



EnR-MOD: Geometric Orbital Spectrum Analysis of Resonant Ion-Mode Interactions and Transport in Tokamaks and Stellarators (GOSARIT)

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The confinement of energetic particles and their collective interaction with modes remain central challenges for the performance of magnetic confinement fusion devices. This project introduces a unified framework for analyzing resonant mode-particle interactions, focusing on energetic ion dynamics, across both tokamak and stellarator configurations.

Our methodology is based on a Hamiltonian formulation of guiding-center dynamics, expressed in canonical Action-Angle variables, building directly on unperturbed particle motion, to:

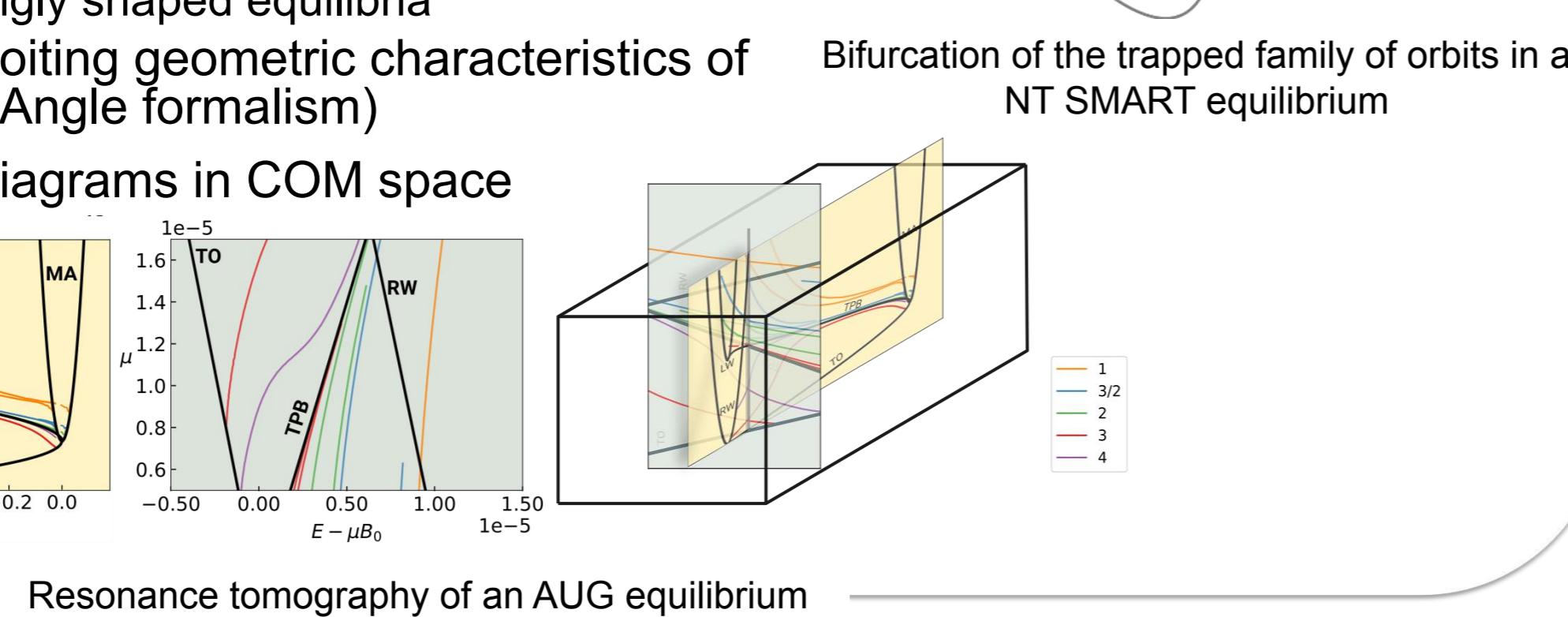
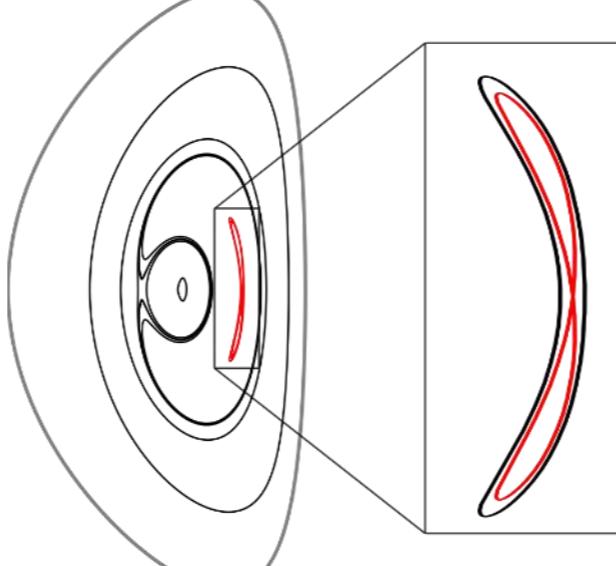
- provide a computationally efficient geometric calculation of orbital frequency spectra
- facilitate a systematic treatment of resonant mode-particle interactions
- expose the direct link between plasma equilibrium shaping and the resonance structure governing transport and stability
- construct predictive resonance diagrams
- determine transport barriers
- characterize island overlap
- provide intuitive understanding of the underlying transport mechanisms under the presence of multi-scale perturbations

WP1. Background Magnetic Field Configuration (EQUIL)

- Compile a collection of magnetic equilibria for tokamaks and stellarators to support the actions of WPs 2-5:
- Representation in Boozer coordinates.
- Extra step for stellarators: Decomposition into integrable and non-integrable parts

WP2. Orbital Frequency Spectrum and its dependence on the shape of the background magnetic field (OFS)

- Analysis on unperturbed dynamics based on integrable background equilibrium field
- Bifurcation analysis
 - Determine domains of families of orbits
 - Reveal bifurcations in strongly shaped equilibria
- Frequency calculation exploiting geometric characteristics of unperturbed orbits (Action-Angle formalism)
- Calculation of resonance diagrams in COM space

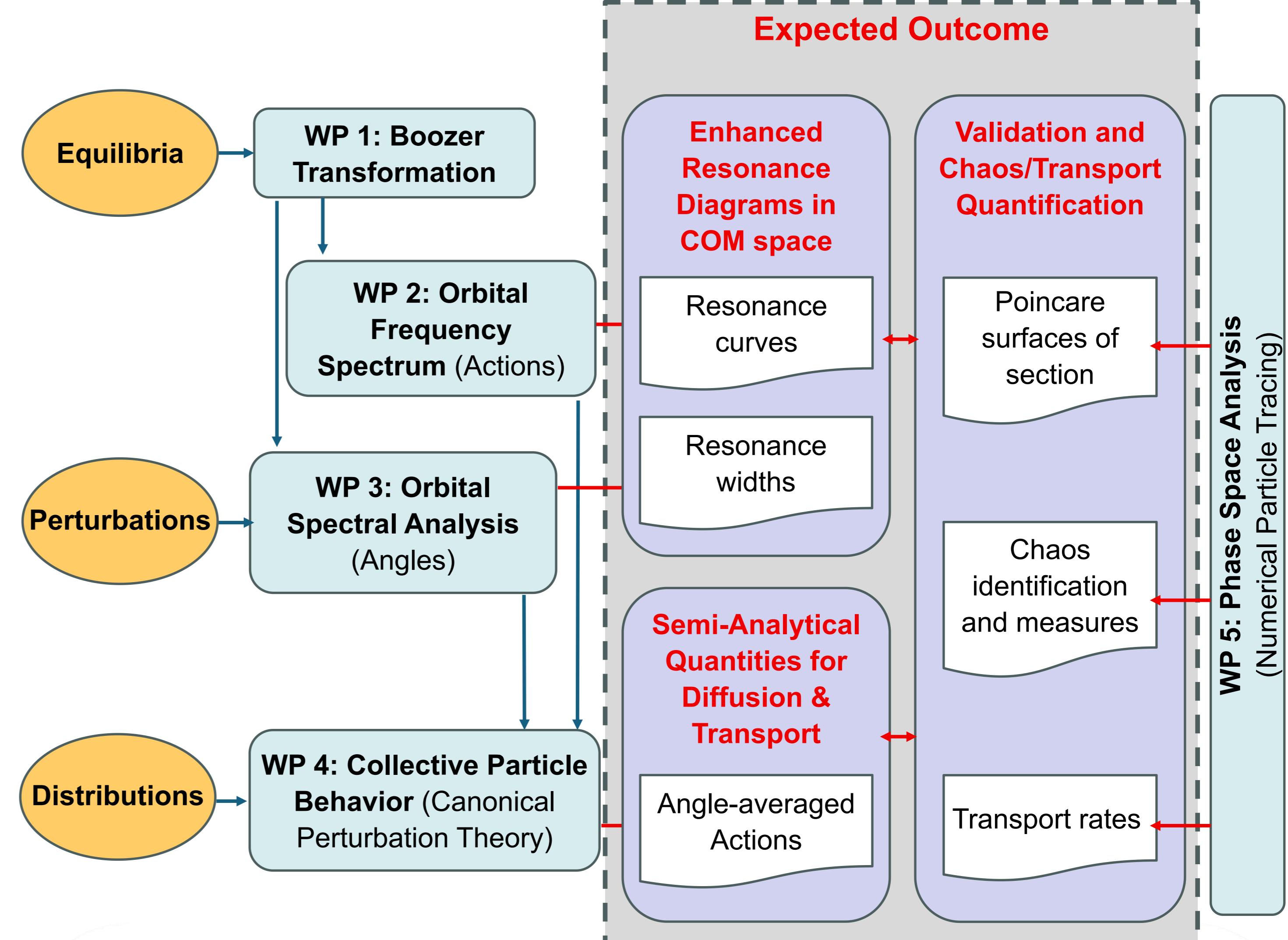


WP4. Quantitative semi-analytic results (Canonical Perturbation Theory) for transport (CPT)

- Semi-analytic calculation of the collective particle behavior in terms of phase-space transport as well as its effect on particle-driven modes
- Based on Canonical Perturbation Theory (CPT)
- Account for particle distribution functions expressed in terms of the Actions (or COM)
- Quantitative results for radial and momentum transport:
 - Angle-averaged Action variations → semi-analytic results for radial and momentum transport.
 - Diffusion and transport coefficients will be calculated without the need of orbit-following numerical algorithms
 - Connection and interfacing with stellarator optimizing algorithms (e.g. STELLOPT suite) will be considered
- Study of particle-driven modes:
 - Calculate driving terms of the mode evolution equations via Angle-averaged Action variations.
 - The particularly important case of EP-driven AEs in burning plasmas will be considered

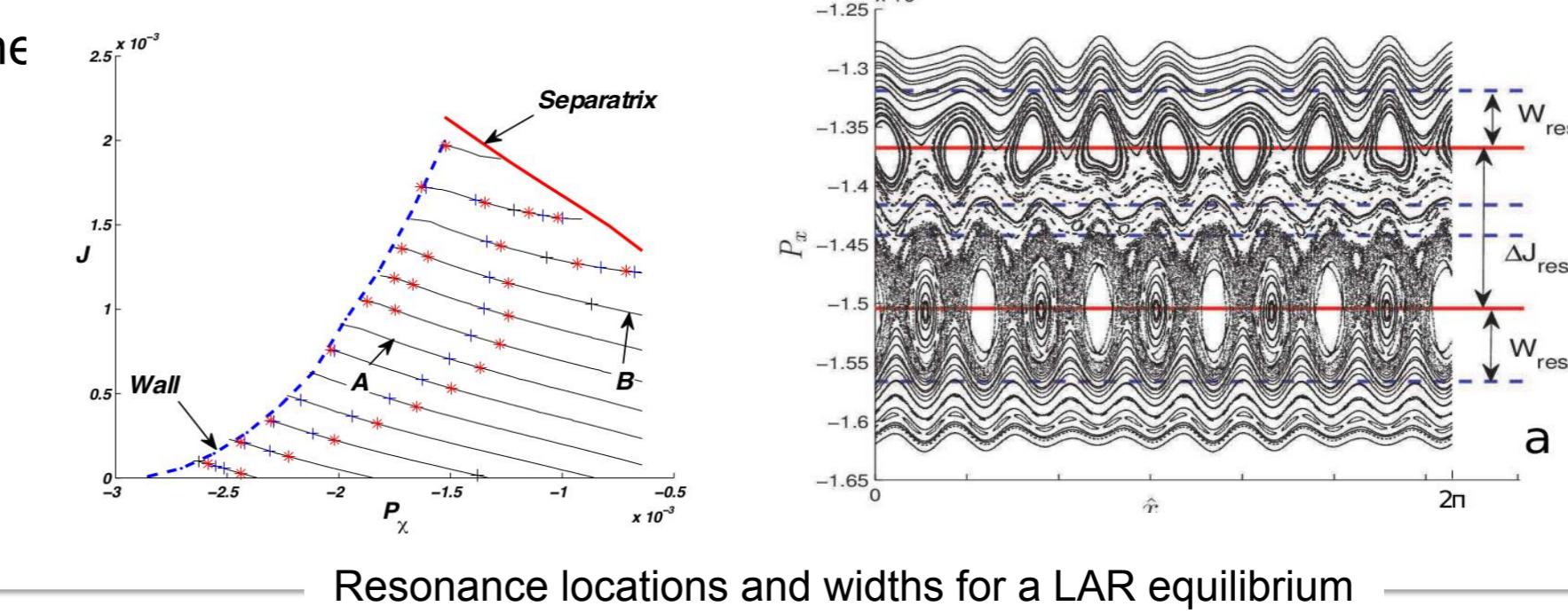
Connection to other E-TASC projects

- TSVV-G:** provides an interpretive layer that links plasma geometry to energetic particle resonance dynamics. Will make use of calculated distribution functions in COM space for the purposes of WP4.
- TSVV-I:** clarifies the impact of shaping and quasi-symmetry breaking on energetic particle confinement and develops reduced models that can in the future be incorporated into stellarator optimization objective functions.
- TSVV-02:** makes use of calculated equilibria for resonance analysis in NT vs PT studies



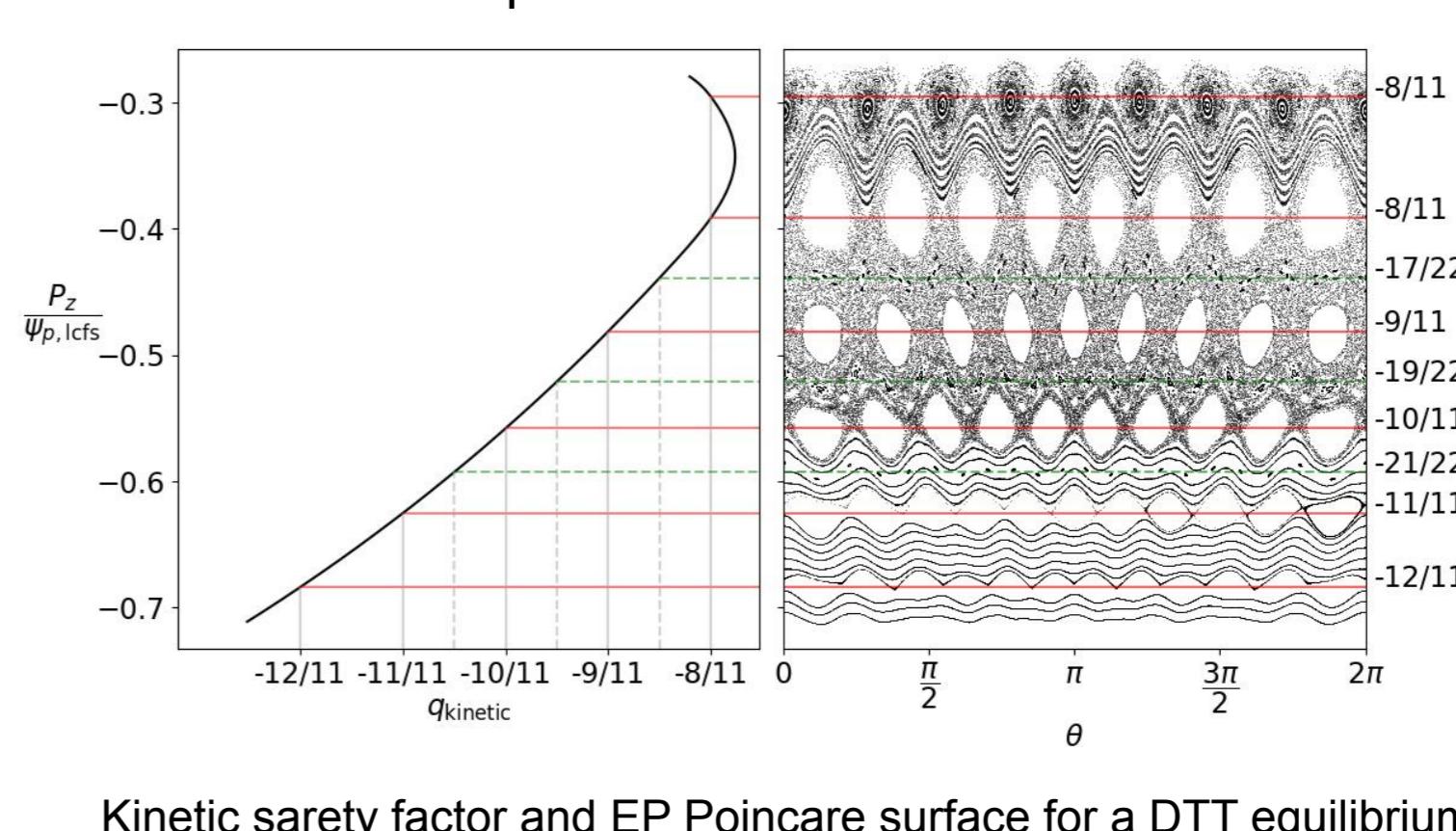
WP3. Orbital Spectrum Analysis of multi-scale perturbations (OSA)

- AA transformation of perturbative modes.
- Determine the perturbation spectrum in the particle unperturbed frame of reference (Fourier analysis in terms of orbital frequencies)
- Island widths and enhanced resonance diagrams
 - Derive resonant island widths
 - Add thickness



WP5. Particle tracing Phase-Space analysis of Transport (PST)

- The only WP requiring numerical integration of the perturbed dynamical system.
- Validate, assess and complement the methods developed in WPs 2-4 through:
 - Poincaré surfaces of section for trajectory visualization,
 - Finite-Time Lyapunov Exponents (FTLE), Smaller Alignment Index (SALI), Lagrangian Descriptors (LD) for chaos quantification and transport structure identification.



Expected outcome

A computationally efficient procedure, applicable to a wide range of equilibria, that not only advances theoretical understanding but also provides practical guidance for engineering plasma configurations tailored to optimize energetic particle confinement, heating efficiency, and stability in next-generation fusion devices.

