Universal behaviour of frequency chirping fluctuations in magnetized plasmas

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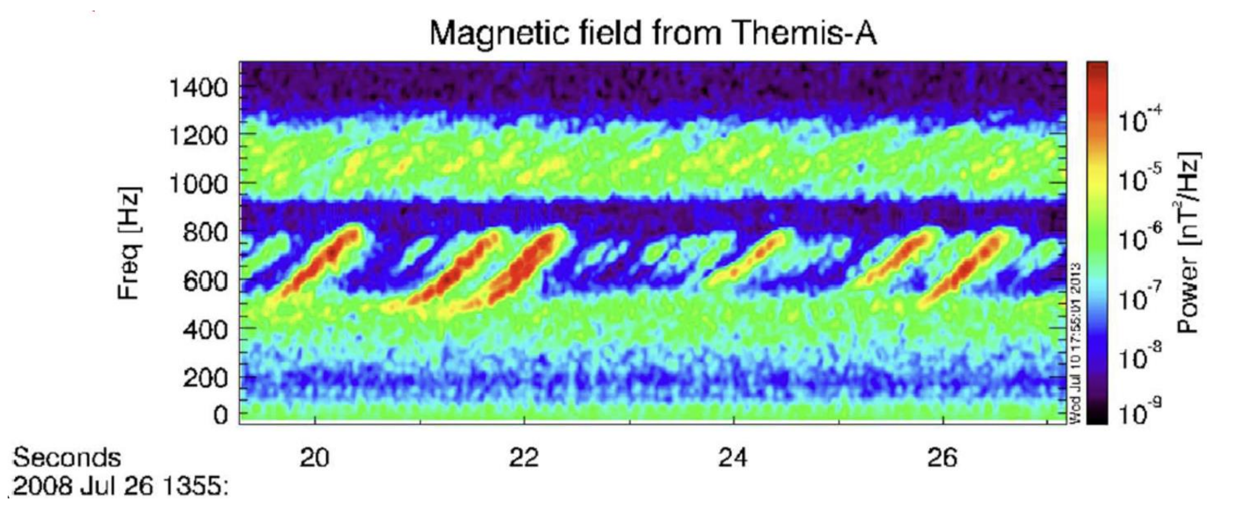
OBSERVATION OF FREQUENCY CHIRPING FLUCTUATIONS

- Frequency-chirping fluctuations are ubiquitous in magnetized plasmas and are routinely observed in space and laboratory environments [1-5]
- Examples are whistler mode chorus [6] and electromagnetic ion cyclotron (EMIC) waves in the Earth's magnetosphere [7]

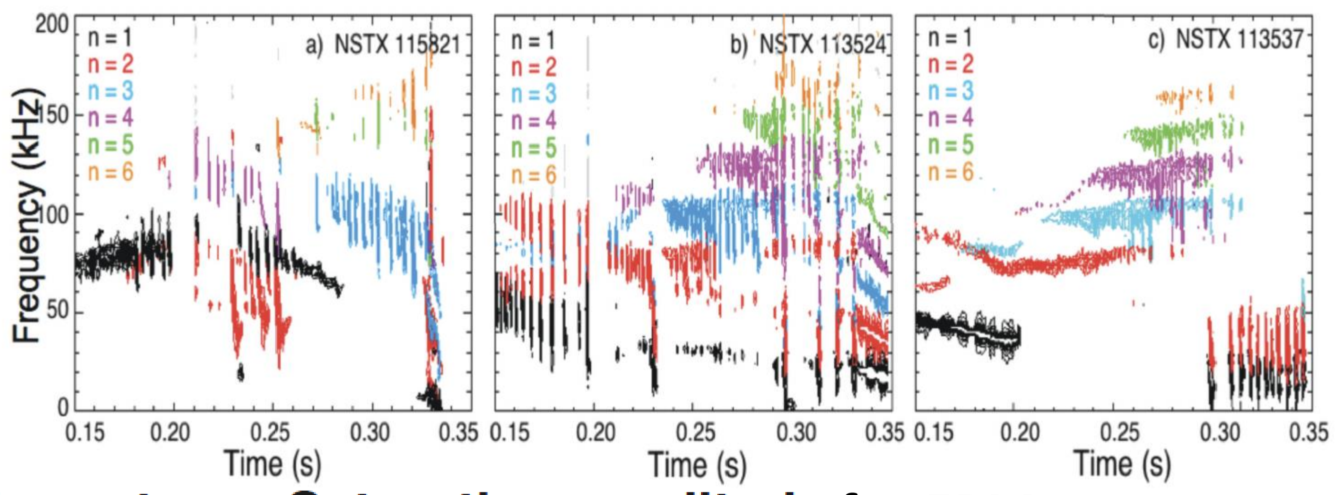


Whistler mode chorus from Themis-A observations in Ref [6].

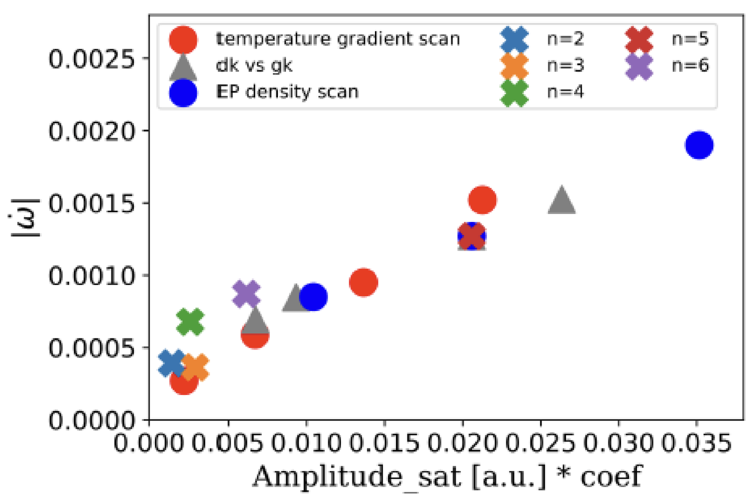
EMIC have been recently interpreted as due to same physics process [7].

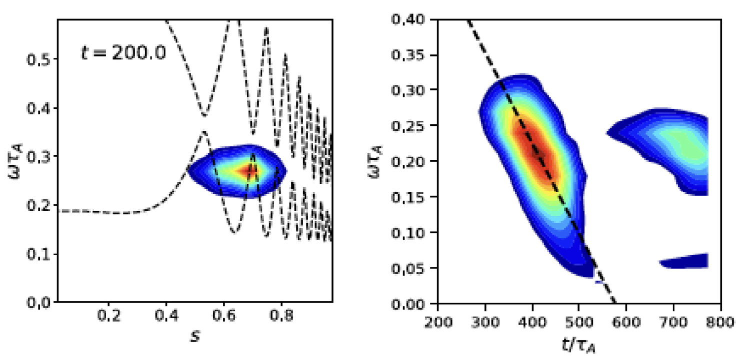
CHIRPING MODES IN LABORATORY. COMPARISON WITH NUMERICAL SIMULATIONS

- Evidence of fishbones and Energetic Particle Modes (EPM) from NSTX [8]



◉ **Chirping rate vs. Saturation amplitude for EPM**





intensity contour plot

$\dot{\omega} \simeq \delta \dot{X}_{\perp} \cdot \nabla \omega_{res}$ 🡐 theory prediction [1]

- PIC simulations of EPM in tokamaks show chirping rate linear scaling with amplitude [9]
- PIC simulations of fishbones show same scaling even in the presence of zonal flows, which, however, may reduce the resonance frequency sweeping in phase space [10].
- Same linear scaling observed for chorus emission & chirping: example from space [2-5].
- **Underlying physics mechanism**: phase locking and maximal wave-particle power transfer (see below).

THEORETICAL ANALYSIS OF CHIRPING RATE

- Based on the general theoretical framework of [1-5], the whistler mode chorus chirping rate has been shown to obey the Vomvoridis espression [11]

$$\frac{\partial \omega}{\partial t} = R \frac{\omega_{tr}^2}{(1 - v_r/v_g)^2}$$

🡐 ω_{tr} wave-particle trapping frequency
v_r resonant particle speed
v_g wave packet group velocity
$R \cong 1/2$ normalized chirping rate

- Same expression can be used to interpret EMIC chirping [7] based on the Trap-Release-Amplify (TaRA) model for chorus [4] and can predict chorus chirping on MARS [12].
- Theoretical analysis is based on the calculation of the renormalized energetic particle response by means of a Dyson-like equation (DSE) and the solution a model equation for the wave packet evolution, similar to the Dyson-Schrödinger Model (DSM) [13] applied to EPM/fishbone fluctuations in tokamaks.
- In this work:
  - The DSE is solved for a generic resonance showing that chirping has the expected role slowing down the detuning of resonant phase space structures (PSZS) [14,15]
  - $R \cong 1/2$ naturally arises from nonlinear evolution of PSZS
  - Vomvoridis expression of chirping rate applies to all resonances, provided the appropriate expression of ω_{tr} is used
- This demonstrates the universal behavior of frequency chirping fluctuations in magnetized plasmas.

SOLUTION OF THE DYSON-LIKE EQUATION

- The whistler chorus DSE (as illustration) reads [3,5]

$$\partial_\tau f_0 = \omega_{tr}^2 \omega/(2k^2) \bar{\partial}_{\mathcal{E}} \partial_\tau \left[(\omega - \omega_{res})^2 + \partial_\tau^2 \right]^{-1} \bar{\partial}_{\mathcal{E}} (\omega_{tr}^2 \omega / k^2) f_0$$

- Here, $\partial_\tau = (1 - v_r/v_g)\partial_t$, $\bar{\partial}_{\mathcal{E}} = (k/\omega)\partial_{v_\parallel} + (1 - k v_\parallel/\omega)/v_\perp \partial_{v_\perp}$ and ω_{res} is the resonance frequency. This equation has 1degree of freedom as $B\omega\dot{\mu} = \Omega\dot{\mathcal{E}}$, with $\mathcal{E} = v^2/2$, and a nonlinear invariant exists.
- From existing theory [1-5], a wave packet solution of the wave equation can be constructed, satisfying the Vomvoridis chirping expression [11], provided that

$$\mathcal{E}_{res} = \mathcal{E}_{res,0} + \int_0^\tau R\omega_{tr}^2 \omega / k^2 d\tau'$$

- The DSE can be solved for weakly varying wave packet amplitude, changing variables from $(\mathcal{E}, \tau)$to (x, T) (moving in the wave packet moving frame)

$$x = \frac{k^2}{\omega \omega_{tr}} \left(\frac{2}{(2 - 4R^2)^{1/2}} \right)^{1/2} \left(\mathcal{E} - \mathcal{E}_{res,0} - \int_0^\tau R\omega_{tr}^2 \omega / k^2 d\tau' \right) \qquad T = \omega_{tr} \tau \left(\frac{(2 - 4R^2)^{1/2}}{2} \right)^{1/2}$$

- The solution is expressed as series of orthonormal Hermite functions $\psi_n(x)$

$$f_0(x, T) = \bar{f}_0 + \sum_{n=0}^{\infty} \{ \kappa_n [\varphi_n(x, T) - \varphi_n(x_0, 0)] + c.c. \}$$

$$\sum_{n=0}^{\infty} (C_{m,n} \kappa_n + c.c.) = \int_{-\infty}^{\infty} \bar{f}_0 \psi_m(x_0) dx_0$$

$$C_{m,n} = \int_{-\infty}^{\infty} \varphi_n(x_0, 0) \psi_m(x_0) dx_0$$

$$\varphi_n(x, T) = \int_0^x \frac{dx'}{2R} \left[\psi_n'(x') - \frac{2Rb_n}{(2 - 4R^2)^{1/2}} \psi_n(x') \right] \times \exp \left[b_n \left(T + \frac{(2 - 4R^2)^{1/2} x}{2R} - \frac{x'}{(2 - 4R^2)^{1/2} R} \right) \right]$$

$$b_n = i(n + 1/2)^{1/2} (2 - 4R^2)^{1/2}$$

- Phase space structure rotation is slowed down by chirping 🡒 PHASE LOCKING
- Wave particle power exchange is maximized for $R \cong 1/2$, consistent with [1-5,11].

UNIVERSAL BEHAVIOR OF FREQUENCY CHIRPING

- Use action angle coordinates for general tokamak geometry: θ_c and ζ_c such that $\omega_b = \dot{\theta}_c$and $\overline{\omega}_d = \dot{\zeta}_c$ are, respectively, the bounce/transit and the magnetic drift precession frequency; $\tilde{\Xi}_c$ parameterizing the equilibrium particle motion as $\zeta = \zeta_c + \tilde{\Xi}_c$ at constant actions (μ, J, P_φ)
- Use the notion of nonlinear equilibrium in the presence of flows[1,14-16] to self-consistently compute wave-particle resonant interaction with EPM/fishbone

$$\dot{P}_\phi = en \left| \overline{e^{-in\zeta - im\bar{\theta}_c + i\bar{Q}} \frac{\omega_{dn}}{\omega} \langle \delta\psi_{ng} \rangle} \right| \sin(\Theta + \beta) \qquad i\bar{Q} = \frac{RB_\phi}{d\psi/dr} \frac{v_\parallel}{\Omega} \frac{\partial}{\partial r} + \tilde{\Xi}_c \frac{\partial}{\partial \zeta};$$

$$\dot{E} = e\omega \left| \overline{e^{-in\zeta - im\bar{\theta}_c + i\bar{Q}} \frac{\omega_{dn}}{\omega} \langle \delta\psi_{ng} \rangle} \right| \sin(\Theta + \beta) \qquad \overline{(\ldots)} = \frac{\omega_b}{2\pi} \oint (\ldots) \frac{d\theta}{\dot{\theta}}$$

- Near resonance of *(m,n)* poloidal harmonics 🡐 phase locking

$$\Theta = n\zeta_c - m\bar{\theta}_c + \frac{1}{\omega_b} \int^{\theta_c} \Delta_1 d\theta_c' - \int^t \omega dt'$$

$$\dot{\Theta} = \omega_{res} - \omega = n\bar{\omega}_d + n\bar{q}\sigma\omega_b - m\dot{\bar{\theta}}_c + \Delta_1 - \omega$$

$$\ddot{\Theta} = -\dot{\omega} + \frac{\partial \omega_{res}}{\partial P_\phi} \dot{P}_\phi + \frac{\partial \omega_{res}}{\partial E} \dot{E} \simeq 0$$

🡐 phase locking

- Predicted frequency chirping for EPM/fishbones scales linearly with fluctuation amplitude. Effect of zonal flows is embedded in Δ_1 [1,14-16] $\Delta_1 = -i \overline{\left[e^{i\bar{Q}} \left(\delta\dot{X}_z \cdot \nabla + \delta\dot{\mathcal{E}}_z \partial_{\mathcal{E}} \right) \right]}$,

$$\dot{\omega} \simeq \omega_{tr}^2/2 = \frac{1}{2} \left| \left(en \frac{\partial \omega_{res}}{\partial P_\phi} + e\omega \frac{\partial \omega_{res}}{\partial E} \right) \overline{e^{-in\zeta - im\bar{\theta}_c + i\bar{Q}} \frac{\omega_{dn}}{\omega} \langle \delta\psi_{ng} \rangle} \right|$$

CONCLUSIONS AND DISCUSSIONS

- Explicit expression of frequency chirping is derived, showing it is a consequence of maximized wave-particle power transfer and phase locking [1-5].
- Explicit expression of frequency chirping illuminates the important role of zonal field structures [10].
- Explicit expression of chirping rate also shows linear scaling with fluctuation amplitude, demonstrating the universal behavior of frequency chirping in space and laboratory plasmas, consistent with the Vomvoridis expression [11].
- Detailed quantitative numerical verifications of these predictions are in progress.

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