Flux-tube simulations of TAE modes (work in progress)

A. Mishchenko

Acknowledgements: D. Brioschi, T. Görler, T. Hayward-Schneider, R. Kleiber, A. Könies, A. Di Siena

MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

This work has been carried out within the framework of the EUROfusion Consortium, funded by the European Union via the Euratom Research and Training Programme (Grant Agreement No 101052200 — EUROfusion). Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or the European Commission. Neither the European Union nor the European Commission can be held responsible for them.

ITPA TAE benchmark: majority EP codes have been compared

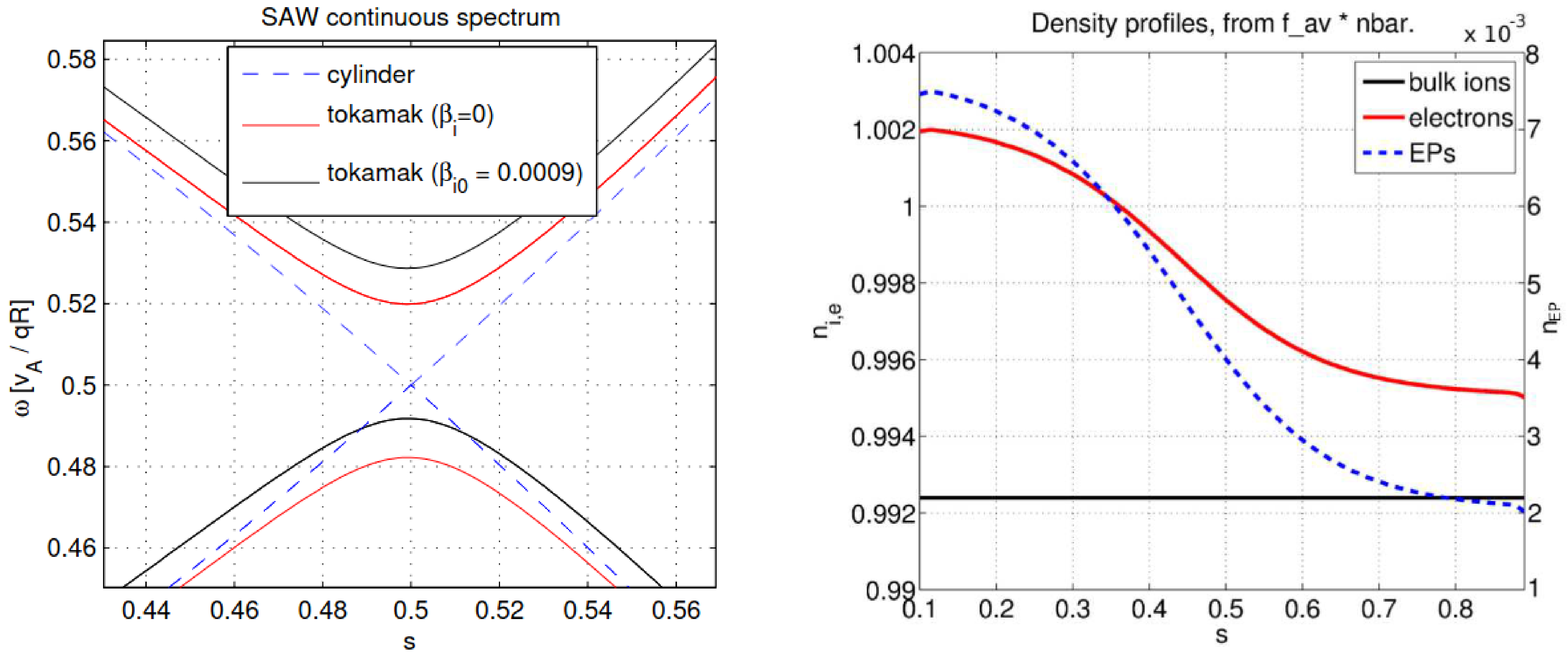
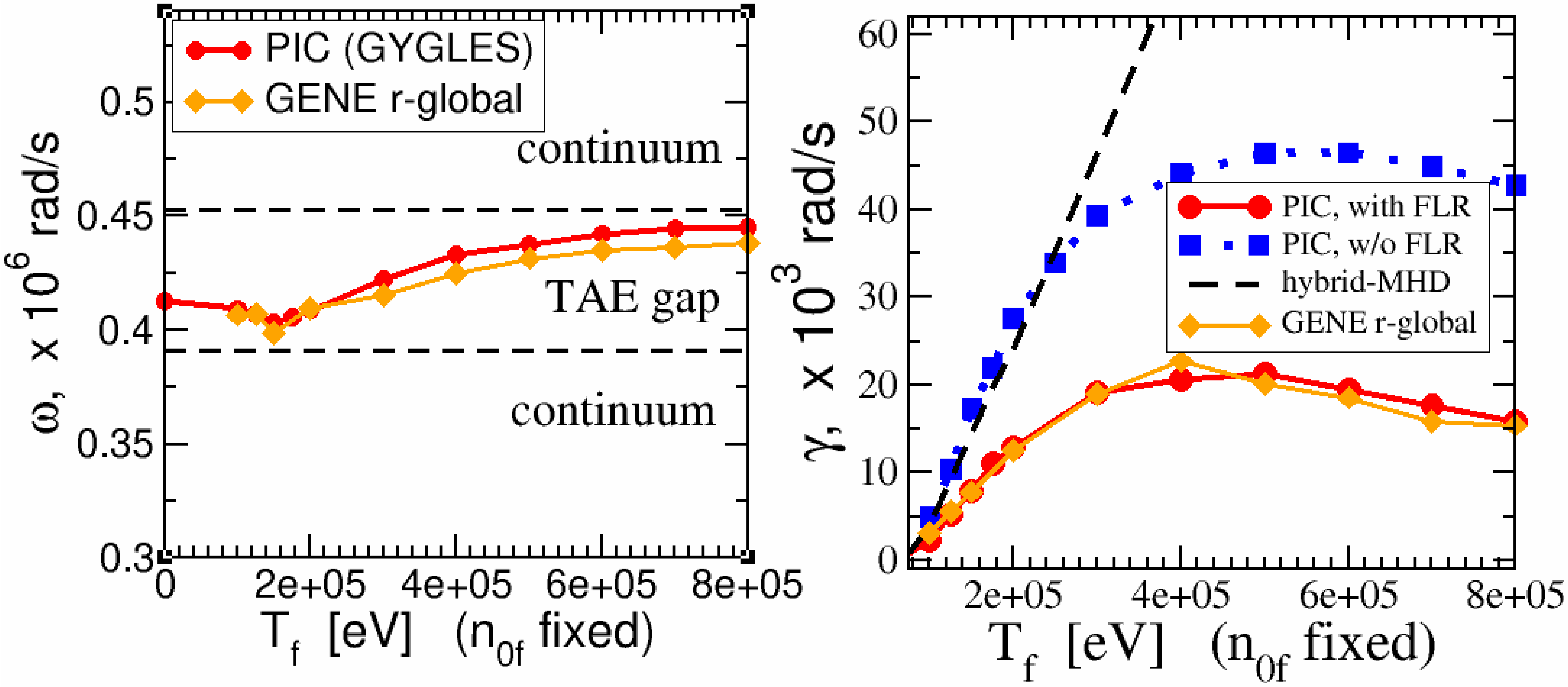


Figure 1: On the left, zoom of the continuous spectrum obtained by neglecting toroidicity and compressibility (blue dashed lines), or by keeping toroidicity only (red line) or by keeping both toroidicity and compressibility (black line), with approximated formula of Ref. [21]. On the right, density profiles in units of $n_{e0} = 2 \cdot 10^{19} m^{-3}$, for a case with $\langle n_{EP} \rangle / n_{e0} = 0.004$.

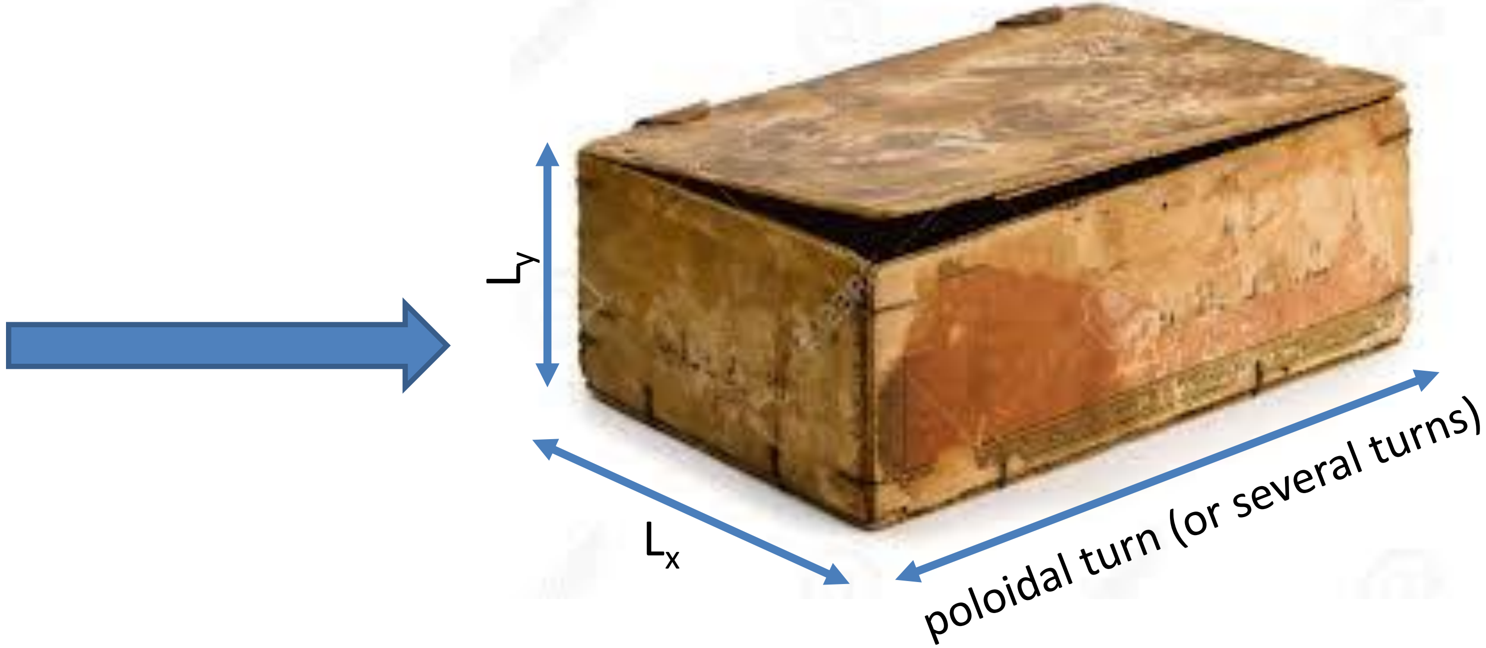
ITPA TAE benchmark: r-global GENE-GYGLES comparison (following D. Brioschi)



Radially-global GENE linear frequency and growth rate compare very well to GYGLES (even maybe slightly better than Davide's comparison with ORB5 performed last year).

GYGLES simulations: [*Phys. Plasmas* 16, 082105 (2009)] ; ITPA benchmark: [A. Könies *et al* 2018 *Nucl. Fusion* **58** 126027]

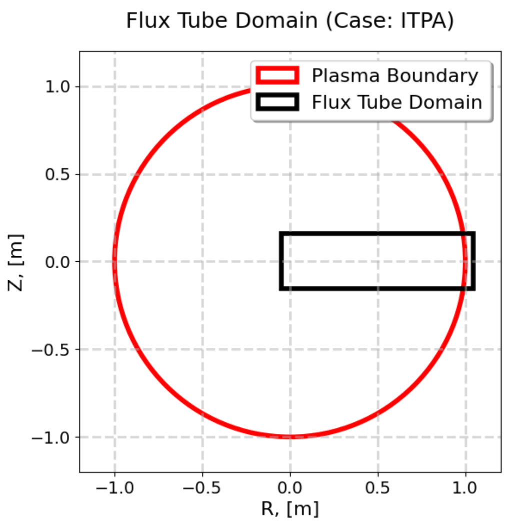
ITPA TAE benchmark: flux-tube GENE-GYGLES comparison

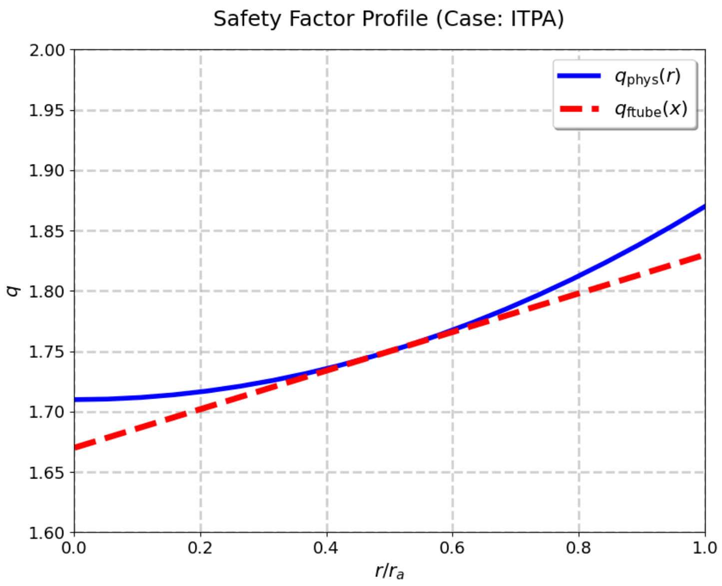


ITPA geometry:
$k_{y,min}\,\rho_s = 0.022$
shear = 0.0457
$L_x = 507.9\,\rho_s$, $N_{pol} = 2$
$L_y = 291.7\,\rho_s$
$r_a / \rho_s = 927.0$

In the flux-tube apporach, one replaces a torus with a box
Poloidal box size is determined by spectral resolution: $L_y = 2\pi / k_{y,min}$
Radial box size is determined by „twist-and-shift“ condition (parallel bc): $L_x = 1/(k_{y,min}\,\text{shear})$

ITPA TAE benchmark: flux-tube GENE



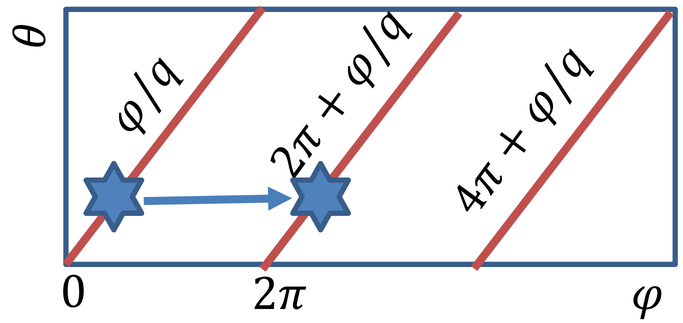


In the flux-tube apporach, one replaces a torus with a box

Poloidal box size is determined by spectral resolution:

$L_y = 2\pi / k_{y,min}$, $k_{y,min}\,\rho_s = m_0\,\rho_*$

Radial box size is determined by „twist-and-shift“ condition (parallel bc): $L_x = 1/(k_{y,min}$ shear $N_{pol})$



ITPA geometry:

$k_{y,min}\,\rho_s = 0.022$

shear = 0.0457

$L_x = 1015.8\ \rho_s$ ($N_{pol} = 1$)

$L_y = 291.7\ \rho_s$

$r_a / \rho_s = 927.0$

Twist-and-shift parallel boundary condition:

Shift: $F(x,\alpha,\theta + 2\pi\, Npol) = F(x, \alpha - 2\pi q Npol, \theta)\,;\, \alpha = q(x)\theta - \varphi\,;\ \ k_y r_0 = m_0$

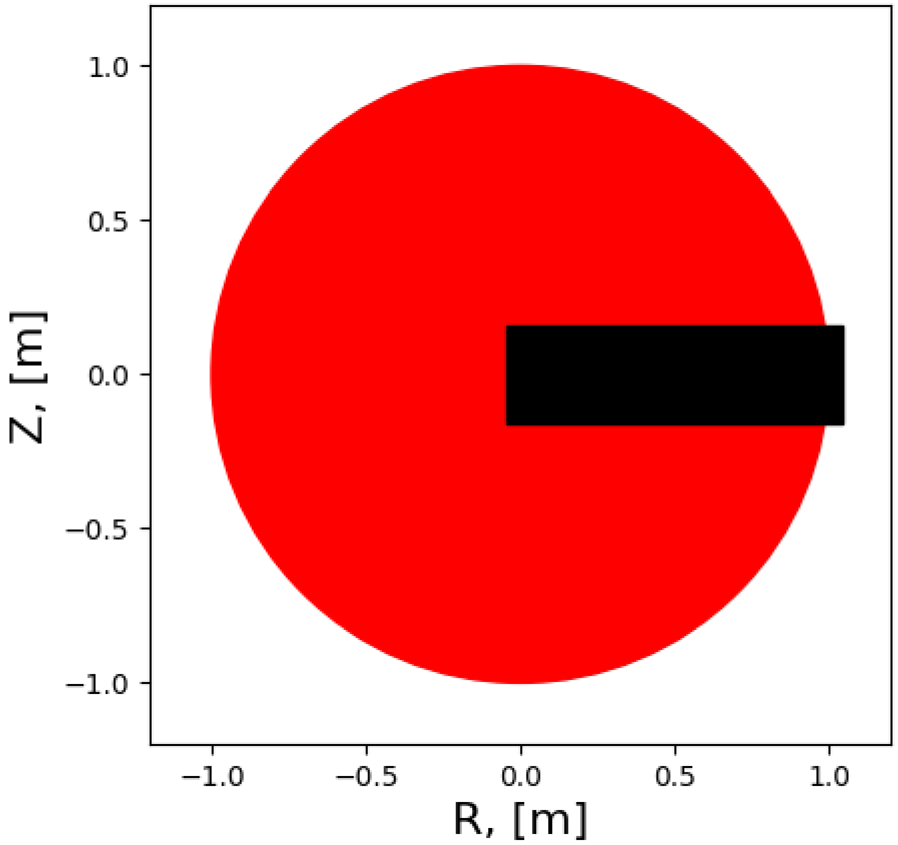
$$\exp(i\, k_x'\, x)\exp\left[ik_y\left(y - \frac{2\pi q(x) r_0}{q_0} N_{pol}\right)\right] \sim \exp\left(-2\pi i\, N_{pol}\, k_y r_0\right)\ \exp\left(ik_x' - 2\pi i\, N_{pol}\, k_y \hat{s}\, x\right)$$

Twist: $k_x' = k_x + 2\pi k_y \hat{s}\, N_{pol} \Rightarrow k_{x,min} = 2\pi N_{pol}\, k_{y,min} \Rightarrow L_x = 1/(k_{y,min}\hat{s}\, N_{pol})$

ITPA TAE benchmark: flux-tube GENE

In the flux-tube apporach, one replaces a torus with a box
Poloidal box size is determined by spectral resolution: $L_y = 2\pi / k_{y,min}$, $k_{y,min}\, \rho_s = m_0\, \rho_*$
Radial box size is determined by „twist-and-shift“ condition (parallel bc): $L_x = 1/(k_{y,min}$ shear $N_{pol})$

ITPA geometry:
$k_{y,min}\, \rho_s = 0.022$
shear = 0.0457
$L_x = 1015.8\, \rho_s$ ($N_{pol} = 1$)
$L_y = 291.7\, \rho_s$
$r_a / \rho_s = 927.0$



Only shear (twist-and-shift bc) and density gradient computed at r_0 for radial (perpendicular) dependencies

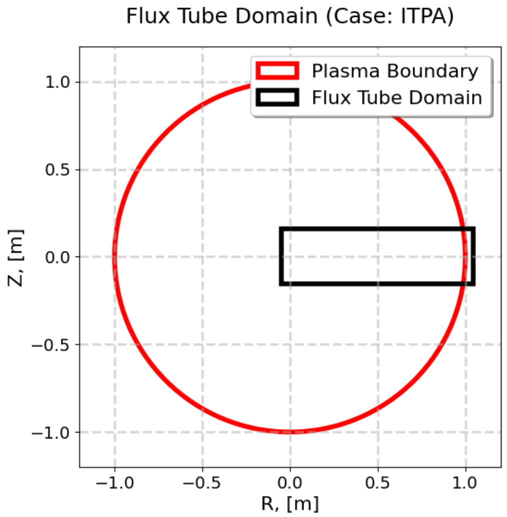
Parallel variation same for all „x“

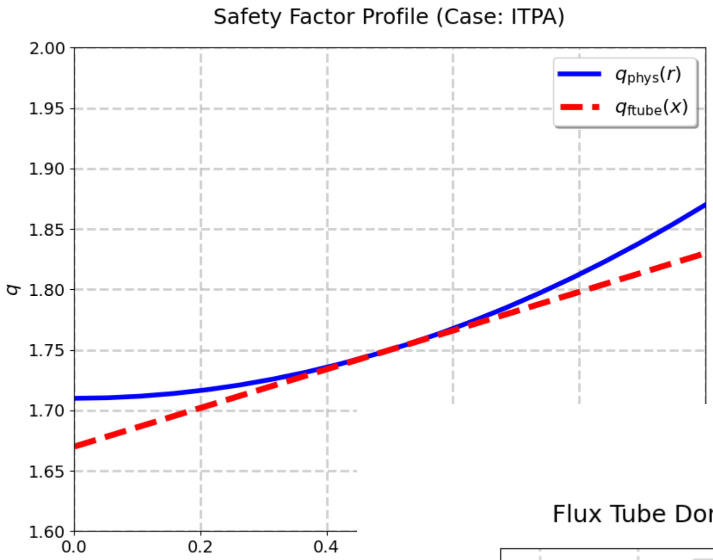
Minimalistic „adapted“ L_x for linear run (nonlinear Lx can be much larger)

EP orbits are „smaller“ than the flux tube

The flux tube is not „small“

ITPA TAE benchmark: flux-tube GENE





In the flux-tube apporach, one replaces a torus with a box

Poloidal box size is determined by spectral resolution:

$L_y = 2\,\pi / k_{y,min}$, $k_{y,min}\,\rho_s = m_0\,\rho_*$

Radial box size is determined by „twist-and-shift“ condition (parallel bc): $L_x = 1/(k_{y,min}$ shear $N_{pol})$

ITPA geometry:

$k_{y,min}\,\rho_s = 0.022$

shear = 0.0457

$L_x = 1015.8\,\rho_s$ ($N_{pol} = 1$)

$L_y = 291.7\,\rho_s$

$r_a / \rho_s = 927.0$

ITPA geometry:

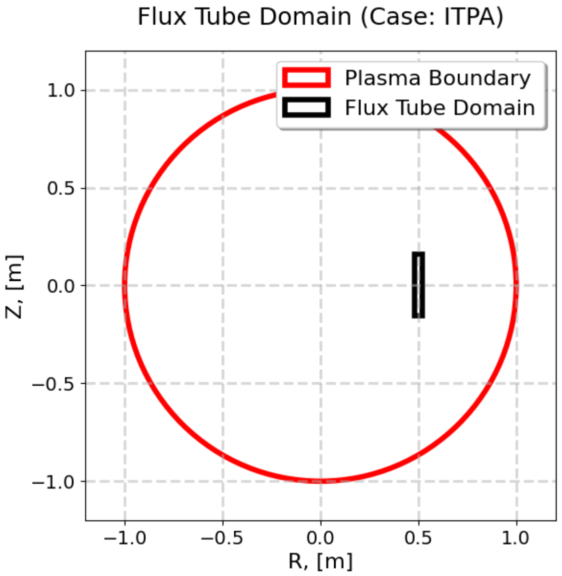
$k_{y,min}\,\rho_s = 0.022$

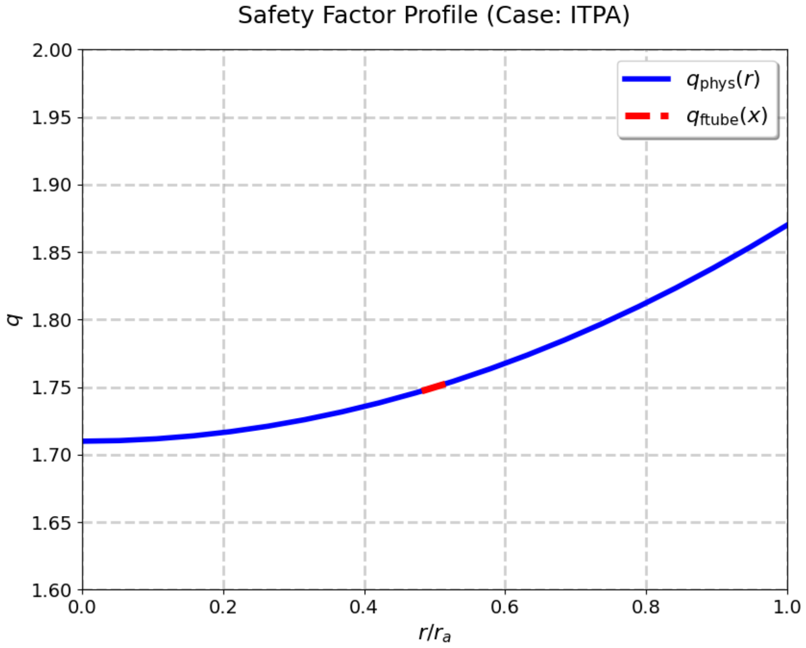
shear = 0.0457

$L_x = 33.8\,\rho_s$ ($N_{pol} = 30$)

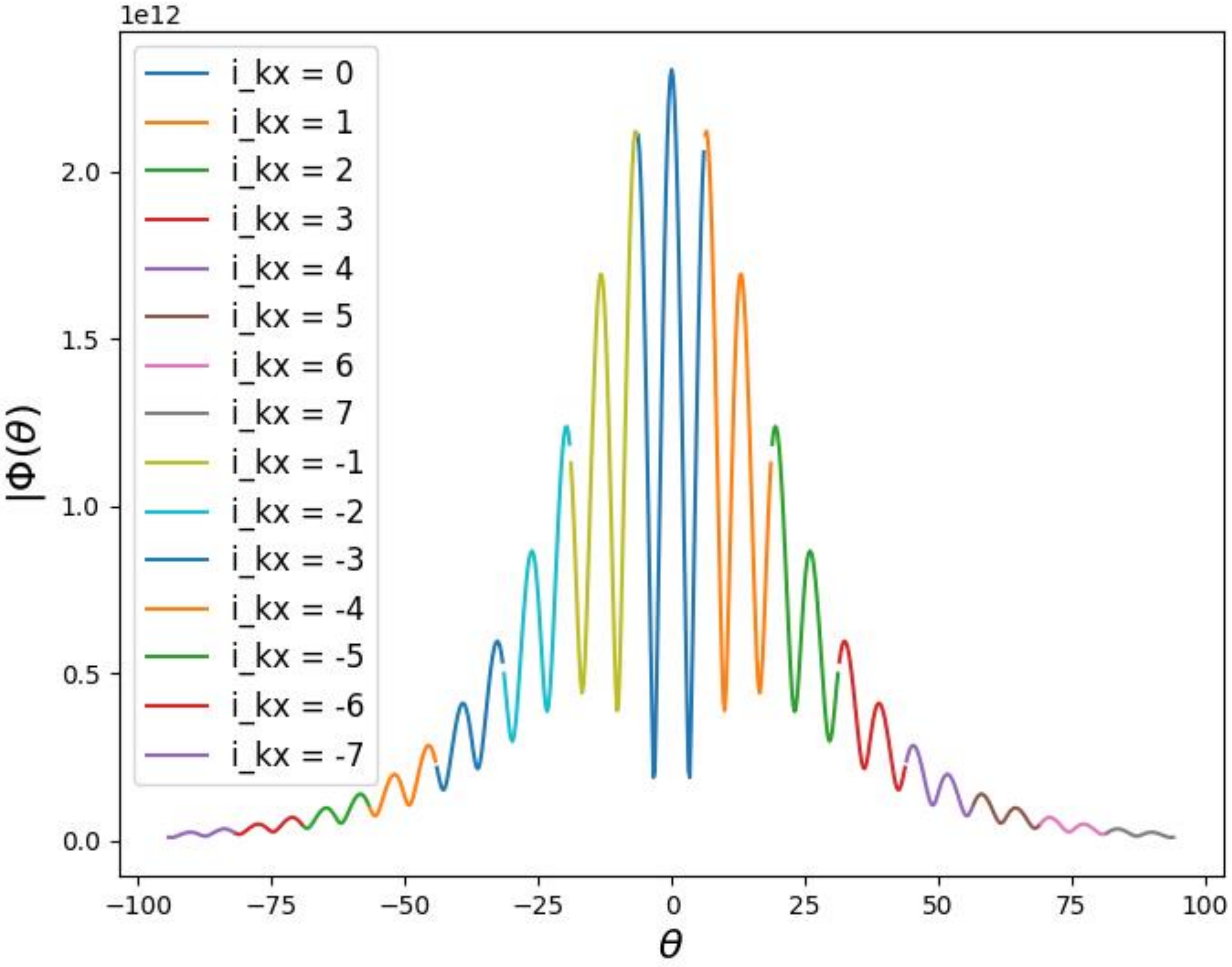
$L_y = 291.7\,\rho_s$

$r_a / \rho_s = 927.0$



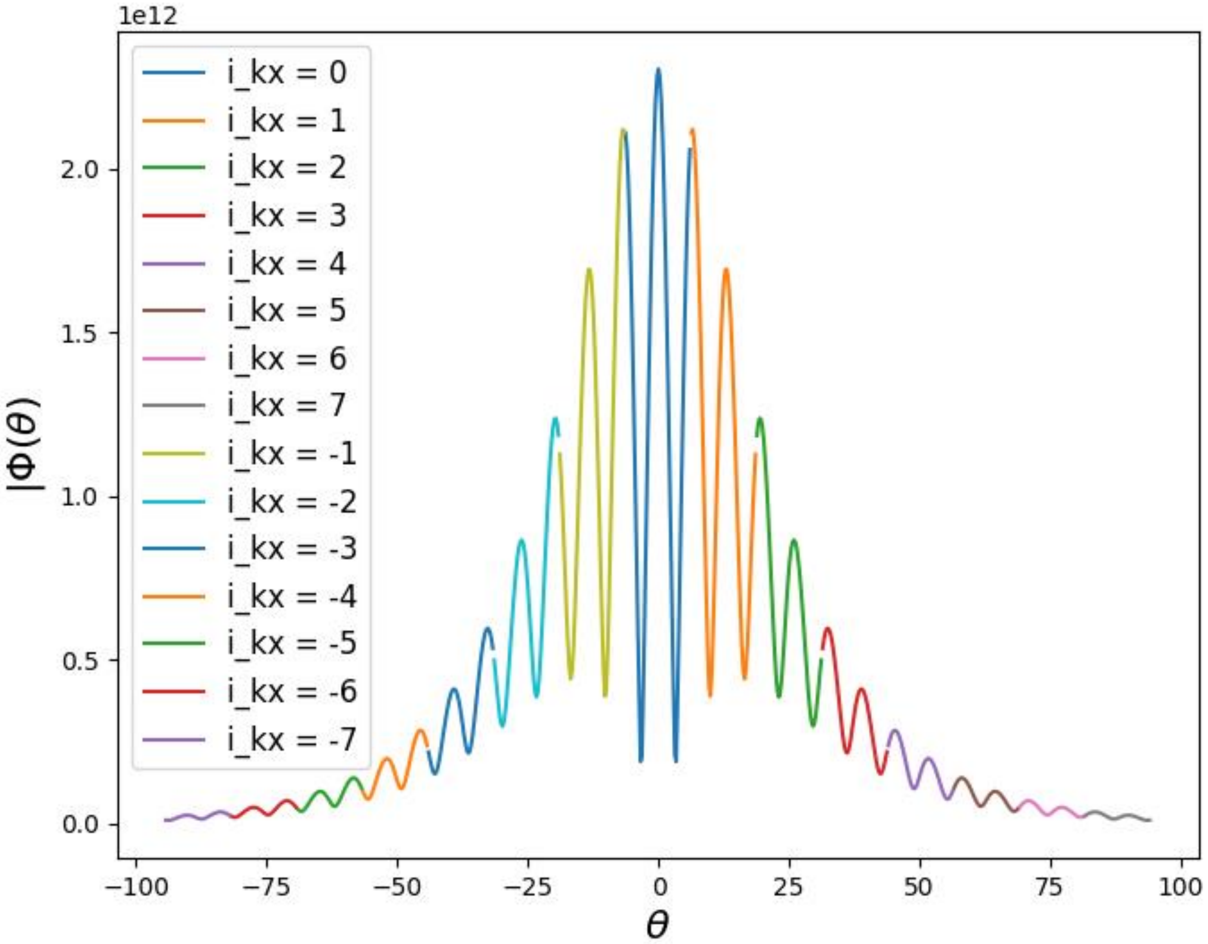


ITPA TAE benchmark: flux-tube GENE – parallel mode structure



TAE instability (physical gap mode)
Around **31 poloidal turns** are needed to resolve the mode: -31π < ϑ < -31π

ITPA TAE benchmark: flux-tube GENE – parallel mode structure



TAE instability (physical gap mode)
Around **31 poloidal turns** are needed to resolve the mode: $-31\pi < \vartheta < -31\pi$

TAE mode results from a specific interaction of two shear Alfvén waves

Multiple-scale parallel mode structure

The width of the continuum mode has to be resolved (implying high kx or, alternatively, large L_x)

For ideal MHD, SAW width is a delta function resulting in infinite domain in the ballooning space

Kinetically, gyro-radius introduces a finite scale ~ ρ_s

ITPA TAE benchmark: flux-tube GENE – poloidal mode structure extraction

Extraction of the poloidal mode structure:

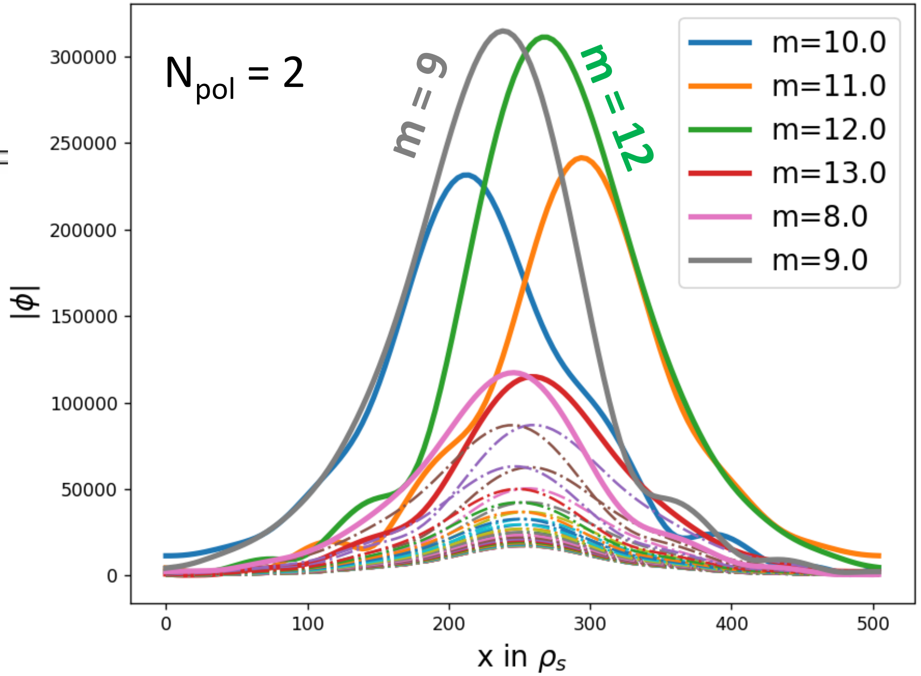
$$x = q_0/(B_0 r_0)(\psi - \psi_0) = r - r_0 \; ; \; y = (r_0/q_0)[q(x)\theta - \zeta] \; ; \; z =$$

$$q = q_0(1 + \hat{s}x/r_0) \; ; \; \hat{s} = (r_0/q_0)dq/dr$$

$$\phi_n = \Phi_n(x,z)\exp(ik_y r_0\theta)\exp(ik_y x\hat{s}\theta)\exp[-ik_y(r_0/q_0)\zeta]$$

$$n_0 = k_y r_0/q_0 \; ; \; m_0 = k_y r_0 = k_y \rho_s/\rho_* \; ; \; \rho_* = \rho_s/r_0$$

$$\phi_n = \sum_{m'}\left[\sum_{k_x}\hat{\Phi}_{k_x m' n}\exp(ik_x x)\right] e^{im'\theta}\exp(ik_y x\hat{s}\theta)\exp(im_0\theta)\exp(-in_0\zeta), \; \theta \in [-N_{pol}\pi, N_{pol}\pi)$$

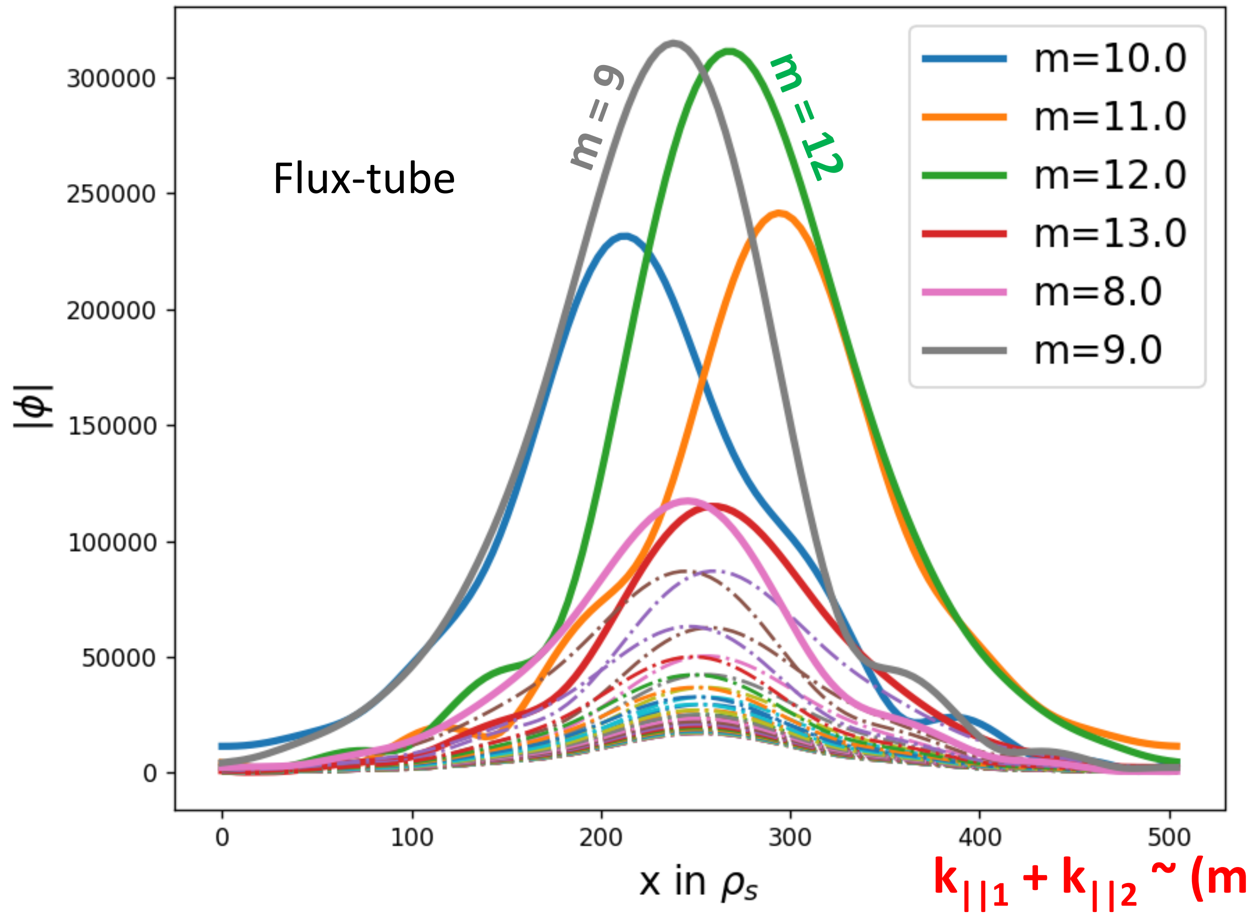


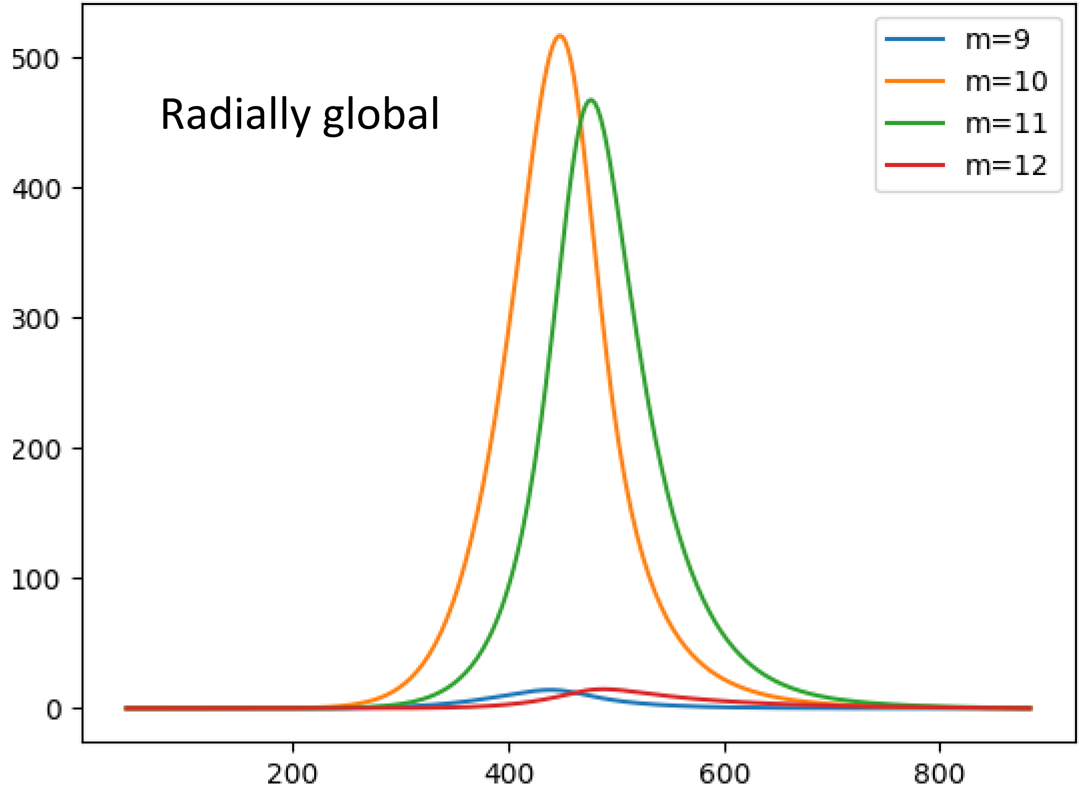
$k_{||1} + k_{||2} \sim (m_1 - n_0 q_0) + (m_2 - n_0 q_0) = 0$

- Mode structure of coupled poloidal harmonics (a “standing wave”) is reproduced:
  $m_1 + m_2 = 2 n_0 q(s_0) = 21$ for $n_0 = 6$ and $q(s_0) = 1.75$
- **In contrast to global result, harmonics with $m_2 - m_1 > 1$ are present and strong!**
- The radial structure of the mode is different : **periodic BC in radial direction (box „multiplied“)**

Toroidal mode number:
$n_0 = k_y r_0/q_0$
Poloidal mode number:
$m_0 = k_y r_0$

ITPA TAE benchmark: flux-tube GENE – radially global GENE comparison





$$k_{||1} + k_{||2} \sim (m_1 - n_0 q_0) + (m_2 - n_0 q_0) = 0$$

- Mode structure of coupled poloidal harmonics (a “standing wave”) is reproduced:

  $m_1 + m_2 = 2\, n_0\, q(s_0) = 21$ for $n_0 = 6$ and $q(s_0) = 1.75$
- **In contrast to global result, harmonics with $m_2 - m_1 > 1$ are present and strong!**
- The radial structure of the mode is different : **periodic BC in radial direction (box „multiplied“)**

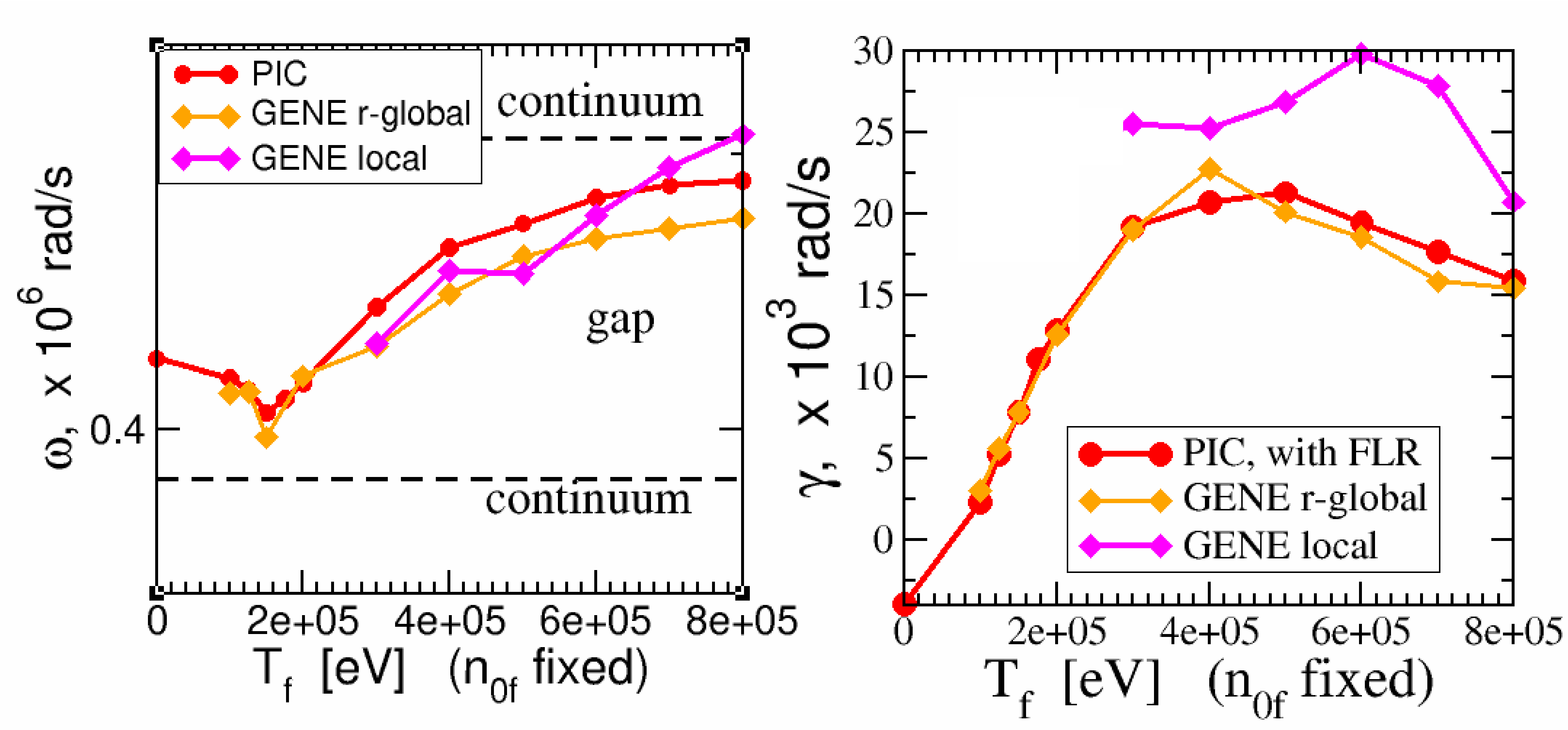
Toroidal mode number:

$n_0 = k_y\, r_0/q_0$

Poloidal mode number:

$m_0 = k_y\, r_0$

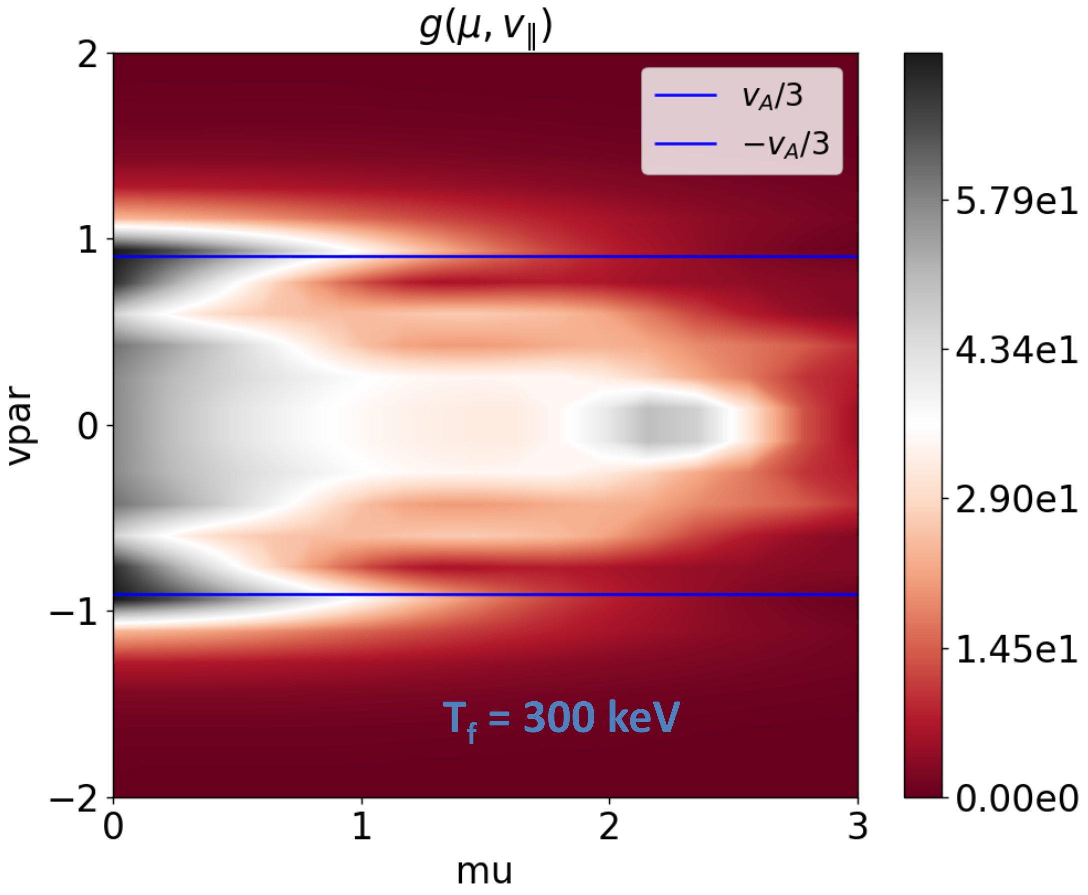
ITPA TAE benchmark: flux-tube GENE – radially global GENE comparison

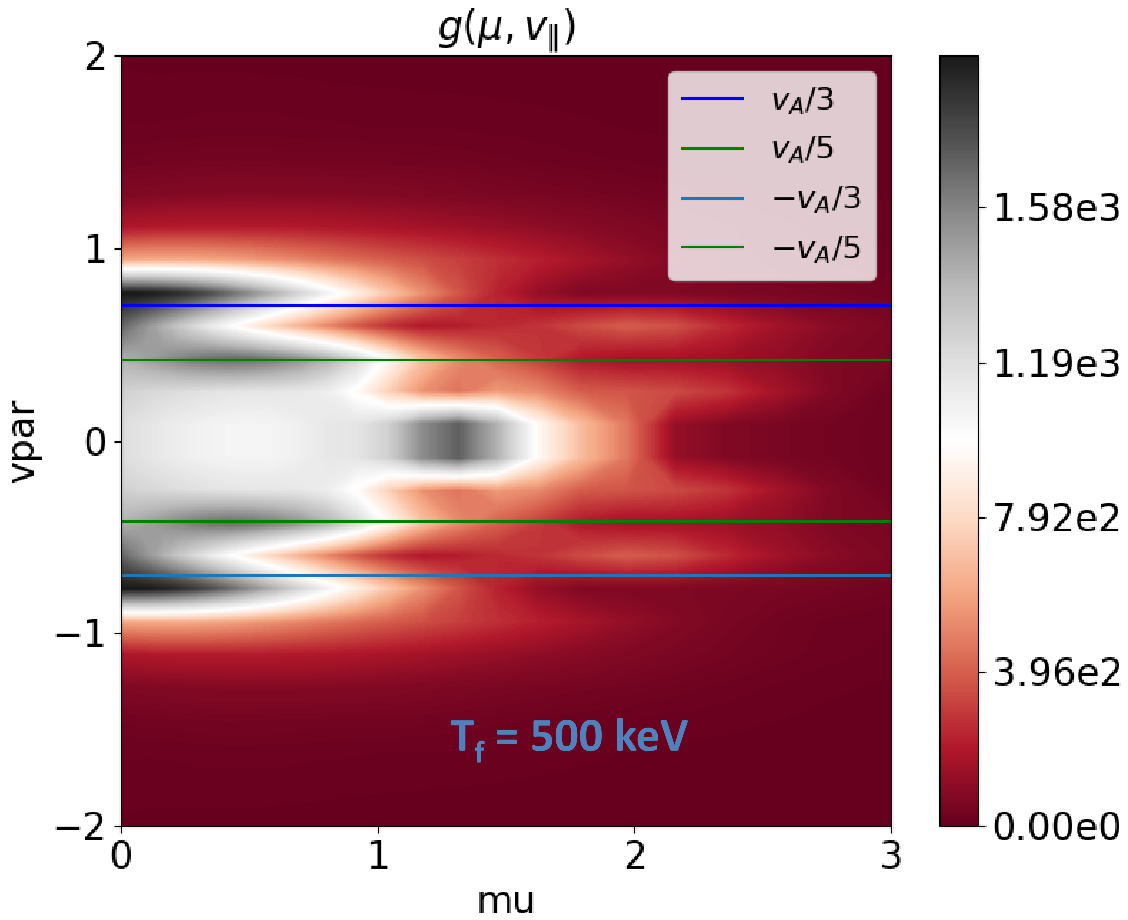


Good agreement in the TAE frequency; comparable numbers in the growth rate

Parallel mode structure at the accumulation point is the key for the mode; included (to some extent) in flux tube

ITPA TAE benchmark: flux-tube GENE – radially global GENE comparison





Energetic-particle velocity space: non-adiabatic distribution function

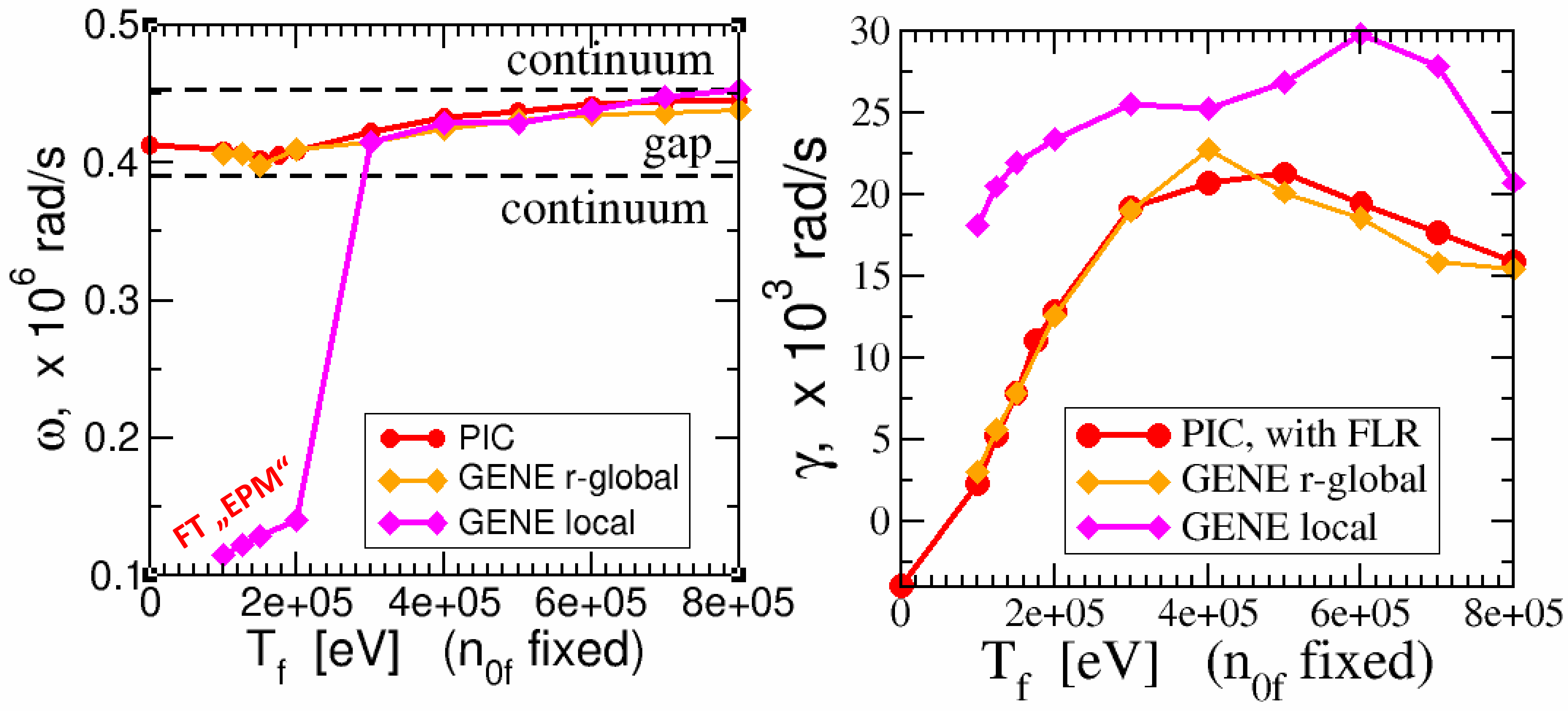
Resonances with energetic passing particles: the main resonance v_A and the „side-band“ resonances $v_A/(2\ s - 1)$: $v_A/3$, **$v_A/5$ (?), $v_A/7$ (?)** …

In global simulations: only $v_A/3$ or v_A are usually relevant

Resonance condition:

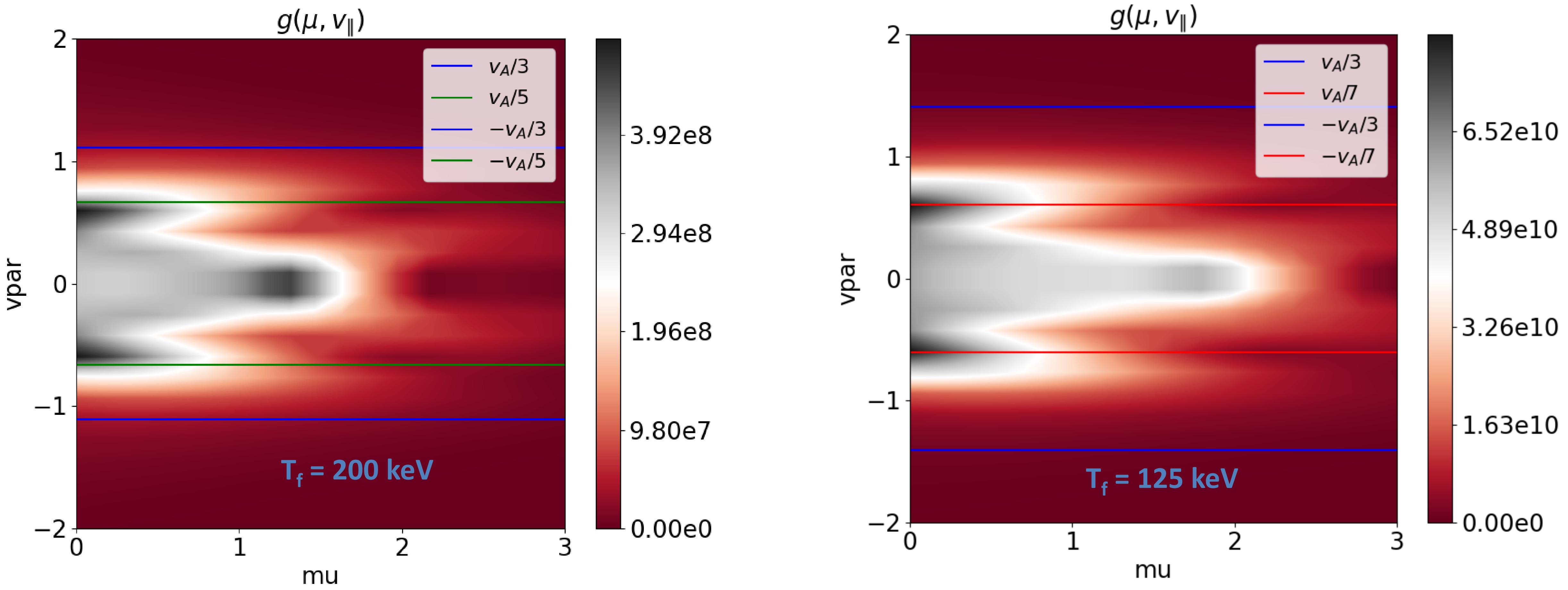
$k_z\ v_A = k_{z1}\ v_{EP}$, k_{z1} /= k_z (generaly)

ITPA TAE benchmark: flux-tube GENE – radially global GENE comparison



For smaller temperatures: jump to an „EPM" in the flux-tube case (deep into the continuum)
The „flux-tube EPM" appears to be well resoved (codewise). Is it real?

ITPA TAE benchmark: flux-tube GENE – radially global GENE comparison



Energetic-particle velocity space: non-adiabatic distribution function

Higher-order „side-band“ resonances are dominant: $v_A/5$, $v_A/7$; lower frequencies better for drive

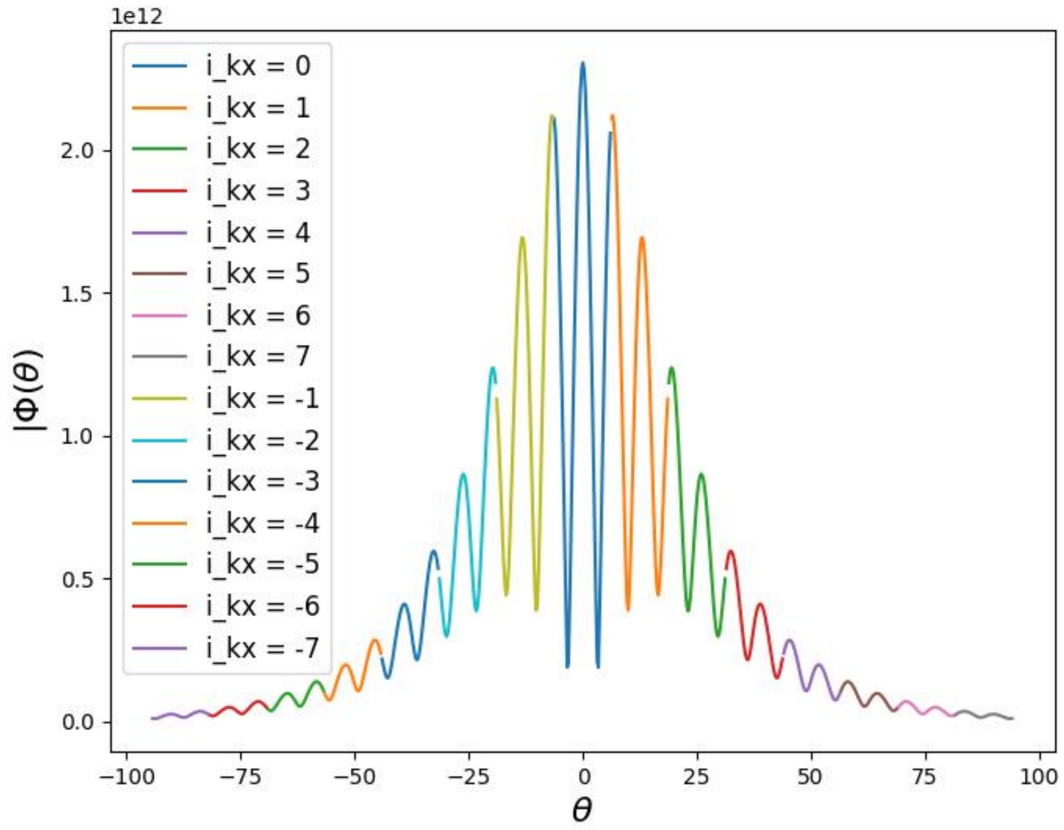
In global simulations: only $v_A/3$ is usually relevant ; frequency remains in the TAE gap

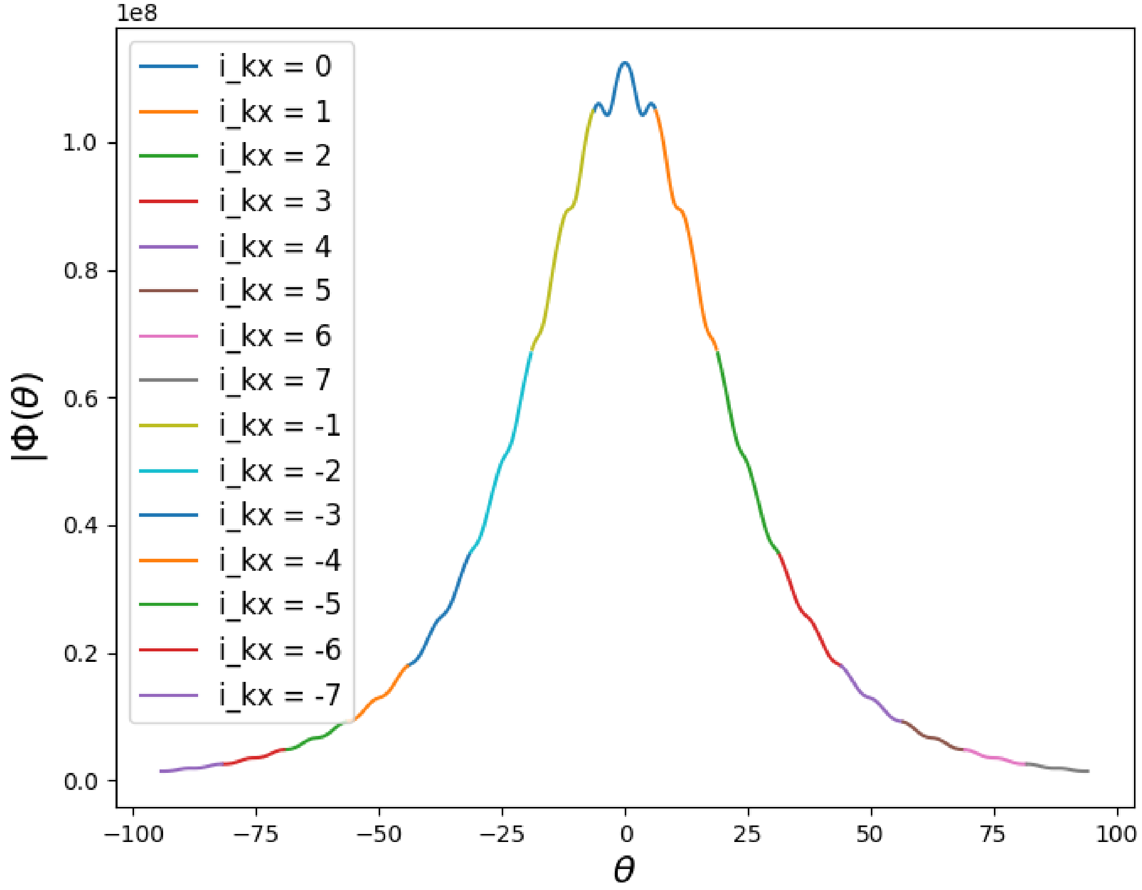
Resonance condition:

$k_z v_A = k_{z1} v_{EP}$, k_{z1} /= k_z (generaly)

Additional „FT“ resonance because of the FT distortions in the mode structure?

ITPA TAE benchmark: flux-tube GENE – parallel mode structure





TAE instability (physical gap mode)
More than **31 poloidal turns** are needed to resolve the mode: $-31\pi < \vartheta < -31\pi$

Flux-tube „EPM". Is it physical?

Parallel and radial mode structures result from the same dataset

Conclusions (not yet final)

- Reasonable agreement in frequency for higher Tf; similar order of growth rates
- Mode structure is very broad in parallel direction; multiple scales (fine-scale component + envelope)
- The radial mode structure is different (additional couplings): may be a consequence of the periodic boundary conditions (radially) effectively „mutiplying“ the box in the radial direction
- Side-band resonances appear (v_A/5, v_A/7) not so present globally: may be a consequence of the issues with the parallel FT mode structure (failure of the „outgoing wave“ boundary condition; reflections)
- These additional resonances drive an „EPM“ at smaller Tf < 300 keV