MAX-PLANCK-INSTITUT
FÜR PLASMAPHYSIK

Investigating the island divertor physics with an extended stellarator two-point model

Wendelstein 7-X

N Maaziz, F Reimold, V R Winters, S O Makarov and Y Feng

EUROfusion

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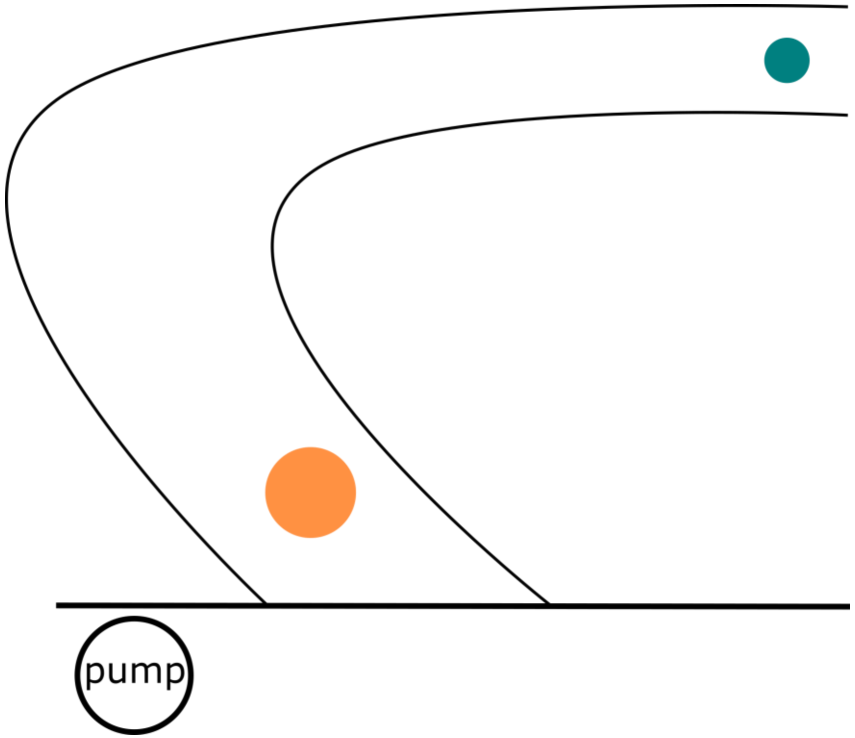
Motivation: Density build up

- Defined as achievable **target density** (downstream) as a function of **separatrix density** (upstream)

$$n_d(n_u)$$



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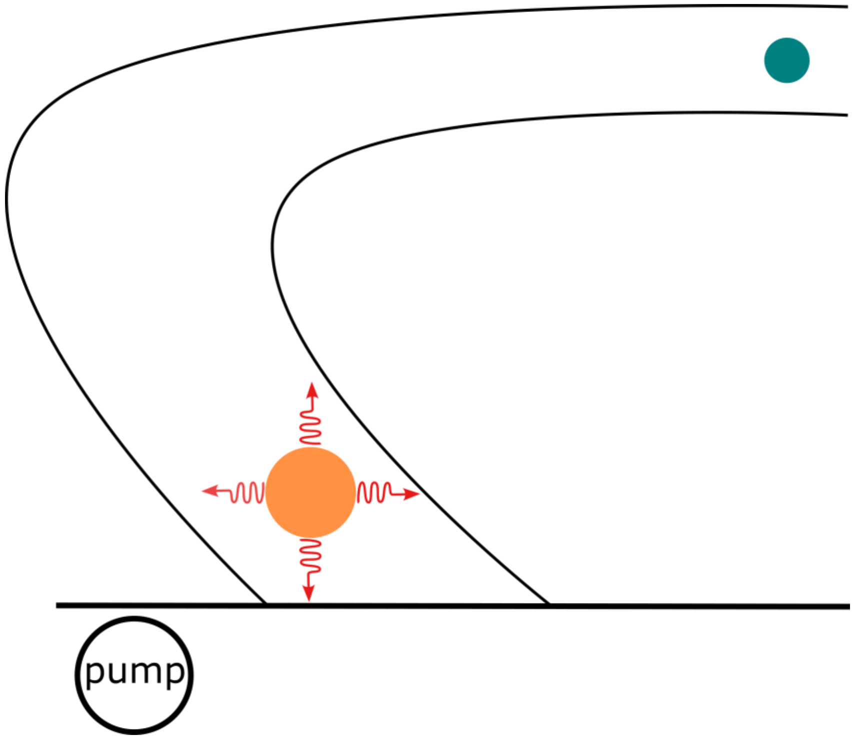
- Improved divertor **performance**:
  - **Power dissipation** via radiation

$$\epsilon = c_z n_d^2 L_Z$$



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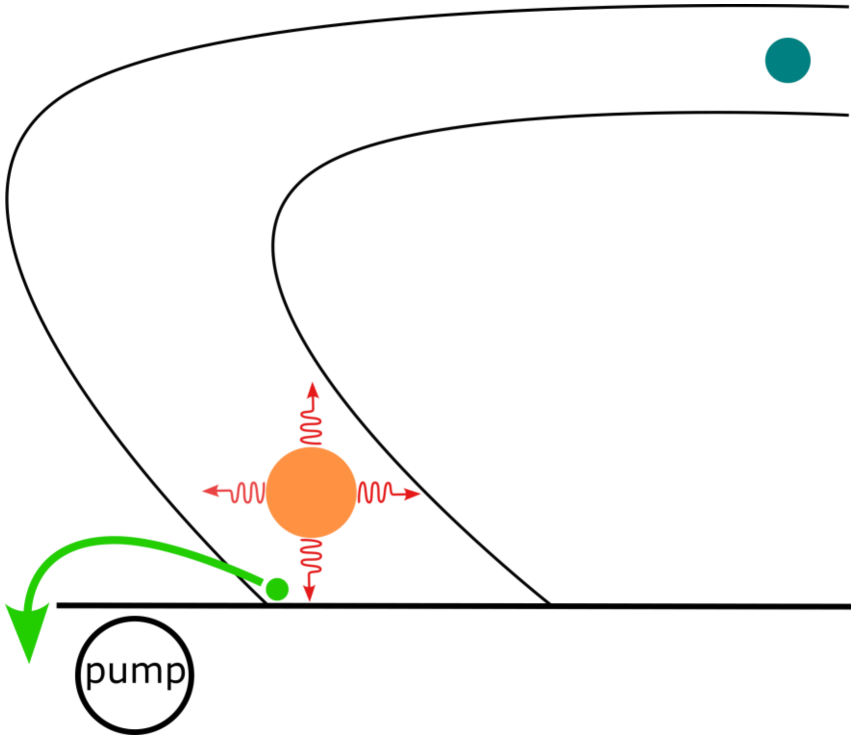
  - **Neutral pressure** via increased recycling flux

$$\Gamma = n_d c_{sd}$$



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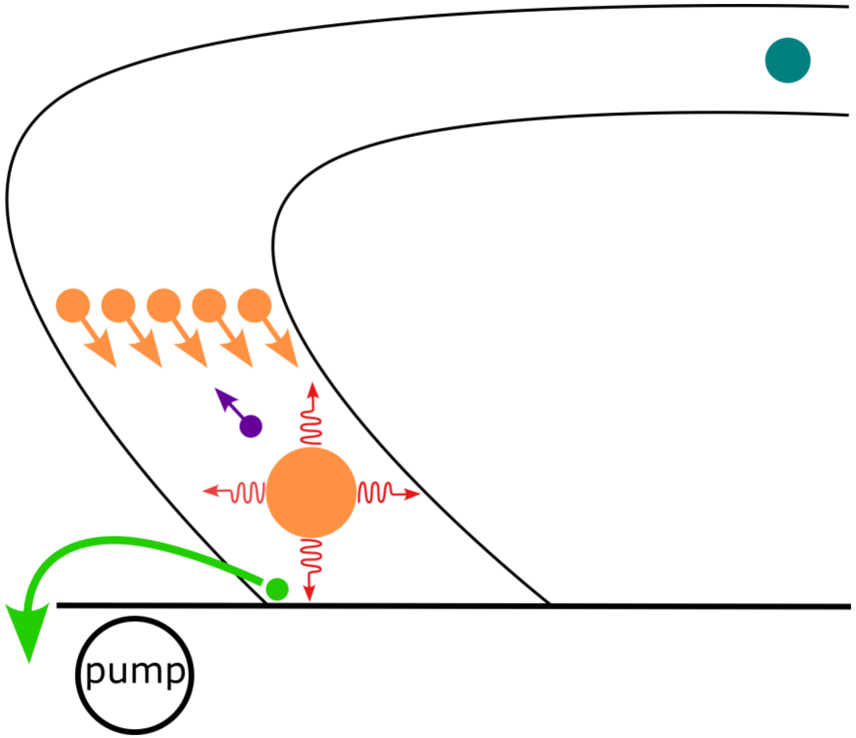
$$\Gamma = n_d c_{sd}$$

  - **Impurity retention** via increased drag force

$$F_D = m_z \frac{v_I - v_Z}{\tau_s}$$

$$\tau_s \propto \frac{1}{n_d}$$



Motivation: Density build up

- Define
  eam)
- Impro
  - **Pow**
  - **Neu**
  - **Imp**

$\boldsymbol{n_d}$

- ITER requires divertor neutral pressure $\boldsymbol{p_n \approx 10}$ **Pa**
- AUG routinely measures $\boldsymbol{p_n \approx 1}$ **Pa**
- So far only limited neutral pressures have been measured in W7-X
  - $\boldsymbol{p_{n,max} \approx 0.18}$ **Pa**
- **Higher recycling conditions are necessary to ensure the reactor-relevance of the island divertor in terms of pumping efficiency.**

Outline

- Extending the stellarator two-point model (STPM)
- Sensitivity of the scalings around a reference set of parameters
- Comparison against EMC3-Eirene
- Predicting the *diffusion-limited regime*
- Target-Shadow-Region-induced pressure step

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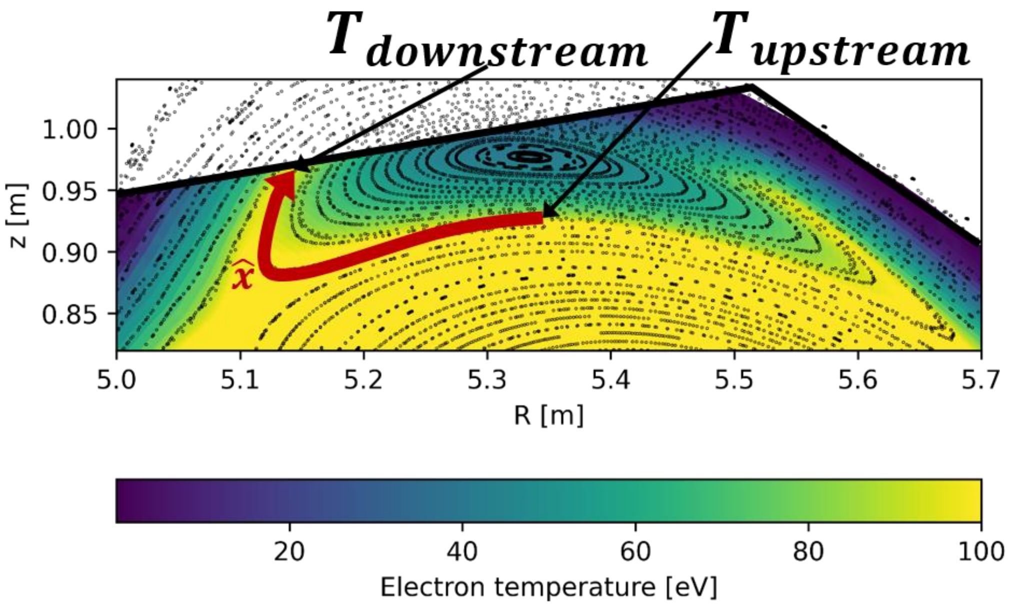
Introduction: Stellarator Two-Point Model (STPM)

Introducing perpendicular transport in the energy equation:

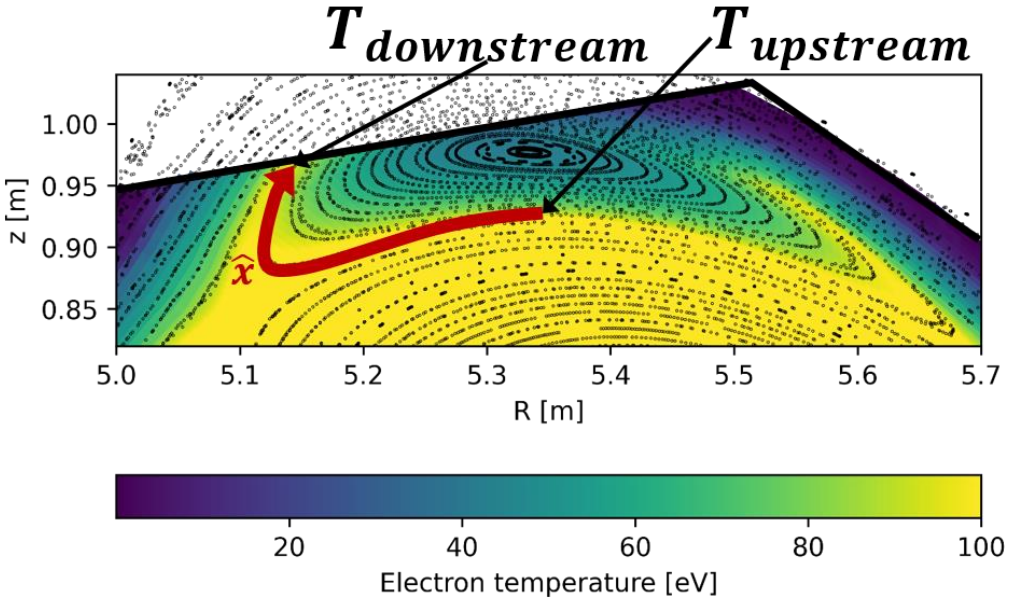
- $\underbrace{\Theta \frac{\mathrm{d}}{dx}\left(-\kappa_{e,i} T_{e,i}^{5/2} \Theta \frac{dT_{e,i}}{dx}\right)}_{\text{parallel heat conduction}} + \underbrace{\boxed{\frac{d}{dx}\left(-\chi_{e,i} n \frac{dT_{e,i}}{dx}\right)}}_{\text{perpendicular heat diffusion}} = 0$
- $\Theta \frac{d}{dx} \underbrace{\left(n v_{i\parallel}^2 + \frac{p}{m_i}\right)}_{\text{total pressure}} = \underbrace{S_{mom}}_{\text{cross-field momentum}}$
- $\underbrace{q_\parallel}_{\text{total power}} = \underbrace{\gamma n_d c_{sd} T_d}_{\text{power to the target}}$



Introduction: Stellarator Two-Point Model (STPM)

Integration gives STPM :

- $\underbrace{T_u^{7/2} = T_d^{7/2} + \frac{7q_\parallel L_C}{2\kappa_e}}_{\text{tokamak two-point model}} \underbrace{\boxed{-\frac{7\chi(n_u+n_d)}{4\kappa_e\Theta^2}(T_u - T_d)}}_{\text{stellarator-specific term}}$
- $p_u = 2p_d\underbrace{(1 + f_m)}_{\text{momentum losses}}$ with $\underbrace{f_m = \frac{\alpha}{\sqrt{T_d}}}_{\text{stems from counter-streaming flows}}$
- $\underbrace{q_\parallel = \gamma n_d c_{sd} T_d}_{\text{unchanged}}$



*[Y. Feng et al 2006 Nucl. Fusion **46** 807]*

Extending the Stellarator Two-Point Model (STPM)

Introducing new parameters related to volumetric sources/sinks:

- Convected power fraction f_{conv}
  - competes with parallel conduction and perpendicular diffusion

- Momentum loss factor f_{mom}
  - more general parametrization

- Target-localized power dissipation fraction f_{cool}
  - e.g., radiation, ionization, charge exchange

- Not included in this work: flux expansion

Extended STPM:

$$T_u^{7/2} = T_d^{7/2} + \frac{7(1 - f_{conv})q_{\parallel}L_C}{2\kappa_e} - \frac{7\chi(n_u + n_d)}{4\kappa_e\Theta^2}(T_u - T_d)$$

$$p_u = \frac{2p_d}{1 - f_{mom}}$$

$$q_{\parallel} = \frac{\gamma n_d c_{sd} T_d}{1 - f_{cool}}$$

Recycling regimes

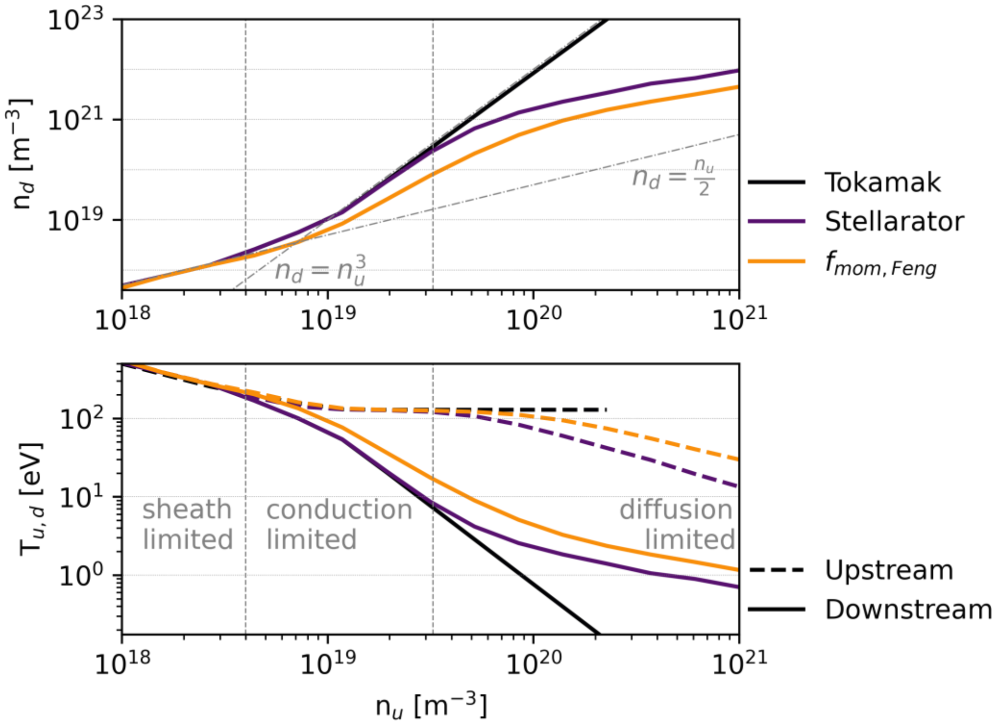
Two recycling regimes observed in tokamaks:

- **Sheath-limited** regime
  - $T_u \sim T_d \rightarrow n_d \propto \frac{n_u}{2}$
- **Conduction-limited** (high recycling) regime
  - T_d drops $\rightarrow n_d \propto n_u^3$

A regime specific to stellarators:

- **Diffusion-limited** regime
  - T_u drops & weaker n_d increase and T_d decrease
  - Transition happens at target temperatrue $T_{d,diff}$

  - **Limits target cooling, density build-up and upstream pressure**
  - (sputtering limit requires $T_d < 5\ \mathrm{eV}$)



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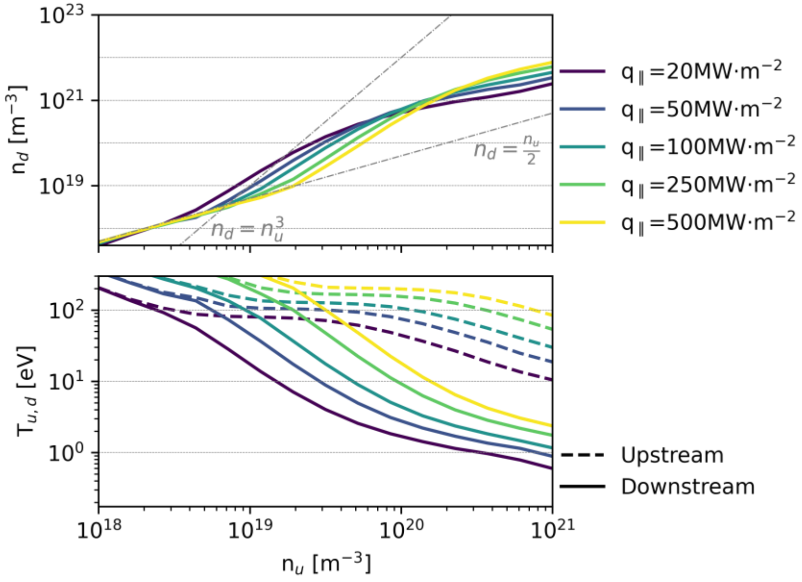
STPM parameter sensitivity around a reference case

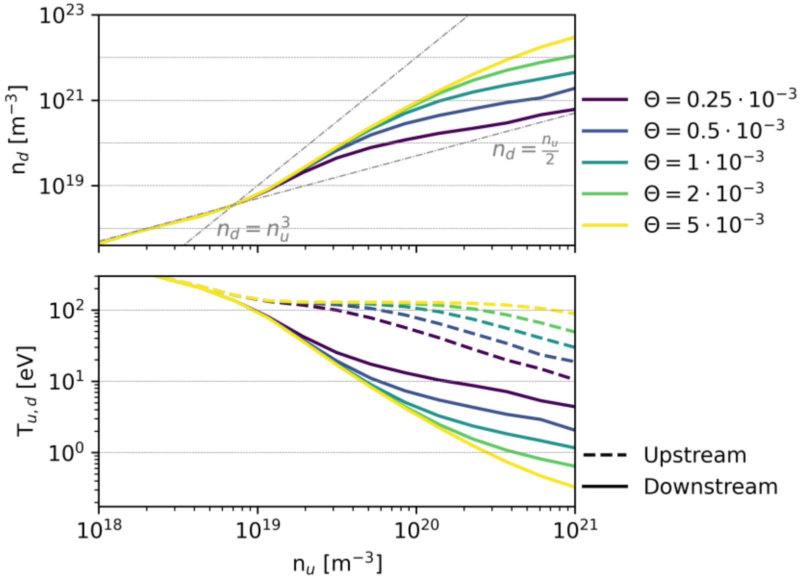
Reference parameters: $q_\parallel = 100\ \mathrm{MW} \cdot \mathrm{m}^{-2}$, $L_C = 180\ \mathrm{m}$, $\chi = 1.5\ \mathrm{m}^2 \cdot \mathrm{s}^{-1}$, and $\Theta = 10^{-3}$

$1 - f_{mom} = \left(1 + \frac{\alpha}{T_d}\right)^{-1}$ with $\alpha = 2$, and $f_{conv} = f_{cool} = 0$

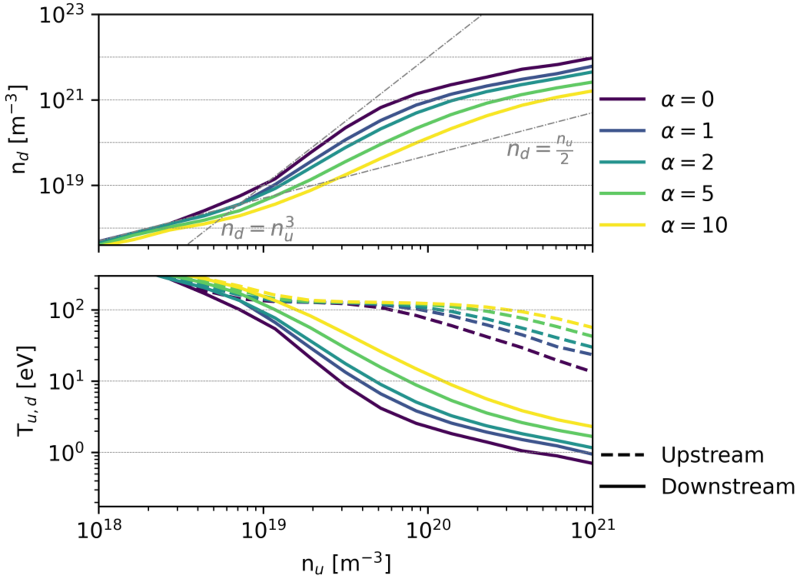
Parallel heat flux $\boldsymbol{q}_\parallel$



Field line pitch $\boldsymbol{\Theta}$



Momentum losses strengh $\boldsymbol{\alpha}$



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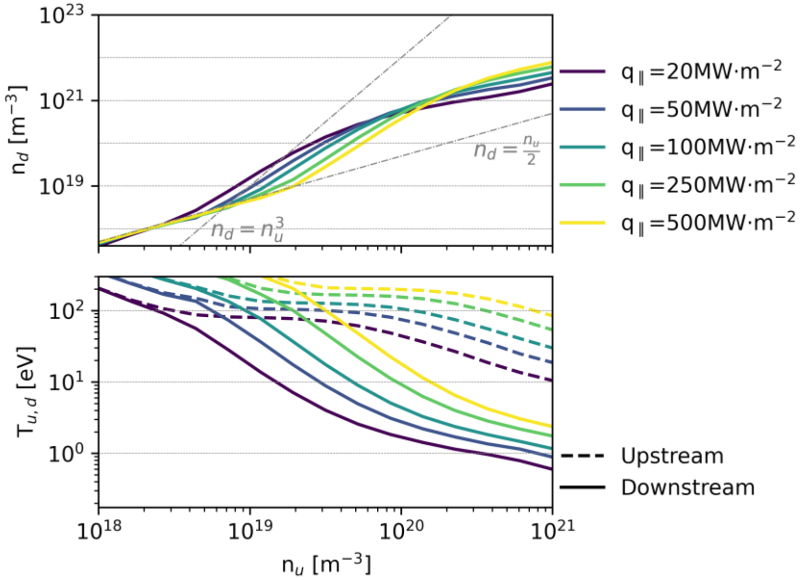

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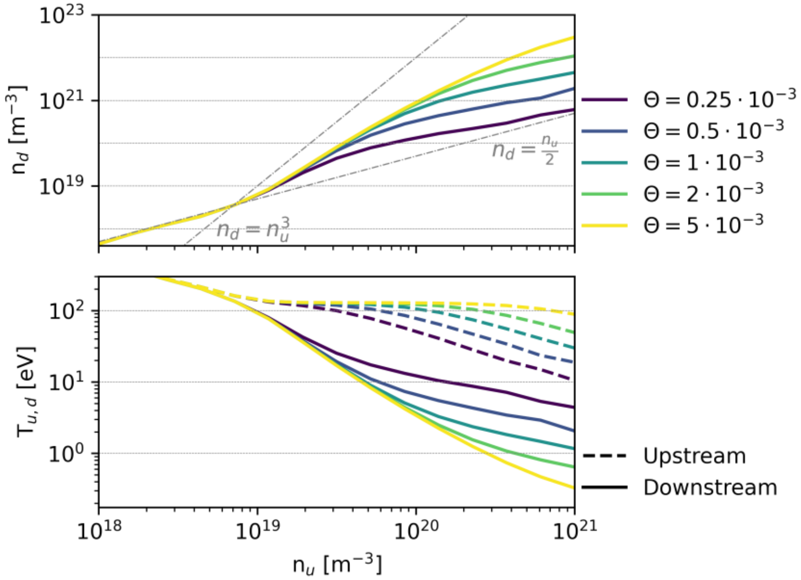

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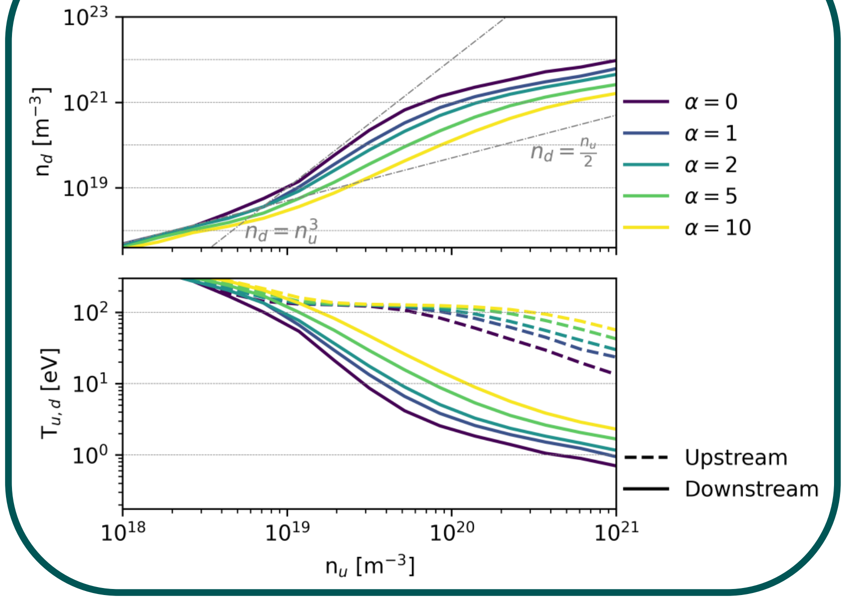
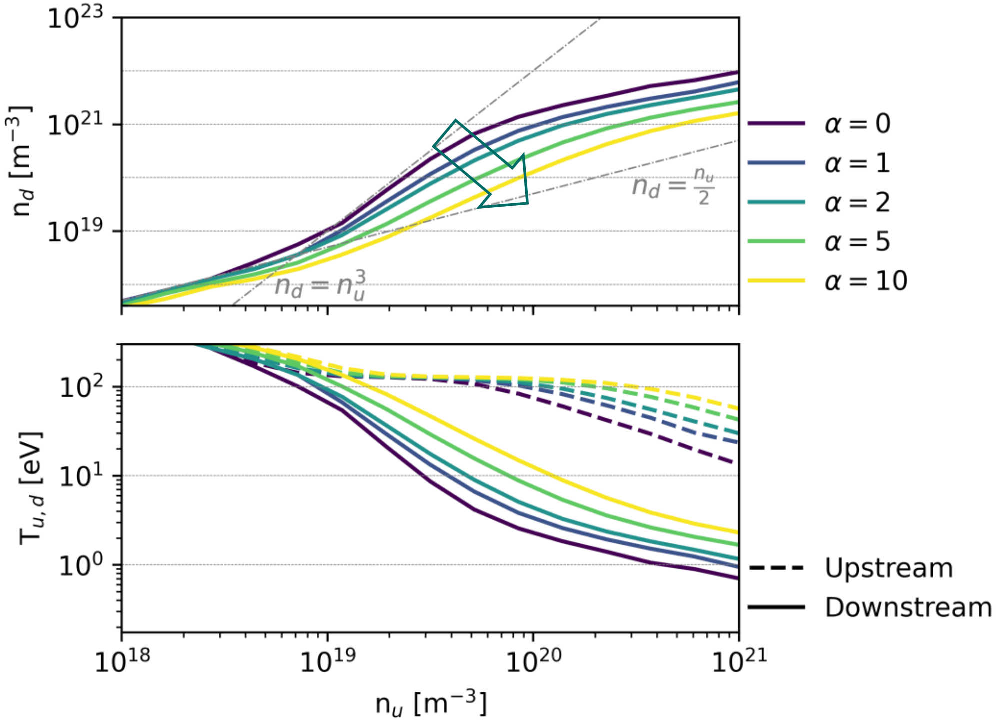


STPM parameter sensitivity: α

- Recycling weakens with increasing α



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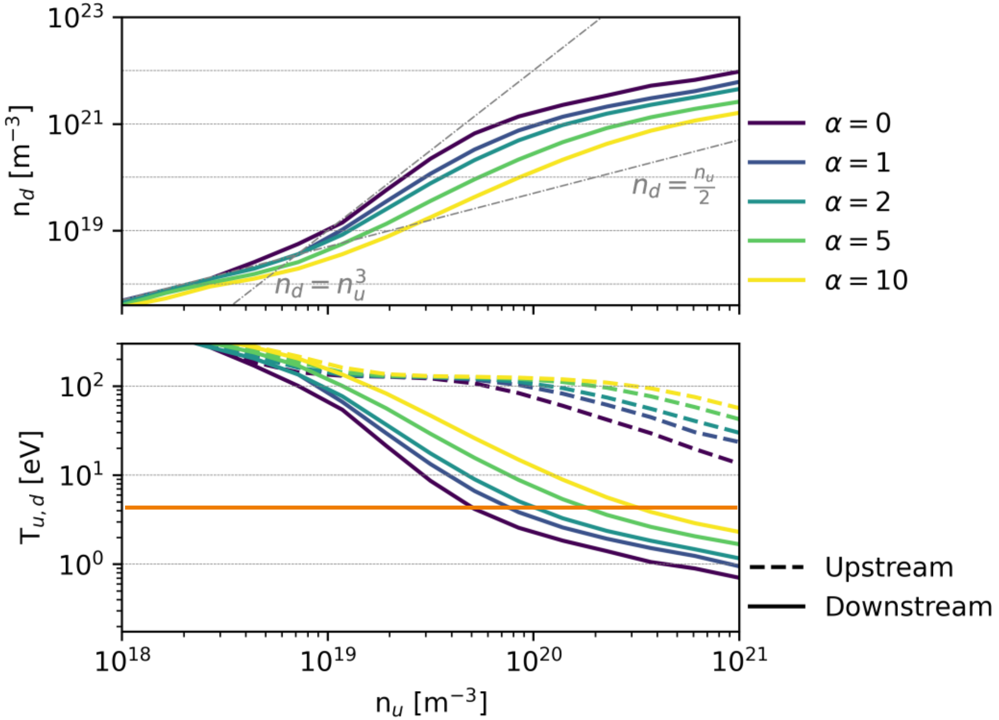
- Downstream temperature triggering diffusion-limited regime similar, here $T_{d,diff} \approx 4$ eV



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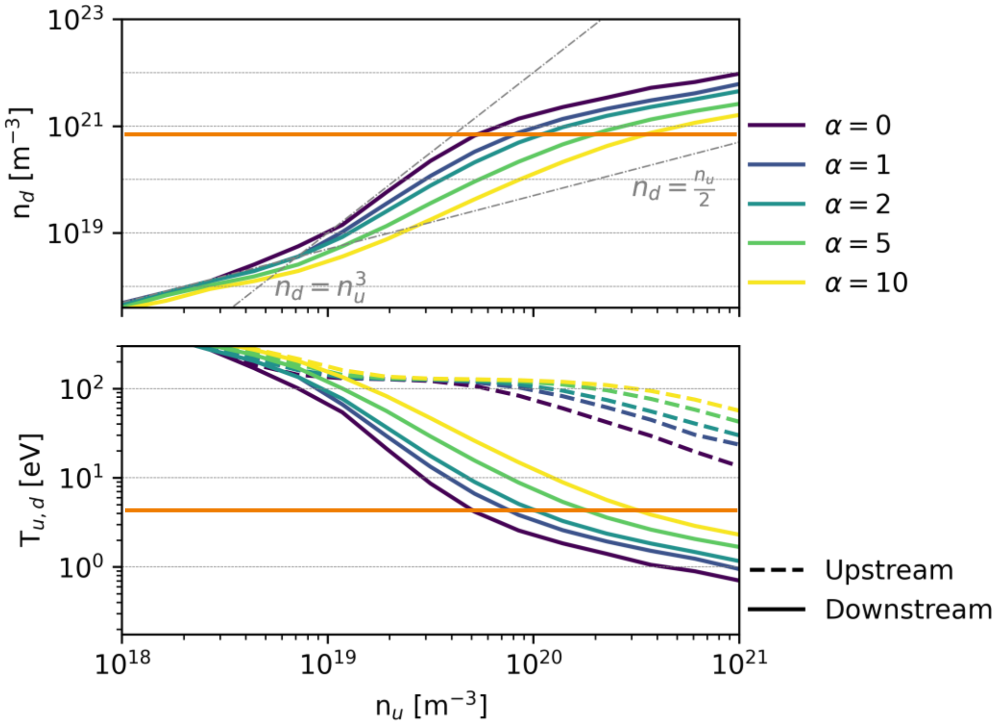
- Recycling weakens with increasing α
- Downstream temperature triggering diffusion-limited regime similar, here $T_{d,diff} \approx 4$ eV
- Associated with similar downstream density

$$n_{d,diff} \approx 10^{21}\ m^{-3}$$



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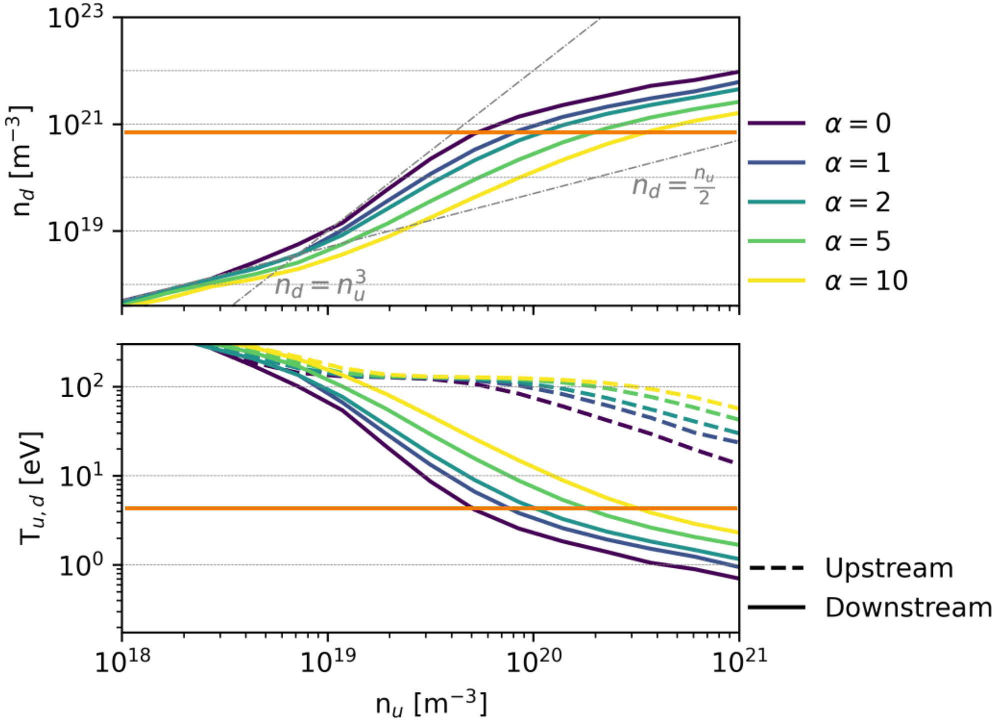
$$n_{d,diff} \approx 10^{21}\ m^{-3}$$

↘ *Limits recycling in the conduction-limited regime*



Extended STPM parameter sensitivity

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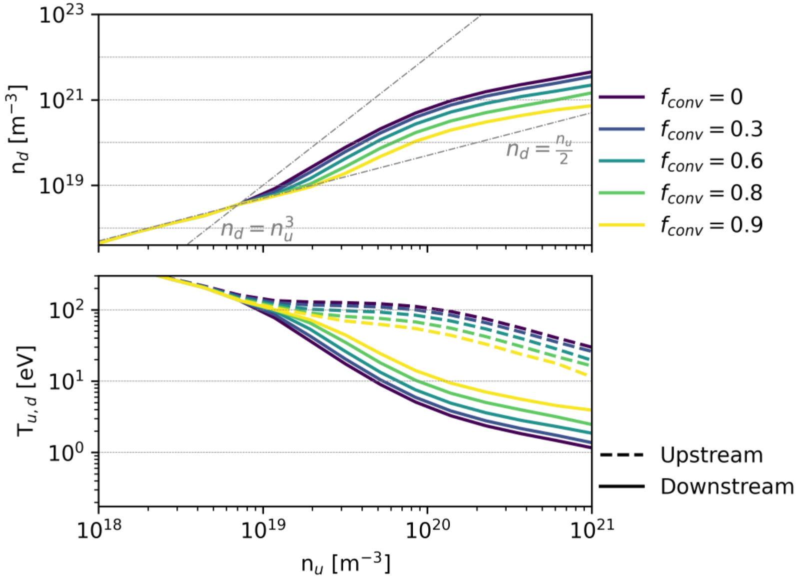
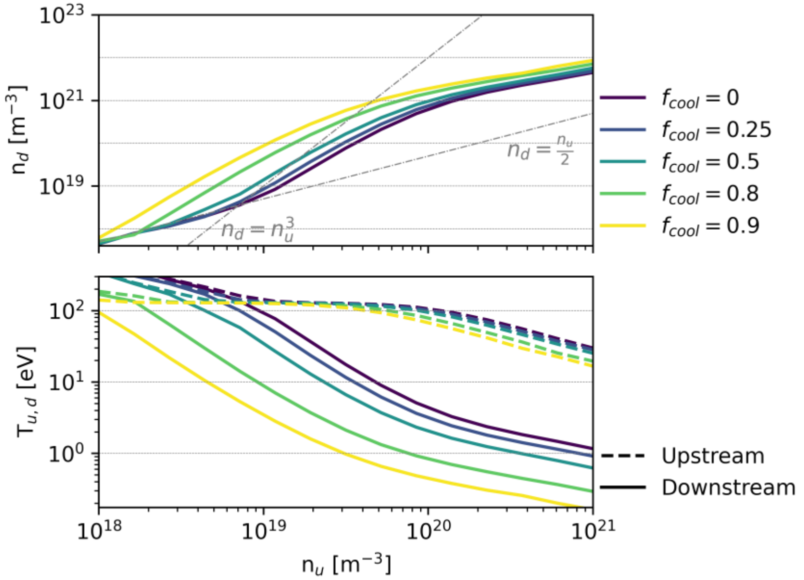
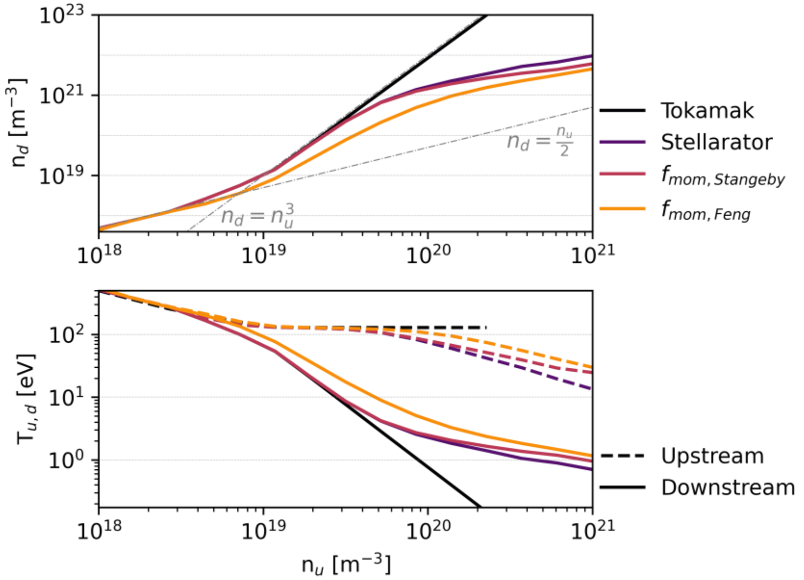
Convected power fraction $\boldsymbol{f_{conv}}$



Power dissipation fraction $\boldsymbol{f_{cool}}$



Momentum loss parametrization $\boldsymbol{f_{mom}}$



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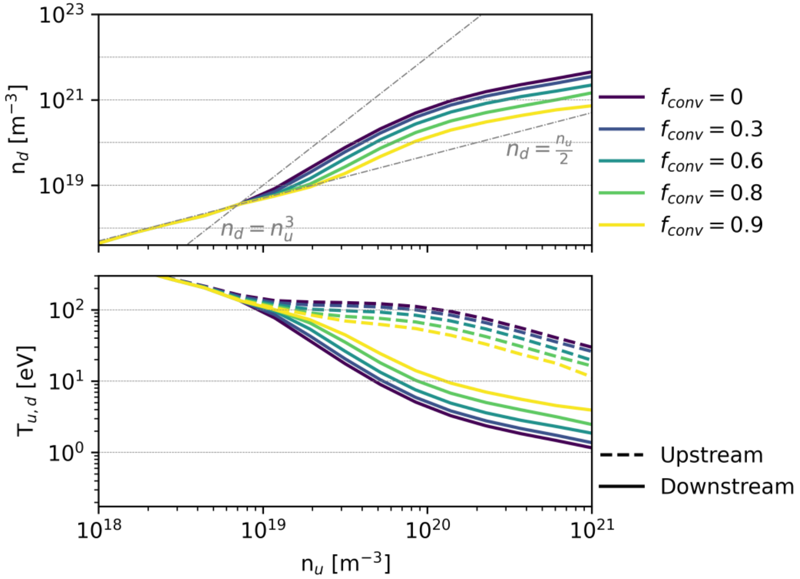
Earlier:

$$1\text{-}f_{mom,Feng} = \left(1 + \frac{\alpha}{\sqrt{T_d}}\right)^{-1}$$

Convected power fraction $\boldsymbol{f_{conv}}$



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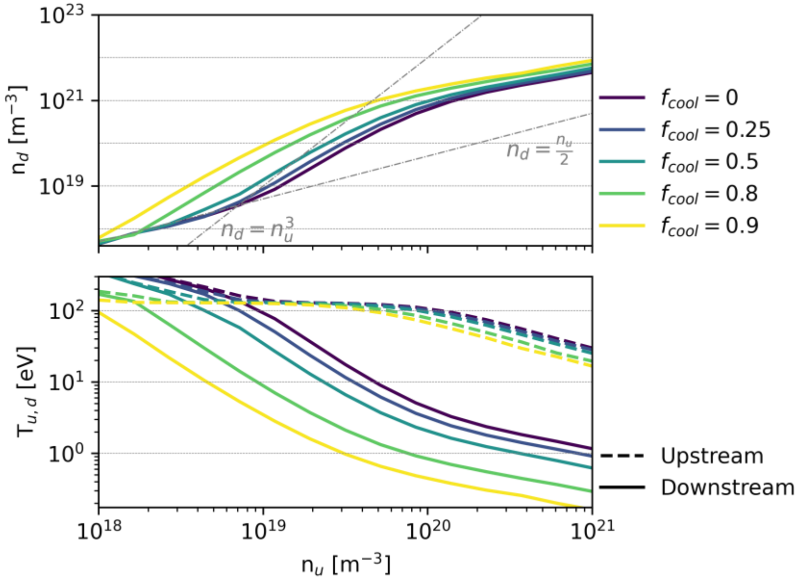

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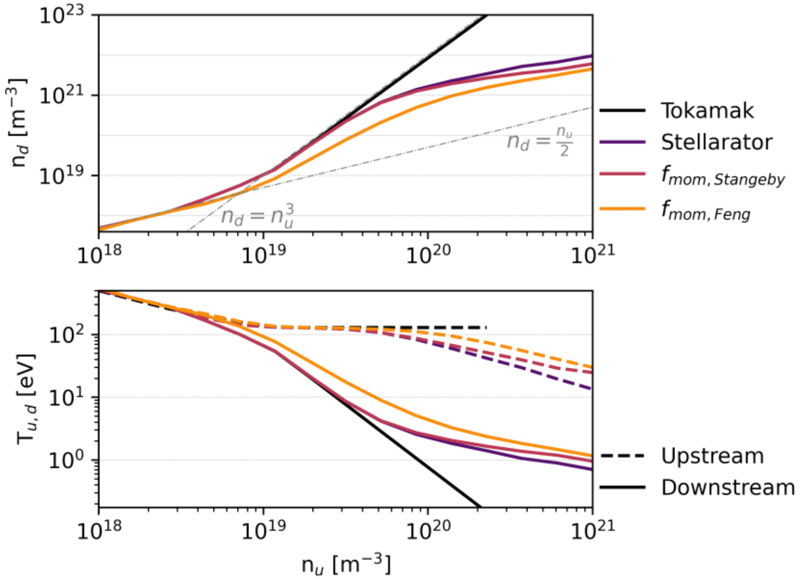

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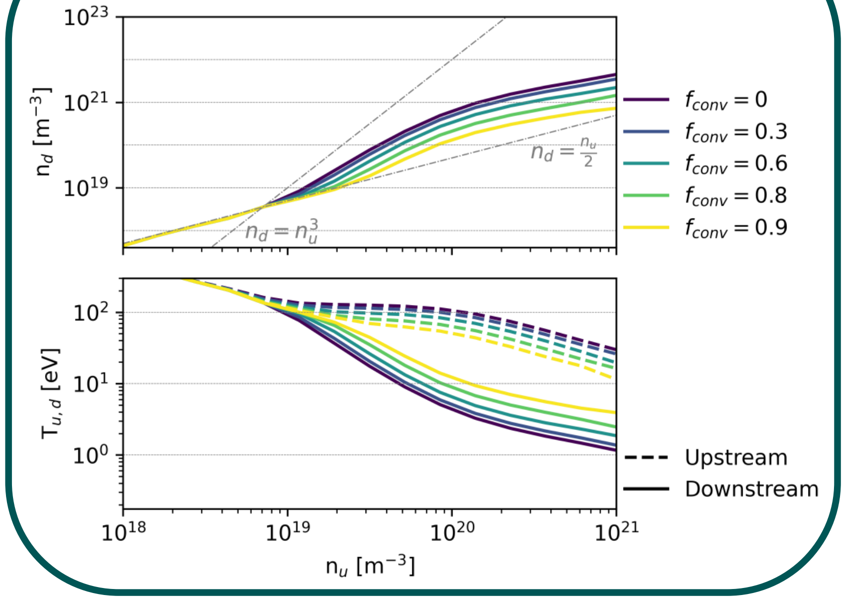

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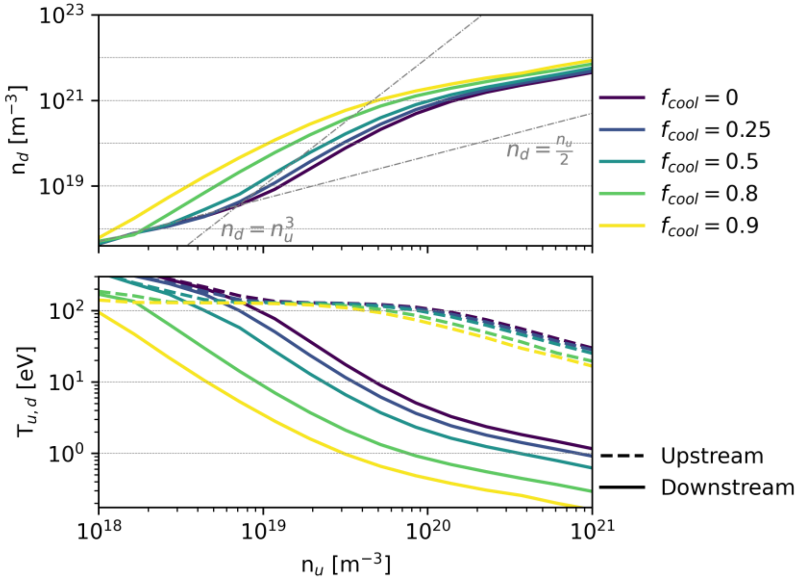

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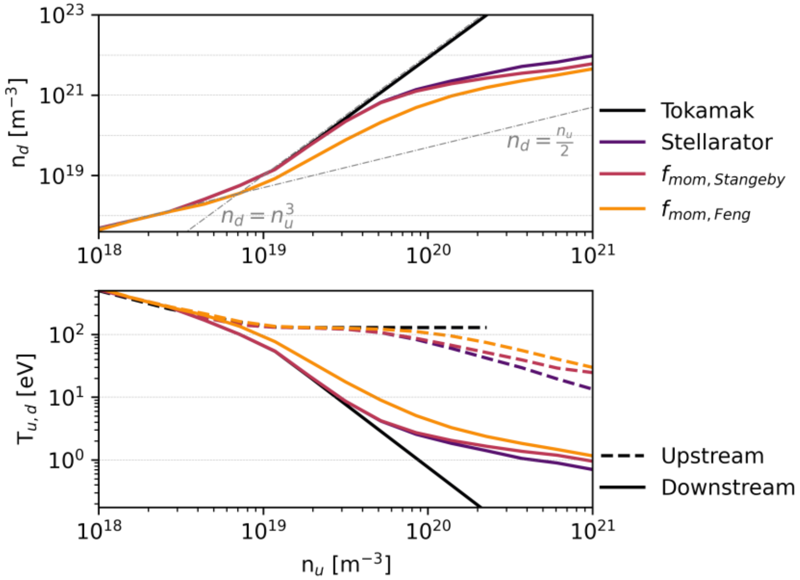
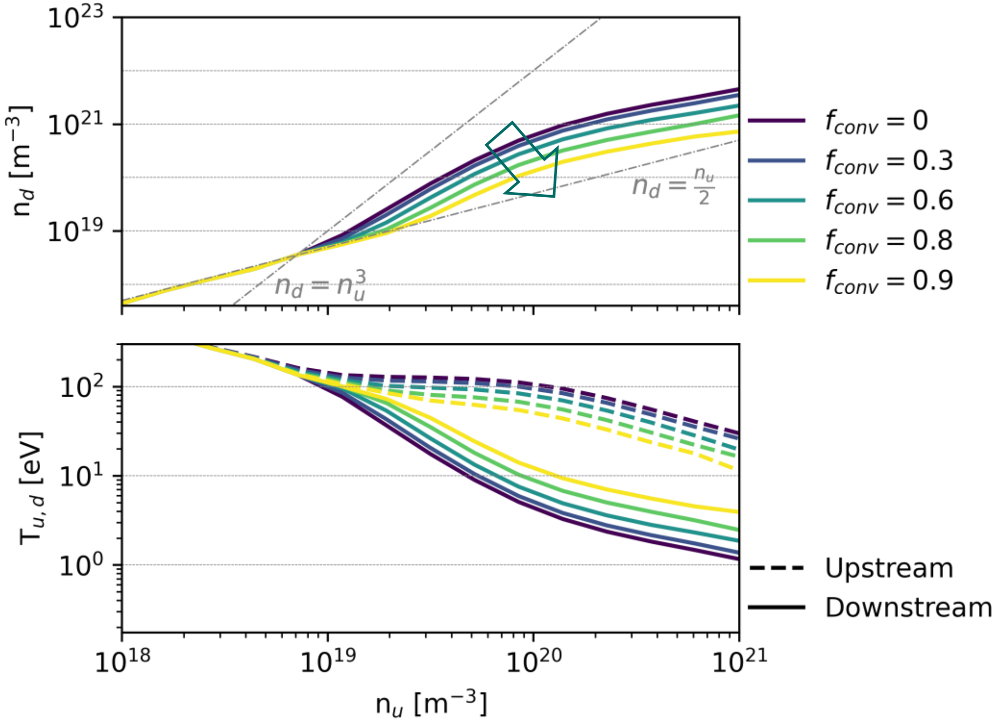


STPM parameter sensitivity: f_{conv}

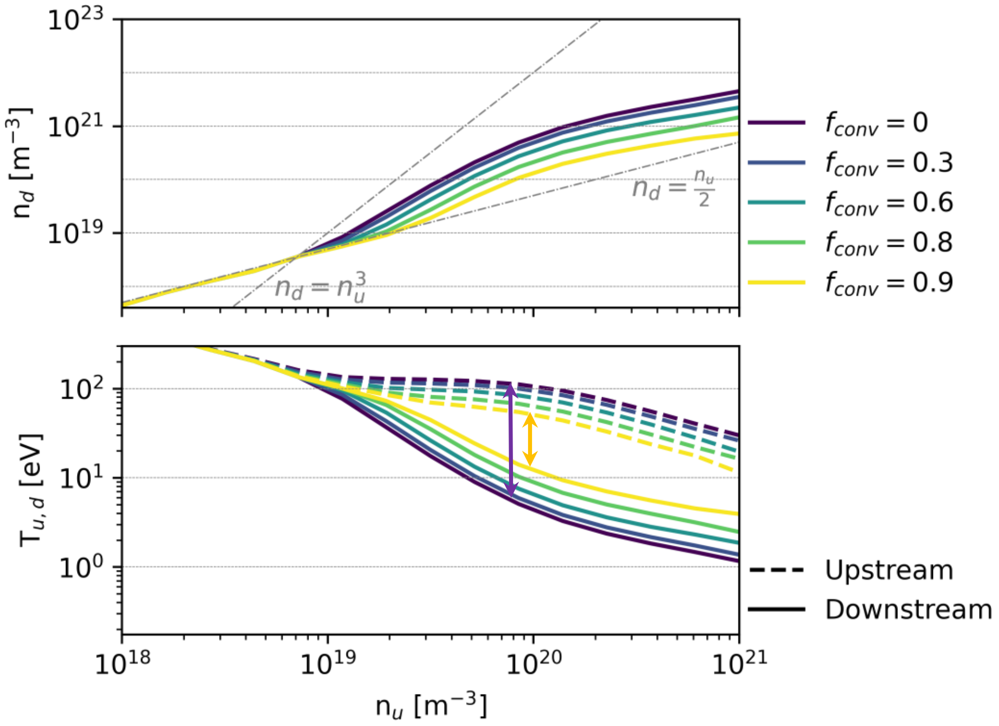
- Recycling decreases with increasing f_{conv}



STPM parameter sensitivity: f_{conv}

- Recycling decreases with increasing f_{conv}

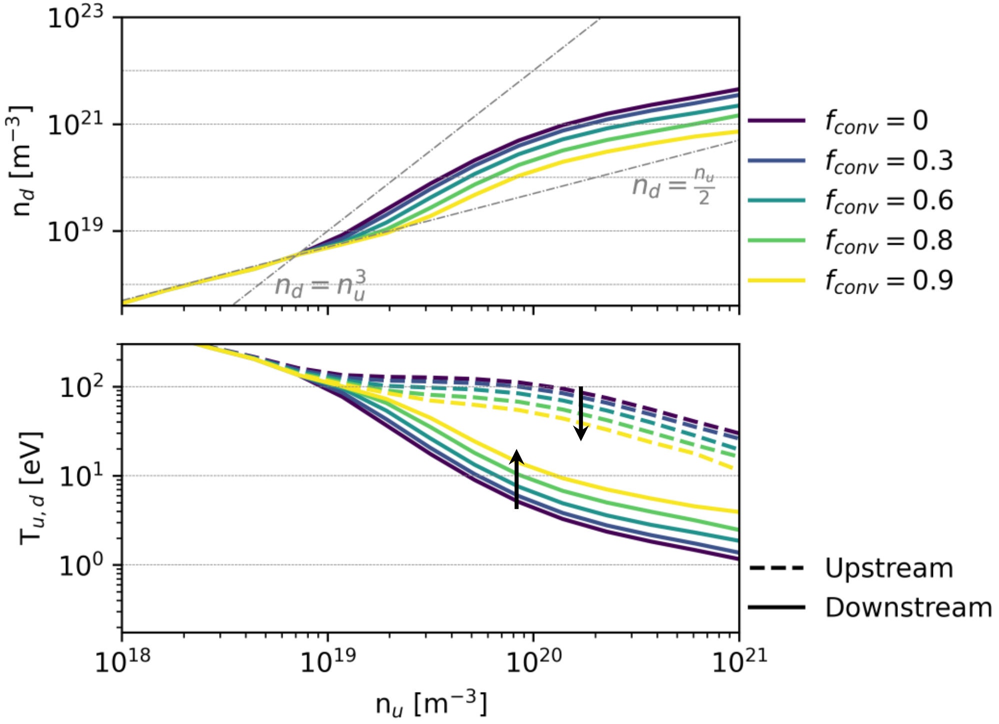
- Gradient drastically reduced



STPM parameter sensitivity: f_{conv}

- Recycling decreases with increasing f_{conv}

- Gradient drastically reduced
  - Higher T_d
  - Lower T_u



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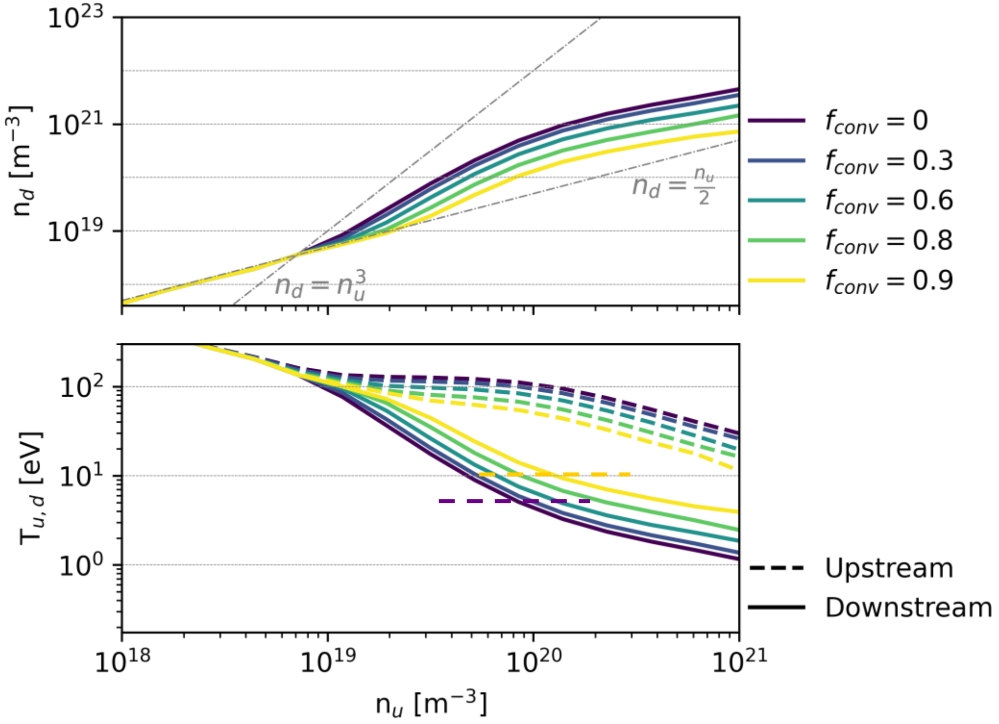
- Increase of $T_{d,diff}$



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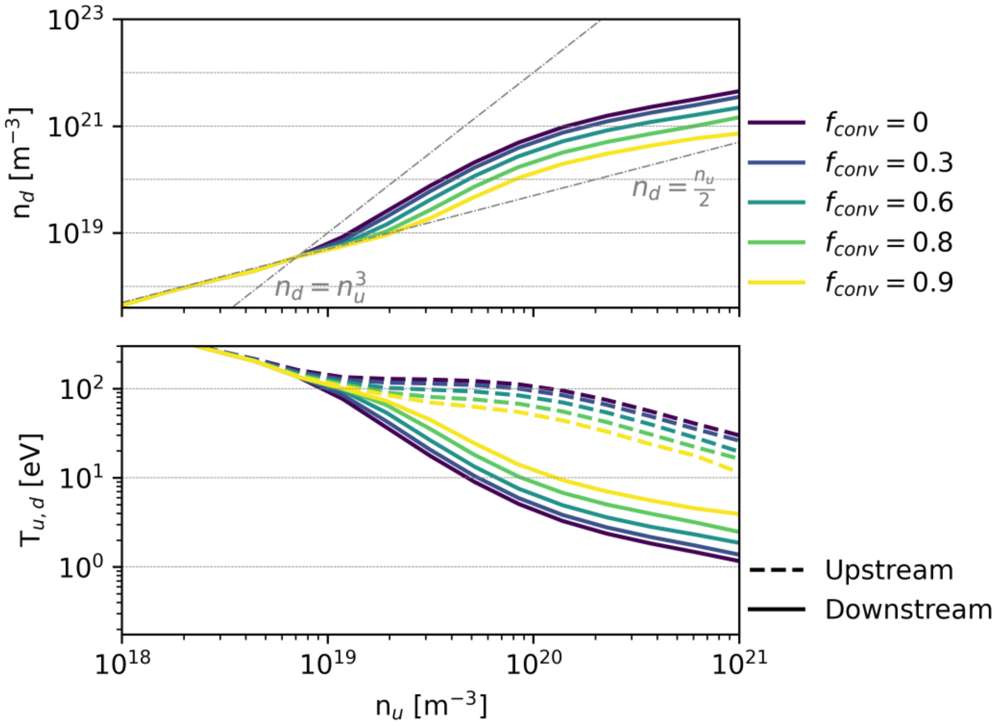
↘ *Limits target cooling, density build-up and upstream pressure*

↘ *Prior to the diffusion-limited regime*

↘ *Transitions to diffusion-limited earlier*



Summary: STPM parameter sensitivity

Significant **momentum losses** (f_{mom}) and **convected power fraction** (f_{conv}) prevent **high recycling** conditions prior to the diffusion-limited regime

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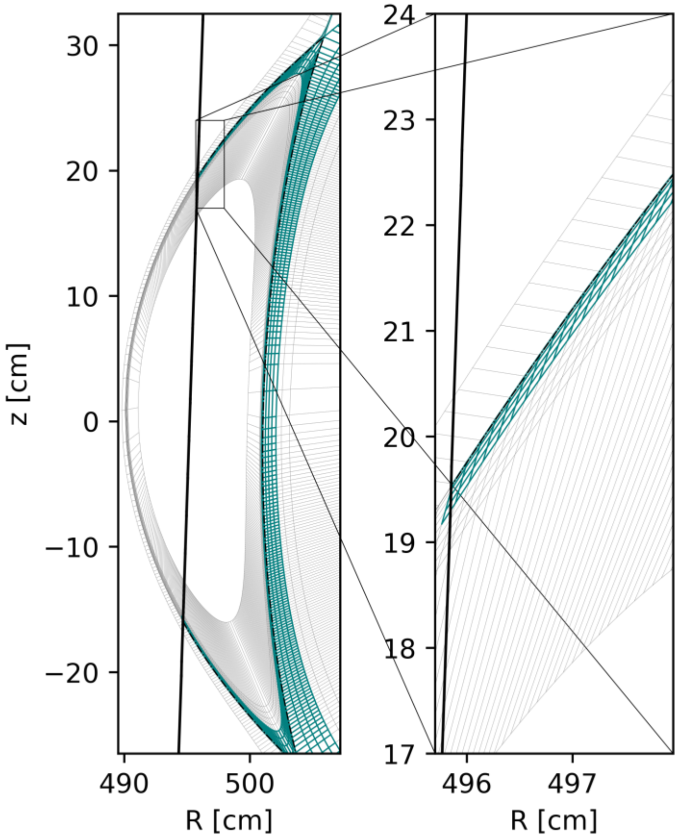
Comparison against EMC3-Eirene

- EMC3-Eirene: 3D fluid code with kinetic neutrals

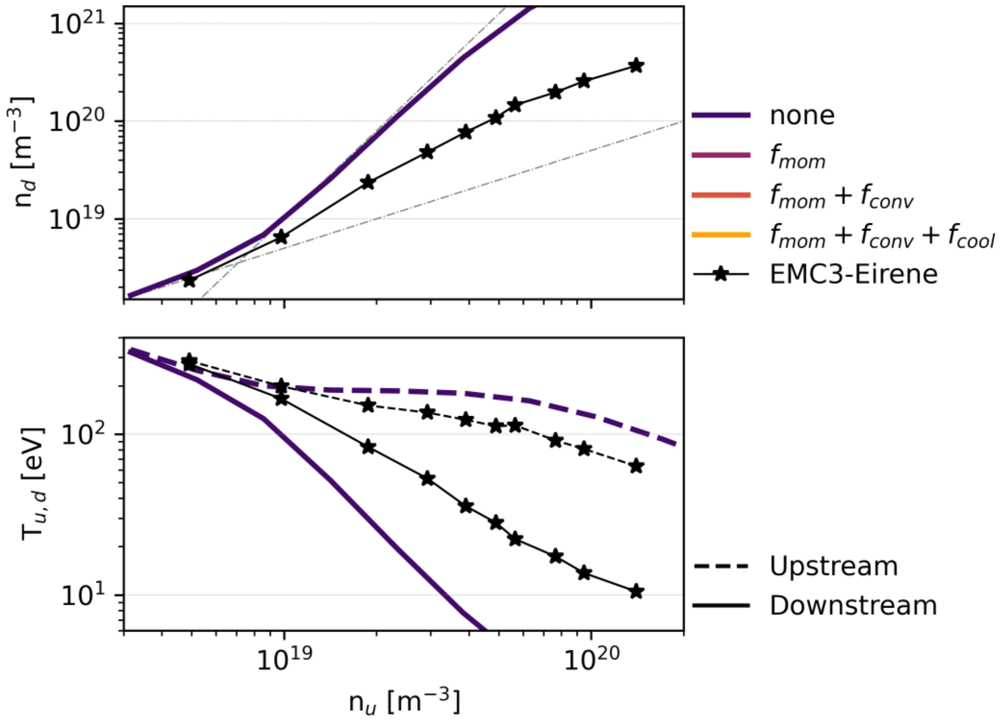
- Simplified island geometry limited to a Power carrying layer (PCL)
  - Suppressed island radial perpendicular transport (no flux)
  - Retains 3D features: target shadow region
- Density scan without impurities
- Parametrization extracted from the simulations for:
  - f_{mom}, f_{conv}, and f_{cool}



Comparison against EMC3-Eirene

Gradual inclusion of the terms:

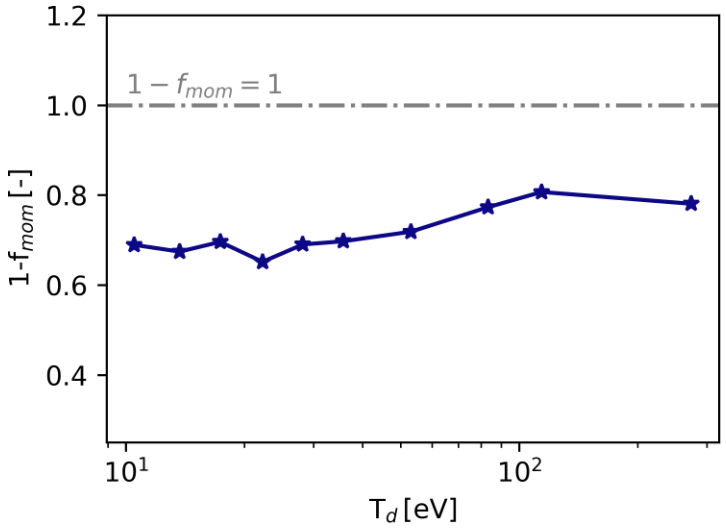
- Poor agreement without correction terms

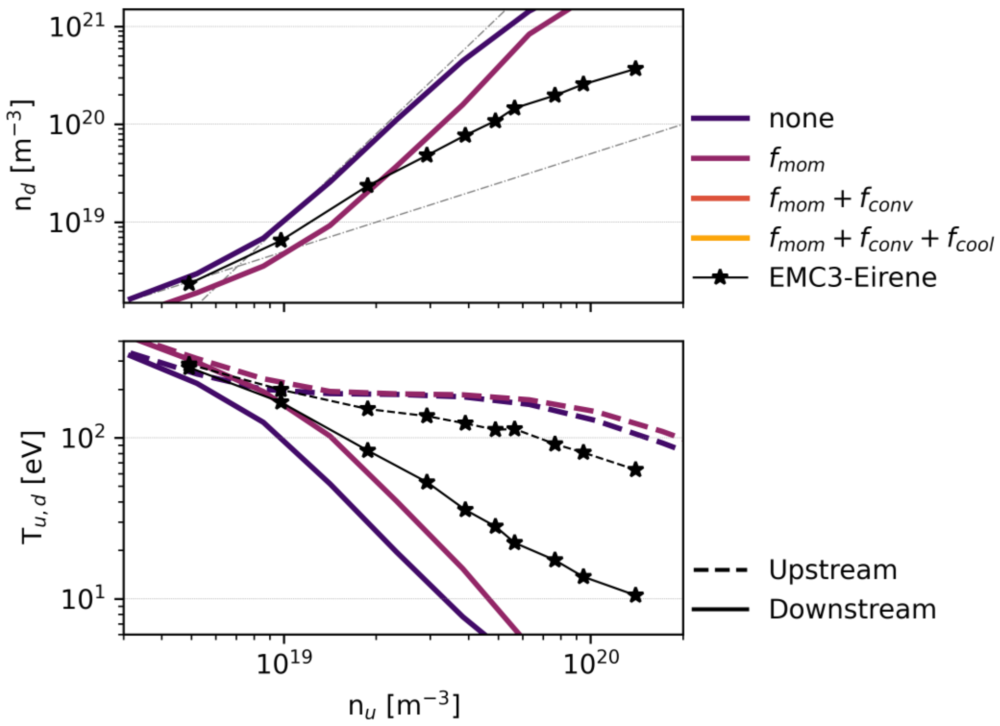


Comparison against EMC3-Eirene

Gradual inclusion of the terms:

- Poor agreement without correction terms
- Transition to conduction-limited captured with f_{mom}





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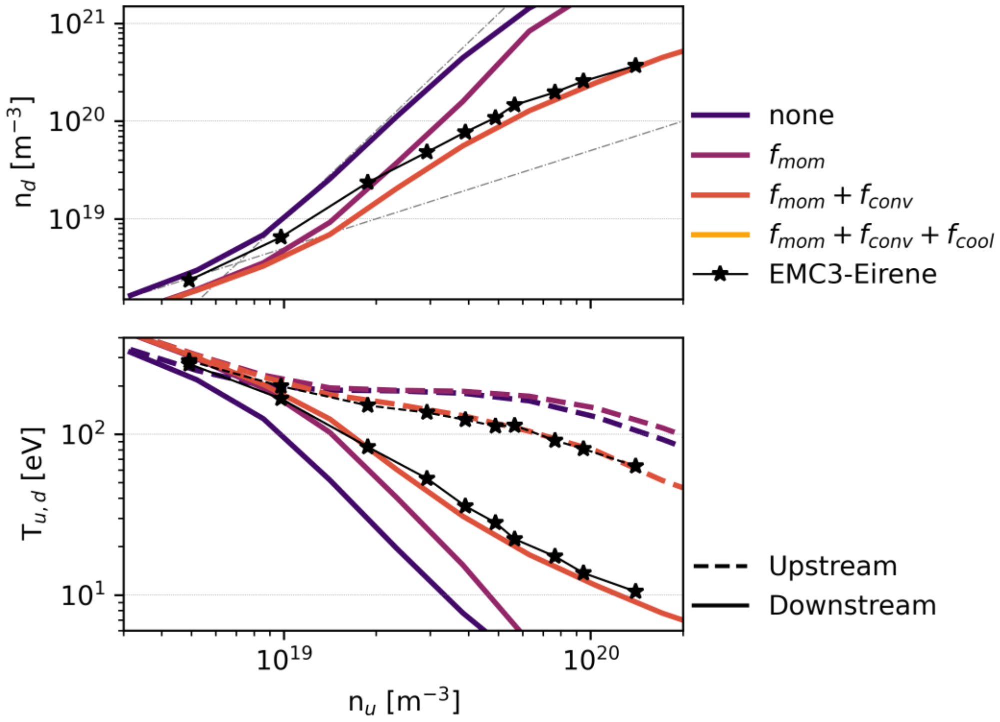
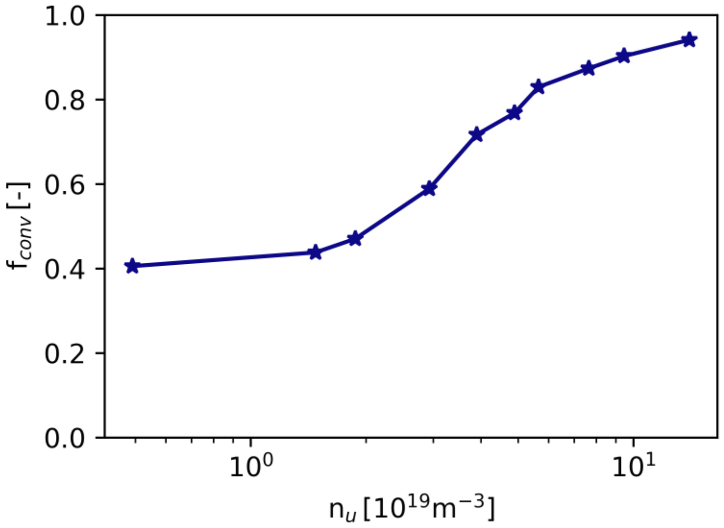
- Temperature gradient match with f_{conv} & large impact n_d





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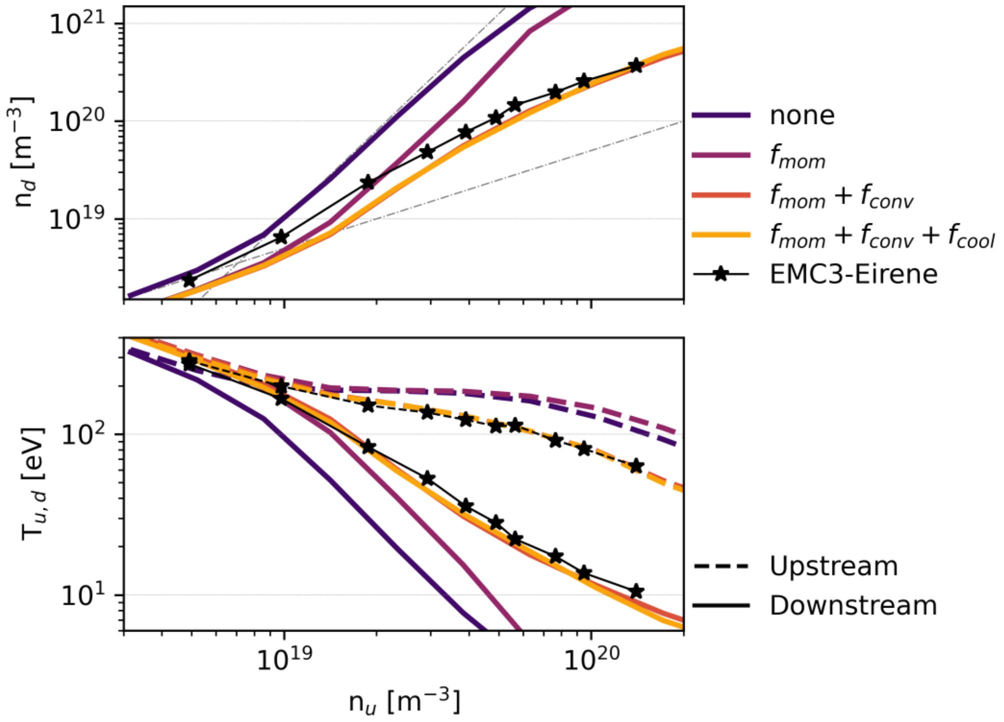
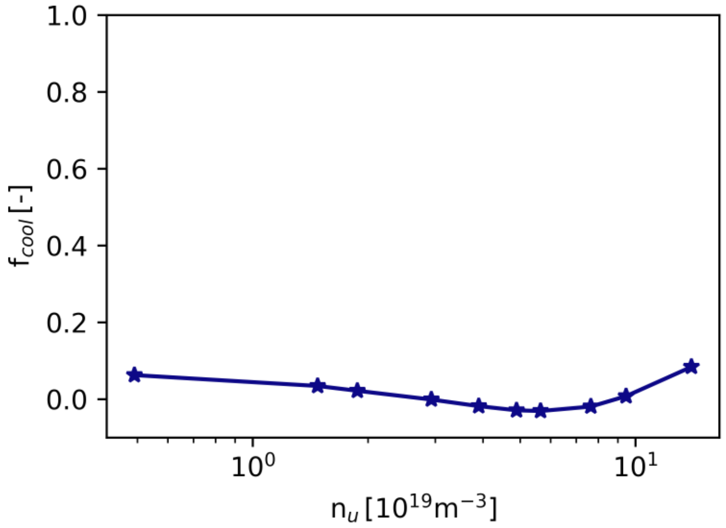
- Temperature gradient match with f_{conv} & large impact n_d
- Negligble impact of f_{cool} here (no impurities)





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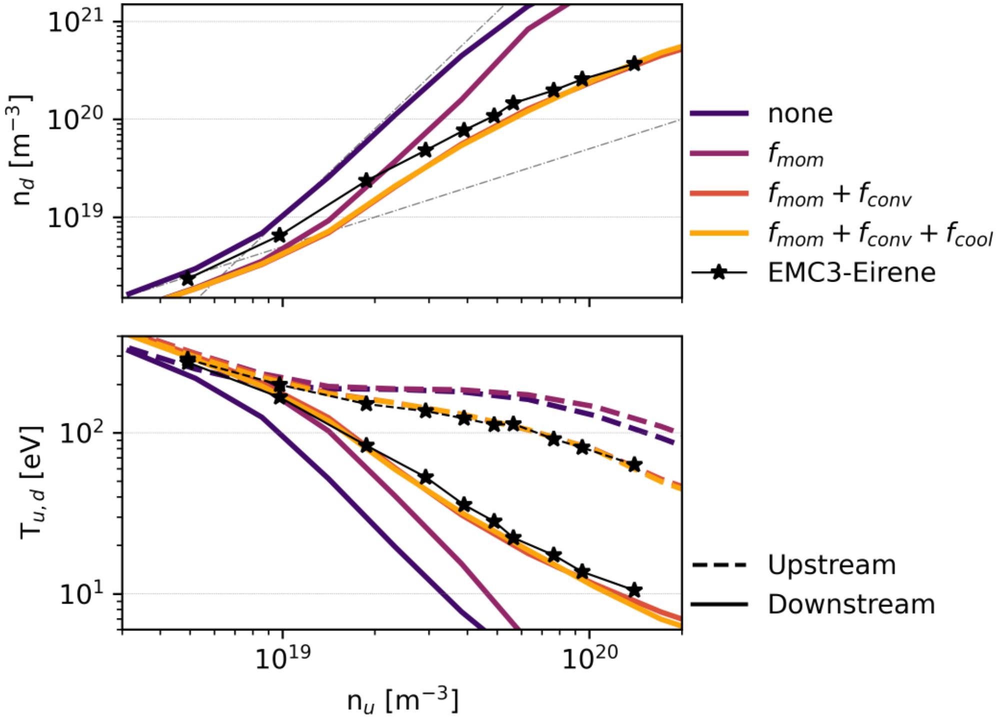

- **„Two-point model formatting“:**

↘ *sheds light on relative importance of each correction factor*

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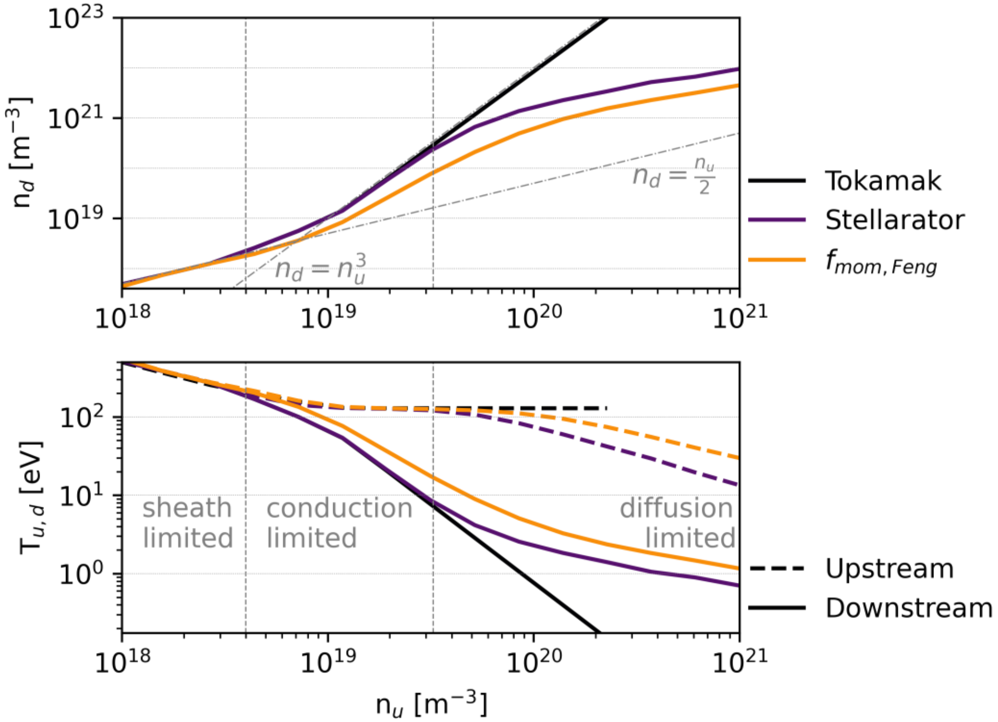
Predicting the diffusion-limited regime

Temperature equation:

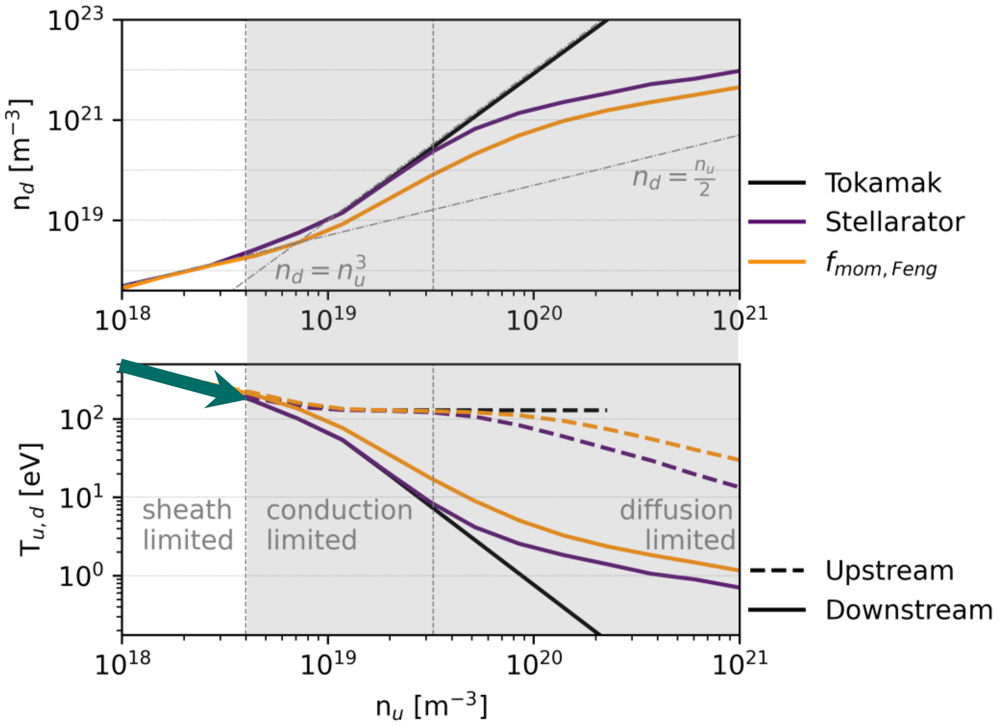
$$T_u^{7/2} = T_d^{7/2} + \frac{7 q_\parallel L_C}{2\kappa_e} - \frac{7\chi(n_u + n_d)}{4\kappa_e \Theta^2}(T_u - T_d)$$



Predicting the diffusion-limited regime

First sheath-limited $T_{u,sl} \approx T_d$

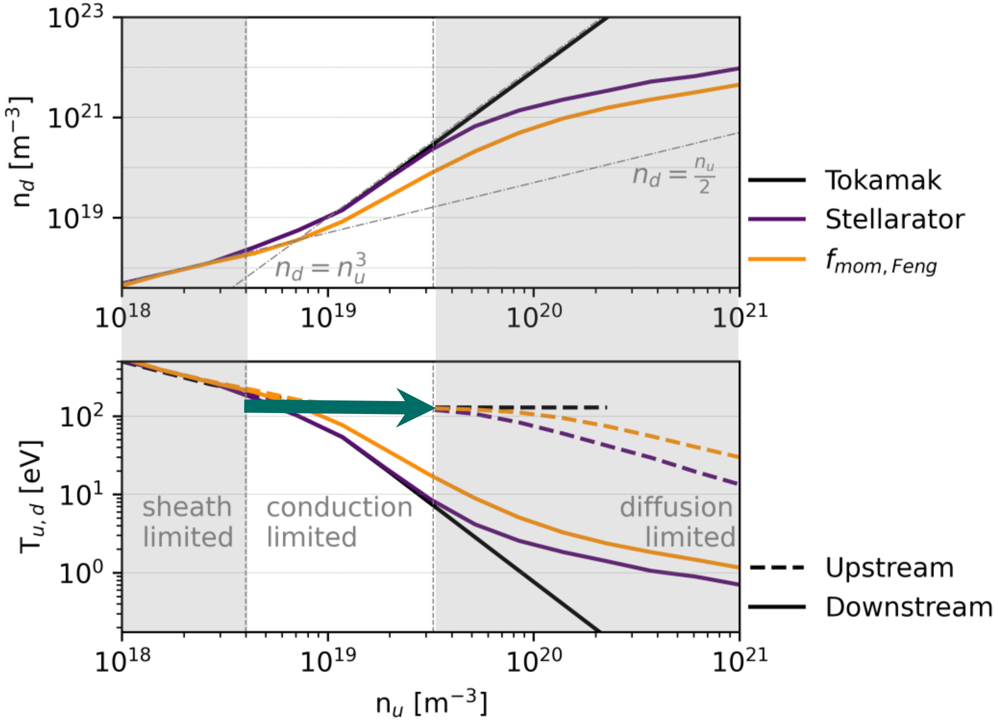
$$T_u^{7/2} = T_d^{7/2} + \frac{7q_\parallel L_C}{2\kappa_e} - \frac{7\chi(n_u + n_d)}{4\kappa_e \Theta^2}(T_u - T_d)$$



Predicting the diffusion-limited regime

Then conduction-limited $T_{u,c-l}^{7/2} \approx \frac{7q_\parallel L_C}{2\kappa_e}$

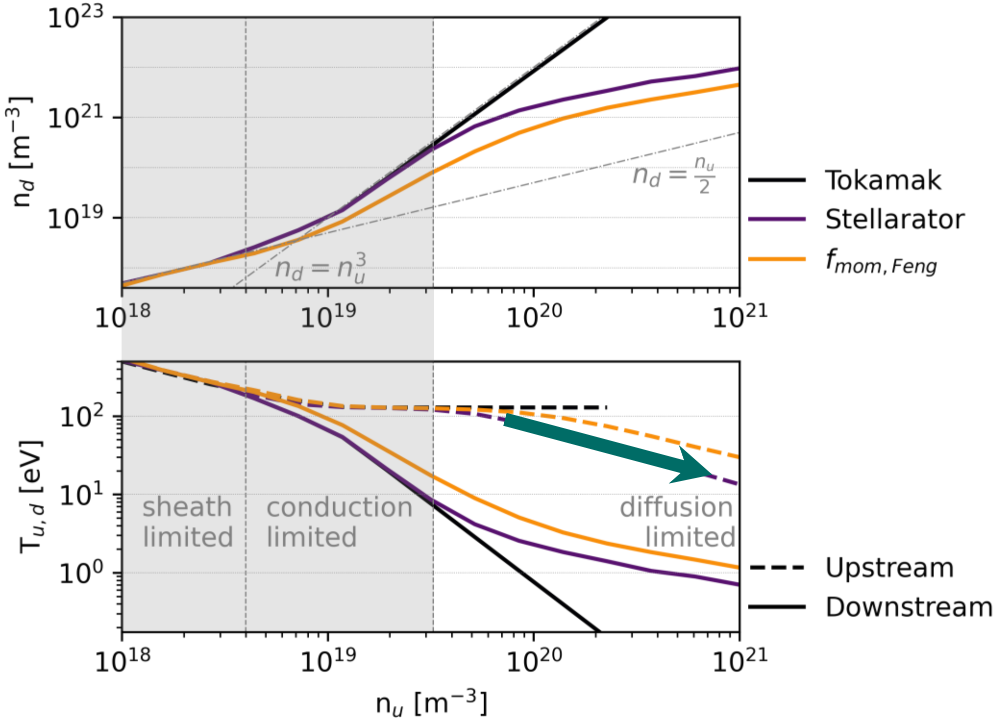
$$T_u^{7/2} = T_d^{7/2} + \frac{7q_\parallel L_C}{2\kappa_e} - \frac{7\chi(n_u + n_d)}{4\kappa_e \Theta^2}(T_u - T_d)$$



Predicting the diffusion-limited regime

Eventually diffusion-limited $T_{u,d-l}$ drops because of the relative importance of the last term

$$T_u^{7/2} = T_d^{7/2} + \frac{7q_\parallel L_C}{2\kappa_e} - \frac{7\chi(n_u + n_d)}{4\kappa_e \Theta^2}(T_u - T_d)$$



Predicting the diffusion-limited regime

Temperature equation:

$$T_u^{7/2} = T_d^{7/2} + \frac{7q_\parallel L_C}{2\kappa_e} - \frac{7\chi(n_u + n_d)}{4\kappa_e \Theta^2}(T_u - T_d)$$

The ratio of the third to the second term is defined as:

$$\xi = \frac{\chi(n_u + n_d)(T_u - T_d)}{2\Theta^2(1 - f_{conv})q_\parallel L_C}$$

which represents a reduction of T_u from $T_{u,c-l}$, more precisely

$$T_u \approx T_{u,c-l}(1 - \xi)^{2/7}$$

Predicting the diffusion-limited regime

Temperature equation:

$$T_u^{7/2} = T_d^{7/2} + \frac{7q_\parallel L_C}{2\kappa_e} - \frac{7\chi(n_u + n_d)}{4\kappa_e \Theta^2}(T_u - T_d)$$

The ratio of the third to the second term is defined as:

$$\xi = \frac{\chi(n_u + n_d)(T_u - T_d)}{2\Theta^2(1 - f_{conv})q_\parallel L_C}$$

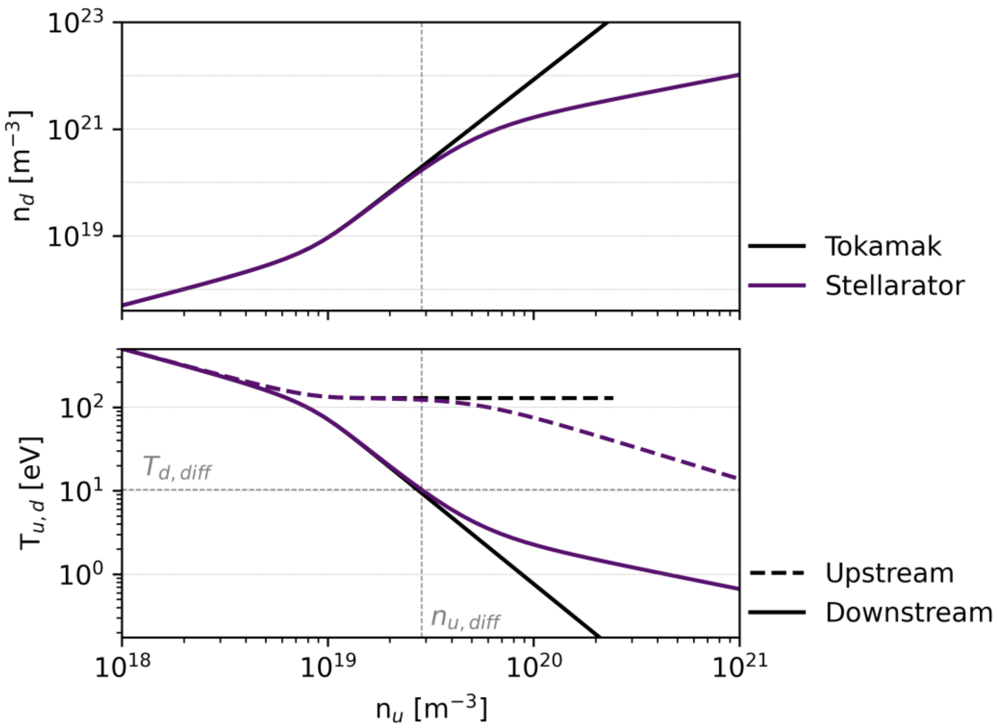
Rearrangement gives for a threshold value ξ_{diff}:

$$T_{d,diff} = T_u - \xi_{diff}\frac{2\Theta^2(1 - f_{conv})q_\parallel L_C}{\chi(n_{u,diff} + n_d)}$$

Predicting the diffusion-limited regime

Rearrangement gives for a threshold value ξ_{diff} (here 0.15):

$$T_{d,diff} = T_u - \xi_{diff} \frac{2\Theta^2(1 - f_{conv})q_\parallel L_C}{\chi(n_{u,diff} + n_d)}$$



Predicting the diffusion-limited regime

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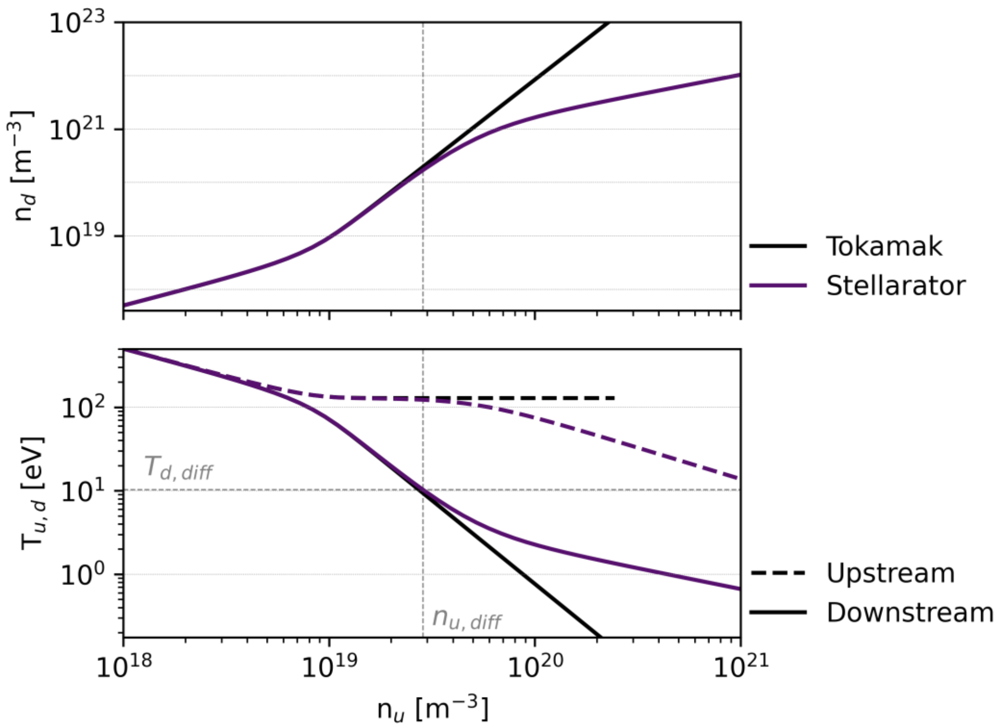
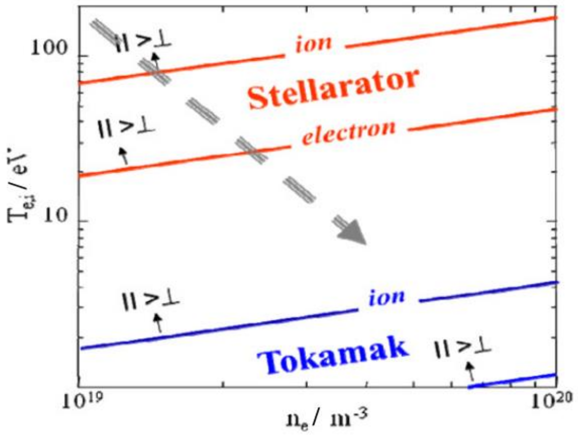




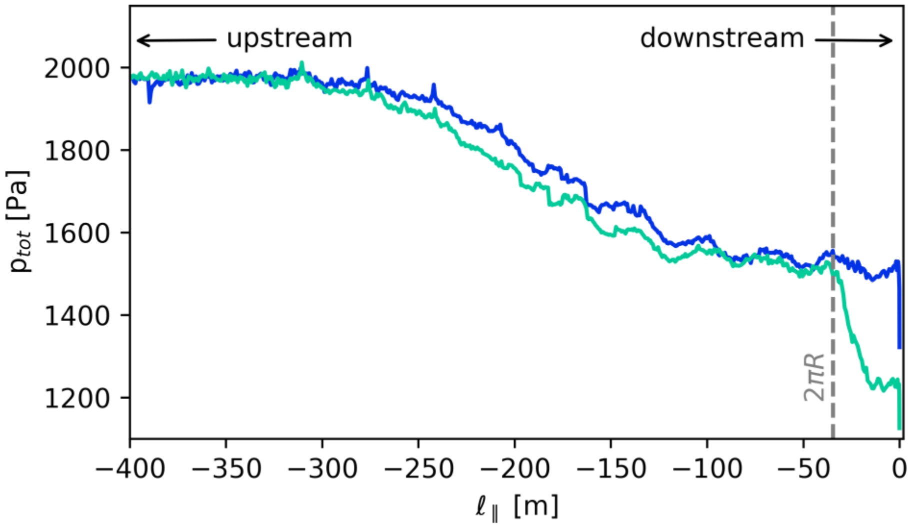
[Y Feng et al 2011 Plasma Phys. Control. Fusion ***53*** *024009]*

Outline

- Extending the stellarator two-point model (STPM)
- Sensitivity of the scalings around a reference set of parameters
- Comparison against EMC3-Eirene
- Predicting the *diffusion-limited regime*
- Target-Shadow-Region-induced pressure step

TSR-induced pressure step in EMC3-Eirene

- Observed for certain field lines, in the vicinity of the TSR in the last turn (≈ 35 m)



- Magnetic field line used in this work does not show this feature

Summary

- Stellarator two-point model includes **perpendicular transport**: leads to ***diffusion-limited regime***

- **Momentum losses** and **heat convection** prevent **high recycling** prior to this detrimental regime

- Comparison against **EMC3-Eirene** simulations of **a Power carrying layer geometry** shows:
  - good agreement with STPM predictions
  - the relative importance of the different correction factors

- Stellarator two-point model **predicts onset** of the *diffusion-limited regime*

- **Target shadow region** leads to **pressure step** along the field line, to be investigated in future work