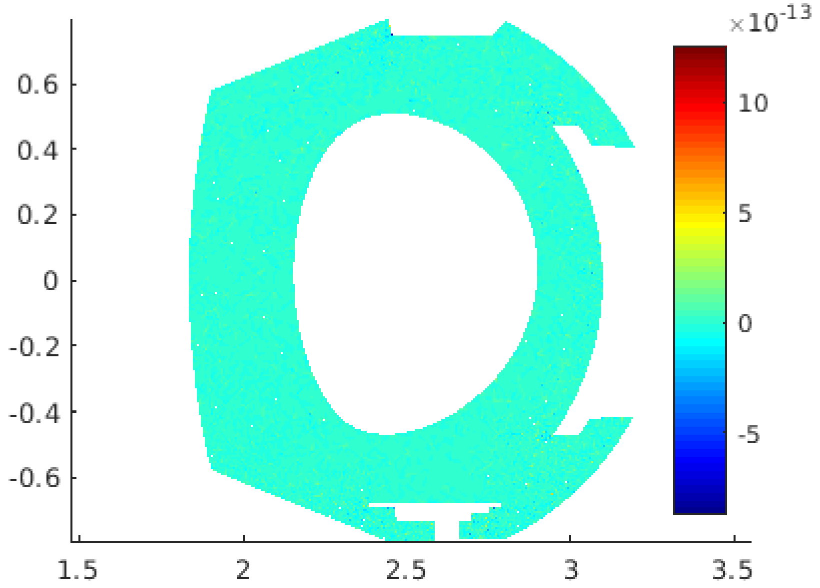
Element order



Solution



Error



Toy interchange model

- 2-field model

$$\partial_t n + \nabla \cdot (n\boldsymbol{u}_E - D_\perp \nabla_\perp n) = 0$$
$$n - \nabla \cdot \left(\frac{\nabla_\perp \phi}{B} \right) = 0$$

- Introduced option to add/remove time dependent term per equation

- MMS tested (Dirichlet BCs)

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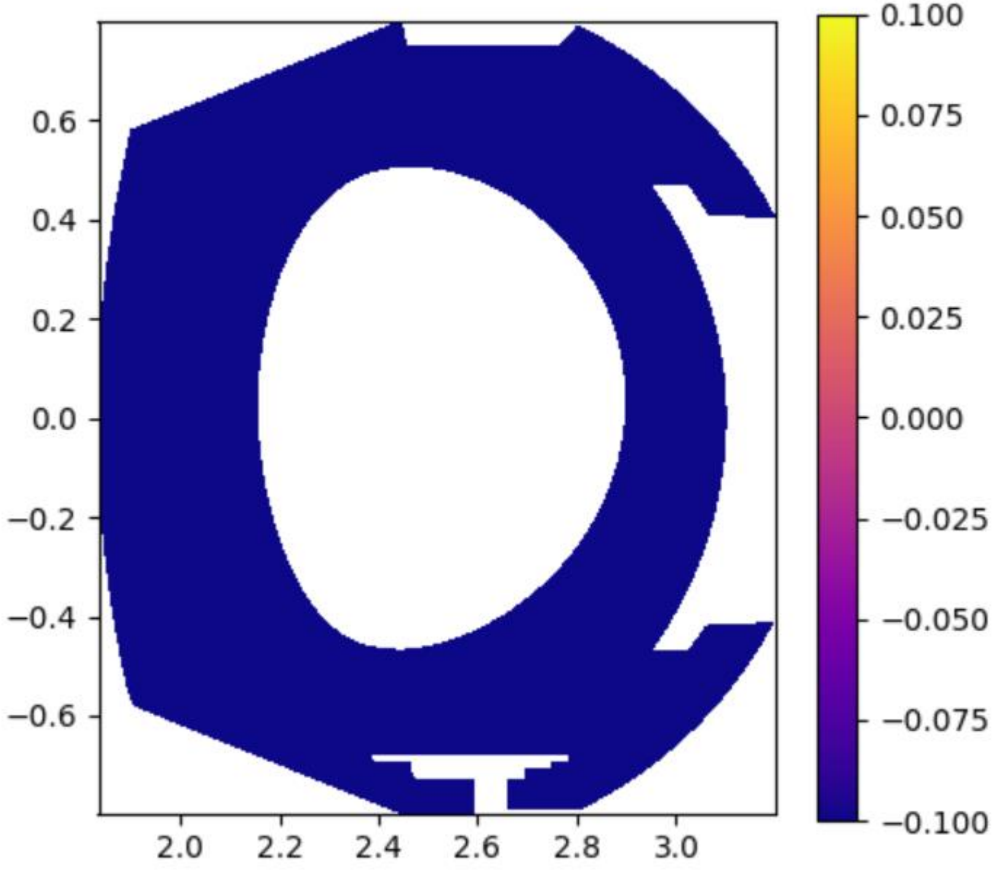
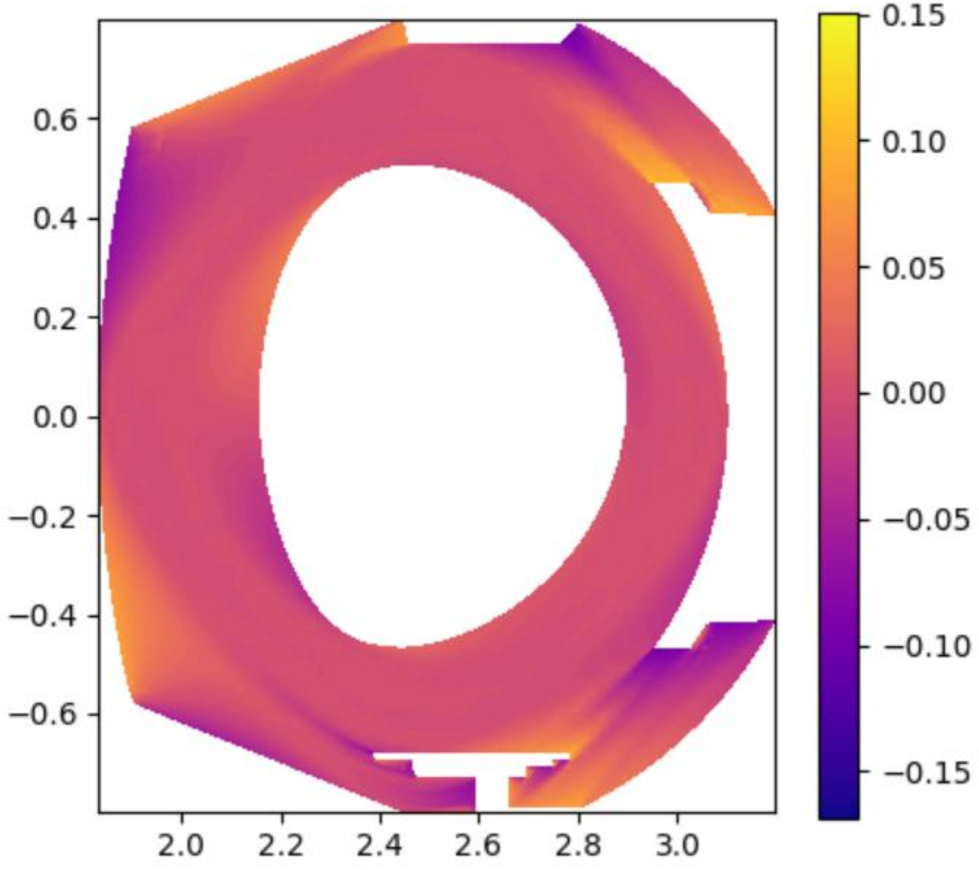
Preliminary results 3-field model, sheath BCs

Initial state



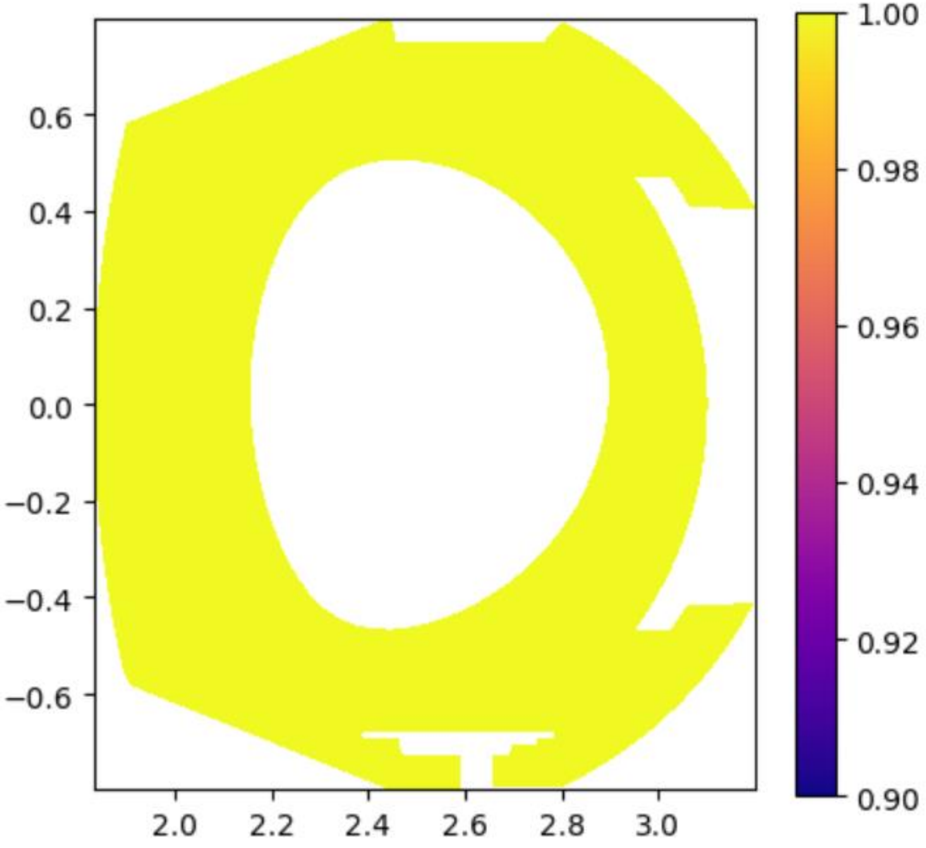
After a few iterations



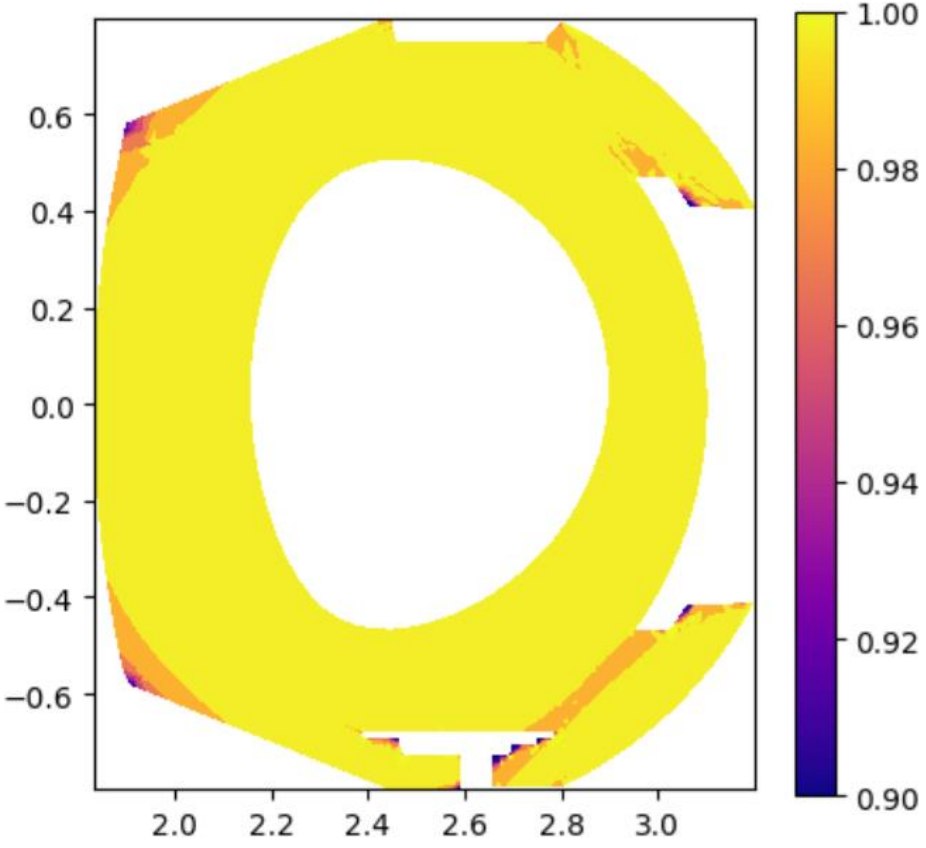
Parallel flux

Preliminary results 3-field model, sheath BCs

Initial state



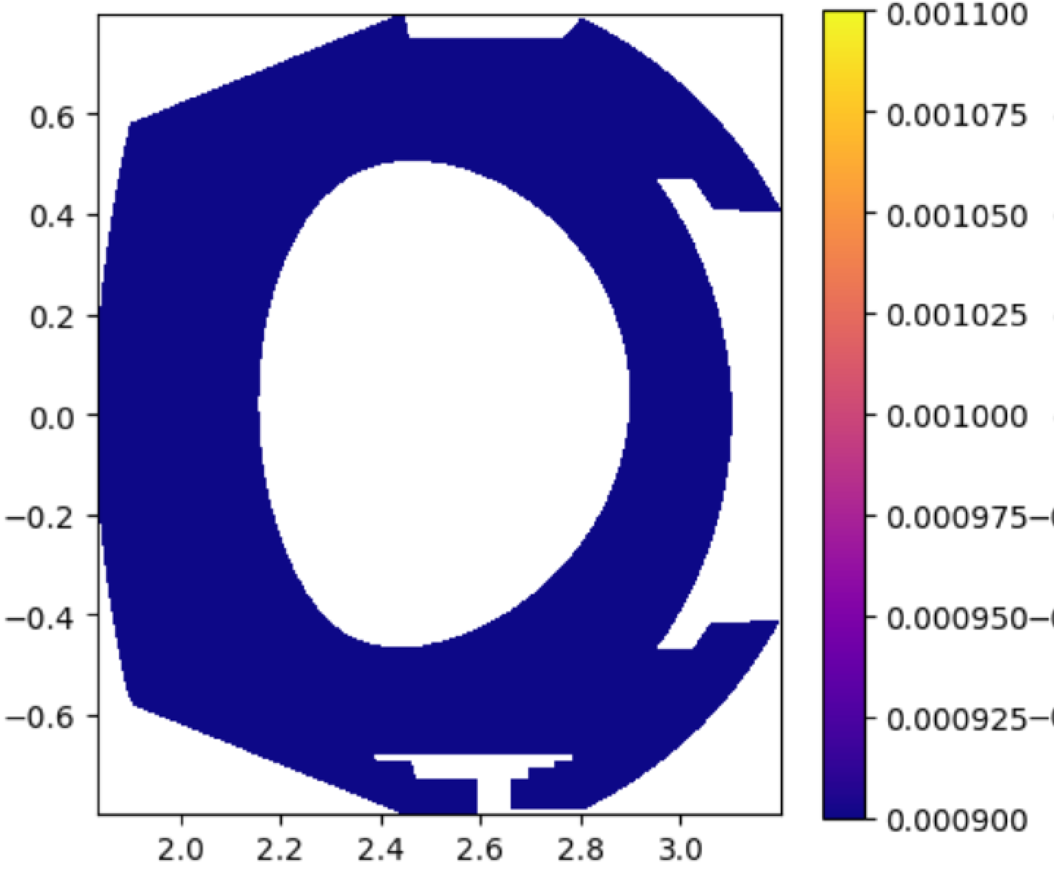
After a few iterations



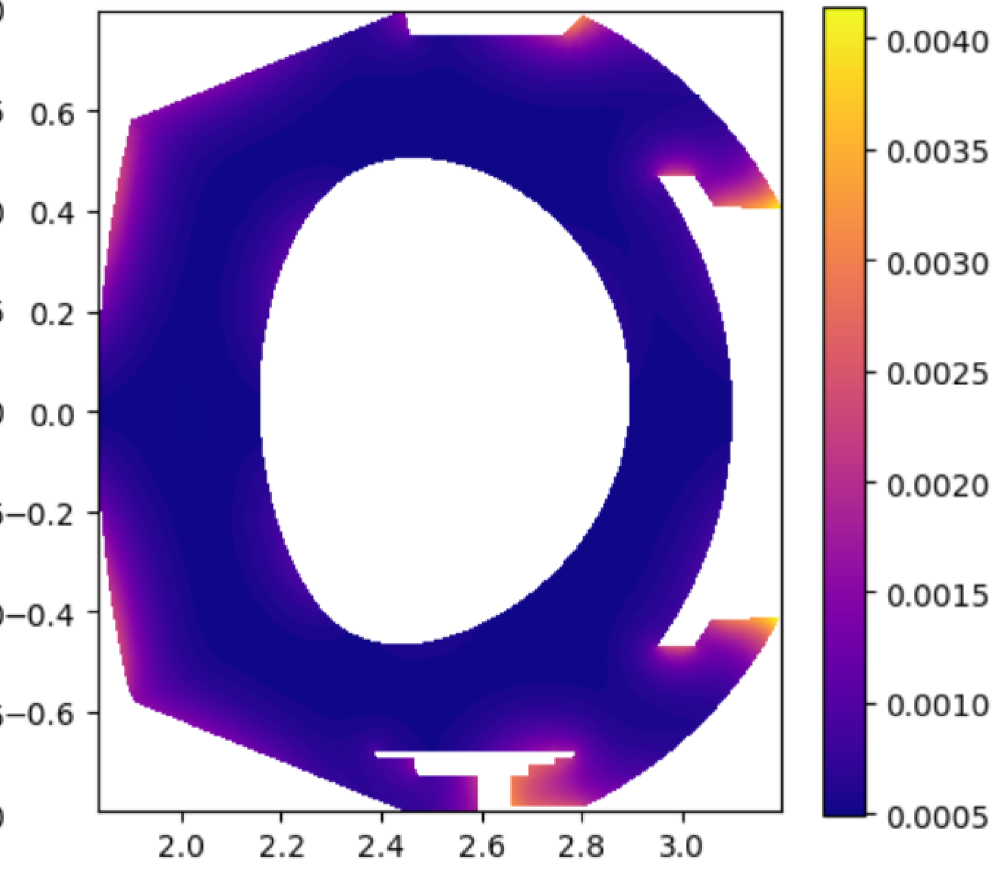
Ion density

Preliminary results 3-field model, sheath BCs

Initial state



After a few iterations



Neutral density

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