Adding parallel magnetic fluctuations to global gyrokinetic codes

TSVV4 Annual Meeting

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Structure of this presentation

- Introduction
- Description of possible models
- Results from GENE
- Summary and Outlook

What are parallel magnetic fluctuations?

$$\mathbf{B} = \mathbf{B_0} + \mathbf{B_{1\perp}} + \mathbf{B_{1||}}$$

What are parallel magnetic fluctuations?

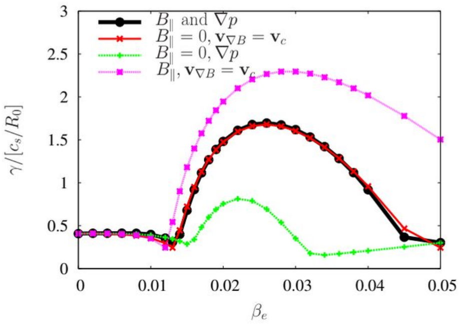
$$\mathbf{B} = \mathbf{B_0} + \mathbf{B_{1\perp}} + \mathbf{B_{1||}}$$

$$[\nabla_\perp \times \mathbf{B}_1]_\perp = \nabla_\perp B_{1,||} \times \mathbf{b} = \frac{4\pi}{c} \sum_\sigma \mathbf{j}_{1,\perp,\sigma}$$

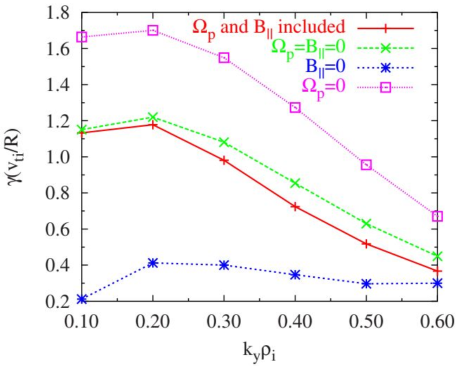
Typically neglected since $B_{1||} \propto \beta$

Why should we care about $B_{1\parallel}$?

- Significant impact on MHD instabilities (KBM)
- Reactor scenarios typically have considerably higher β
- Higher fidelity is always nice… Especially if the cost is low!

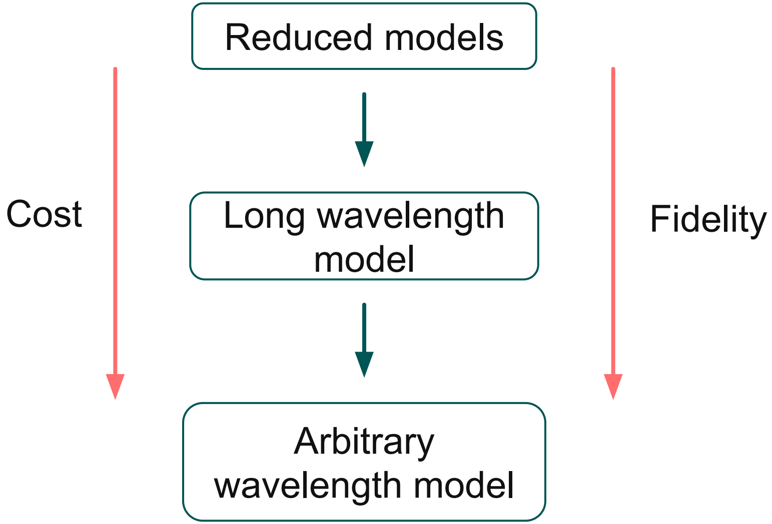


[T. Görler, 2013]



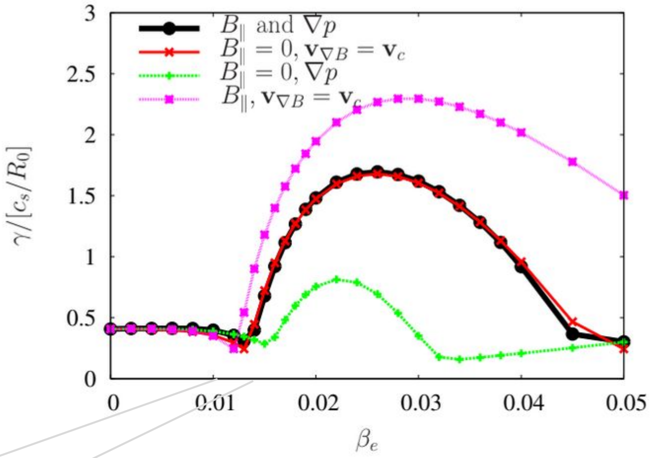
[N. Joiner, PoP, 2010]

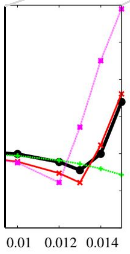
Descriptions for $B_{1\parallel}$



Descriptions for $B_{1\|}$: Reduced models

- Easiest to implement
- Very low/trivial cost with decent performance
- Most common option*: $\boxed{v_{\nabla B}=v_c}$
- Non negligible discrepancies at the KBM β threshold





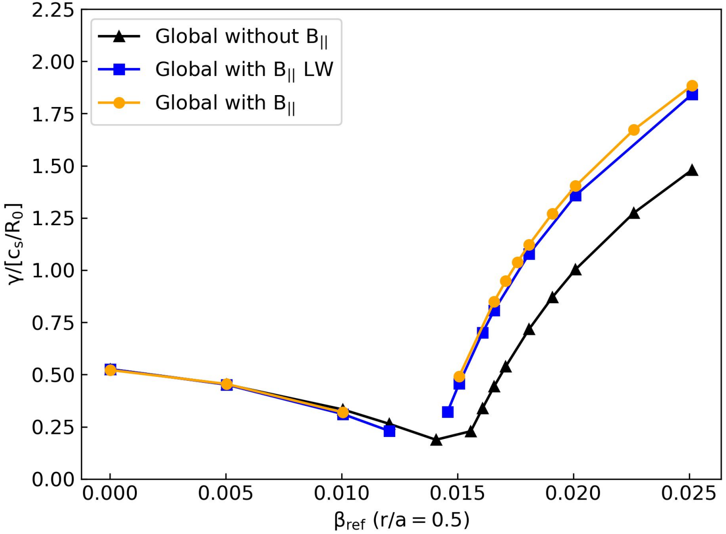
[T. Görler, 2013]

*[Joiner et al., PoP (2010)]
[Waltz et al, PoP (1999)]
[Sharma et al, PoP (2023)]
[Graves et al, PPCF (2019)]
[Scott, arXiv (2024)]

Descriptions for $B_{1||}$: Long wavelength model ($k_\perp \rho_s$<<1)

- Straightforward implementation
- Good physics at low cost
- What is long wavelength?

$$\hat{B}_{1,||} = \frac{-\frac{\pi}{2}\beta_{\mathrm{ref}} \sum_\sigma \hat{n}_{0,\sigma}\hat{T}_{0,\sigma}\hat{B}_0 \int_{-\infty}^{\infty}\int_{0}^{\infty} d\hat{v}_{||} d\hat{\mu}\, \hat{\mu}\hat{F}_{1,\sigma}}{1+\beta_{\mathrm{ref}}\sum_\sigma \frac{\hat{n}_{0,\sigma}\hat{T}_{0,\sigma}}{\hat{B}_0^2}\hat{n}_{\mathrm{p},\sigma}\hat{T}_{\mathrm{p},\sigma}}$$



[Sheffield, et al., PPCF (2025)]

$$\beta_{\mathrm{ref}} = \frac{8\pi\, n_{\mathrm{ref}} T_{\mathrm{ref}}}{B_{\mathrm{ref}}^2}$$

Descriptions for $B_{1||}$: Full wavelength model

- Coupled system with $\boldsymbol{\phi}$
- Higher cost and highest fidelity

$$\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \begin{pmatrix} \phi_1 \\ B_{1,||} \end{pmatrix} = \sum_\sigma \frac{8\pi^2}{m_\sigma} \int_{-\infty}^{\infty} \int_0^{\infty} \begin{pmatrix} q_\sigma \mathcal{K}\left\{B_0 F_{1,\sigma}\right\} \\ -\mathcal{K}_{\mathrm{D}}\left\{\left(B_0 F_{1,\sigma}\right)\right\} \end{pmatrix} dv_{||} d\mu$$

$$\mathcal{G}_{\mathrm{D}}\{u\} \equiv \frac{q_\sigma \Omega_\sigma}{2\pi c} \int_0^{2\pi} \int_0^{\rho} u(\mathbf{X} + \rho'(\Theta))\rho' d\rho' d\Theta$$

$$\mathcal{K}_{\mathrm{D}}\{u\} \equiv \frac{q_\sigma \Omega_\sigma}{2\pi c} \int_0^{2\pi} \int_0^{\rho} u(\mathbf{x} - \rho'(\Theta))\rho' d\rho' d\Theta$$

$$C_{11}\phi_1 = -\nabla_\perp^2 \phi_1 + 8\pi^2 \sum_\sigma \frac{q_\sigma^2}{m_\sigma} \int_{-\infty}^{\infty} \int_0^{\infty} \left(\frac{F_{0,\sigma} B_0}{T_{0,\sigma}} \phi_1 - \mathcal{K}\left\{ \frac{F_{0,\sigma} B_0}{T_{0,\sigma}} \mathcal{G}\{\phi_1\} \right\} \right) dv_{||} d\mu$$

$$C_{12} B_{1,||} = -8\pi^2 \sum_\sigma \frac{q_\sigma}{m_\sigma} \int_{-\infty}^{\infty} \int_0^{\infty} \left[\mathcal{K}\left\{ \frac{F_{0,\sigma} B_0}{T_{0,\sigma}} \mathcal{G}_{\mathrm{D}}\left\{B_{1,||}\right\} \right\} \right] dv_{||} d\mu$$

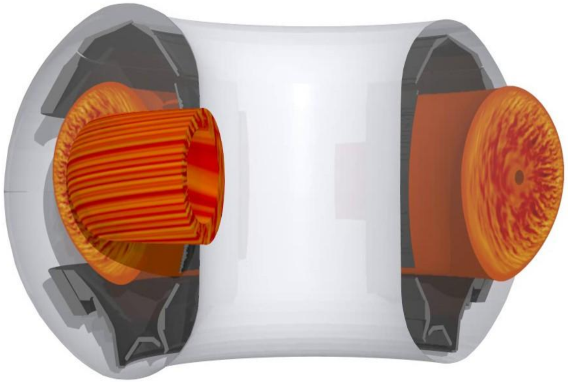
$$C_{21}\phi_1 = 8\pi^2 \sum_\sigma \frac{q_\sigma}{m_\sigma} \int_{-\infty}^{\infty} \int_0^{\infty} \left[\mathcal{K}_{\mathrm{D}}\left\{ \left(\frac{F_{0,\sigma} B_0}{T_{0,\sigma}} \mathcal{G}\{\phi_1\} \right) \right\} \right] dv_{||} d\mu$$

$$C_{22} B_{1,||} = B_{1,||} + 8\pi^2 \sum_\sigma \frac{1}{m_\sigma} \int_{-\infty}^{\infty} \int_0^{\infty} \mathcal{K}_{\mathrm{D}}\left\{ \left(\frac{F_{0,\sigma} B_0}{T_{0,\sigma}} \mathcal{G}_{\mathrm{D}}\left\{B_{1,||}\right\} \right) \right\} dv_{||} d\mu.$$

[Wilms, Merlo et al., CPC (2025)]

Parallel magnetic fluctuations in GENE

- Gyrokinetic delta-f code
- Extensively validated in the core
- Field aligned coordinates; can't cross the separatrix
- Capable of running linear and and nonlinear simulations in local and global settings.



[genecode.org]

Parallel magnetic fluctuations in GENE global

Long wavelength model gave promising results, but what **is** long wavelength?

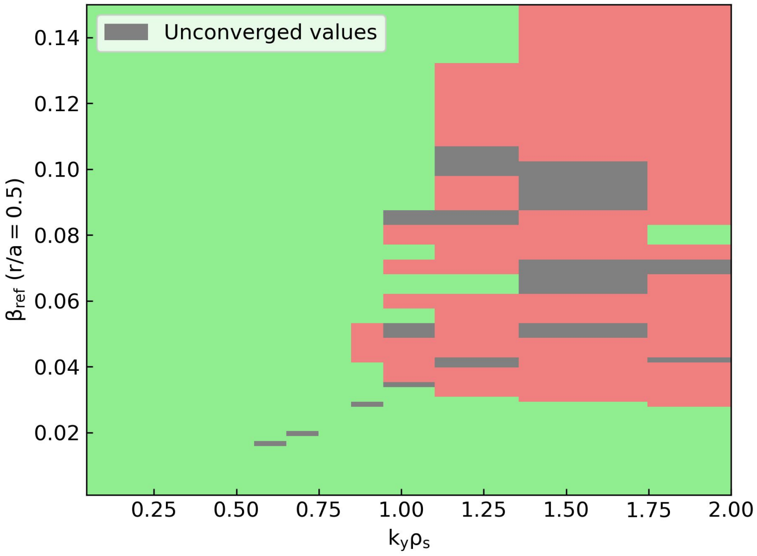
We compare the full, LW and reduced models

Perform 2D scans to find validity region using Miller geometry

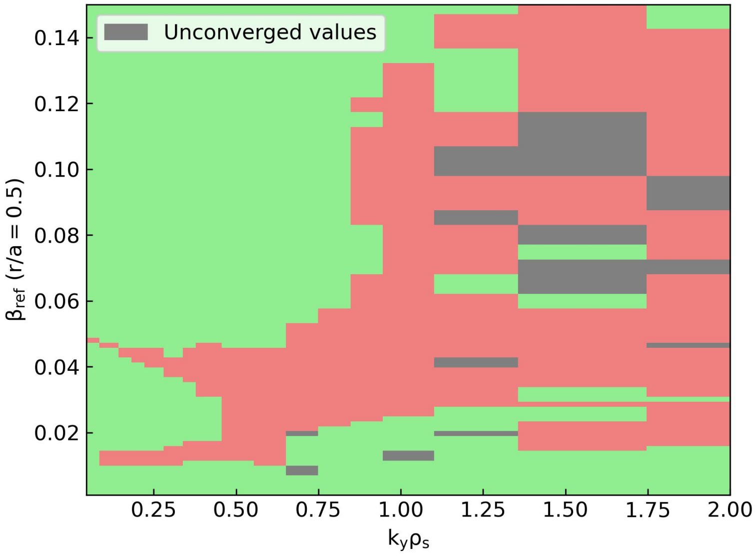
Parallel magnetic fluctuations in GENE global

Long wavelength model gave promising results, but what **is** long wavelength?

[Sheffield, et al., PPCF (2025)]



Full model vs LW



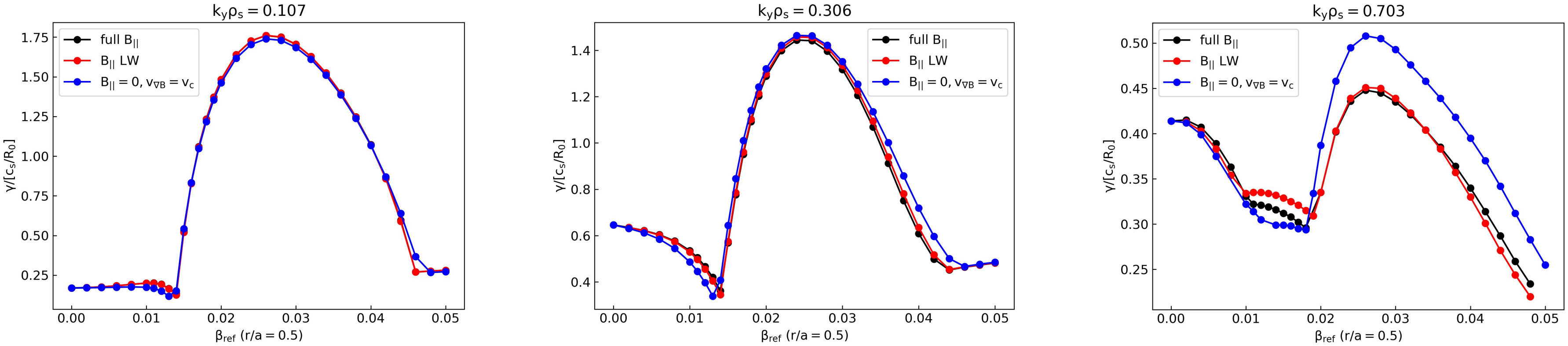
Full model vs reduced model

Criteria: Relative difference $|\gamma_1/\gamma_2-1|>0.1$

Parallel magnetic fluctuations in GENE global

[Sheffield, et al., PPCF (2025)]

What is $k_\perp \rho_s \ll 1$ in this case?



Parallel magnetic fluctuations in GENE global

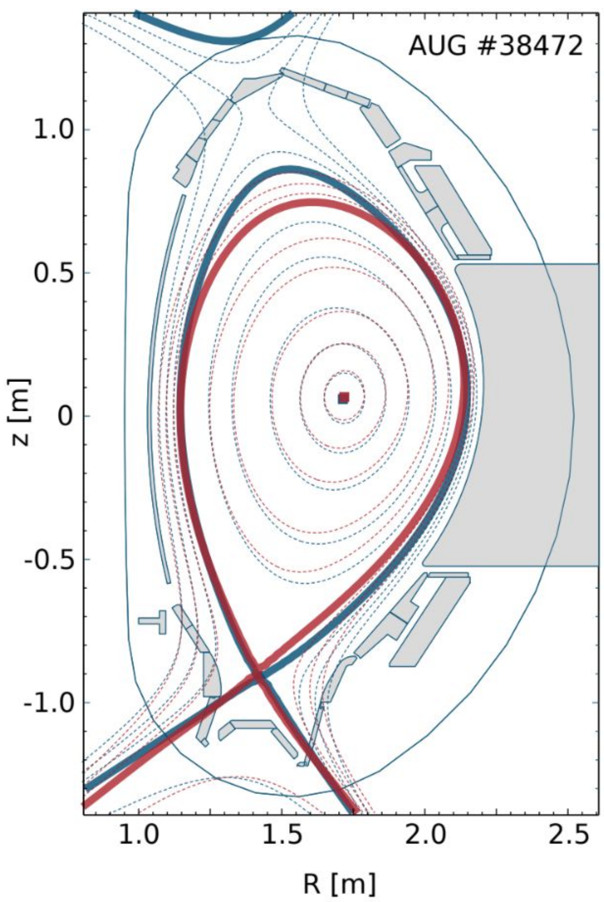
[L. Radovanovic et al., NF (2025)]

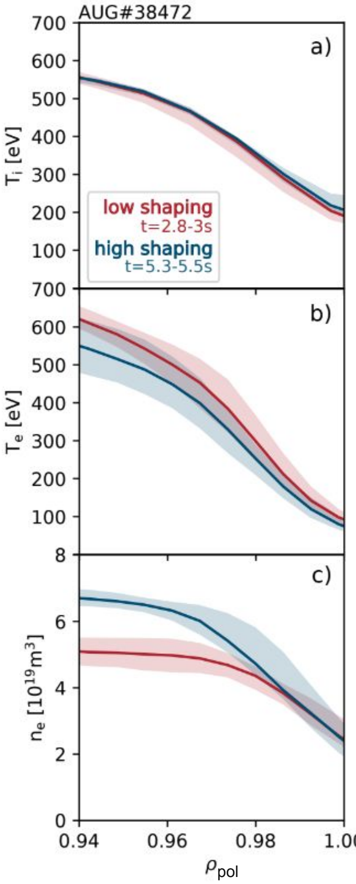
And how about a more realistic case?

Low shaping δ_{up}=0.10, κ=1.6

Fix β_{pol} while reducing heating

High shaping δ_{up}=0.25, κ=1.7





Parallel magnetic fluctuations in GENE global

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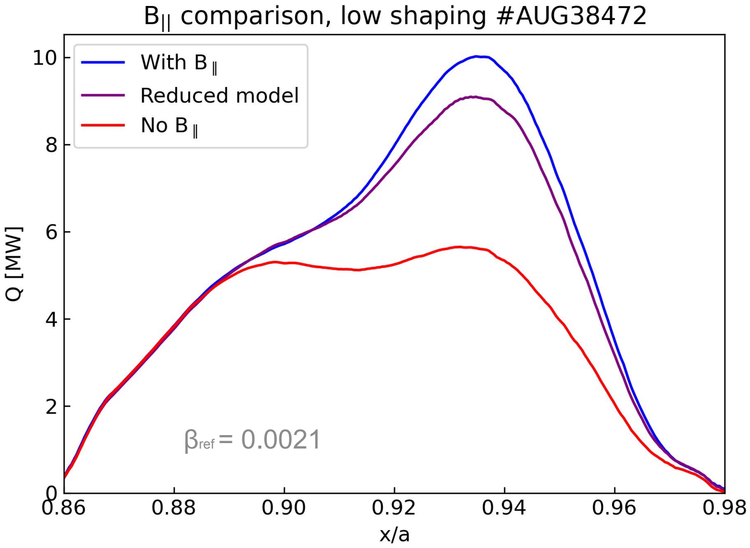
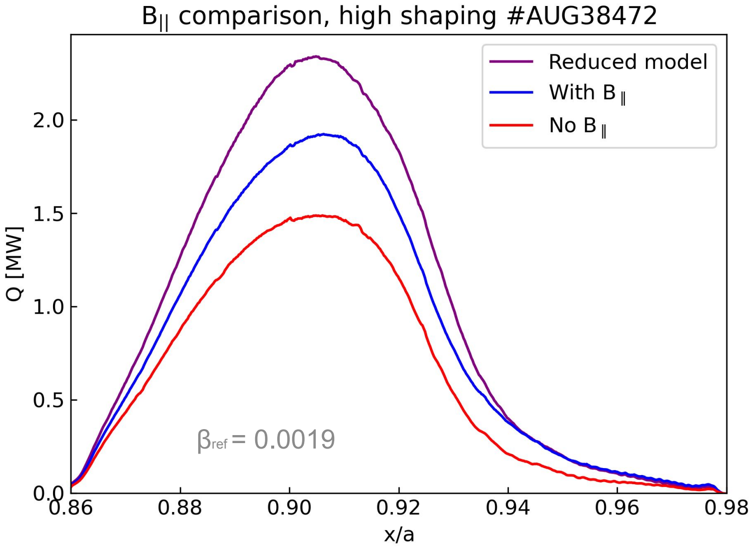


Up to ~100% increase
vs no $B_{1||}$



Up to ~20% increase
vs no $B_{1||}$

Impact of $B_{1||}$ can be substantial and seems case dependant

pol

Summary and Outlook

- ★ Several approaches can be used to model parallel magnetic fluctuations in global codes.
- ★ Mutual agreement between long wavelength and arbitrary wavelength models.
- ★ Long wavelength model valid up to considerably high $k_y\rho_s$ values.
- ★ Simulations on realistic scenarios reveal $B_{1||}$ can have substantial impact depending on the case.

- ➢ $B_{1||}$ implementation on GENE-X currently ongoing
- ➢ Code benchmarking (ORB5) is being pursued

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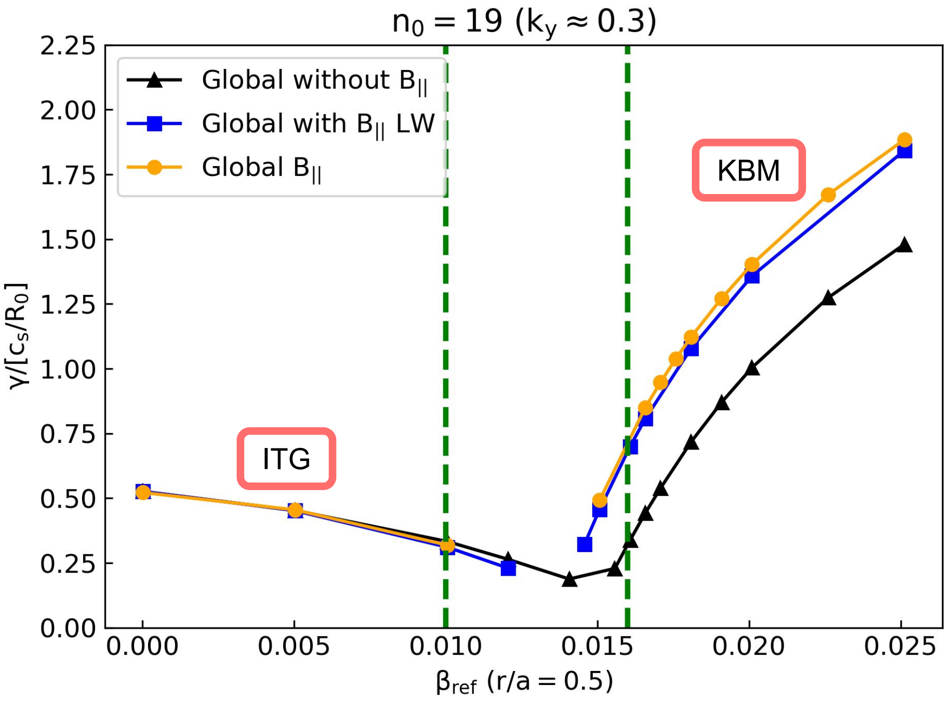
Thank you!

Parallel magnetic fluctuations in GENE global

Nonlinear simulations → What is the nonlinear impact of $B_{||}$ on transport?
We would expect similar behaviour to linear results

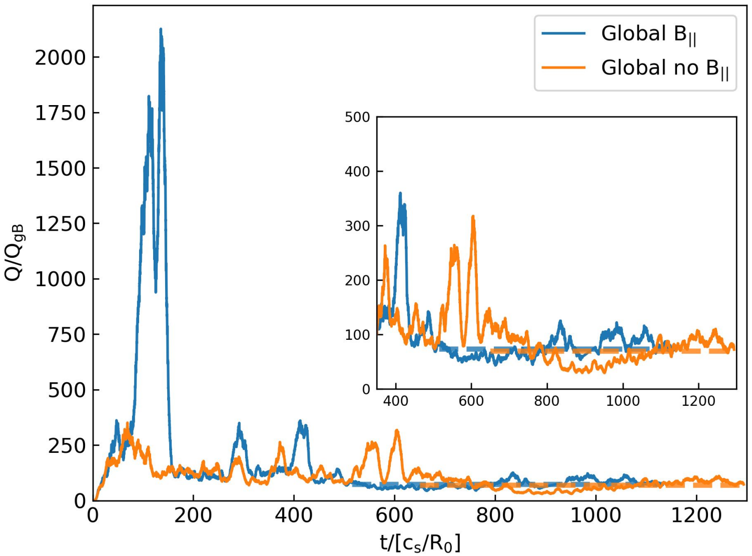


We use the LW model to avoid increased computational costs

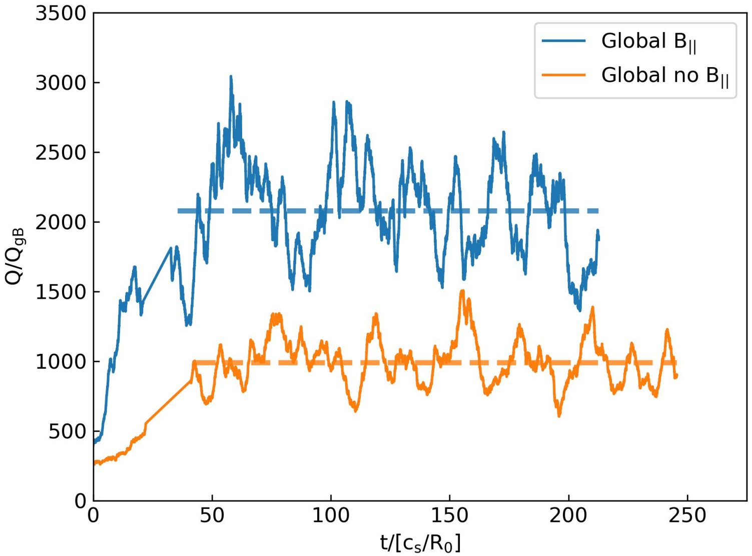
Parallel magnetic fluctuations in GENE global

Nonlinear simulations

β = 0.01



β = 0.016



No significant impact of $B_{1||}$ at low β

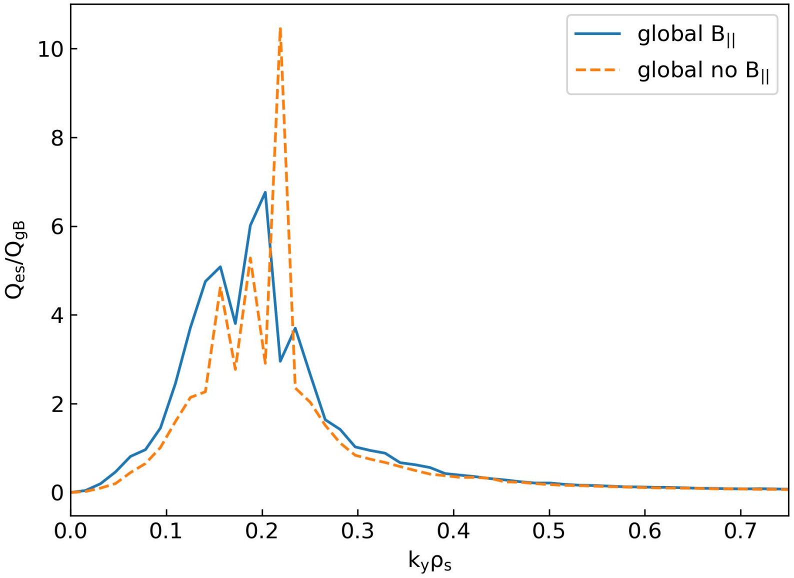
Transport doubles with $B_{1||}$ at high β

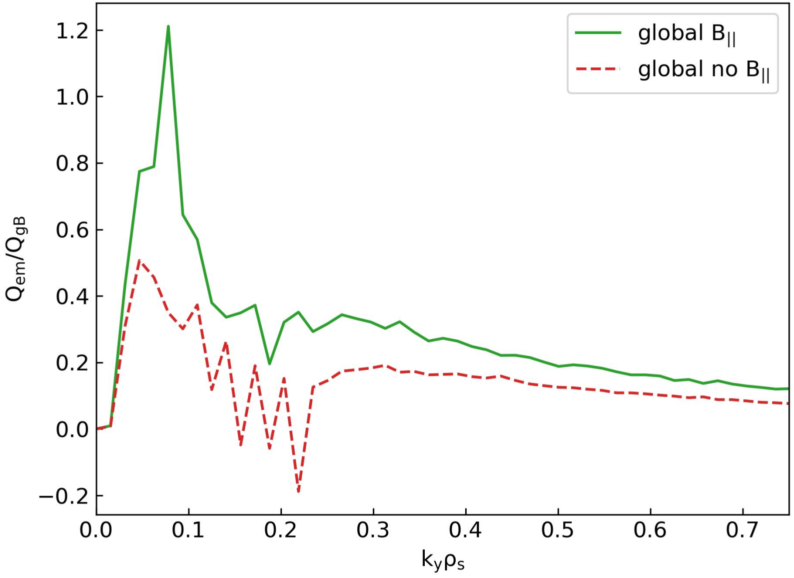
Correlates well with linear findings

First nonlinear results

$\beta = 0.01$

[Sheffield, et al., PPCF (2025)]





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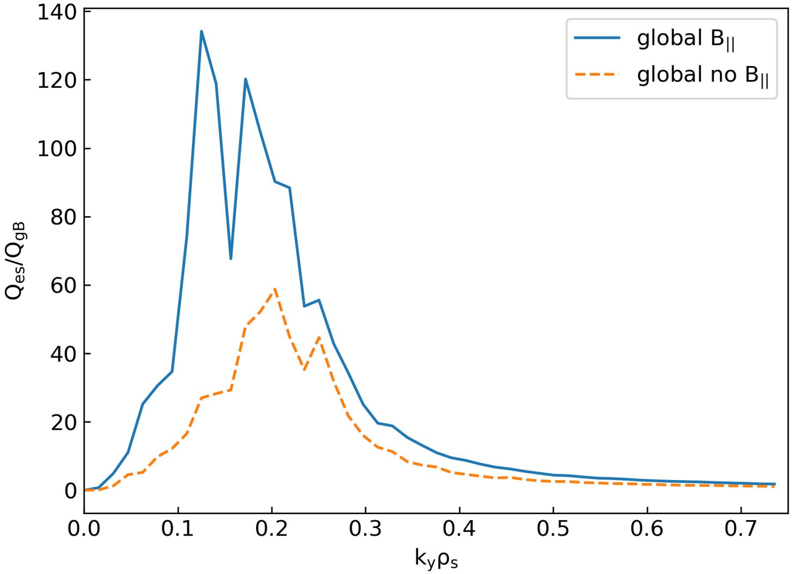
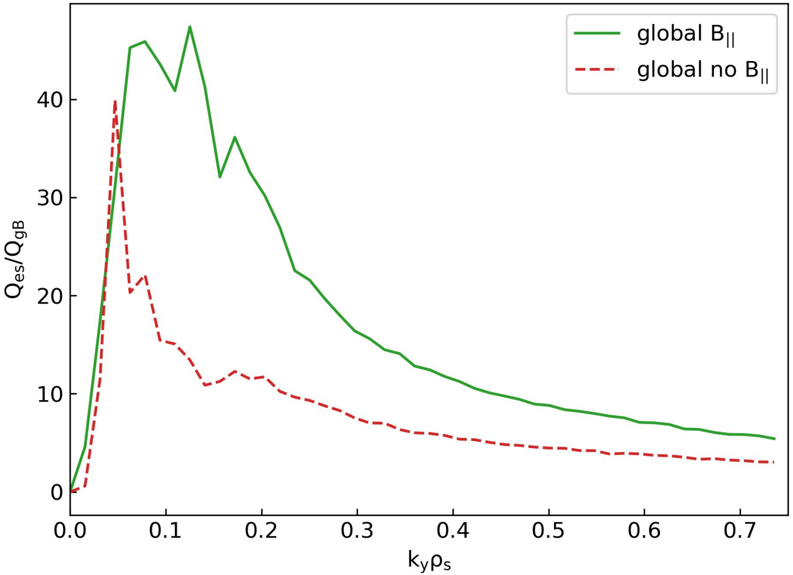




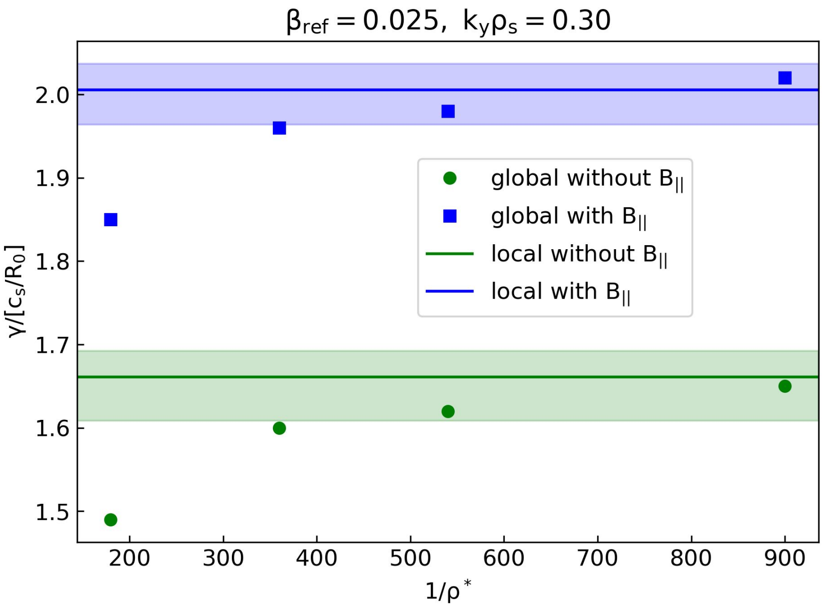
Transport doubles with $B_{1||}$ at high β

n_0=1 was required for these simulations

Verification - Long wavelength model

[Sheffield, et al., PPCF (2025)]

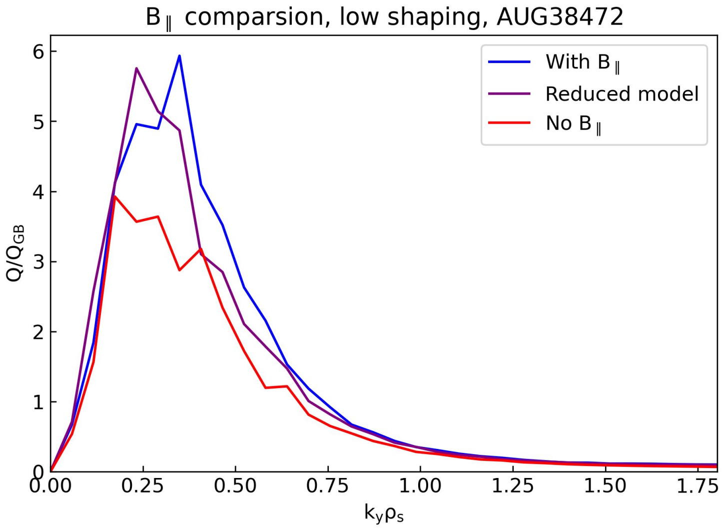
Verification done by taking to local limit as $\varrho^{*} \to 0$

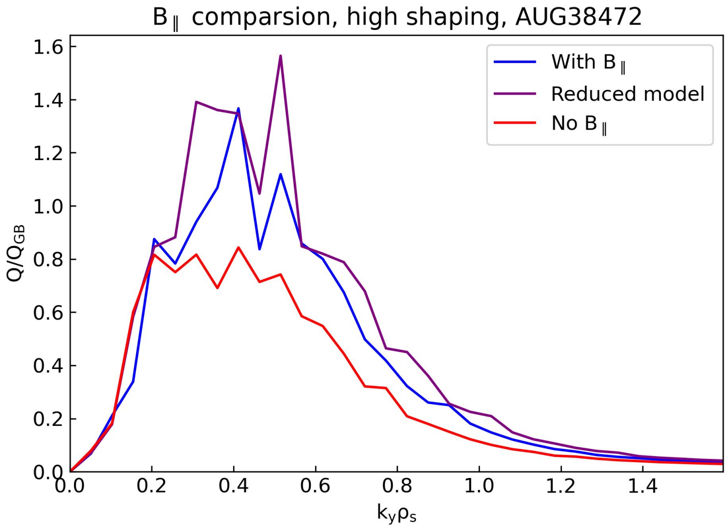


LW model follows the expected trend

First nonlinear results

Spectra for AUG comparisons



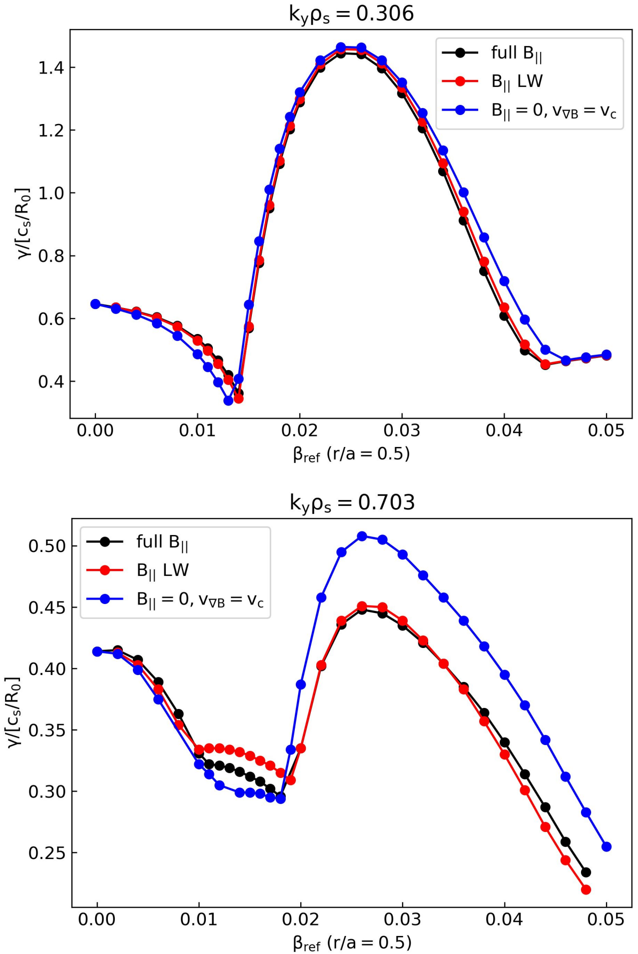


Additional information

On the fluid approximation for $B_{||}$:

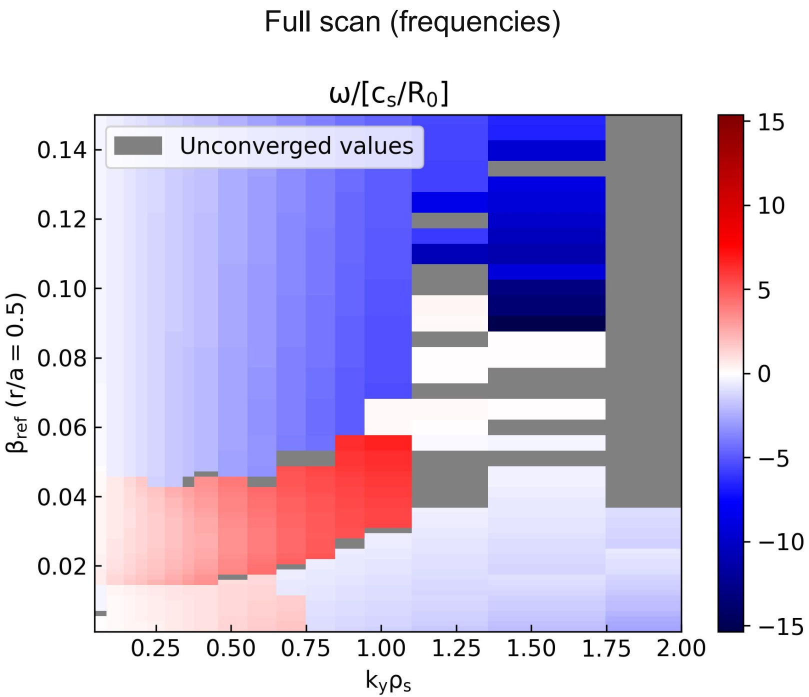
$$\mathbf{v}_c = \frac{v_{\|}^2}{\Omega_\sigma}\left(\hat{\mathbf{b}}_0 \times \left[\frac{\nabla B_0}{B_0} + \frac{\beta_p}{2}\frac{\nabla p_0}{p_0}\right]\right)$$

$$\mathbf{v}_{\nabla B_0} = \frac{\mu c}{q_\sigma B_0^2}\mathbf{B}_0 \times \nabla B_0$$



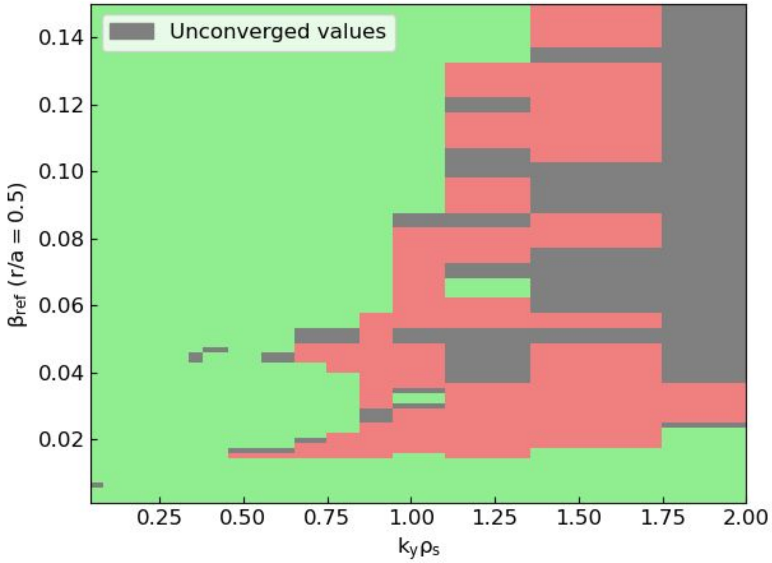
Additional information

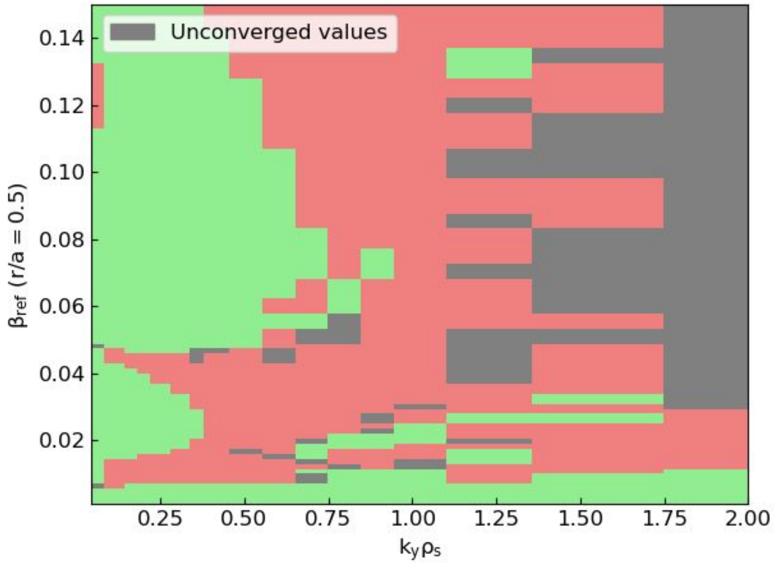
Frequencies of 2D scan:



Additional information

Validity region, new threshold (5%)





Additional information

[Wilms, Merlo et al., CPC (2025)]

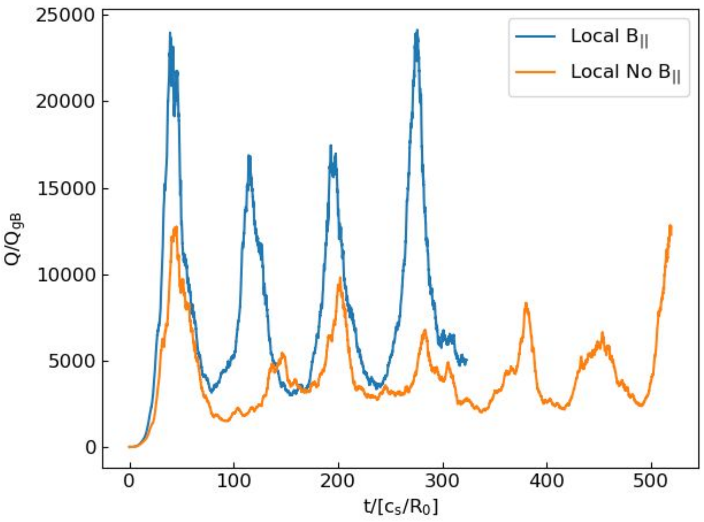
Numerical implementation of the gyrodisk matrices:

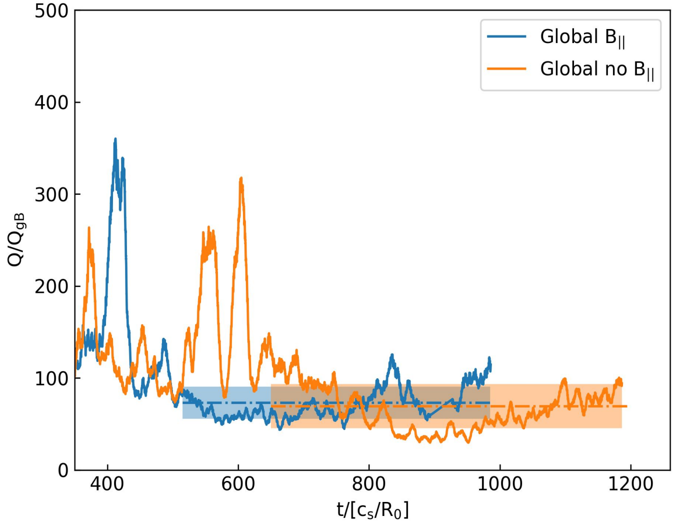
$$
\begin{aligned}
&\mathcal{G}\{u\}(x, y, z) \\
&\approx \sum_{k_y, n} \hat{u}_{k_y, n}(z) e^{i k_{\mathrm{y}} y(\mathbf{X})} \frac{1}{2\pi} \int_0^{2\pi} \Lambda_{\mathrm{n}}(x(\mathbf{X}) + \boldsymbol{\rho}(\Theta) \cdot \nabla x(\mathbf{X})) e^{i k_{\mathrm{y}} [\boldsymbol{\rho}(\Theta) \cdot \nabla y(\mathbf{X})]} \, d\Theta \\
&\equiv \sum_{k_y} e^{i k_{\mathrm{y}} y(\mathbf{X})} \mathcal{G}_{k_y} \cdot \hat{\mathbf{u}}_{k_y},
\end{aligned} \tag{34}
$$

$$
\mathcal{G}_{k_y, ij} = \frac{1}{2\pi} \int_0^{2\pi} \Lambda_{\mathrm{i}}(x_{\mathrm{j}} + \boldsymbol{\rho}(\Theta) \cdot \nabla x_{\mathrm{j}}) e^{i k_{\mathrm{y}} [\boldsymbol{\rho}(\Theta) \cdot \nabla y]} \, d\Theta
\qquad
\mathcal{G}_{\mathrm{D,k_y}} = \int_0^{\mu} \mathcal{G}_{k_y}(\mu') \, d\mu'
$$

Additional information

And what about local simulations?





Local simulations present much more heat flux than global → Not ideal for KBM studies!

Additional information

The gyrokinetic ordering:

$$\frac{f_1}{F_0} \sim \frac{q\phi_1}{T_e} \sim q\frac{v_{\|}}{c}\frac{A_{1\|}}{T_e} \sim \frac{B_{1\|}}{B} \sim \epsilon_\delta$$

$$\frac{\rho_i \nabla F_0}{F_0} \sim \epsilon_F$$

$$\frac{\rho_i \nabla B}{B} \sim \epsilon_B$$

$$\frac{k_{\|}}{k_{\perp}} \sim \epsilon_{\|} \qquad \frac{\omega}{\Omega} \sim \epsilon_\omega$$

.Why delta f works even at very high values?

We compare n_1 not Q. n_1 is small due to the sources fixing the profiles.

Vlasov equation

$$\begin{aligned}\frac{\partial F_{1,\sigma}}{\partial t} = &- \left[v_{\|}\mathbf{b}_0 + (\mathbf{v}_\chi + \mathbf{v}_{\nabla B} + \mathbf{v}_c)\right] \cdot \nabla F_{1,\sigma} + \frac{\mu}{m_\sigma}\mathbf{b}_0 \cdot \nabla B_0 \frac{\partial F_{1,\sigma}}{\partial v_{\|}} \\ &- \mathbf{v}_\chi \cdot \left[\nabla \ln(n_{0,\sigma}) + \nabla \ln(T_{0,\sigma}) \left(\frac{m_\sigma v_{\|}^2/2 + \mu B_0}{T_{0,\sigma}} - \frac{3}{2}\right)\right] F_{\mathrm{M},\sigma} \\ &- \frac{q_\sigma F_{\mathrm{M},\sigma}}{T_{0,\sigma}} \left[v_{\|}\mathbf{b}_0 + (\mathbf{v}_\chi + \mathbf{v}_{\nabla B} + \mathbf{v}_c)\right] \cdot \nabla \mathcal{G}\{\psi_1\} \\ &- \frac{q_\sigma v_{\|}}{c}\frac{F_{\mathrm{M},\sigma}}{T_{0,\sigma}}\frac{\partial \mathcal{G}\{A_{1,\|}\}}{\partial t} - \mathbf{v}_{E_0} \cdot \left(\nabla F_{1,\sigma} + \frac{q_\sigma v_{\|}}{c}\nabla \mathcal{G}\{A_{1,\|}\}\right) \\ &- (\mathbf{v}_{\nabla B} + \mathbf{v}_c) \cdot \left[\nabla \ln(n_{0,\sigma}) + \nabla \ln(T_{0,\sigma}) \left(\frac{m_\sigma v_{\|}^2/2 + \mu B_0}{T_{0,\sigma}} - \frac{3}{2}\right)\right] F_{\mathrm{M},\sigma}.\end{aligned} \qquad (2.7.3)$$

[F. Wilms, PhD thesis (2024)]

Additional information

Some normalized units definitions

$$m_\sigma = m_{\mathrm{ref}}\hat{m}_\sigma; \qquad q_\sigma = e\hat{q}_\sigma; \qquad \rho^*_{\mathrm{ref}} = \frac{\rho_{\mathrm{ref}}}{L_{\mathrm{ref}}}; \qquad \beta_{\mathrm{ref}} = \frac{8\pi\, n_{\mathrm{ref}} T_{\mathrm{ref}}}{B^2_{\mathrm{ref}}};$$

$$n_{0,\sigma}(x) = n_{\mathrm{ref}}\hat{n}_{0,\sigma}(x_0)\,\hat{n}_{p,\sigma}(x); \qquad T_{0,\sigma}(x) = T_{\mathrm{ref}}\hat{T}_{0,\sigma}(x_0)\,\hat{T}_{p,\sigma}(x);$$

$$\mu = \frac{T_{\mathrm{ref}}}{B_{\mathrm{ref}}}\hat{T}_{0,\sigma}\hat{\mu}; \qquad v_{||} = c_{\mathrm{ref}}\hat{v}_{\mathrm{th},\sigma}\hat{v}_{||}; \qquad c_{\mathrm{ref}} = \sqrt{\frac{T_{\mathrm{ref}}}{m_{\mathrm{ref}}}}; \qquad \hat{v}_{\mathrm{th},\sigma} = \sqrt{\frac{2\hat{T}_{0,\sigma}}{\hat{m}_\sigma}};$$

$$\hat{\mathcal{G}}_{\mathrm{D}} = \frac{B_{\mathrm{ref}}}{T_{\mathrm{ref}}\hat{T}_{0,\sigma}(x_0)}\mathcal{G}_{\mathrm{D}}; \qquad \hat{\mathcal{K}}_{\mathrm{D}} = \frac{B_{\mathrm{ref}}}{T_{\mathrm{ref}}\hat{T}_{0,\sigma}(x_0)}\mathcal{K}_{\mathrm{D}}; \qquad \mathcal{G} = \hat{\mathcal{G}}; \qquad \mathcal{K} = \hat{\mathcal{K}};$$

$$F_{\mathrm{M},\sigma} = \frac{n_{\mathrm{ref}}}{c^3_{\mathrm{ref}}}\frac{\hat{n}_{0,\sigma}}{\hat{v}^3_{\mathrm{th},\sigma}}\hat{F}_{\mathrm{M},\sigma}; \qquad F_{1,\sigma} = \rho^*_{\mathrm{ref}}\frac{n_{\mathrm{ref}}}{c^3_{\mathrm{ref}}}\frac{\hat{n}_{0,\sigma}}{\hat{v}^3_{\mathrm{th},\sigma}}\hat{F}_{1,\sigma};$$

$$\phi_1 = \rho^*_{\mathrm{ref}}\frac{T_{\mathrm{ref}}}{e}\hat{\phi}_1; \qquad B_{1,||} = \rho^*_{\mathrm{ref}}B_{\mathrm{ref}}\hat{B}_{1,||};$$

$$\frac{B_0}{m_\sigma}\mathrm{d}v_{||}\mathrm{d}\mu = \frac{c^3_{\mathrm{ref}}\hat{v}^3_{\mathrm{th},\sigma}}{2}\hat{B}_0\mathrm{d}\hat{v}_{||}\mathrm{d}\hat{\mu},$$

Additional information

Some additional results