Integral kernel approach to modelling wave heating of stellarator plasmas: theory and numerical implementation (ENR-MOD.02.LPP-ERM-KMS)

Intermediate report for 2024

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Outline

- Introduction to project ENR-MOD.02.LPP-ERM-KMS
- Evolution of its organization in 2024
- Summary of the 2024 activities and status of the scientific deliverables
- Task planning for 2025

Introduction

- The traditional approach to realistic full-wave modelling of Ion Cyclotron Resonance Heating (ICRH) in tokamaks and stellarators is based on Fourier expansions of the radiofrequency (RF) fields along 2 or 3 spatial coordinates.

⇒ Allows convenient theoretical treatment of wave dispersion along the curved equilibrium magnetic field;
Leads to mixed spectral-finite element or fully spectral numerical formulations;
Plasma described by a dielectric tensor formulated in Fourier space (involving the well-known plasma dispersion function for Maxwellians).

- Our alternative approach formulates the plasma dielectric response as an integral operator in physical space, involving new ‘kernel dispersion functions’.

Advantages of our approach

- Enables using finite element methods (FEM), in 2D and 3D, to model wave propagation and absorption in hot inhomogeneous fusion plasmas;

- Enables local mesh refinements (which are ruled out with spectral methods);

- Better suited RF field representations to address finite Larmor radius effects in toroidal geometry;

- Straightforward connection of the plasma model with advanced RF antenna models, themselves based on the FEM.

$\Rightarrow$ Main goals of the project:

- Extend the theory to stellarators,
- Efficiently implement this non-conventional approach in a new full-wave code,
- Validate it & demonstrate its attractiveness to model RF heating in tokamaks & stellarators.

Our reference set of equations

- Maxwell-Vlasov system, weak form (frequency domain; $\boldsymbol{E}$: RF electric field, $\boldsymbol{F}$: test function):

$$\frac{\mathrm{i}}{2}\int_{\mathcal{V}}\left[\frac{1}{\omega\,\mu_0}(\nabla\times\boldsymbol{F})^*.(\nabla\times\boldsymbol{E})-\omega\,\varepsilon_0\,\boldsymbol{F}^*.\boldsymbol{E}\right]\mathrm{d}r^3+\sum_\beta\mathcal{W}_{\boldsymbol{F}\boldsymbol{E}\beta}=-\frac{1}{2}\int_{\mathcal{V}}\boldsymbol{F}^*.\boldsymbol{j}_S\,\mathrm{d}r^3$$

- Plasma response, showing 0th order FLR: involves a nonlocal integral (s') along magnetic field lines

$$\mathcal{W}_{\boldsymbol{F}\boldsymbol{E}\beta}=-\frac{\mathrm{i}\varepsilon_0}{2}\sum_{L=-1}^{1}\delta_{L,p}\,2^{\alpha/2}\int\mathrm{d}r_\perp^2\,\frac{\omega_\mathrm{p}^2}{v_\mathrm{T}}\int\mathrm{d}s\int\mathrm{d}s'\;F_{\mathcal{L}}^*(s')\;\Upsilon_\alpha\left[\hat{\xi}_p(s,s')\right]\;E_{\mathcal{L}}(s)\qquad(\alpha=2\delta_{L,0})$$

$$\hat{\xi}_p(s,s')=\frac{\omega-p\omega_\mathrm{c}\left(\frac{s+s'}{2}\right)}{v_\mathrm{T}}\;\frac{|s'-s|}{2}$$

- Kernel dispersion functions (α = resp. 0, 2):

$$\Upsilon_\alpha(\xi)=\frac{1}{2\sqrt{\pi}}\int_{\to 0}^{+\infty}\mathrm{e}^{-u+2\mathrm{i}\xi/\sqrt{u}}\left\{\begin{array}{c}\mathrm{i}\\ \xi/\sqrt{u}\end{array}\right\}\frac{\mathrm{d}u}{u}$$

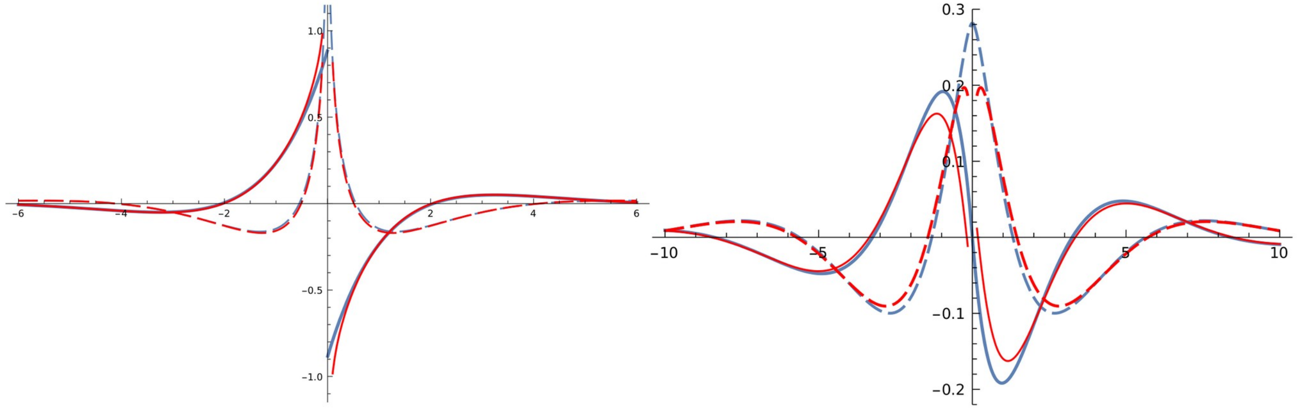


Figure 1: Plasma dielectric kernel functions $\Upsilon_0(x)$ (left) and $\Upsilon_2(x)$ (right) vs x. Continuous lines: real parts; dashed lines: imaginary parts. Blue: series expansion; red: asymptotic expansion. $\mathrm{Re}\,\Upsilon_0$ is discontinuous and $\mathrm{Im}\,\Upsilon_0$ has an (integrable) logarithmic singularity at 0, whereas Υ_2 and its first derivative are regular.

(To fix ideas, comparison between spectral and configuration space formulations)

- Maxwell-Vlasov system, weak form:

$$\frac{\mathrm{i}}{2}\int_{\mathcal{V}}\left[\frac{1}{\omega\,\mu_0}(\nabla\times\boldsymbol{F})^*.(\nabla\times\boldsymbol{E})-\omega\,\varepsilon_0\,\boldsymbol{F}^*.\boldsymbol{E}\right]\,\mathrm{d}r^3+\sum_{\beta}\mathcal{W}_{\boldsymbol{F}\boldsymbol{E}\beta}=-\frac{1}{2}\int_{\mathcal{V}}\boldsymbol{F}^*.\boldsymbol{j}_S\,\mathrm{d}r^3$$

- Plasma response, 0th order FLR, spectral formulation (showing homogeneous plasma case):

$$\mathcal{W}_{\boldsymbol{F}\boldsymbol{E}\beta}=-\mathrm{i}\pi\varepsilon_0\sum_{L=-1}^{1}\delta_{L,p}\,2^{\alpha/2}\int\mathrm{d}r_\perp^2$$
$$\int_{-\infty}^{+\infty}\mathrm{d}k_{//}\;\tilde{F}^*_{\mathcal{L}}(k_{//})\,\frac{\omega_\mathrm{p}^2}{k_{//}v_\mathrm{T}}\;Z^{\{\alpha\}}\!\left(\frac{\omega-p\omega_\mathrm{c}}{k_{//}v_\mathrm{T}}\right)\,\tilde{E}_{\mathcal{L}}(k_{//})\qquad(\alpha=2\delta_{L,0})$$

- Plasma response, 0th order FLR, our configuration space formulation:

$$\mathcal{W}_{\boldsymbol{F}\boldsymbol{E}\beta}=-\frac{\mathrm{i}\varepsilon_0}{2}\sum_{L=-1}^{1}\delta_{L,p}\,2^{\alpha/2}\int\mathrm{d}r_\perp^2\,\frac{\omega_\mathrm{p}^2}{v_\mathrm{T}}\int\mathrm{d}s\int\mathrm{d}s'\;F^*_{\mathcal{L}}(s')\;\Upsilon_\alpha\left[\hat{\xi}_p(s,s')\right]\;E_{\mathcal{L}}(s)\qquad(\alpha=2\delta_{L,0})$$

$$\hat{\xi}_p(s,s')=\frac{\omega-p\omega_\mathrm{c}(\frac{s+s'}{2})}{v_\mathrm{T}}\,\frac{|s'-s|}{2}$$

Evolution of our organization in 2024

- Short notice change of staff: 1 departure in late 2023, but skilled new recruit started in April 2024, highly beneficial to the project.

- Exploratory period of the numerous open-source finite element softwares: not designed to address nonlocal media, very limited documentation, constant interaction with code developers essential.

- Hence decision to pursue code developments both in-house from first principles, and via external collaborations with experts in numerical methods.

Two associated collaborations were launched in 2024

- NMPP Division at Max-Planck-Institut für Plasmaphysik Garching (E. Sonnendrücker & Team):

  Goal: implementing our approach in their Psydac code (FEM, B-splines).

- Dept. of Electrical Engineering and Computer Science, University of Liège (Chr. Geuzaine):

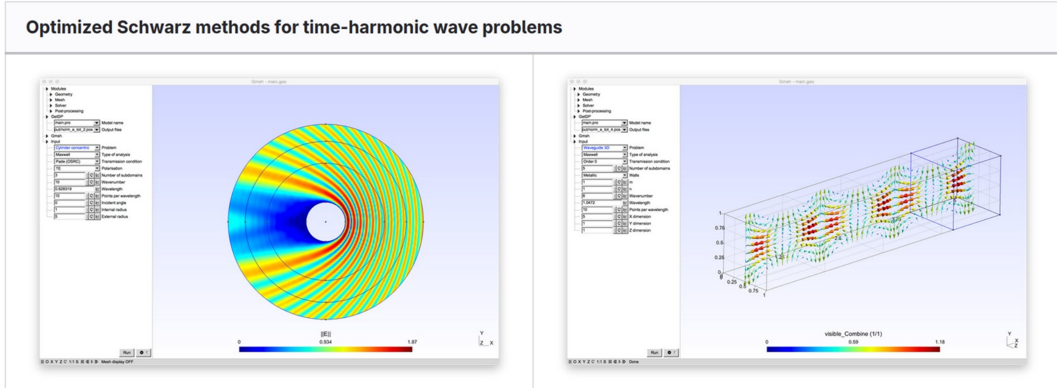
  - Succesful 4-year PhD fellowship application, J. Zaleski started in October (PI is co-promoter),

    “Scalable Calculation of Electromagnetic Fields to Simulate Plasma Wave Heating for Fusion Energy Research”

  Goals:

  - Implementing our approach in GetDP/GmshFEM codes (FEM, high degree polynomials).

  - Enabling / optimizing very large scale computing: preconditioning the large linear system, domain decomposition methods, iterative methods (innovative for Maxwell’s equations).



Summary of the 2024 activities: theory

- Obtained the RF integral kernel expressions to be used for Maxwellian plasmas in all geometries, including stellarators (i.e. the basis of the project).
- Detailed mathematical study in view of their numerical computation: singularities, series and asymptotic expansions

- Demonstrated that our integral approach will be able to exploit stellarator symmetry

  $\Rightarrow$ strong reduction of computing effort (which grows polynomially with the simulation volume): the RF fields in a stellarator with N-fold symmetry can be deduced from N simulations on a single period - i.e. 1/Nth of the volume.

- Special kernels derived for simply-periodic and stellarator-periodic plasmas.

- Simplified expressions have also been derived for an initial proof of principle of our new approach in slab geometry, both for unbounded and periodic plasmas.

- The all-orders FLR theory is also available for future use (but several approaches to FLR are possible), likely post-2025.

Summary of the 2024 activities: numerical methods

- Implemented efficient numerical evaluation of the KDFs.
- Derived bespoke high-order numerical quadrature formulas to deal with the logarithmic singularity of our integral kernel; they are implemented in the IDK code and will soon be tested in detail.
- The numerical mesh exploits the fact that our kernel is only nonlocal along the equilibrium magnetic field lines to minimize the system banwidth.
- We have been exploring 3D meshing strategies to prepare the forthcoming modelling of realistic tokamak and stellarator geometries.
  - The kernel structure makes field-aligned meshes the likely option
  - We plan to handle realistic 3D geometries by means of curvilinear coordinate transformations between the finite element space and physical space.

Summary of the 2024 activities: code development

- Given the novelty of our approach, we are pursuing a staged development.

- Very first implementation: development of the in-house 2.5D finite element code IDK:

  - Solves Maxwell's equations in slab geometry (Nédélec basis functions);

  - Implements the warm plasma integral kernel to model wave dispersion and absorption effects (fundamental cyclotron and Landau damping),

  - i.e. at this stage, relevant to modelling ICRH minority heating scenarios.

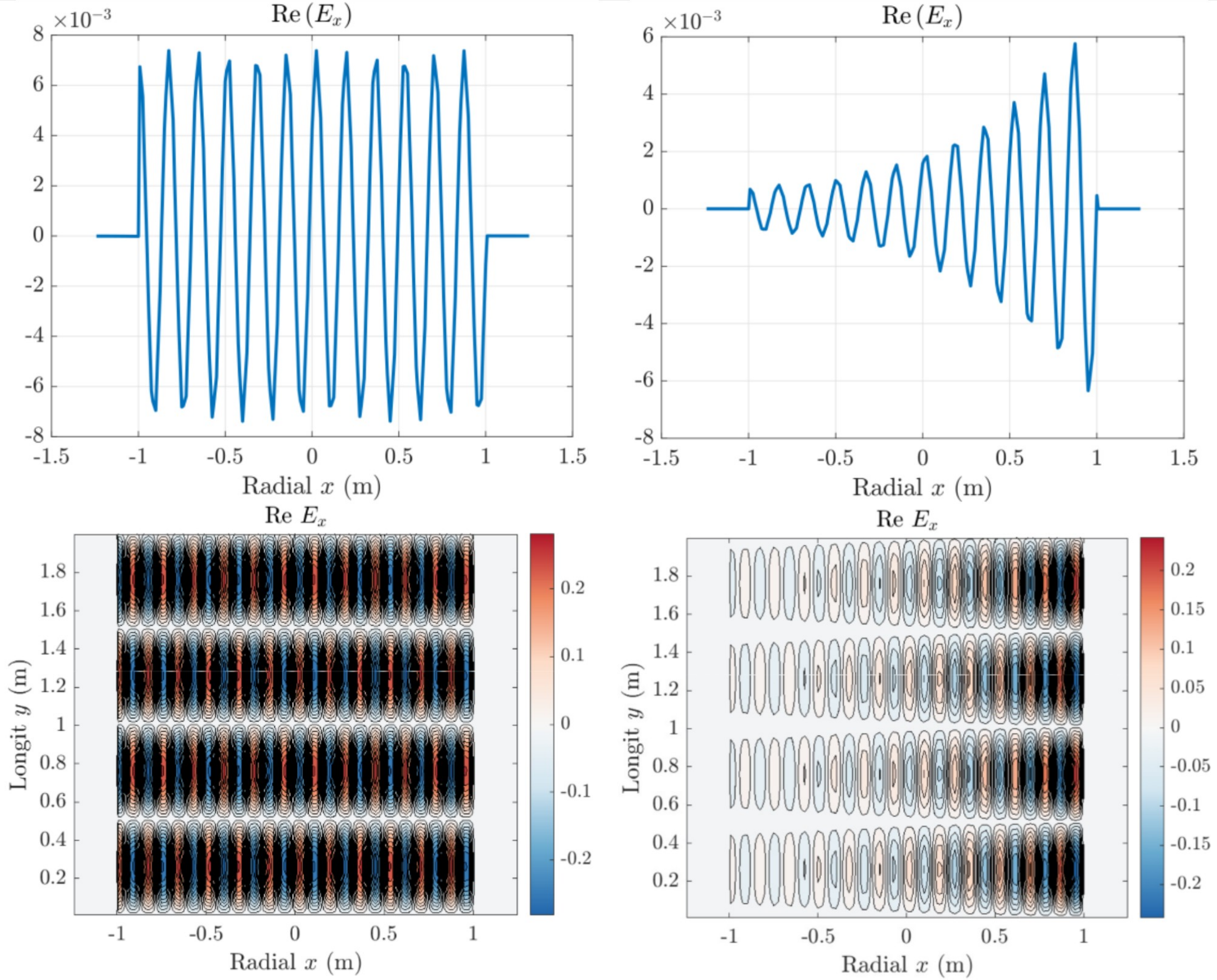
  - Code first validated using a cold plasma model;

  - First results obtained with the integral kernel, currently under intensive testing.

- Independent investigation under way (collaboration with IPP Garching NMPP Team) to implement the integral kernel into the PsyDac FEM code.

- PhD collaboration with University of Liège actively proceeding $\Rightarrow$ implementing our integral kernel in their GetDP / GmshFEM FEM suites.

First runs with the in-house 2.5D IDK code



1- and 2-D RF field patterns

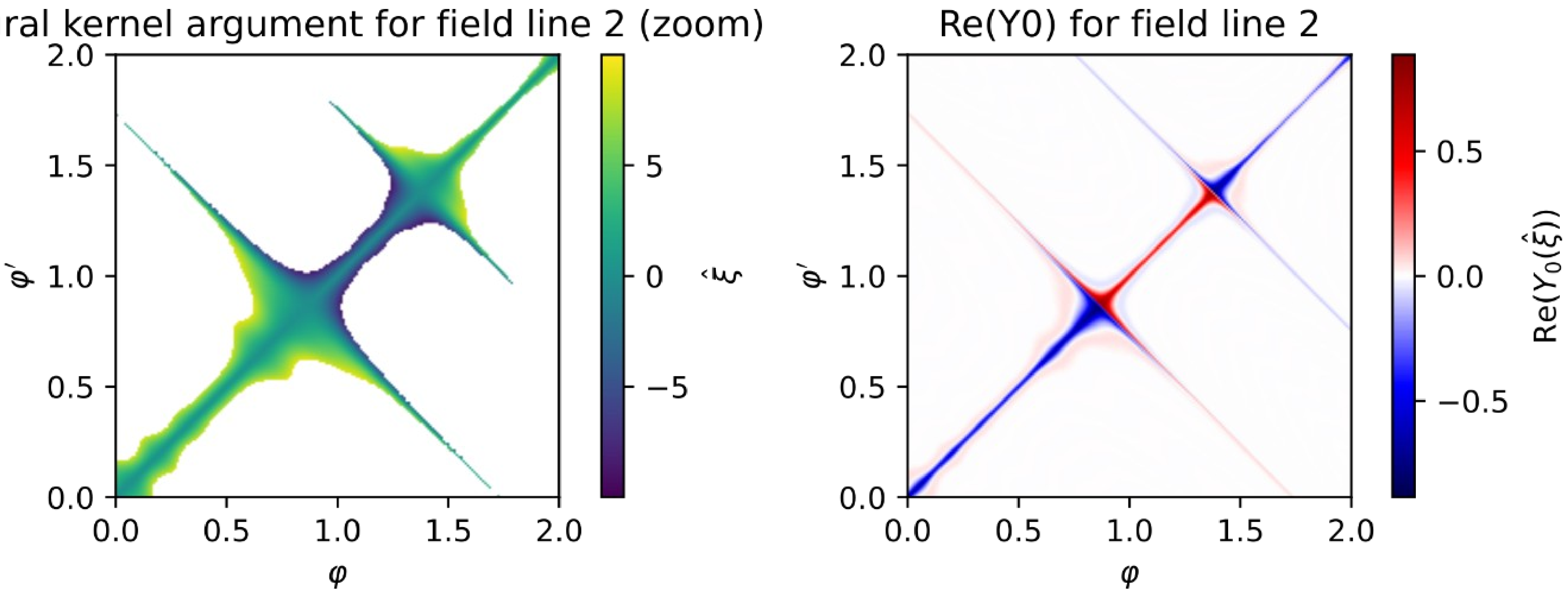
Equilibrium magnetic field along Oy.
Homogeneous plasma (95%D, 5%H, -1m<x<1m)
N= 3×10^{19} m^{-3}.
Vacuum regions 1<|x|<1.25m.
Metallic conditions are imposed at the x boundaries
Periodicity along y.

Test case using a 60MHz sine antenna current along z located at x=1.125m.

Left: cold plasma model showing standing fast wave;

Right: warm plasma, integral dielectric kernel model (T_D=T_H=3keV). Exhibits wave attenuation associated with cyclotron damping.

Preparing the next steps: kernel dispersion function on a W7-X equilibrium



Integral kernel argument (left) and real part of KDF vs toroidal angles of two points located on a common field line, for realistic W7-X parameters.

- Interface to W7-X field-line tracer web-service developed.
- Interaction mostly involves close neighbour points, except near cyclotron resonance layer.
- Will assist determining the sparsity pattern of the system matrix for 3D stellarator geometry and in selecting the most efficient method of resolution.

Overall milestones & deliverables announced for 2024

The project milestones:

No	Title	Description	Expected completion date
M1	Theoretical model	Obtain expression of plasma dielectric response integral kernel in stellarator geometry, valid for arbitrary representation of the RF field (i.e. free from Fourier expansions)	September 2024
M2	Numerical methods	Demonstrate efficient numerical evaluation of integral kernels of the expected form, within finite element methods. Select numerical approach to finite Larmor radius effects.	December 2024
M3	New wave code: design and initial development	Build a general-purpose wave equation solver anticipating later inclusion of consolidated integral kernel expression. Develop code interfaces (e.g. with plasma equilibrium and antenna codes).	December 2024
M4	New kernel numerical implementati on	Numerical implementation in code of the kernel approach produced under Milestone M1.	August 2025
M5	Code validation & early exploitation	Progressive testing / benchmarking / validation of the code. First exploitation of the new code to model ICRH scenarios in use or foreseen for W7-X.	December 2025

2024 deliverables:

Year	Description
2024	For Milestone M1: D1 - Report on the theoretical developments and results. Associated papers to be submitted for publication and for conference presentation. For Milestone M2: D2 - Progress report on specific numerical methods For Milestone M3: D3.1 - Progress report on code development status. D3.2 - Most recent code version and documentation placed under version control.

The 2024 deliverables

Scientific deliverable *(annual scientific deliverables as specified in the Task Agreement)*	**Achieved: Fully/Partly/Not**	**Evidence for achievement, brief reason for partial or non-achievement**
D1 - Report on the theoretical developments and results. Associated papers to be submitted for publication and for conference presentation.	Fully for 2024 scope, except journal publication (submissions planned over 2025)	- Poster presentation at the 2024 Varenna theory workshop, see list of publications. - Theory memoranda on Gitlab: Integral_kernel_theory_memos
D2 - Progress report on specific numerical methods	Fully for 2024 scope	Gitlab: Integral_Dielectric_Kernel_Slab_Summary Gitlab: Open-source code explorations
D3.1 - Progress report on code development status.	Fully for 2024 scope	Gitlab: Integral_Dielectric_Kernel_Slab_Summary Gitlab: W7-X_ICRH_Overview
D3.2 - Most recent code version and documentation placed under version control.	Fully for 2024 scope	Our 2.5D code is under version control on Gitlab: https://gitlab.com/cgbr36/IDK_dev

(The Project memoranda, progress reports and software are stored on a Gitlab repository private to the Project. Unpublished material accessible to reviewers on request, to be released for publication or as open-source software as soon as sufficient maturity reached.)

Our 2025 work plan

- The theoretical developments have reached sufficient maturity to meet the initial goal (modelling ICRF minority heating scenarios).
- The assumptions underlying the theory are so far the same as in traditional spectral approaches. They will be critically reviewed in a contribution at the 25th Topical Conference on RF Power in Plasmas (May 2025).
- The planned activities of ENR-MOD.02.LPP-ERM-KMS for 2025 follow the reference plan:
  - In-house ‘core’ code development and progressive testing/benchmarking activities will take place during the first three quarters of 2025. This will build upon the simple demonstration IDK code developed in 2024 by extending it to realistic tokamak and stellarator geometries.
  - In parallel we will pursue code development activities within two collaborations initiated in 2024: NMPP team at IPP Garching, Dept. of Electrical Engineering and Computer Science at Liège University. This will allow the project to benefit from these institutes’ expertise and extensive finite element libraries.
  - First code exploitation in realistic geometry is planned in the second half of 2025.
- Total project manpower remains as planned, i.e. 31pm for 2025, including 6pm allocated to the Max-Planck-Institut für Plasmaphysik Greifswald beneficiary and 25pm allocated to LPP-ERM-KMS.
- There is ample scope to vigorously continue this line of research beyond 2025.

Forthcoming ENR publications in 2025

- Large theory manuscript (detailing and completing pre-ENR research),

  “High-frequency dielectric integral kernels for Maxwellian tokamak plasmas”, soon to be submitted

- 25th Topical Conference on RF Power in Plasmas (Germany, May 2025):
  - “Integral dielectric kernel approach to modelling RF heating in toroidal plasmas” (likely invited)
  - Another contribution reporting on our 3 concurrent numerical approaches

- Manuscript(s) in preparation to cover
  - extension of the theory to stellarators,
  - comprehensive properties of our ‘kernel dispersion functions’,
  - new ‘super-kernels’ exploiting stellarator and tokamak symmetries
  - specific behaviour of the RF dielectric response at the rational flux surfaces
  - first numerical demonstration of our approach

Thank you for your attention!