MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Characterisation of X- and O- points in Wendelstein 7-X with respect to coil currents

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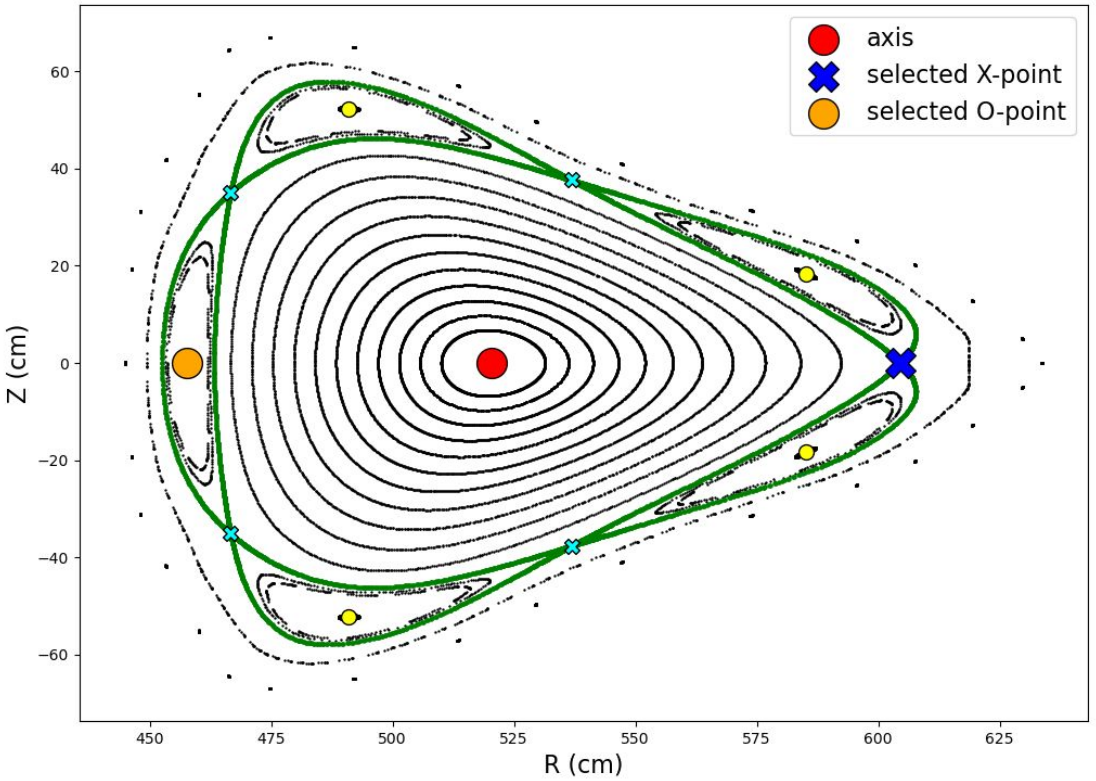
EUROfusion

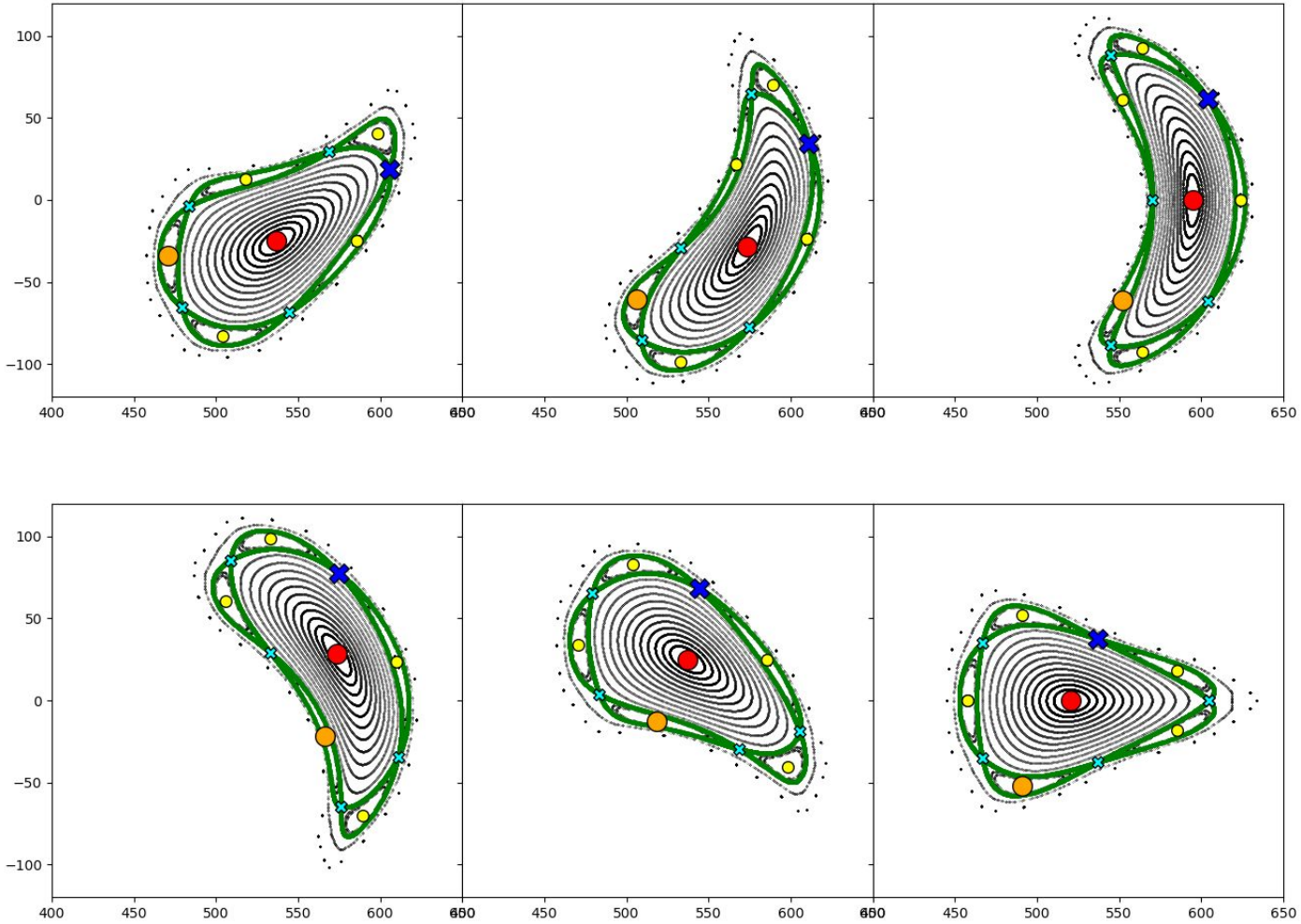
This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.

Summary:

- We can characterise the magnetic axis and edge island chains using fast tools, which allow us to characterise the operational space of W7-X in vacuum configurations (zero plasma beta, zero current)
- We find (and quantify) some expected results:
  - **Planar coils mostly control inwards/outwards shift and shift iota profile**
  - **Control coil mostly controls island chain properties**
- We find some relatively novel phenomena:
  - **Control coil very differently affects X- and O-points in island chain (localised effects?)**
  - **We find a fast proxy for island size -> possibly useful for finding configurations with internal island chains**

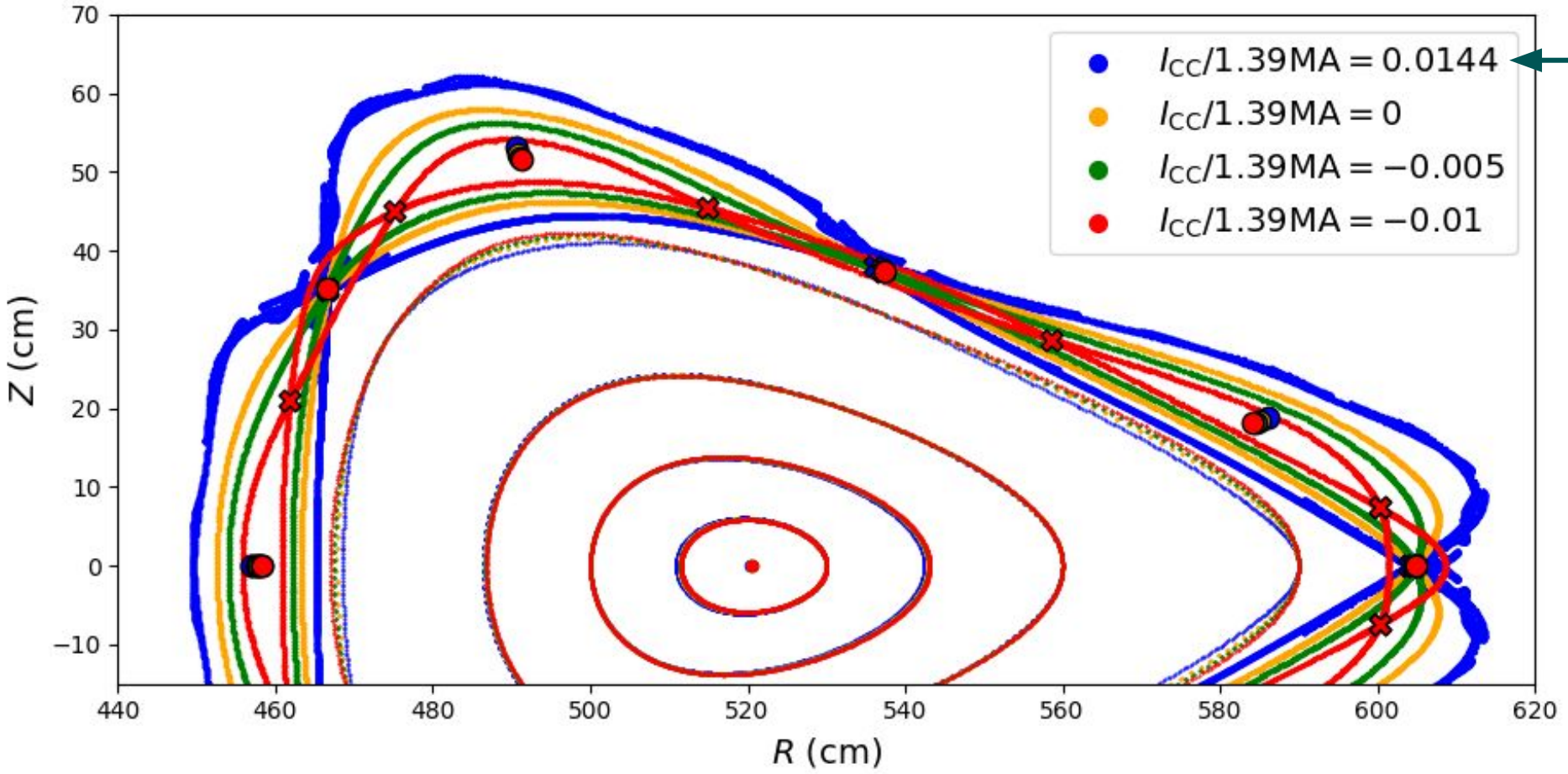
Motivation: Fixed points in W7-X





“standard configuration”
5/5 island chain

Example: 5/5 -> 10/10 transition with control coil



0.0144 corresponds to 20kA filament current (2.5kA winding current)

Can we mathematically represent these changes in a fast way?

Theory/Method

- Fixed points are closed field lines; they map to themselves, after a certain number of field periods n

- Near a fixed point, the behaviour of magnetic field lines is described by a 2x2 matrix, M (“the Jacobian”)

- The ***trace*** of M is important:
  - trace > 2: X-point
  - -2 < trace < 2: O-point

$$R_m \simeq R_0 + M_{11} * (R-R_0) + M_{12} * (Z-Z_0) \quad \text{(eq. 1)}$$

$$Z_m \simeq Z_0 + M_{21} * (R-R_0) + M_{22} * (Z-Z_0) \quad \text{(eq. 2)}$$

trace = $\mathrm{Tr}(M) = M_{11} + M_{22}$

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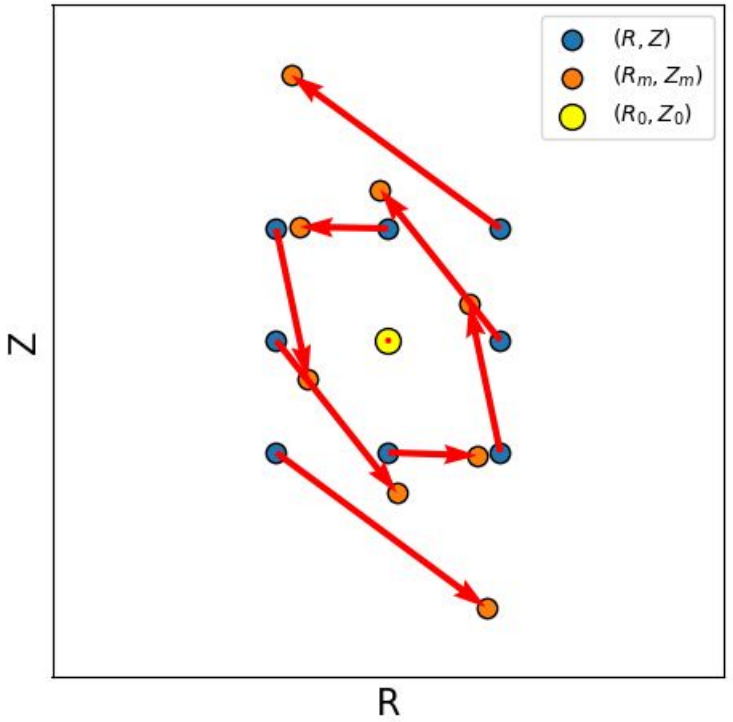
O-point
trace = 0.9



X-point
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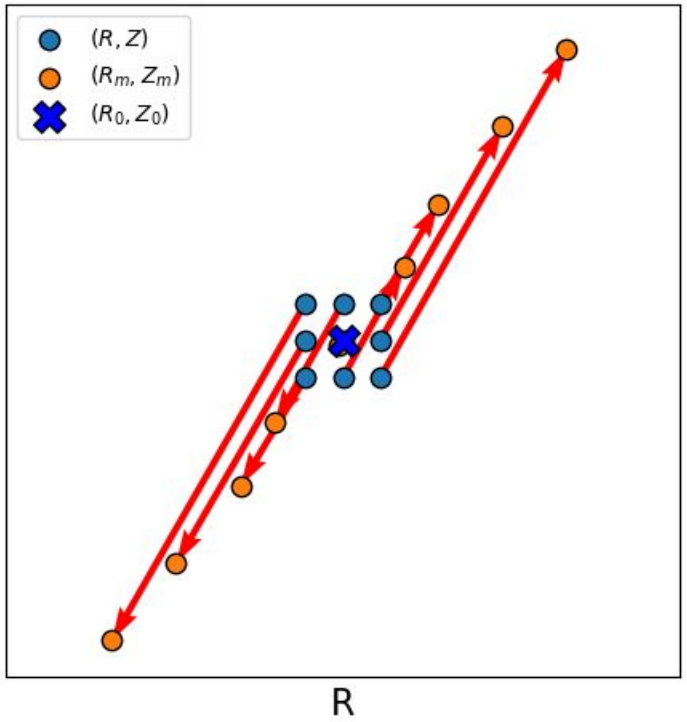

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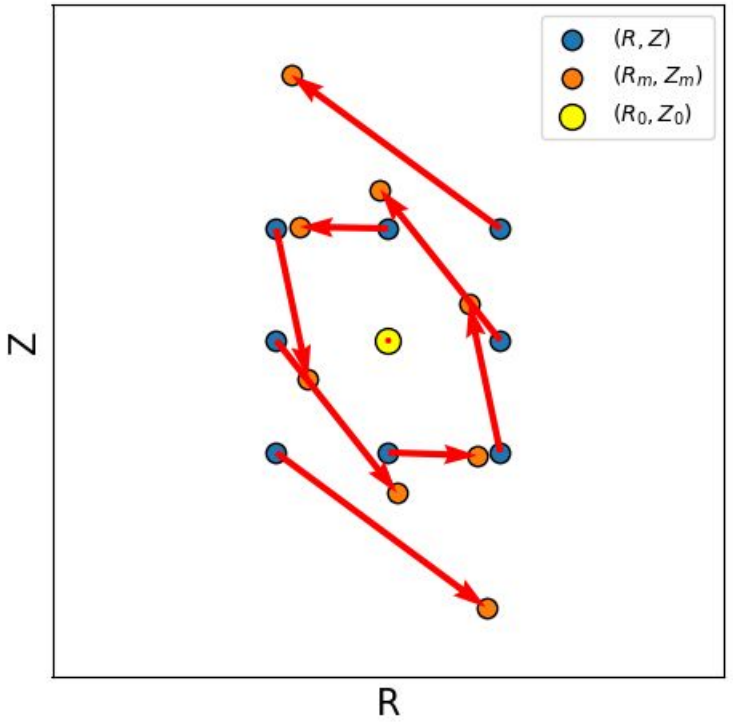

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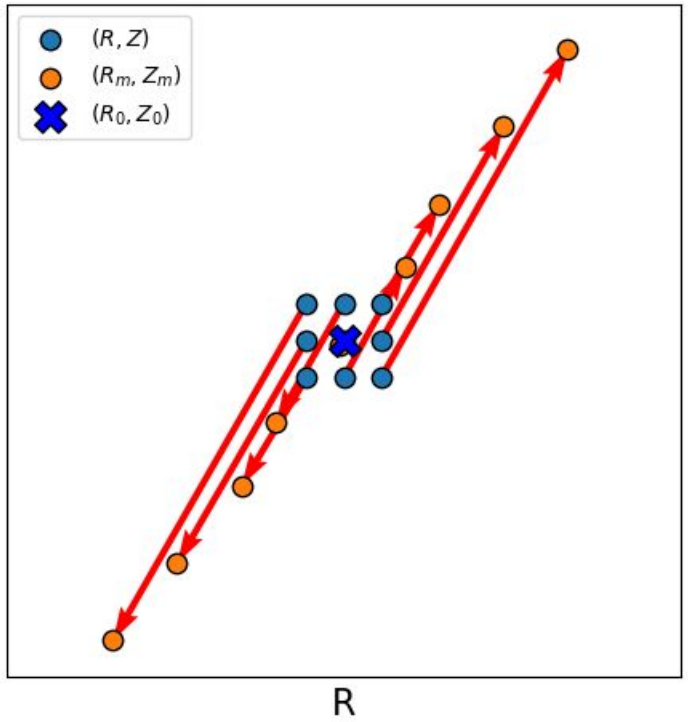

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- For a given configuration, we find the fixed points and the trace of the fixed points. Computational cost: 0.26 seconds per configuration

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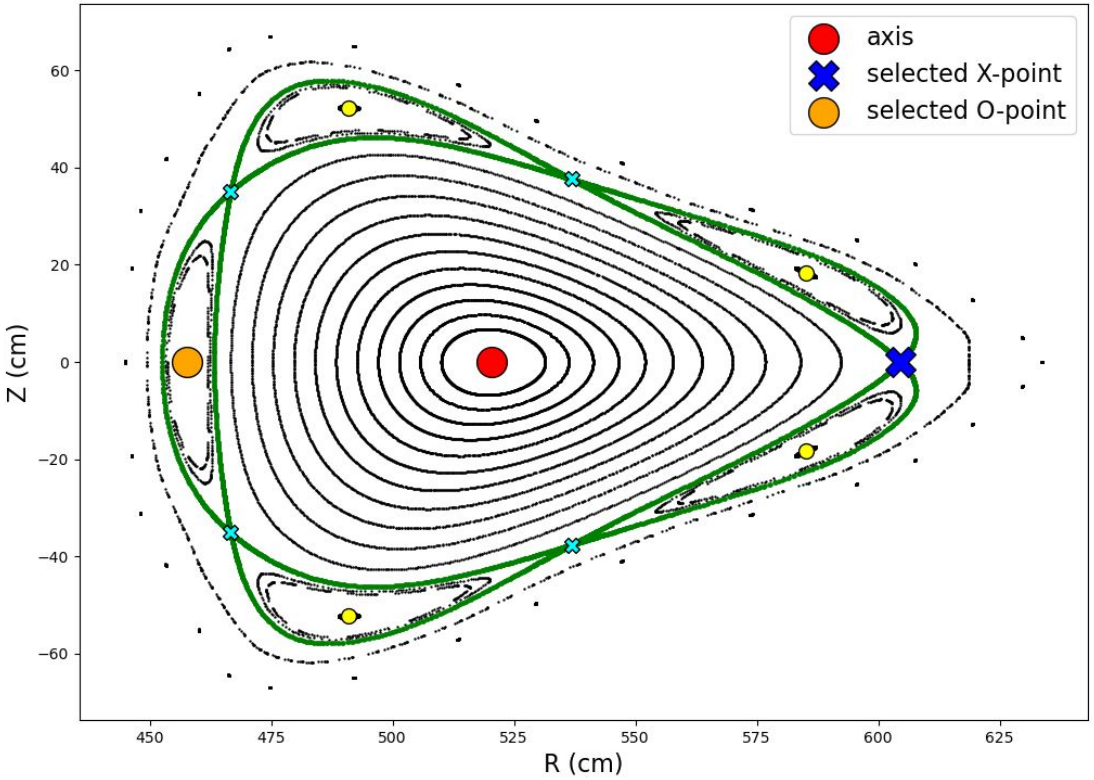
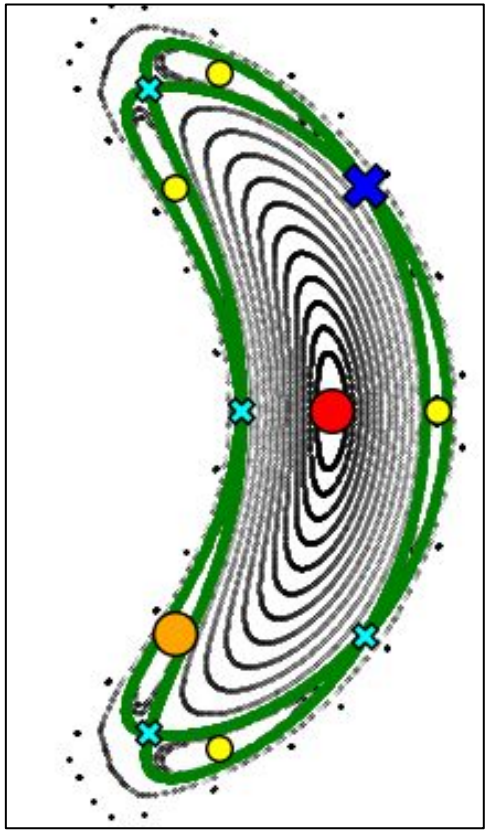
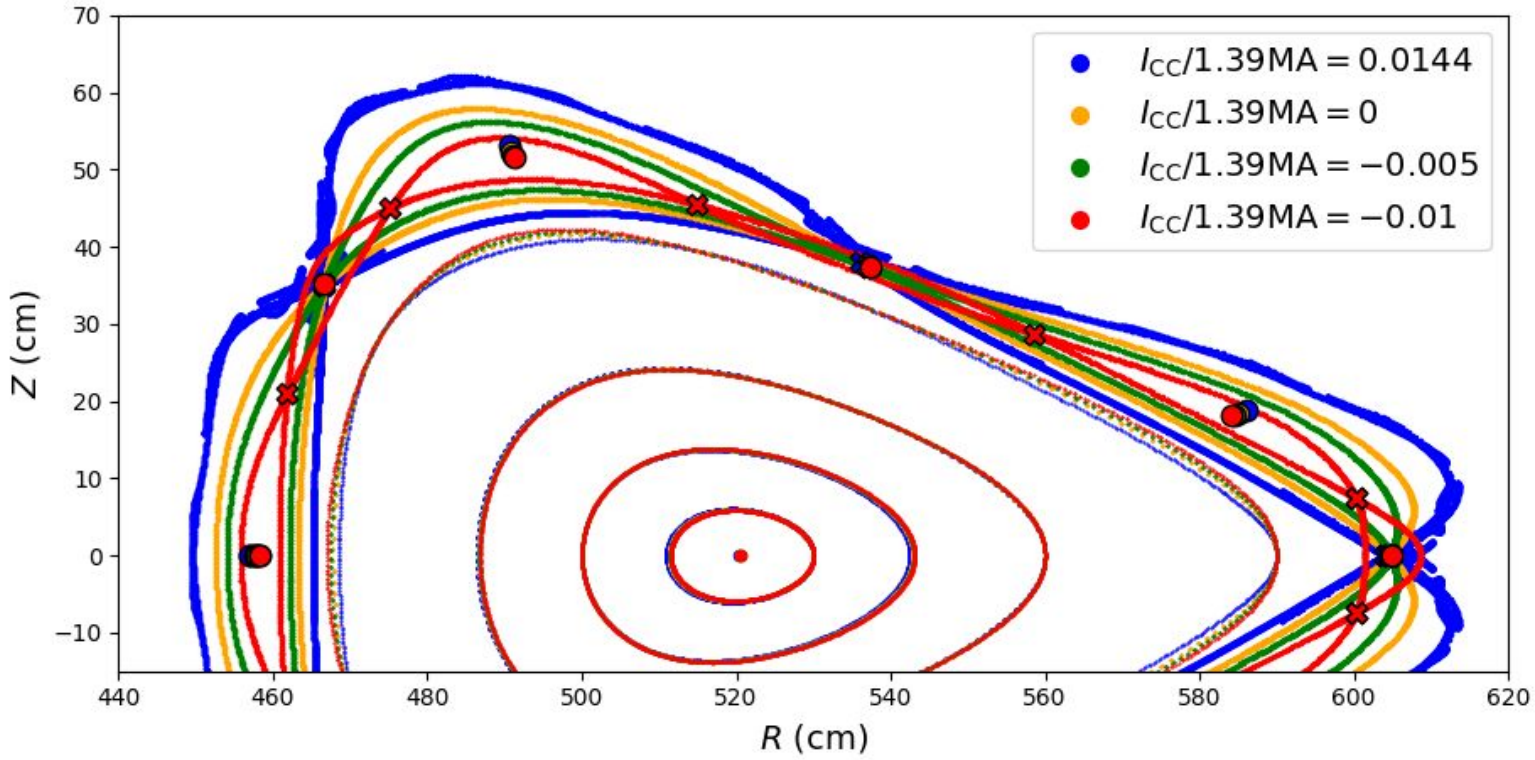
- For a given configuration, we find the fixed points and the trace of the fixed points. Computational cost: 0.26 seconds per configuration
- Disclaimers:
  - Only finds fixed points living at Z=0 at phi=0° or phi=36° are found
  - Finds X- and O-points which are periodic with 1, 4, 5, 6 field periods (e.g. magnetic axis, 5/4, 5/5, 5/6 island chains)

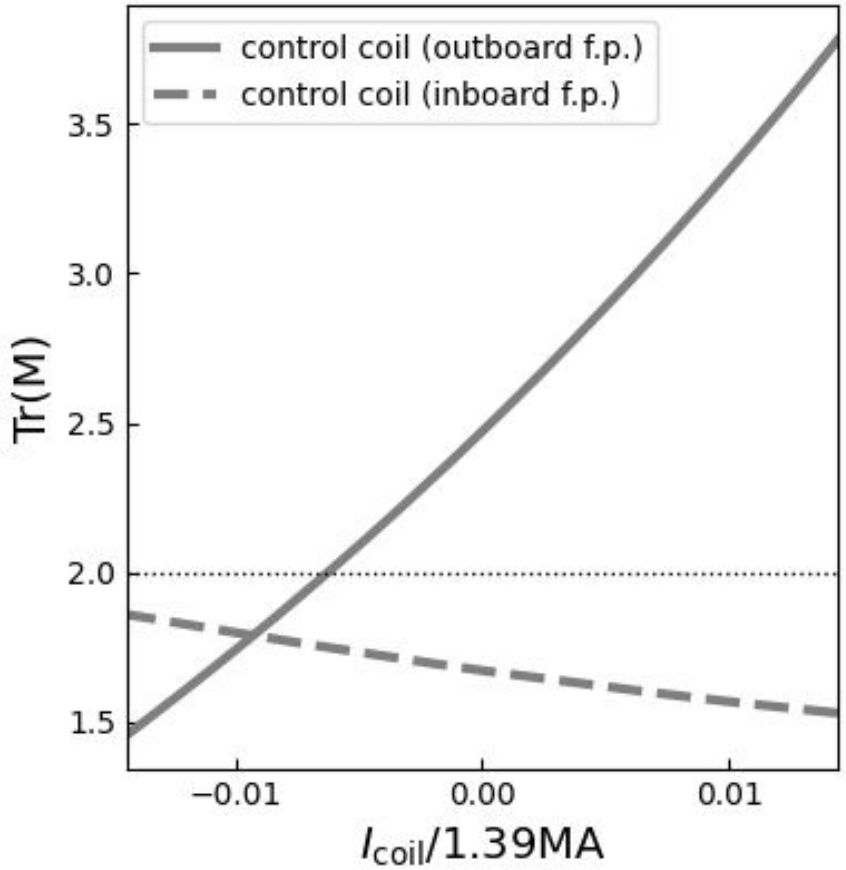




shown here: 5 field period fixed points

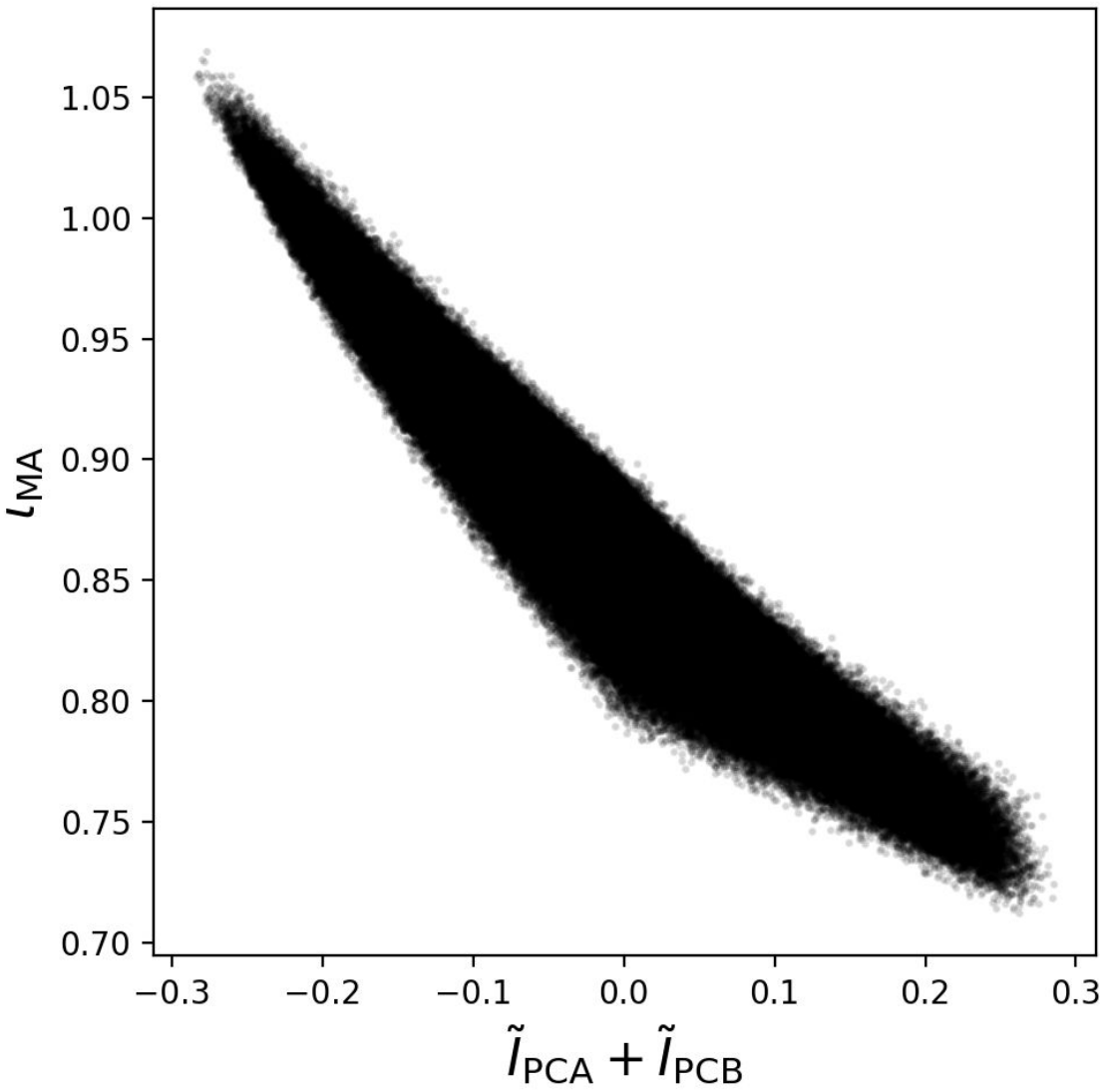
Island chain: 5/5 -> 10/10 transition with control coil

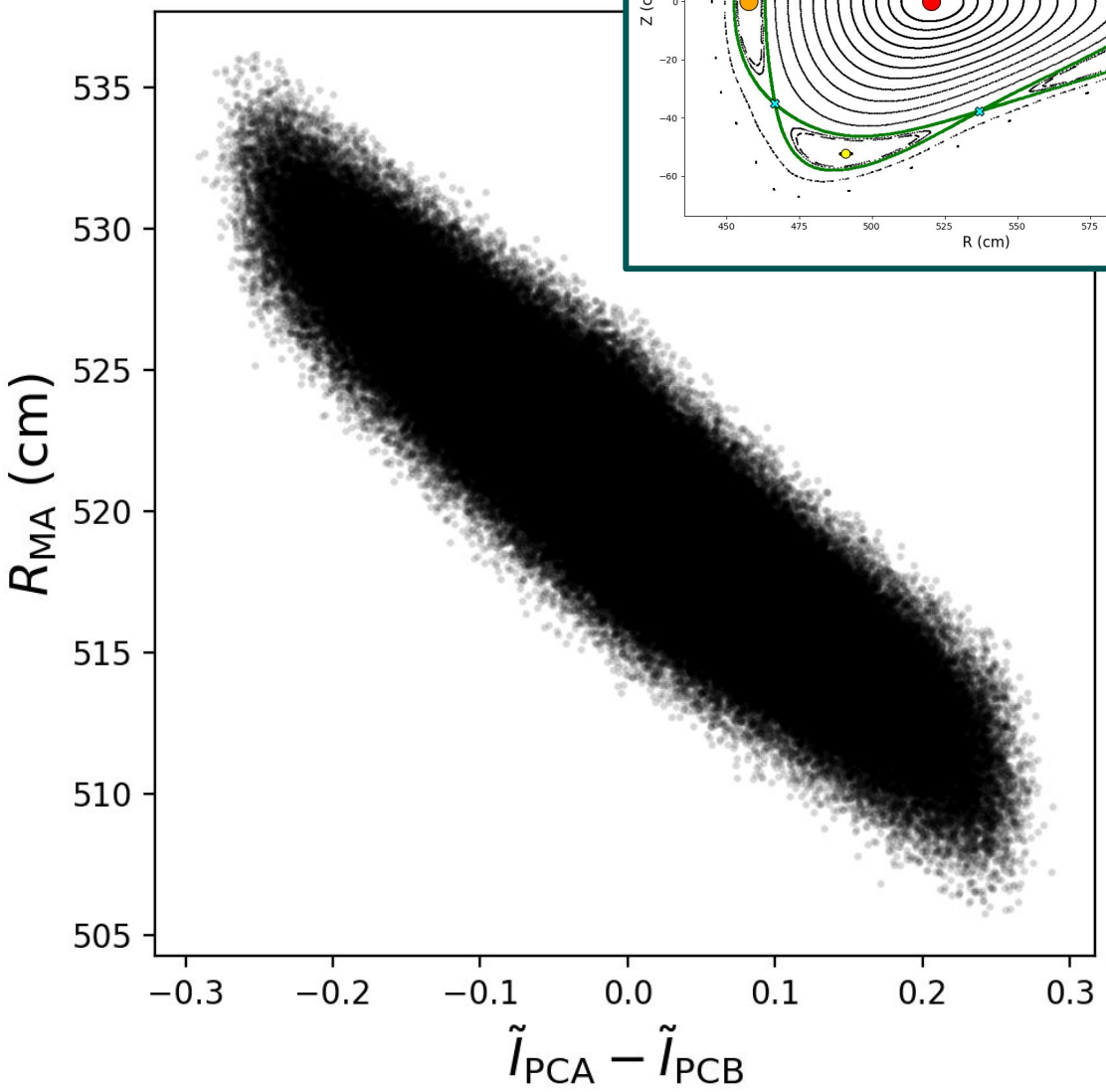


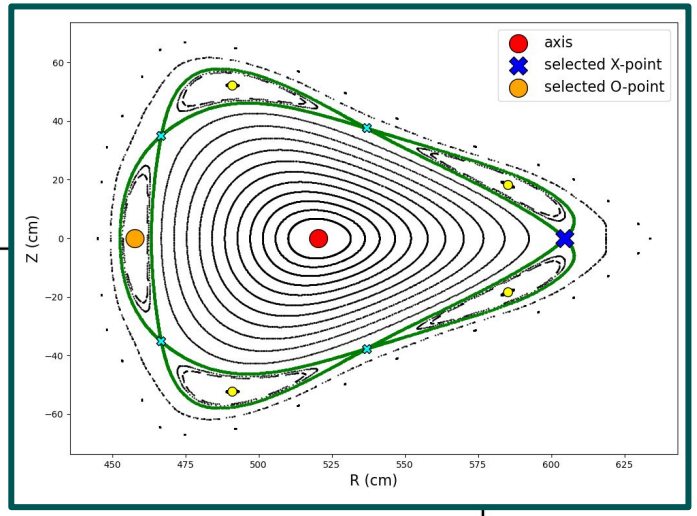


- Fast scheme can show how coils affects island chains ✔
- Control coil ***differently*** affects the X- and O-point
- What about a big parameter scan?

Role of planar coil current: *scan of > 2x10^5 configurations*







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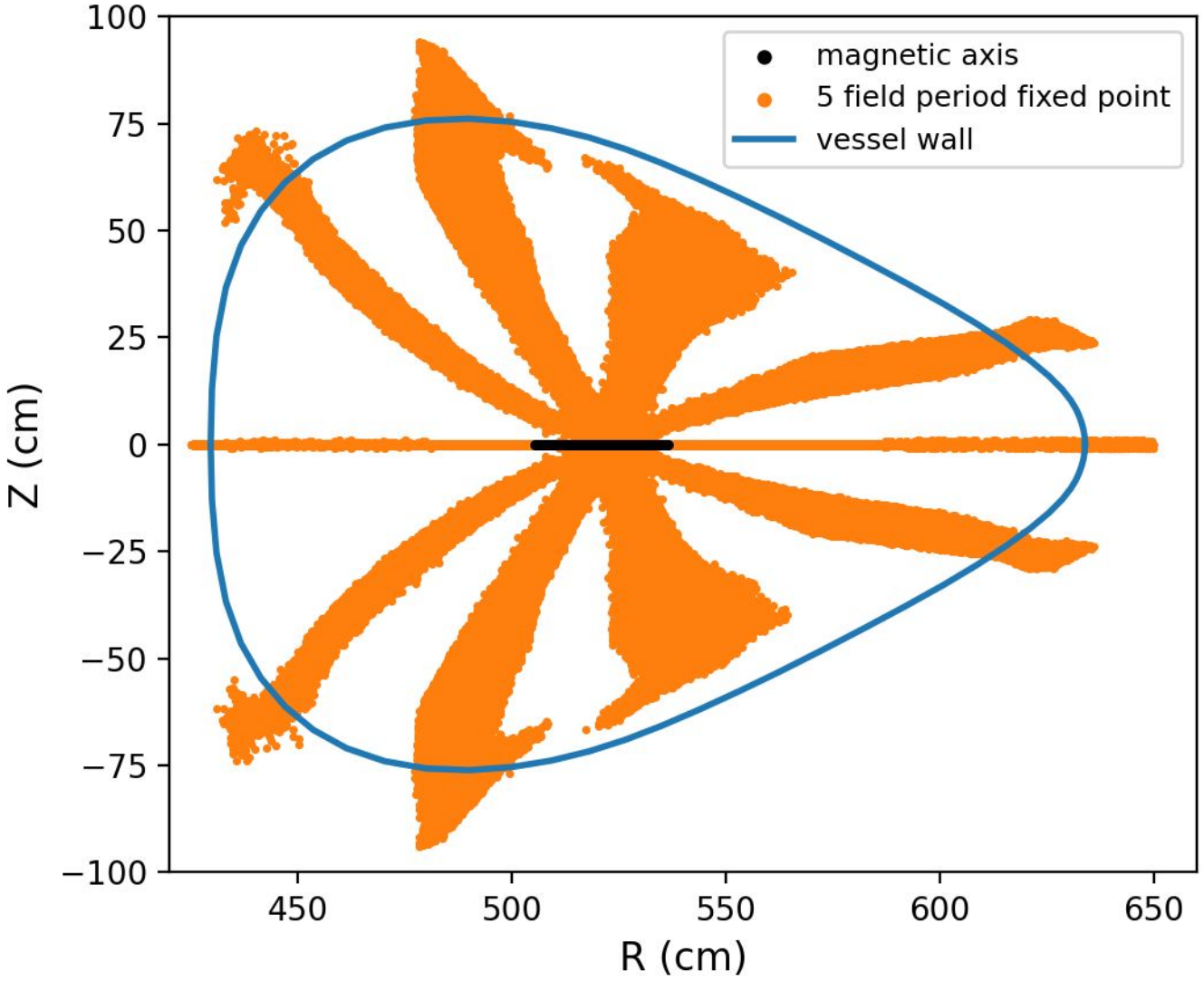
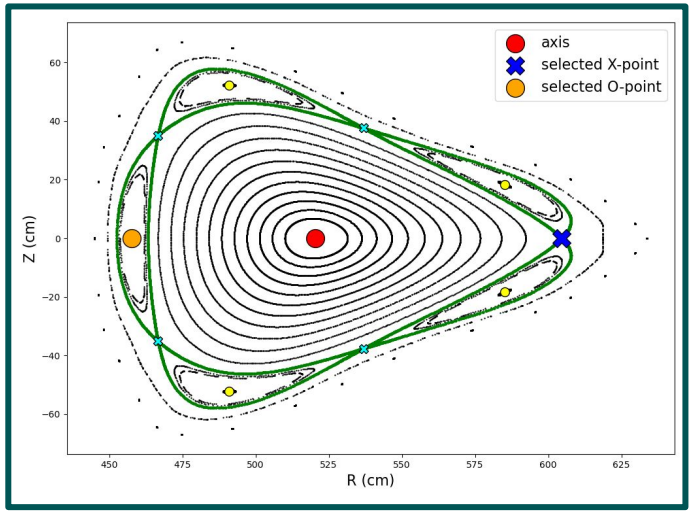
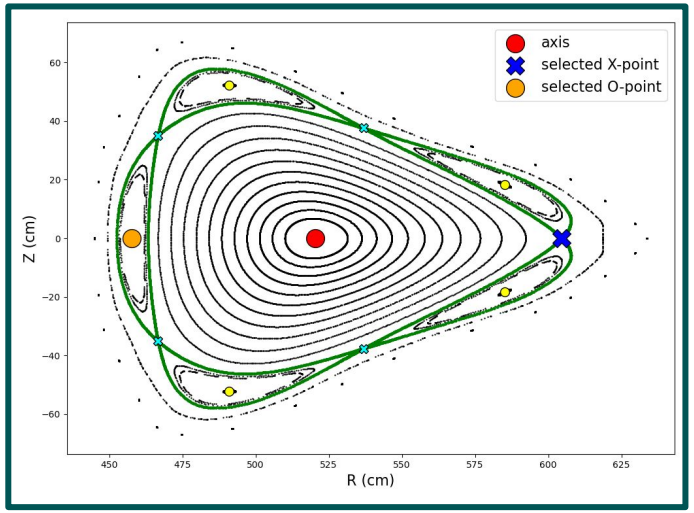
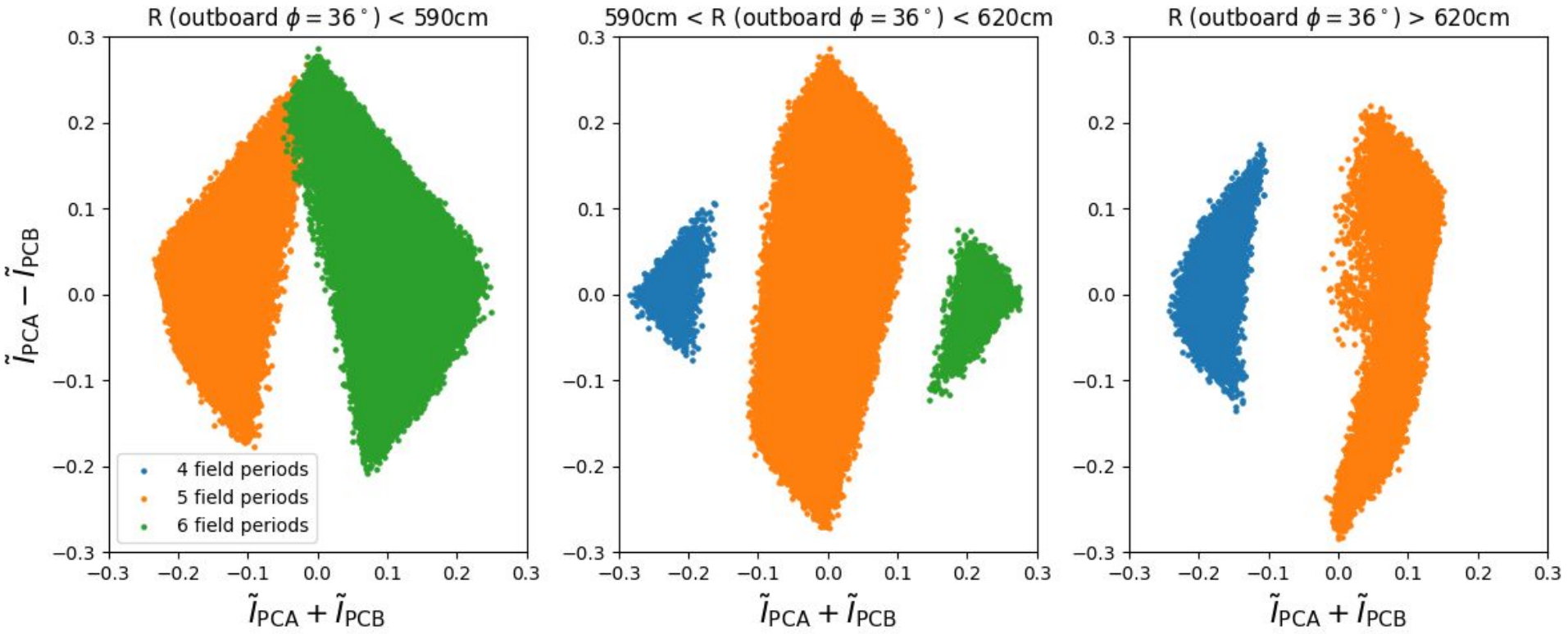




Role of planar coil current: *scan of > 2x10^5 configurations*

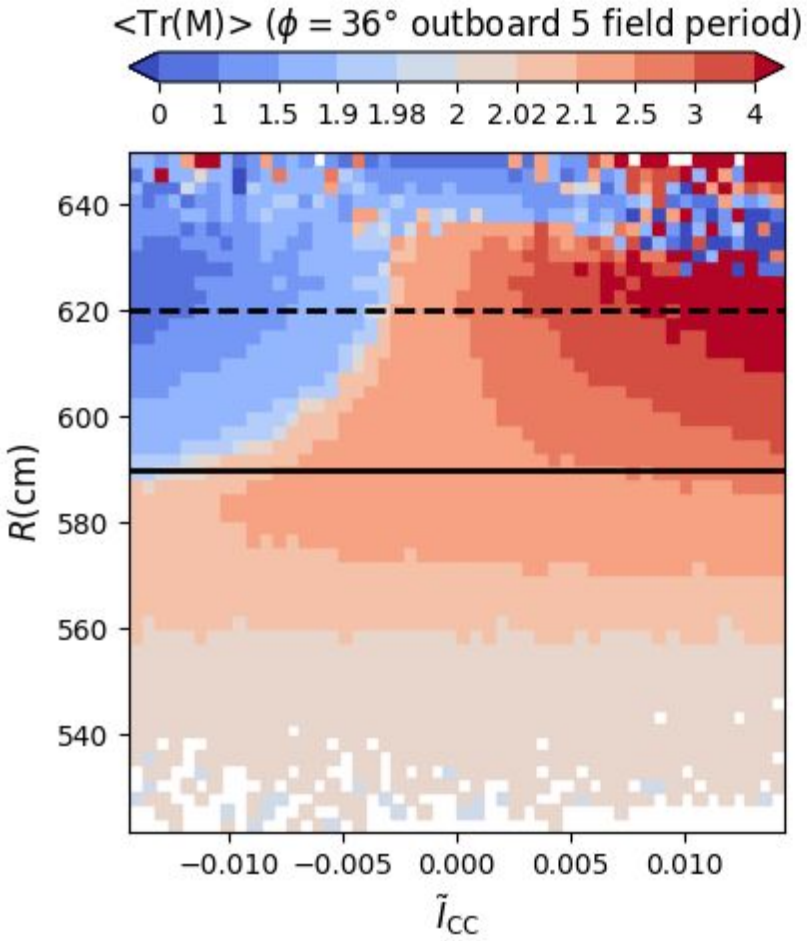
- Planar coils control iota profile; hence location and periodicity of fixed points

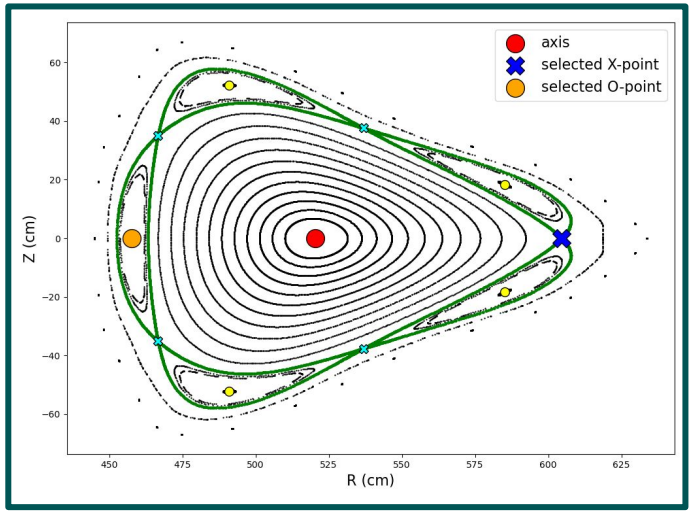




Role of control coil: *scan of > 2x10^5 configurations*

- Control coils affect fixed point trace
- Bigger effect at larger R (closer to the coils)





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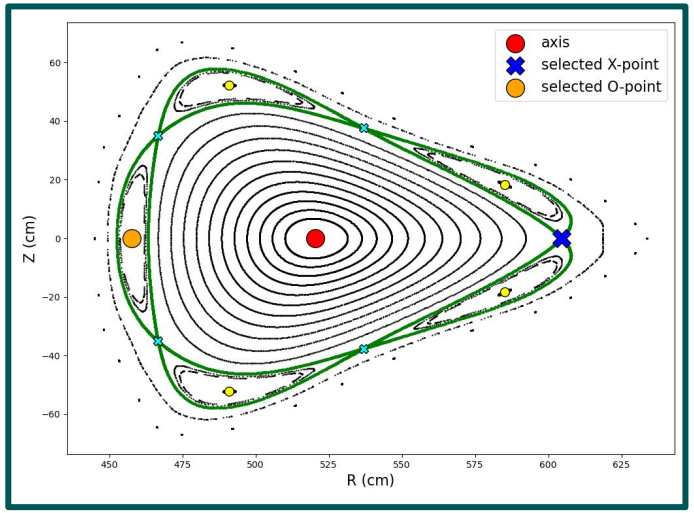
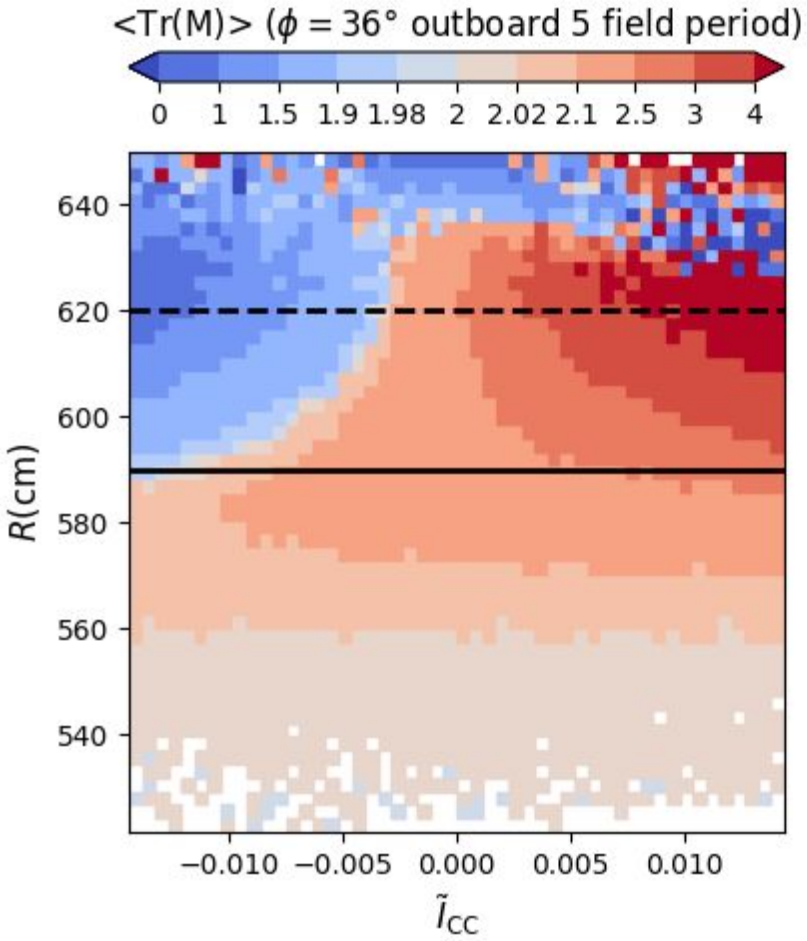
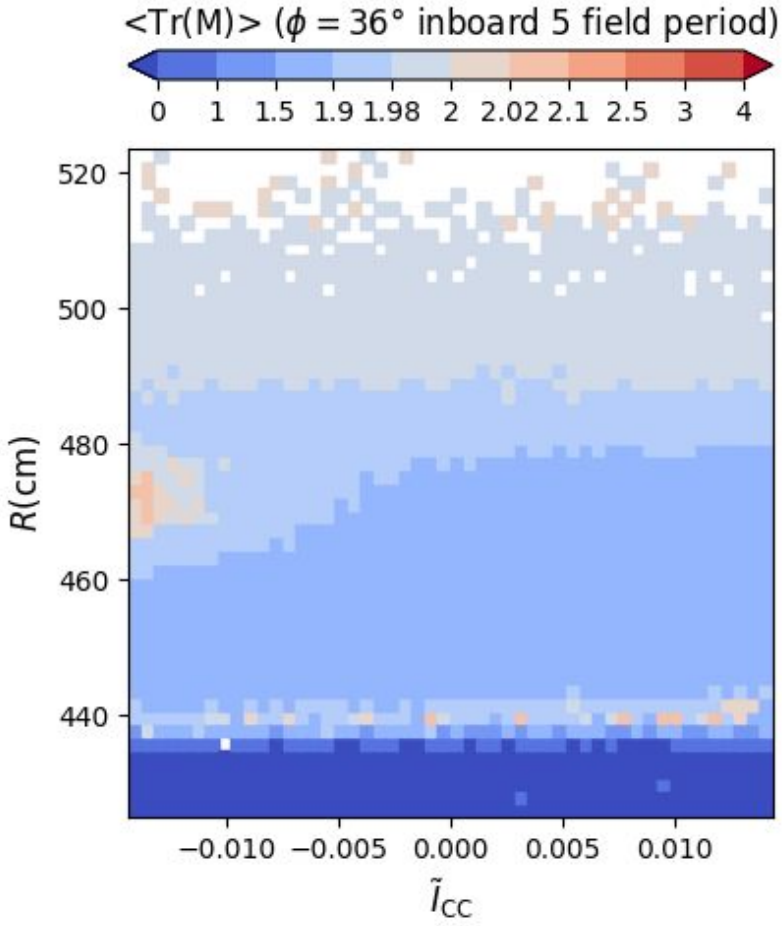
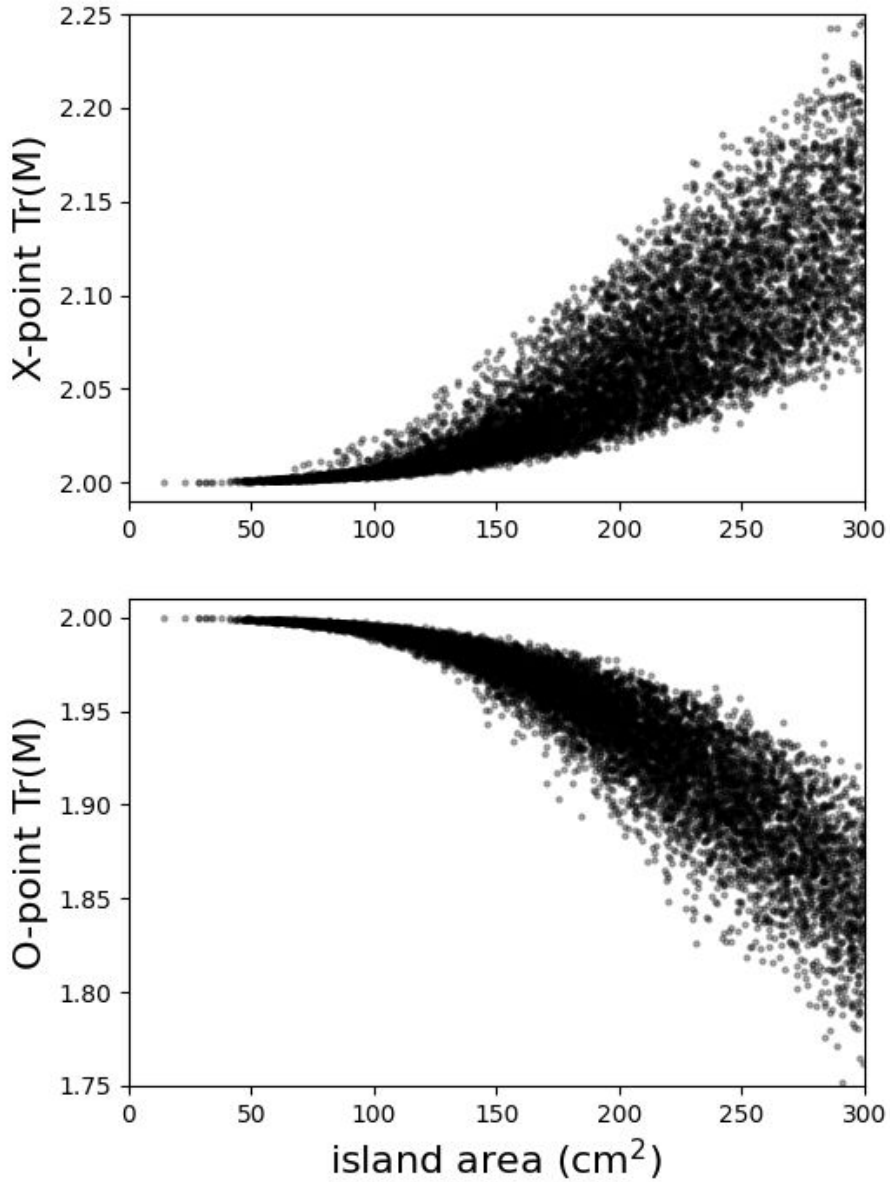
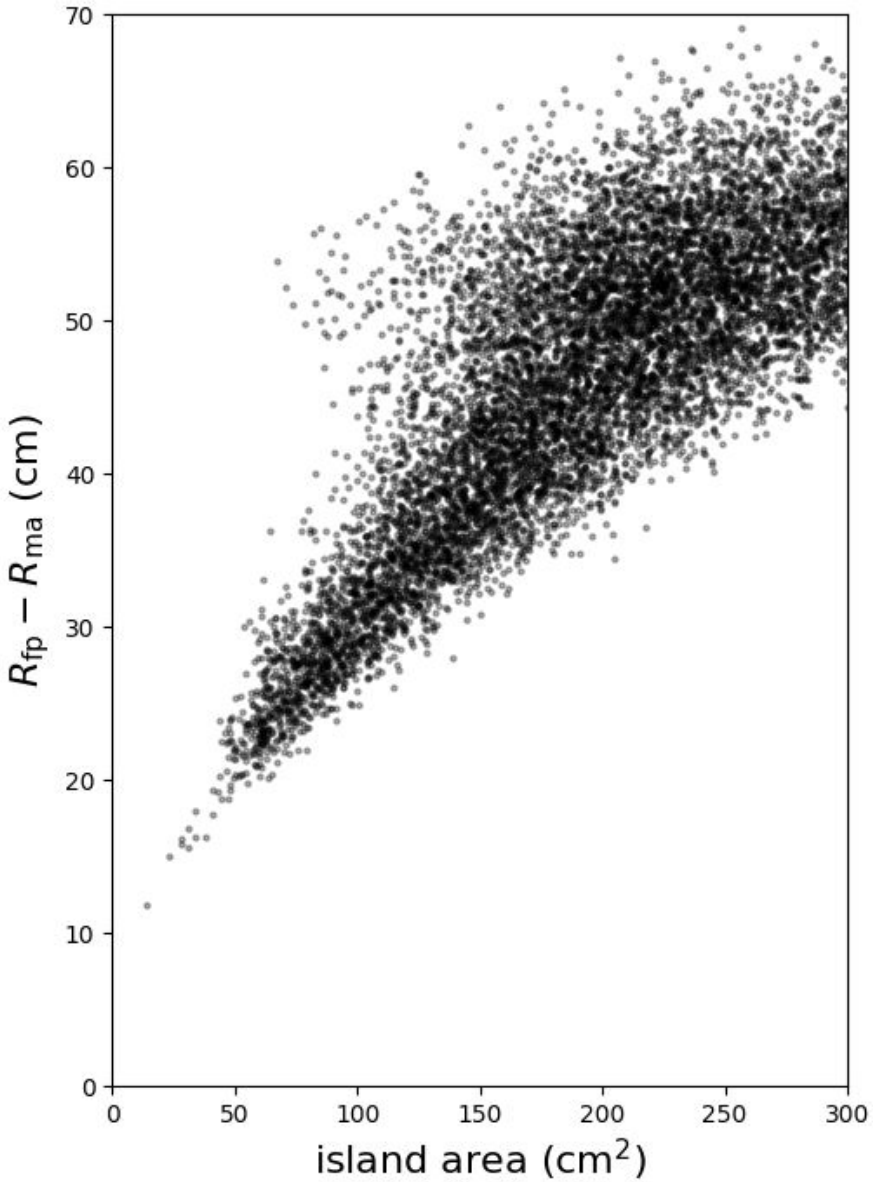






Island area vs trace:





Summary & future work

- We can characterise the magnetic axis and edge island chains using fast tools, which allow us to characterise the operational space of W7-X in vacuum configurations (zero plasma beta, zero current)
- We find (and quantify) some expected results:
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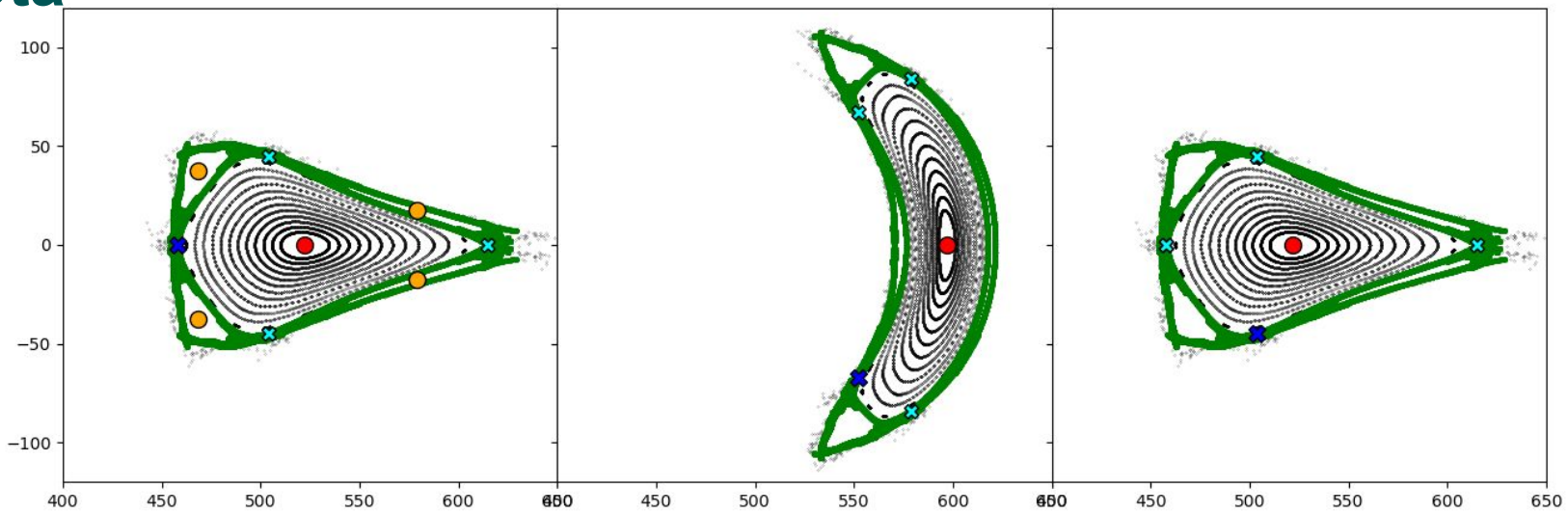
- We find some relatively novel phenomena:
  - **Control coil very differently affects X- and O-points in island chain (localised effects?)**
  - **We find a fast proxy for island size -> possibly useful for finding configurations with internal island chains**
- Future work:
  - **Explaining the “novel phenomena”**
  - **Topological optimisation: finely controlling island divertor magnetic geometry**
  - **Extensions to finite plasma current/beta**

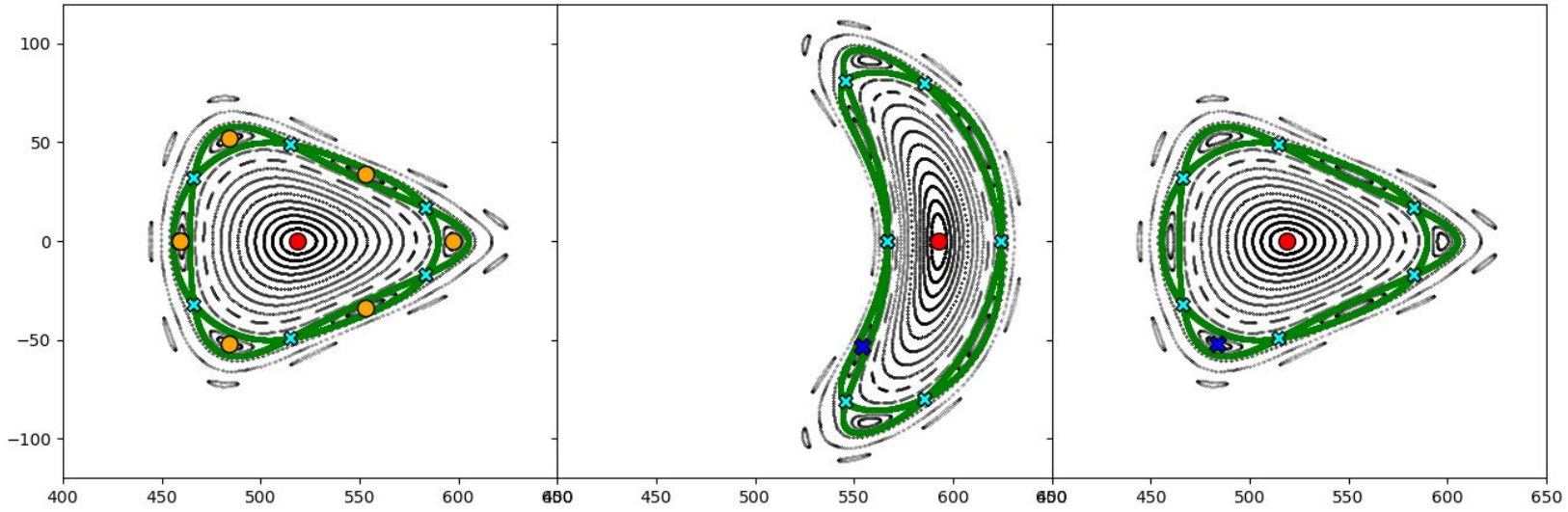
APPENDIX

high and low iota

5/4 island chain
(“high iota”)



5/4 island chain
(“low iota”)



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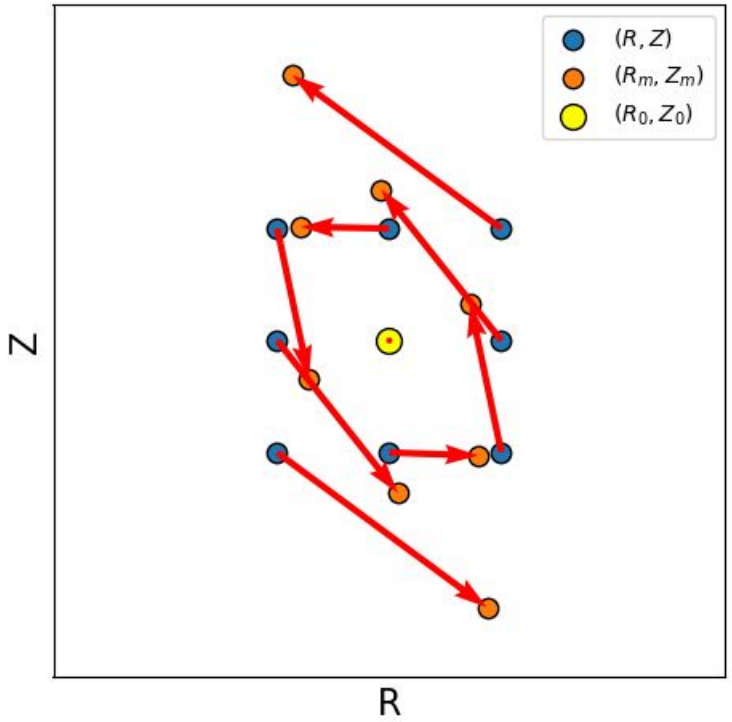

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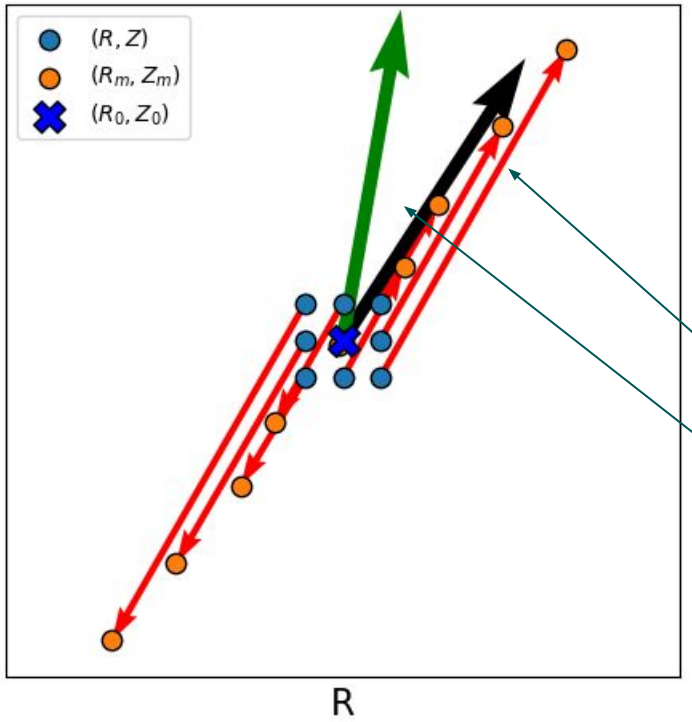

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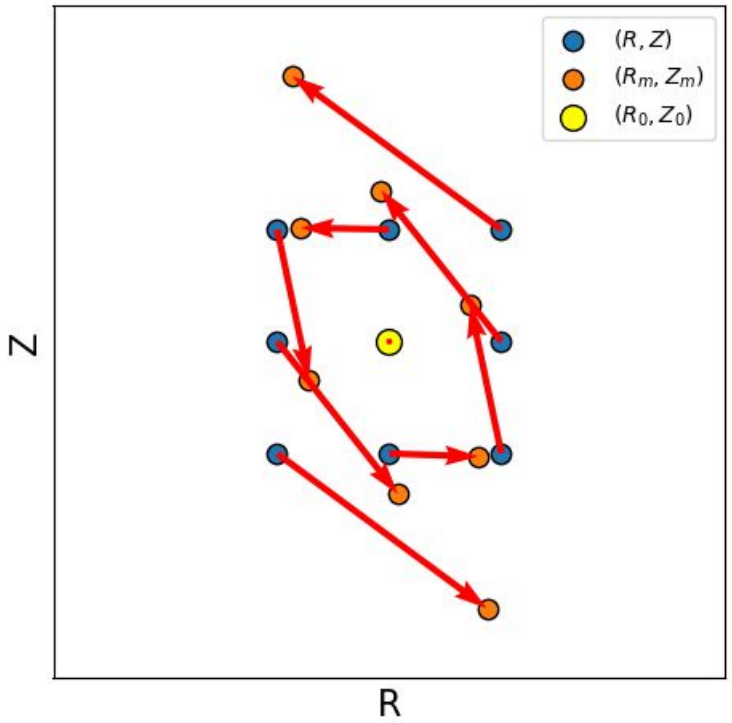
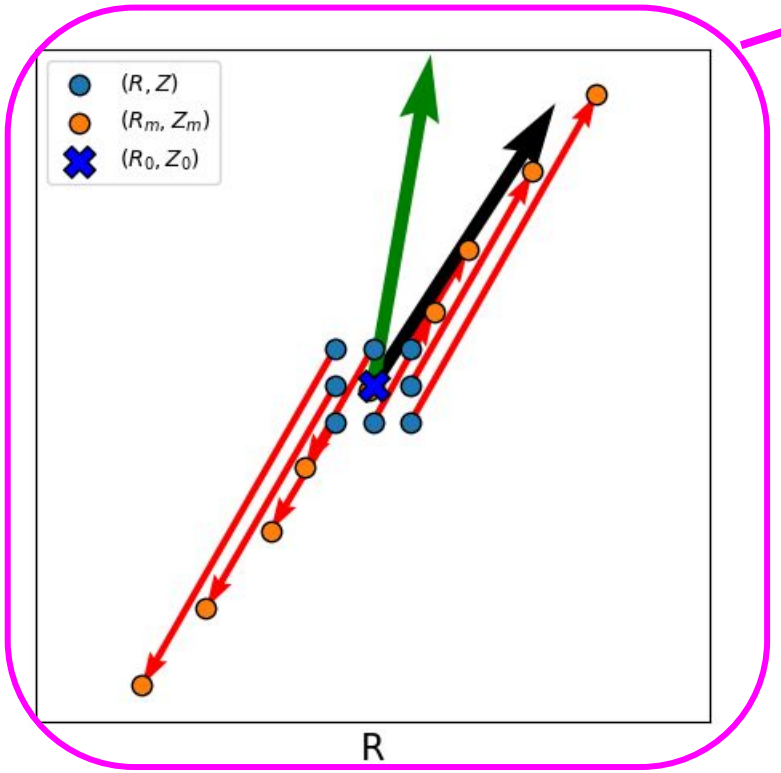
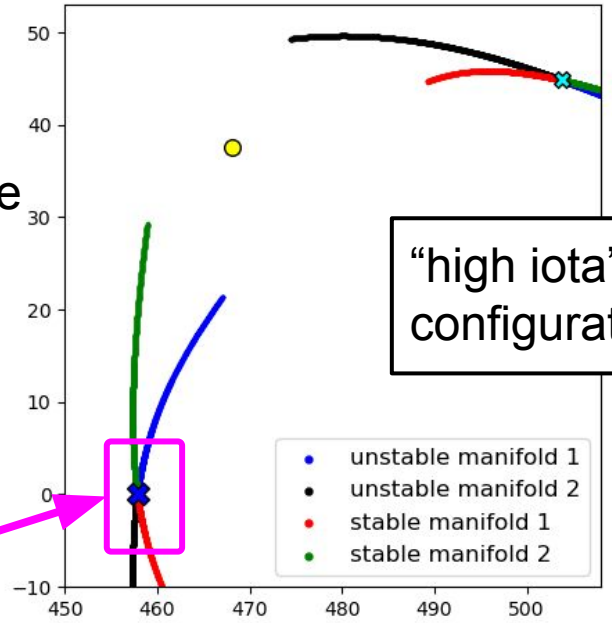
Directions of approach/departure from X-point
(in general, not orthogonal)
(NB these are the eigenvectors of M)

Theory

- By sampling points around the X-point and tracing them, we can find the **island separatrix** (more precisely, the manifolds of the X-point)



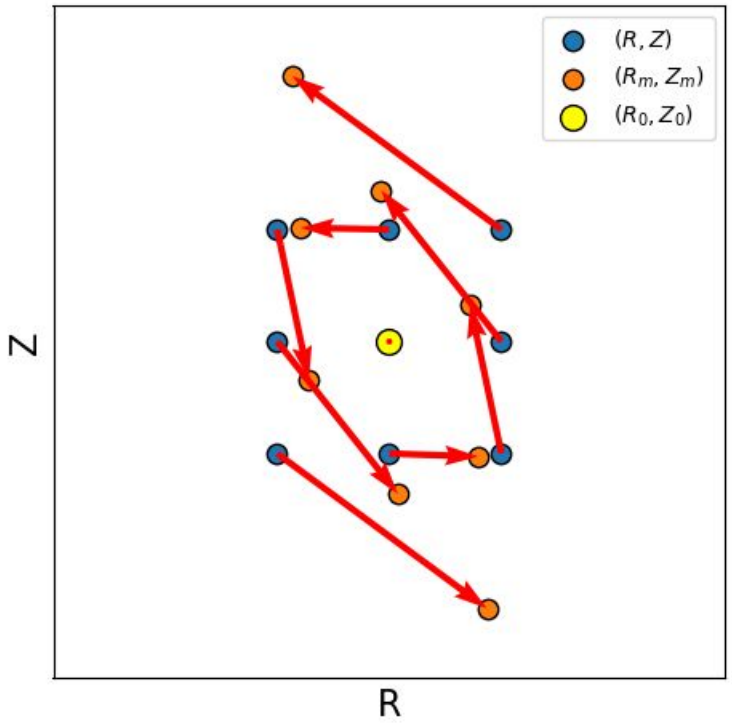


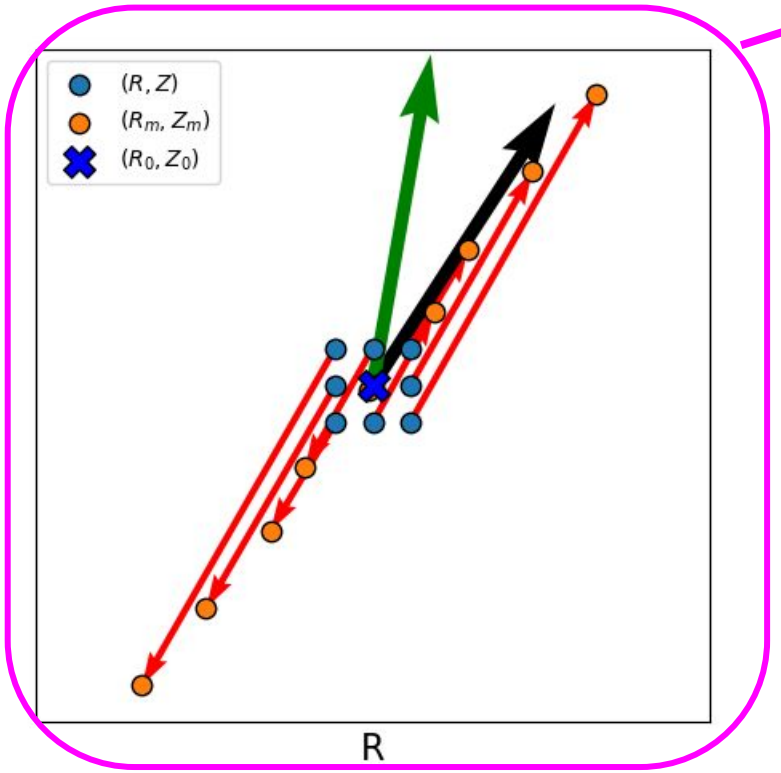


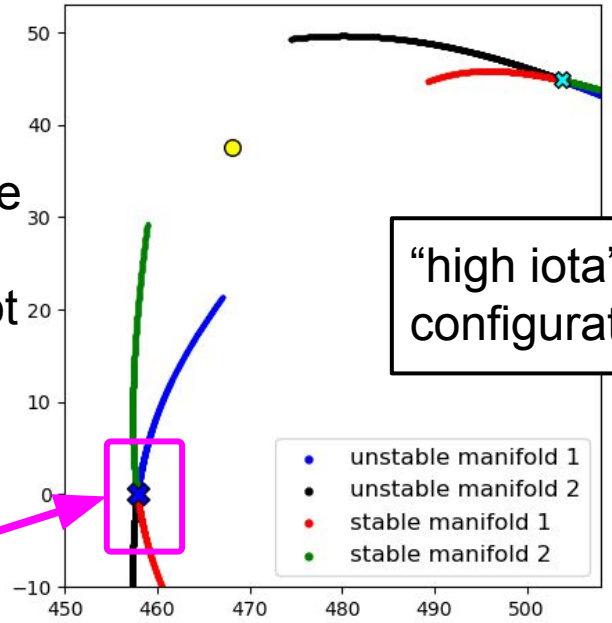
"high iota" configuration

Theory

- By sampling points around the X-point and tracing them, we can find the **island separatrix** (more precisely, the manifolds of the X-point)
- Non-overlapping manifolds = magnetic chaos (good flux surfaces do not exist)







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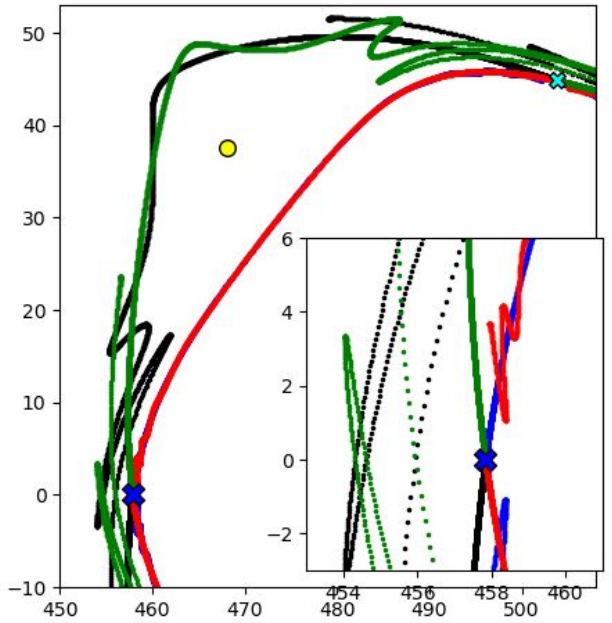
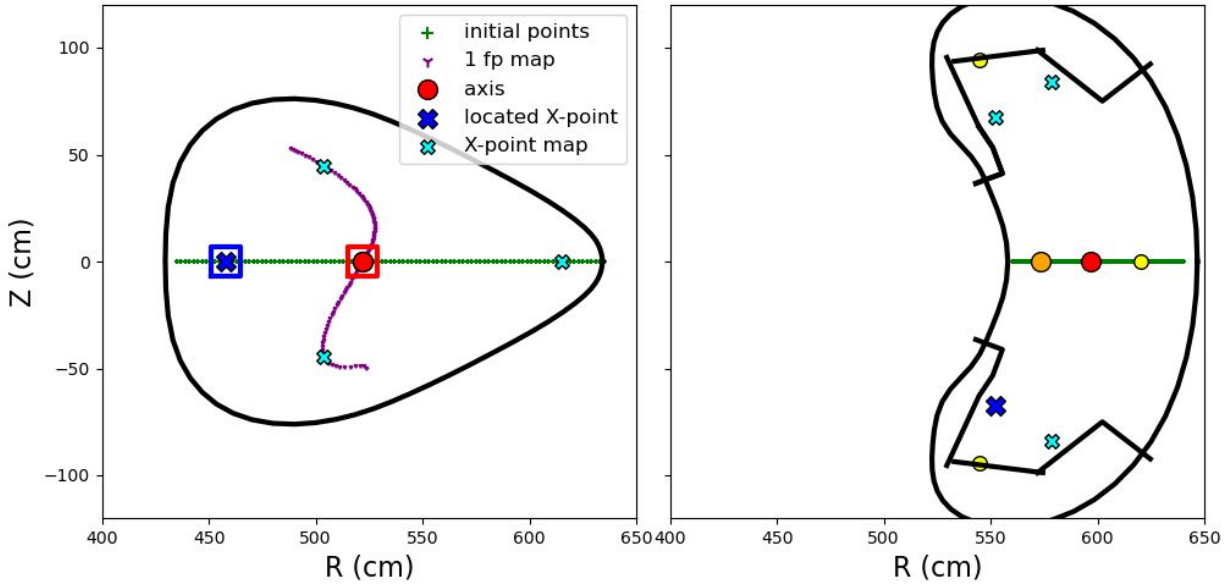
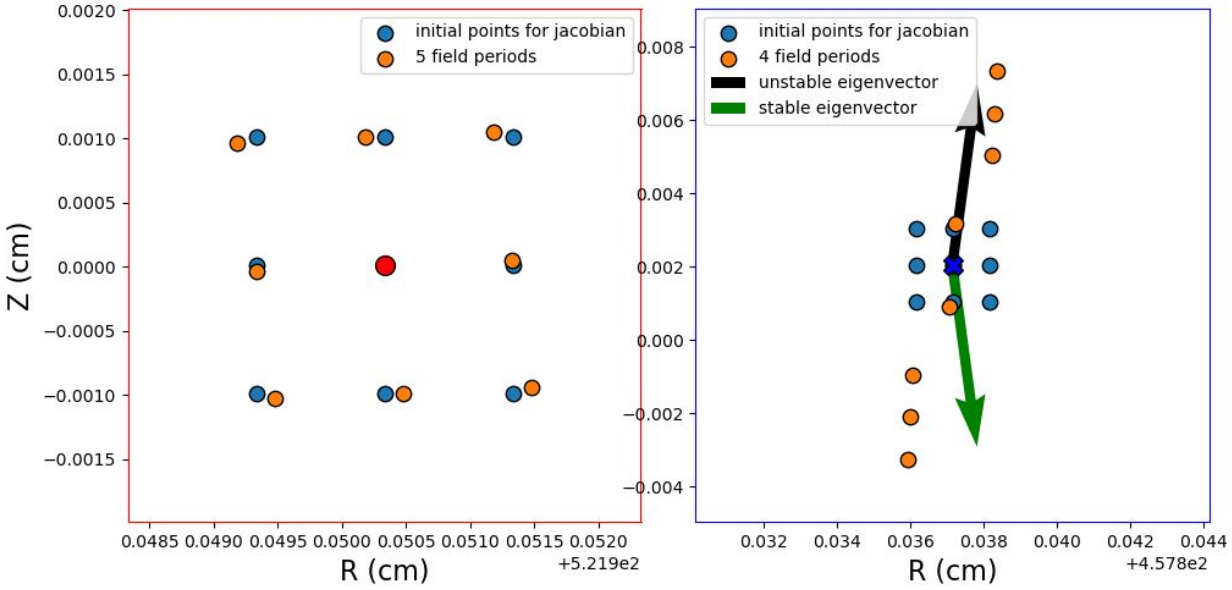
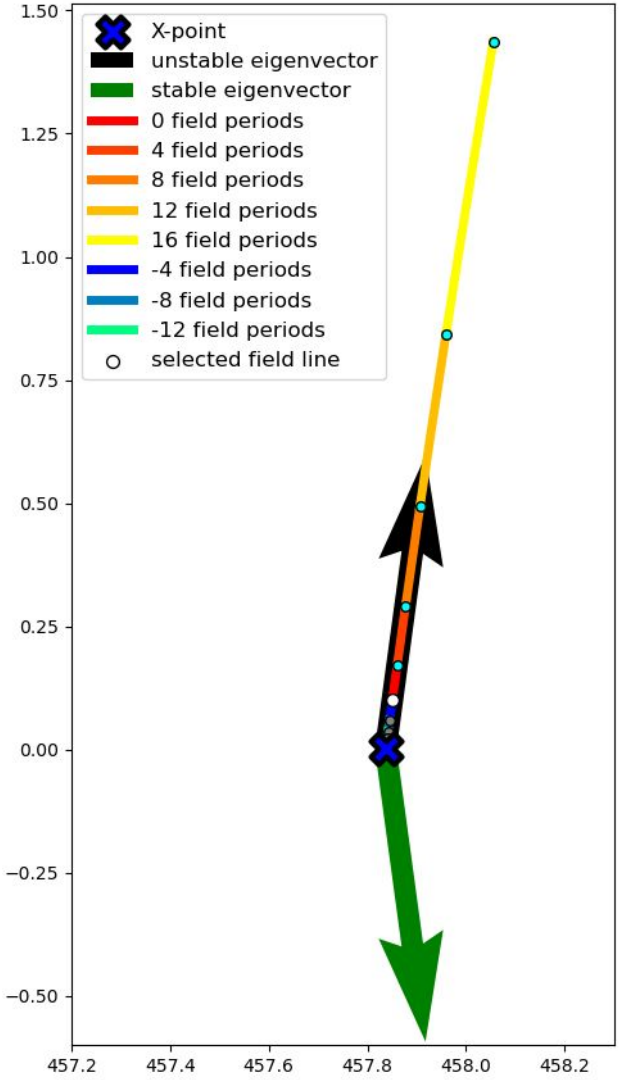
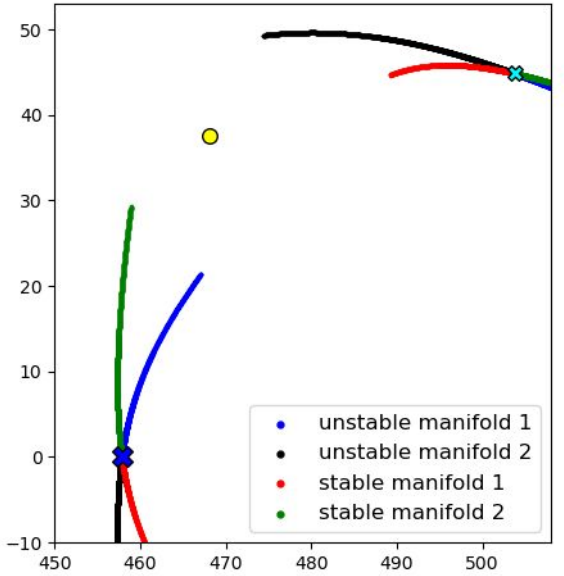


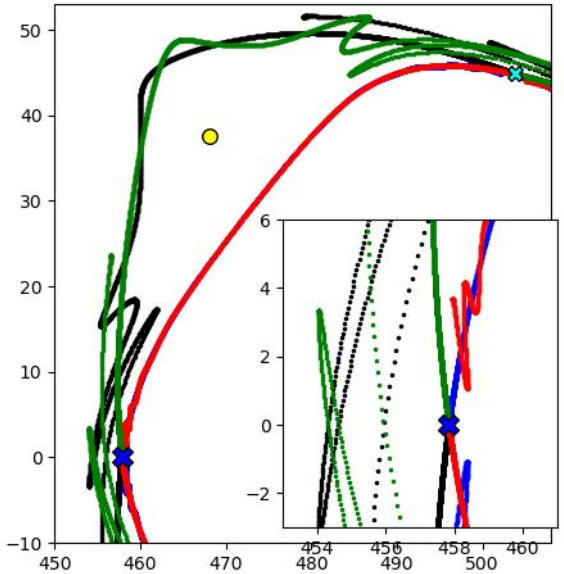
A fast automated analysis: the algorithm (with pictures)









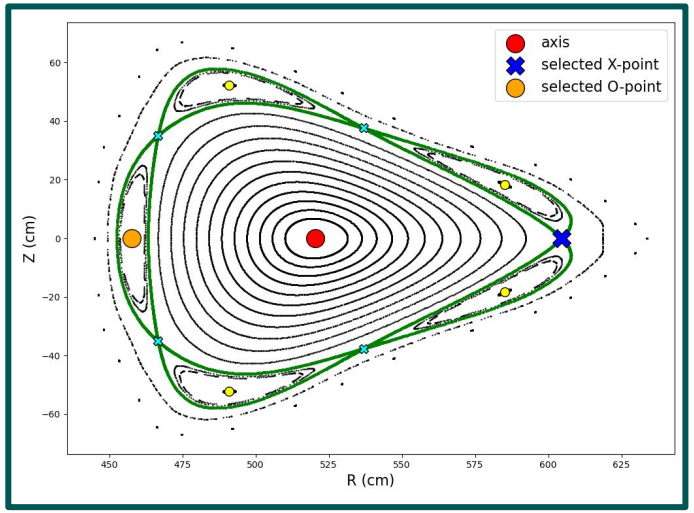


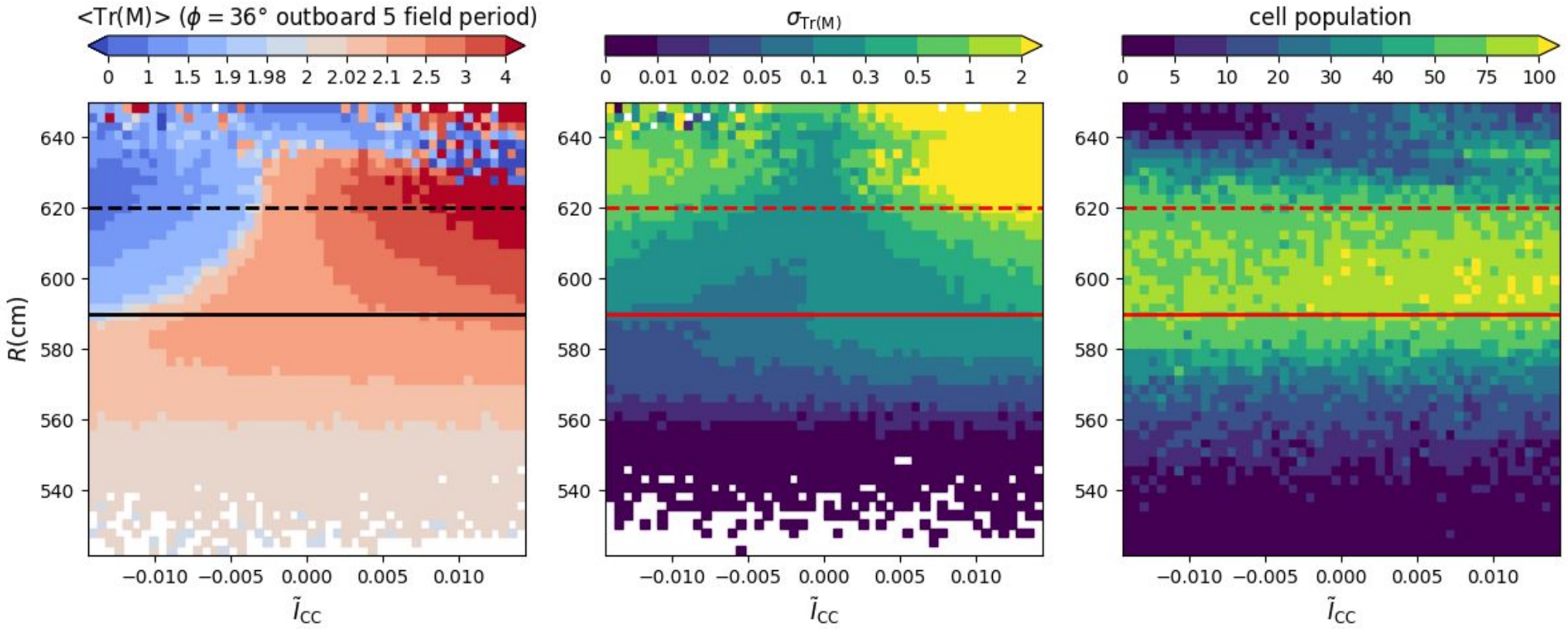
A fast automated analysis: the algorithm (verbose)

1. Specify coil currents
2. Calculate B field on an (R,Z,phi) grid (use precomputed fields for speed)
3. Field line trace a set of points to get (1) an initial guess for the magnetic axis (2) find out if periodicity-4,5,6 fixed points exist (3) if so, an initial guess for their location
    a. **Starting points: a line of points from R=Rmin to R=Rmax with spacing delta R at Z=0 in a stellarator-symmetric toroidal location. Stellarator symmetries forces magnetic axis and X- and O-points to be at Z=0**
    b. **Each point gets traced 6 field periods. For each #field period, we have (R,Z) of the point in this toroidal location, and hence its displacement from initial point**
    c. **Magnetic axis is fixed point of the 1-field period map so maps so displacement after 1 field period is zero. We find the point with minimum displacement and use as the initial guess for magnetic axis**
    d. **Same principle for 4,5,6-fp map.**
4. Find the magnetic axis and its jacobian
    a. **Use scipy's root finder to find the (R,Z) for which (dR, dZ)=(0,0)**
    b. **Trace a bunch of points around the magnetic axis for 1 field period and use this to calculate the jacobian (linear approximation of the Poincare map around the fixed point). This tells us the on-axis rotational transform**
5. Repeat step 4 for the 4,5,6-fp fixed points, if any
6. For X-points, calculate the eigenvectors of the jacobian and use this to trace out the island chain
    a. **Near an X-point, the eigenvectors coincide with the manifolds of the X-point i.e. the separatrix**

Role of control coil: *scan of > 2x10^5 configurations*

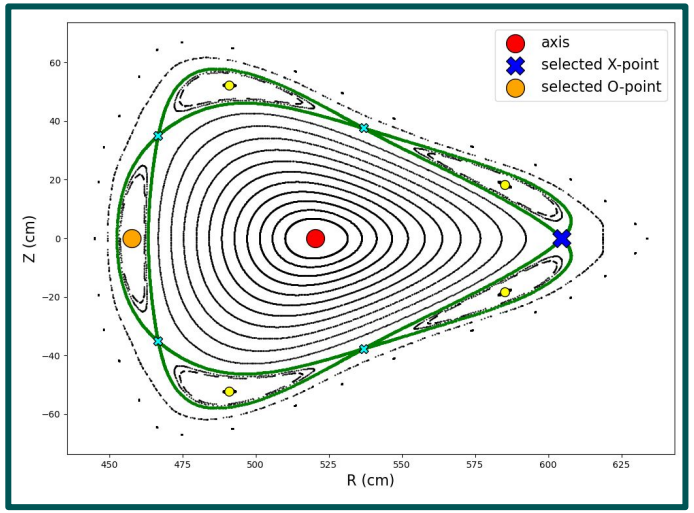
- Control coils affect fixed point trace
- Bigger effect at larger R (closer to the coils)

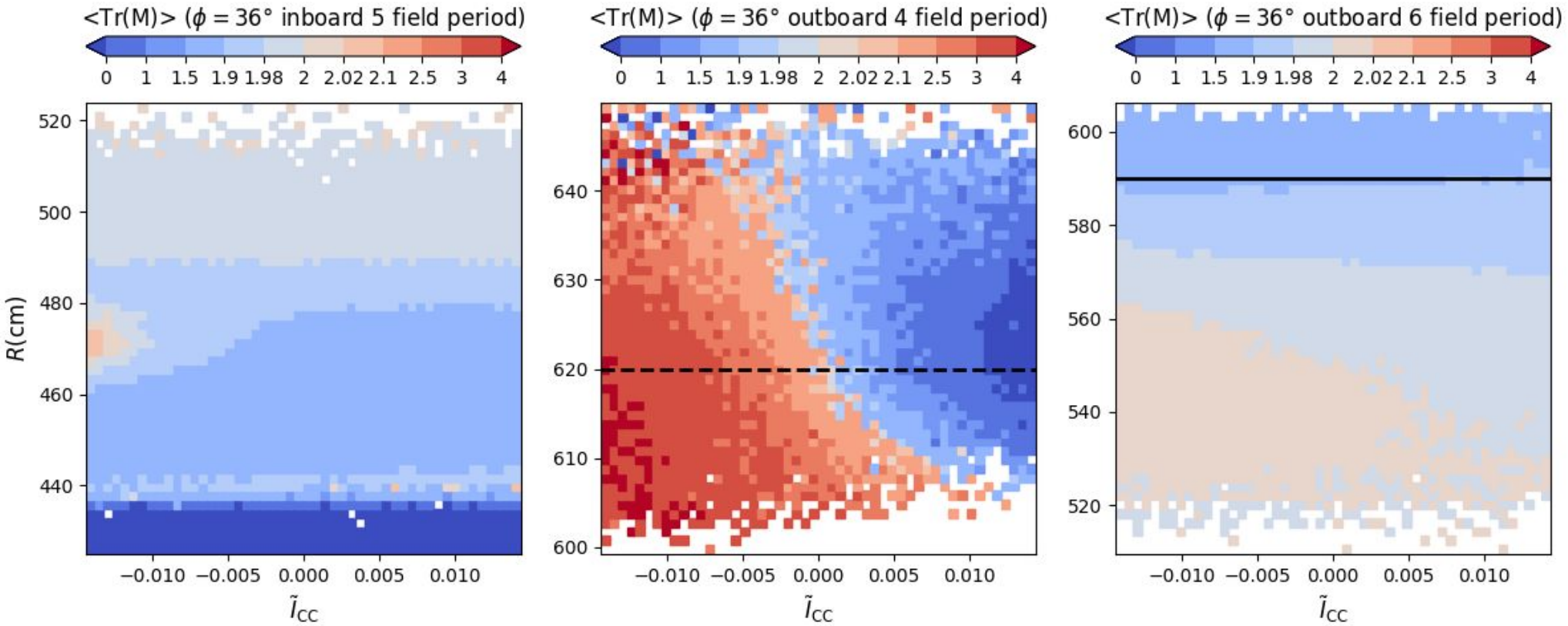




Role of control coil: *scan of > 2x10^5 configurations*

- Control coils affect fixed point trace
- Bigger effect at larger R (closer to the coils)





Selected configurations