Electromagnetic version of the full-F global gyrokinetic code GYSELA

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TSVV-G meeting – 23/02/2026

Contents – EM GYSELA

- Background
- Equations and numerical schemes
- Linear benchmarks
- Nonlinear ITG simulation with finite beta
- Ongoing works
- Summary

Background – instabilities and turbulence

- Instabilities in plasma grow. With several instability modes, the plasma becomes turbulent.
- Cross-field transport is dominated by turbulent transport.
- Turbulence vortices mix density/temperature on different radii during advection.
- Energetic particles and Alfvenic modes interact with turbulence.

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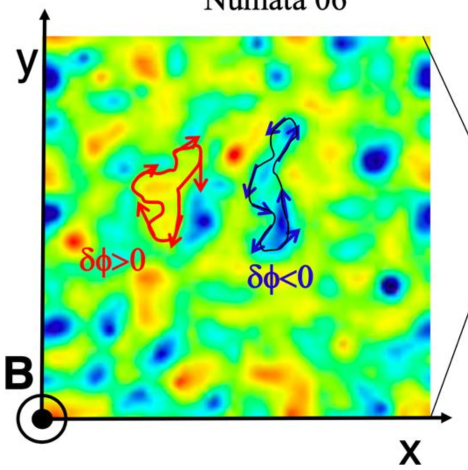
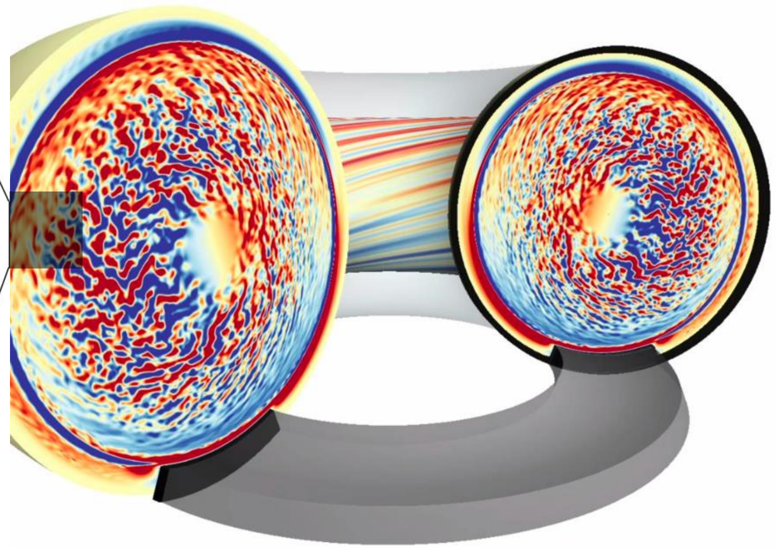


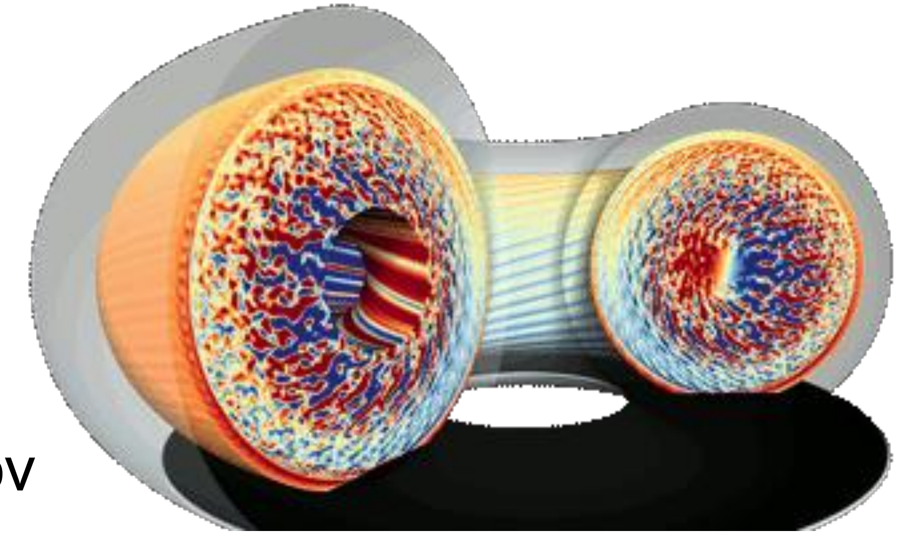
Iso-lines electric potential ϕ



Grandgirard GYSELA-X 2020

Features of GYSELA

[Grandgirard CPC 2016, Qu hal-05342386]



- **Global, flux-driven, full-F** gyrokinetic code
- Semi-Lagrangian scheme for solving the Vlasov
- Field solvers: FFT, Spline
- Three kinds of electron models: adiabatic, trapped-kinetic, fully-kinetic
- Limiter SOL condition / diffusive buffer region.
- Circular, Culham (analytical), External (Chease, GVEC, …, under dev)
- Historically, the code was electrostatic. A preliminary electromagnetic version ($A_{||}$ only) has been included during the PhD of Camille Gillot (2020).
- This work: correct scheme, recent developments and research plan

This presentation

Research plans for EM version of GYSELA

- Constant-of-Motion (CoM) space initialisation & diagnostics
- Alfvén eigenmodes with full-F, Phase Space Zonal Structures (PSZS)
- Add shaping & realistic geometry
- Tearing modes
- Kinetic ballooning modes
- MHD instabilities, $\delta B_{\parallel}$
- EP-AE-turbulence interaction
- RMP
- …

TSVV-G tasks

The gyrokinetic Vlasov-Maxwell system

- Electromagnetic ($\Phi, A_\parallel = A_{\parallel S} + A_{\parallel H}$, no $\tilde{B}_\parallel$)
- Poisson:

$$-\sum_s \nabla \cdot \frac{m_s n_{s0}}{B^2} \nabla_\perp \Phi = \sum_s q_s \bar{n}_s, \quad \bar{n}_s = \int d\mu dp_\parallel J^T [B^*_{\parallel s} \bar{F}_s],$$

- Mixed-variable Ampère equation [Mishchenko PoP 2017, CPC 2019]

$$-\nabla_\perp^2 A_{\parallel H} + \sum_s \frac{\mu_0 q_s^2 n_{0,s}}{m_s} A_{\parallel H} = \mu_0 \sum_s \delta \bar{J}_{\parallel s} + \nabla_\perp^2 A_{\parallel S} \quad \sum_s \delta \bar{J}_{\parallel,s} = \sum_s q_s \int d\mu dp_\parallel J^T [p_\parallel B^*_{\parallel s} \bar{F}_s] - J_{\parallel,0},$$

$$p_\parallel = v_\parallel + \frac{q_s}{m_s}(A_{\parallel H} + A_{\parallel S} - \overline{A_{\parallel S}}),$$

$A_{\parallel H}$ should be small to avoid cancellation problem!

- Ideal MHD predictor $\partial_t A_{\parallel S} = -\nabla_\parallel \Phi,$

The gyrokinetic Vlasov-Maxwell system

- Mixed-variable gyrokinetic Vlasov Eq. (first order) [Brizard and Hahm RMP 2007]

$$\partial_t(B^*_{\|s}\bar{F}_s) + \nabla\cdot(B^*_{\|s}\dot{\mathbf{X}}\bar{F}_s) + \partial_{p_\|}(B^*_{\|s}\dot{p_\|}\bar{F}_s) = B^*_{\|s}\mathcal{R}(\bar{F}_s),$$

$$B^*_{\|s}\dot{\mathbf{X}} = \frac{\mathbf{B}^*_s}{m_s}\partial_{p_\|}\bar{H} - \frac{\nabla\bar{H}\times\mathbf{B}}{q_s B},$$

$$m_s B^*_{\|s}\dot{p_\|} = -\mathbf{B}^*_s\cdot\nabla\bar{H} - q_s B^*_{\|s}\partial_t\overline{A_{\|S}}.$$

$$\partial_t A_{\|S} = -\nabla_\|\Phi,$$

$$\overline{H}_0 = \frac{m_s}{2}p_\|^2 + \mu B,$$

$$\overline{H}_1 = q_s(\overline{\Phi} - p_\|\overline{A_{\|H}}),$$

- Pullback scheme to reduce $A_{\|H}$ at the end of time step

$$A_{\|S,\mathrm{new}} = A_{\|S,\mathrm{old}} + A_{\|H,\mathrm{old}}, \quad A_{\|H,\mathrm{new}} = 0, \qquad p_{\|,\mathrm{new}} = p_{\|,\mathrm{old}} - \frac{q_s}{m_s}\overline{A_{\|H,\mathrm{old}}}.$$

A non-ideal predictor for $A_{//}$: reducing $A_{\parallel H}$ further

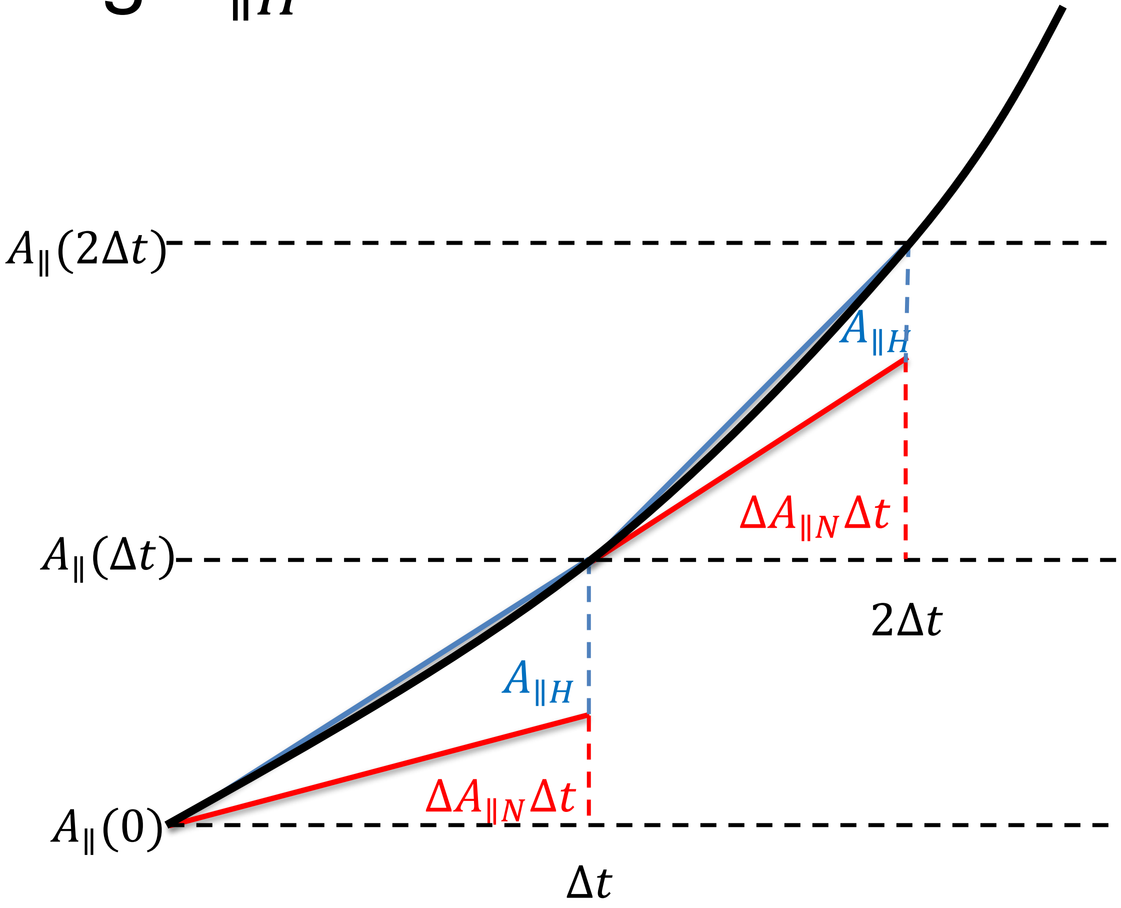
- To further reduce $A_{\parallel H}$, we add a non-ideal predictor

$$\partial_t A_{\parallel s} = -\nabla_\parallel \Phi + \partial_t A_{\parallel N}$$

- $A_{\parallel N}$ is a linear function $A_{\parallel N} = \Delta A_{\parallel N} t$
- $\Delta A_{\parallel N}$ is updated at end of the step to absorb all the slope due to non-ideal effects.

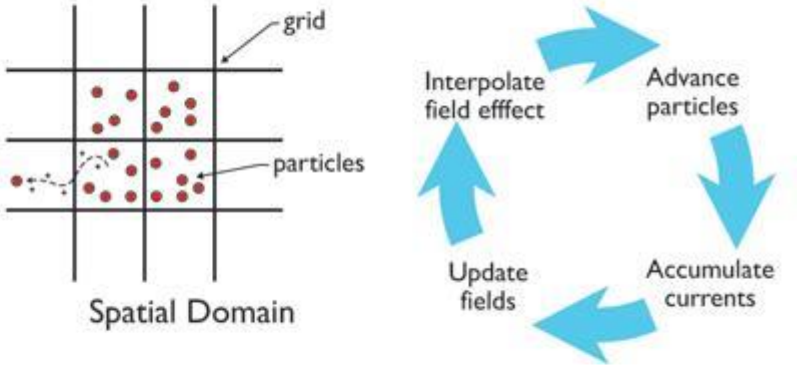
$$\Delta A_{\parallel N,new} = \Delta A_{\parallel N,old} + \frac{A_{\parallel H}}{\Delta t}$$

- The remaining $A_{\parallel H} \sim O(\Delta t^2)$

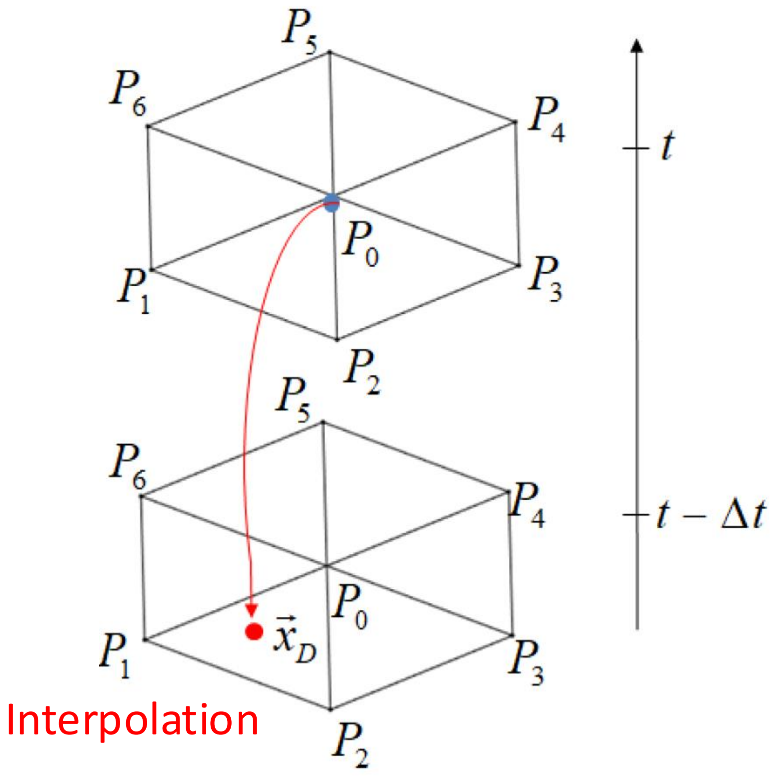


The semi-Lagrangian scheme

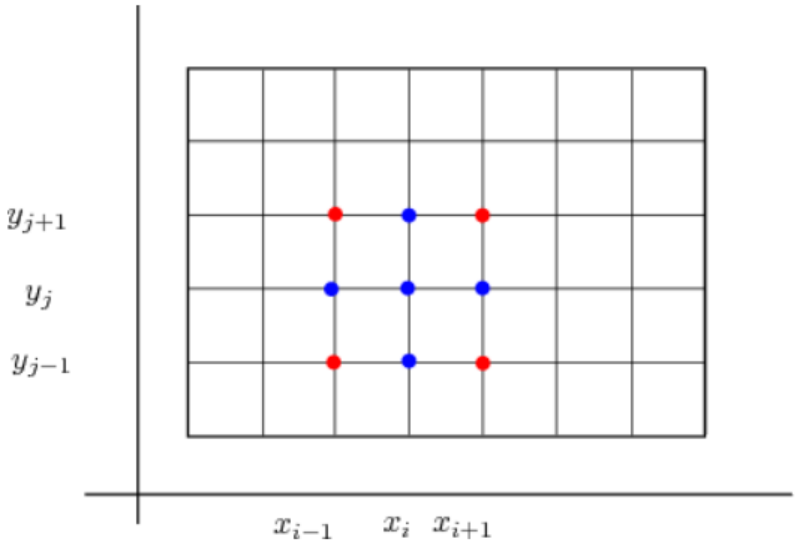
- The Vlasov equation $\frac{dF}{dt} = C(F)$ has three different numerical schemes.



Lagrangian PIC (ORB5, GTC, XGC, etc)

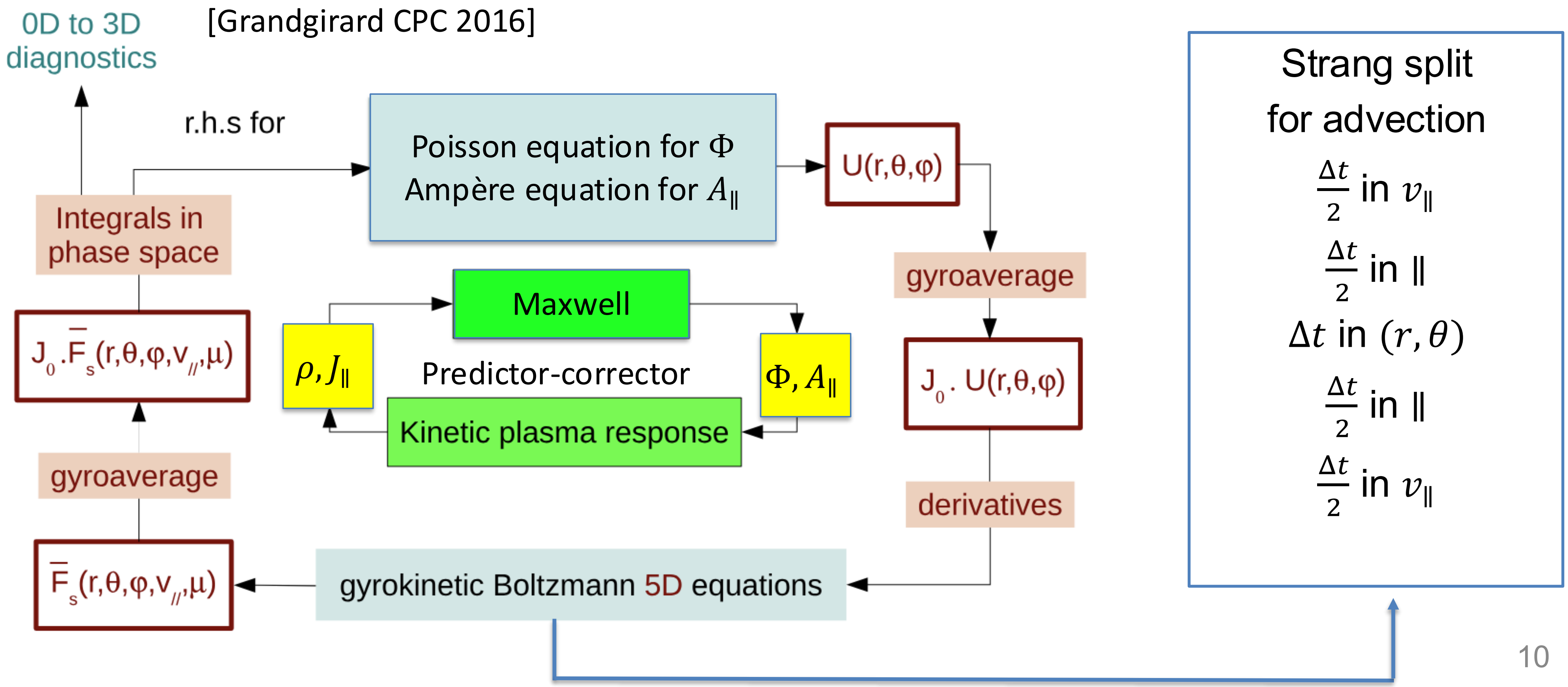


Semi-Lagrangian (GYSELA)



Eulerian grid-based (GENE, GKW, GKNET, etc)

The semi-Lagrangian scheme in GYSELA

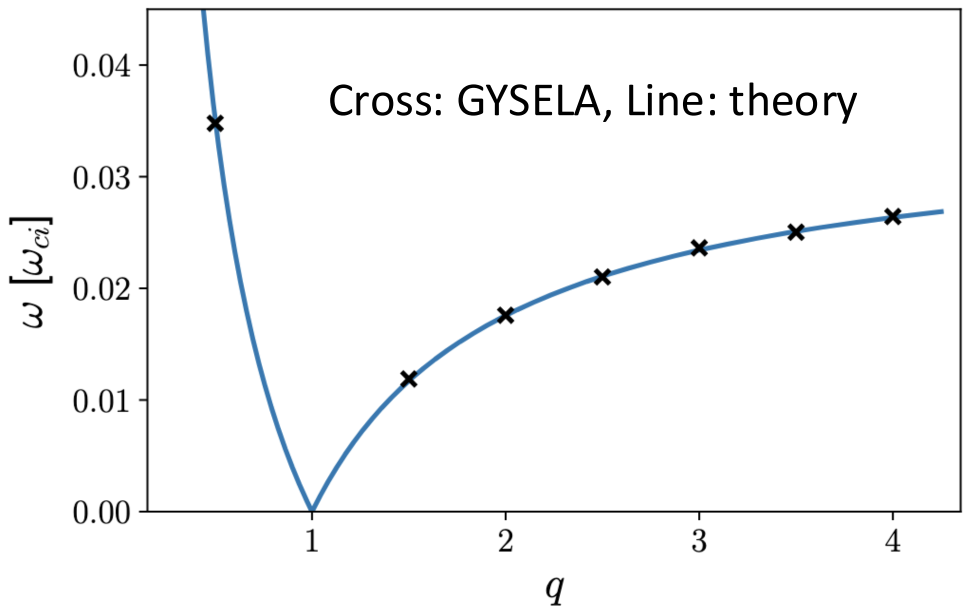


Linear benchmarks - Alfvén wave test in large aspect ratio tokamak, flat profiles

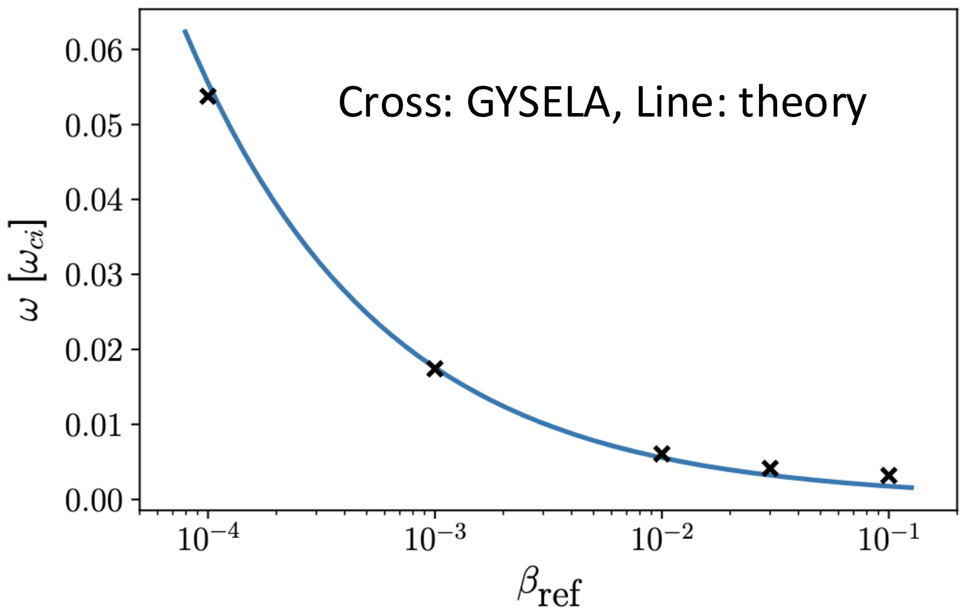
Credit: M Go

- Good match of the frequency of the $m = n = 1$ Alfvén wave $\omega = k_{\parallel} v_A$.

$$\omega = \left(\frac{m}{q} - n\right)\frac{B_0}{R_0\sqrt{\mu_0 m_i n_i}} = \left(\frac{m}{q} - n\right)\frac{\epsilon\rho^*}{\sqrt{\beta_{\mathrm{ref}}}},$$



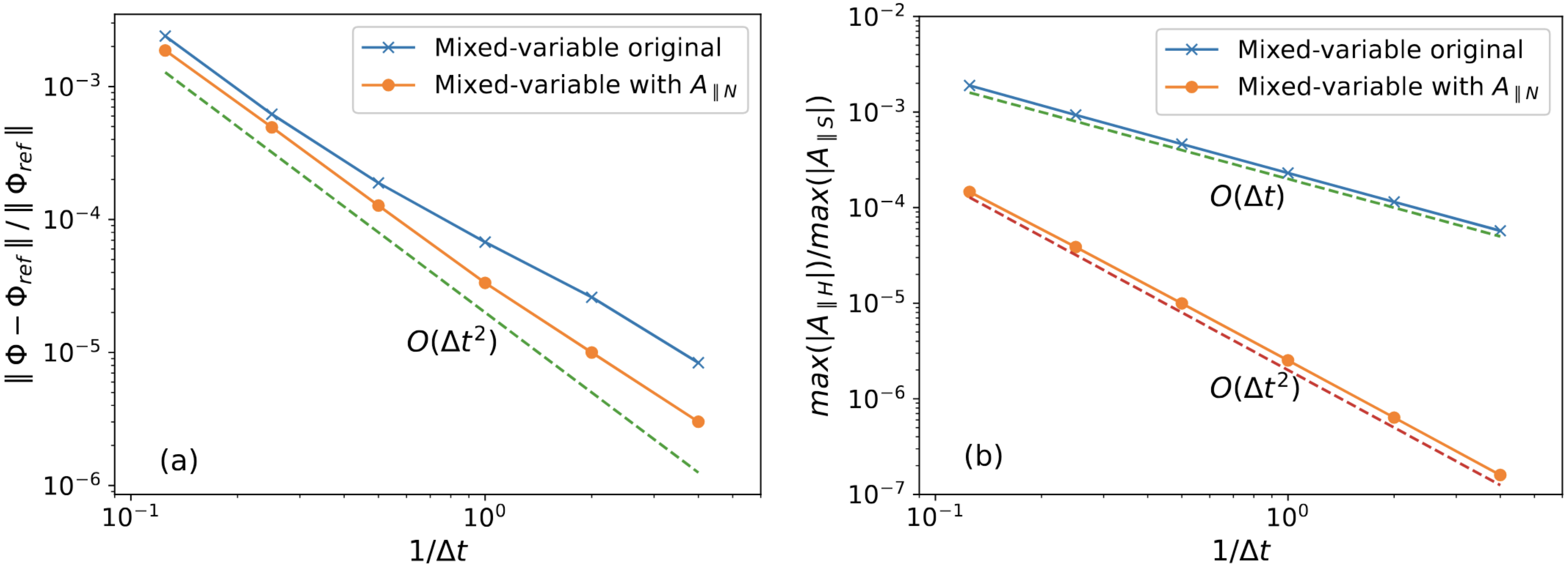
Fixed beta, changing q



Fixed q, changing beta

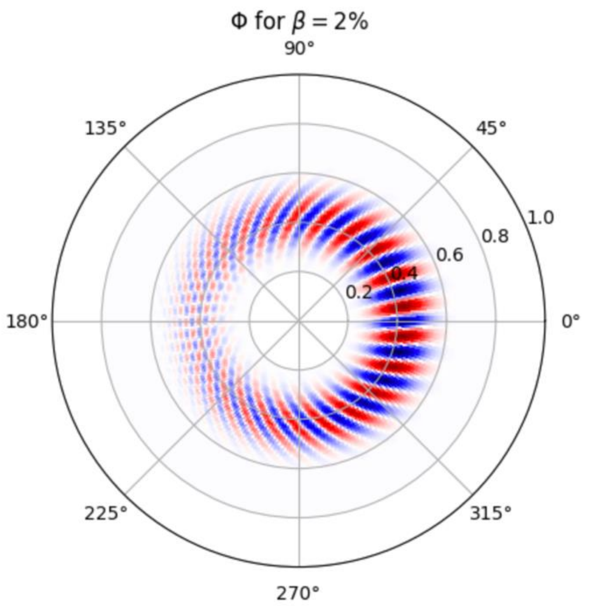
Linear benchmarks - Alfvén wave test

- Convergence test showing the scheme is second-order in time.
  - The scheme with the non-ideal predictor works slightly better
  - It also reduces $A_{\parallel H}$ from $O(\Delta t)$ to $O(\Delta t^2)$

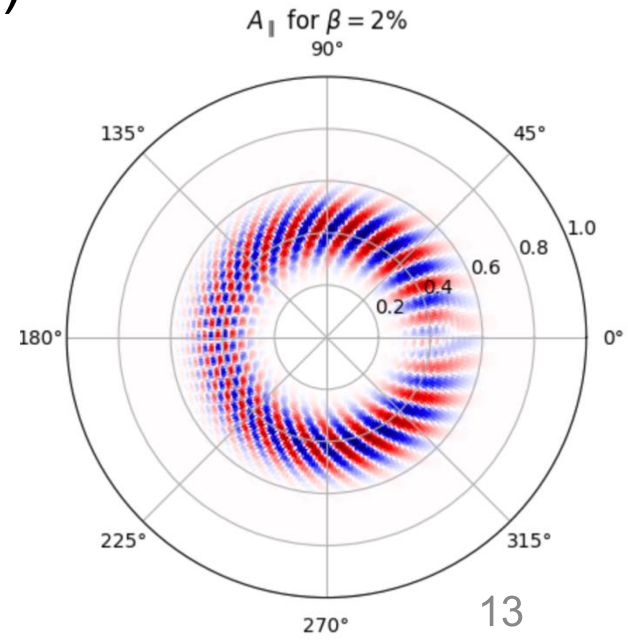


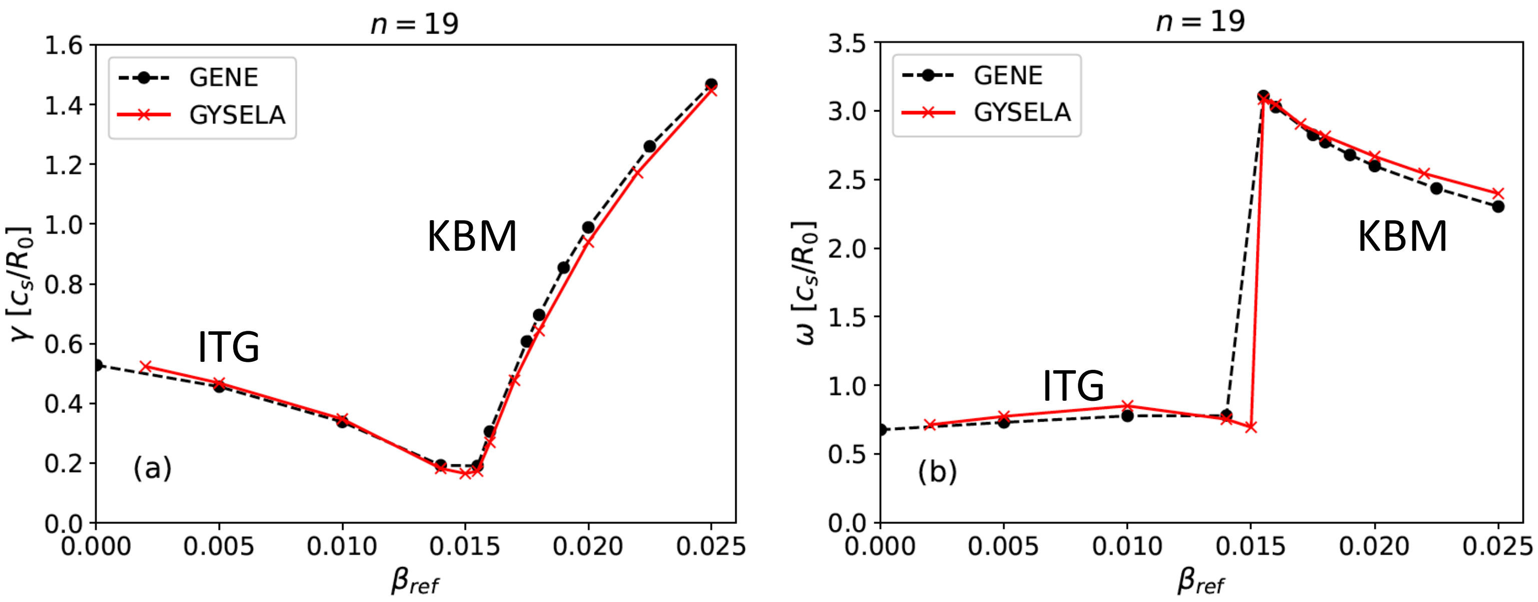
Linear benchmarks - ITG-KBM transition test

ITG=Ion temperature gradient mode, KBM=Kinetic ballooning mode   Credit: CK Chai



- Cyclone-based case for global GK benchmark [Görler PoP 2016]
- Good match with GENE for n=19 mode
- Good match for fixed beta with different n (not shown here)



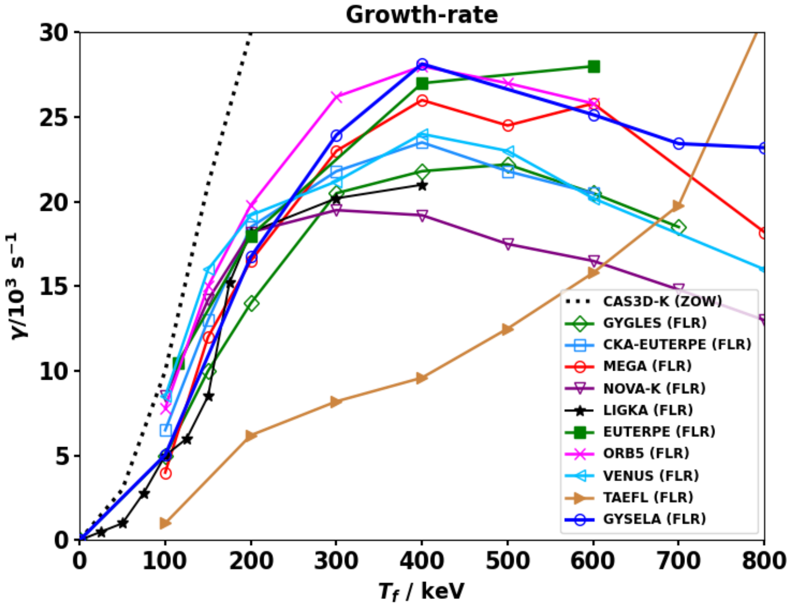


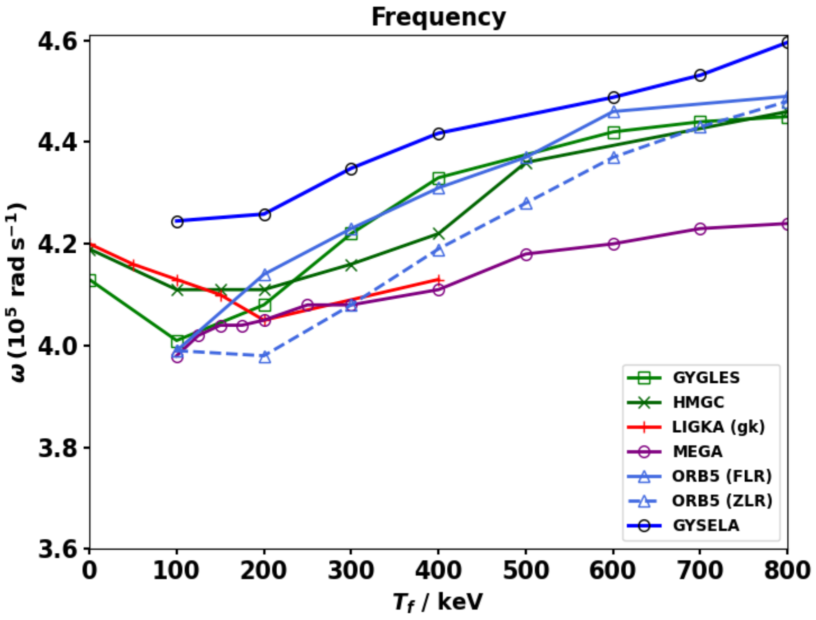
Linear benchmark ITPA-TAE test

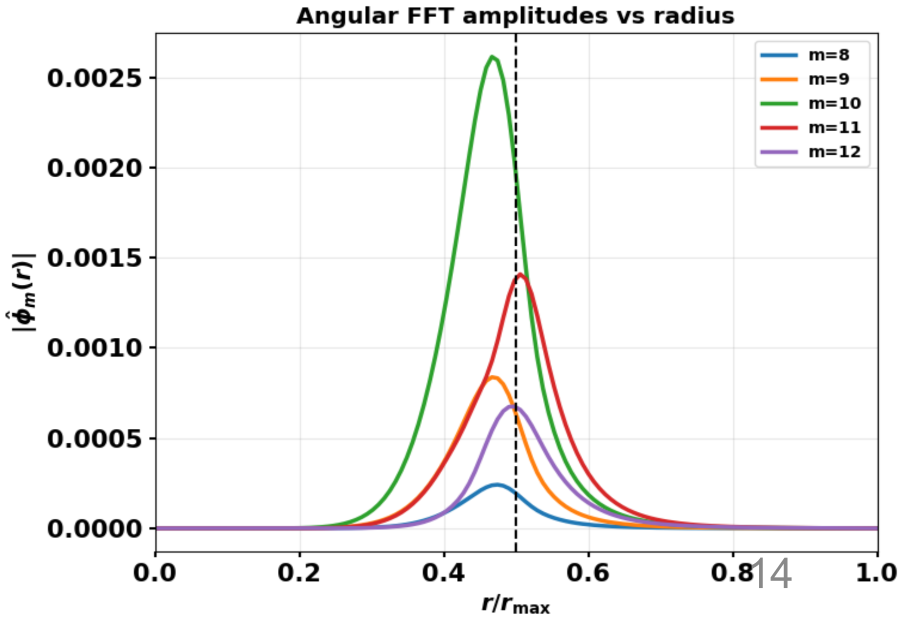
Credit: S Raj

TAE=Toroidal Alfven Eigenmode

- Standard benchmark case of energetic particle driven instabilities, scan EP temperature -> change of growth rate and frequency of TAE.
- Growth rate and frequency matches OK with other codes.
- Global profile relaxation makes the benchmark complicated... [Lu PPCF 2023]



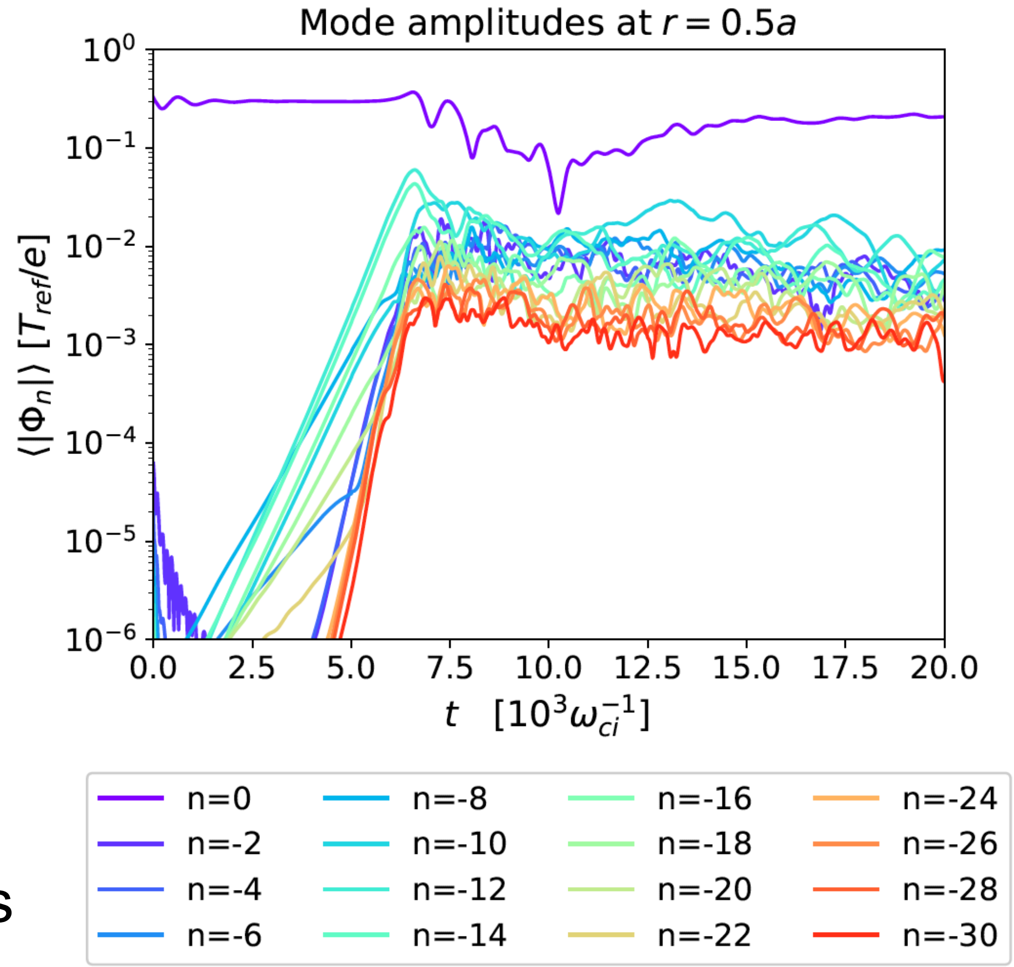




Mode structure for n=6

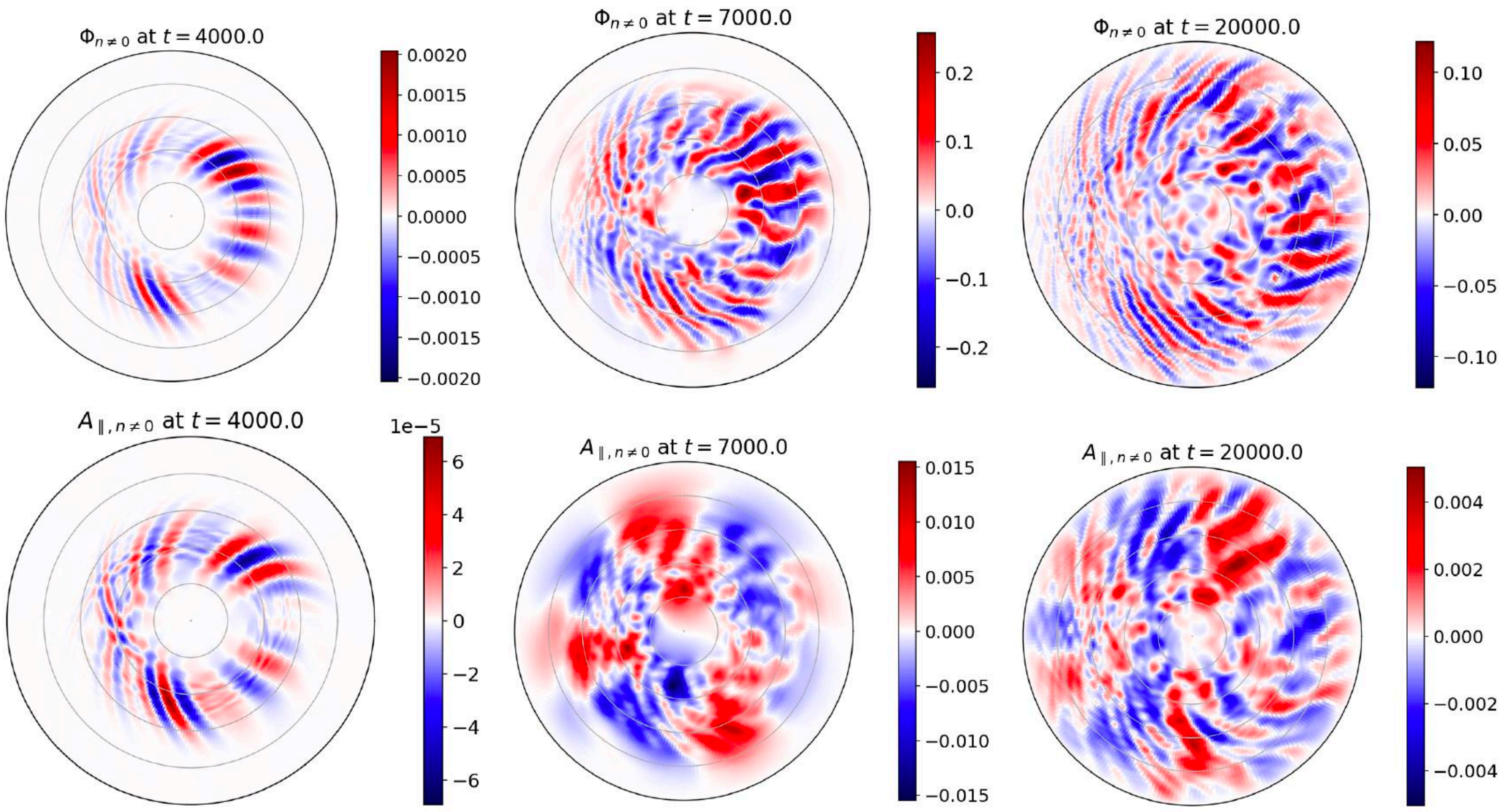
Nonlinear ITG simulation

- "Economic CBC case" used by GKNET and TRIMEG-GKX [Ishizawa PoP 2019, Lu CPC 2026]
  - $\frac{m_i}{m_e} = 100, \rho^* = \frac{1}{100}, \beta_e = 0.5\%$, circular geometry
  - Small grid size $N_r = N_\theta = 256, N_\varphi = 32,\ N_{v\|} = 128, N_\mu = 64$, half torus
  - $\Delta t = 1\omega_{ci}^{-1}, \Delta m = 20$
- Heat source and collision enabled. Diffusive buffer at the boundary.
- Computation cost ~ 0.5M CPU hrs on IRENE-Rome: 64 nodes * 128 CPUs/node @ 2.5 days



Nonlinear ITG simulation

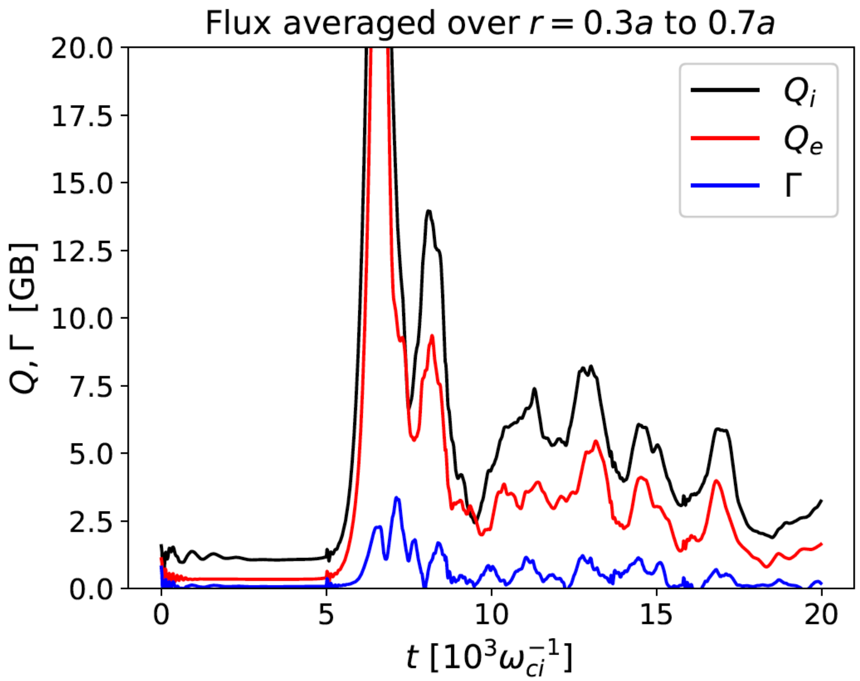
- Fully developed turbulence in the nonlinear stage across the device
- Φ is small scale, but $A_{\|}$ is larger scale, consistent with GKNET [Ishizawa PoP 2019]

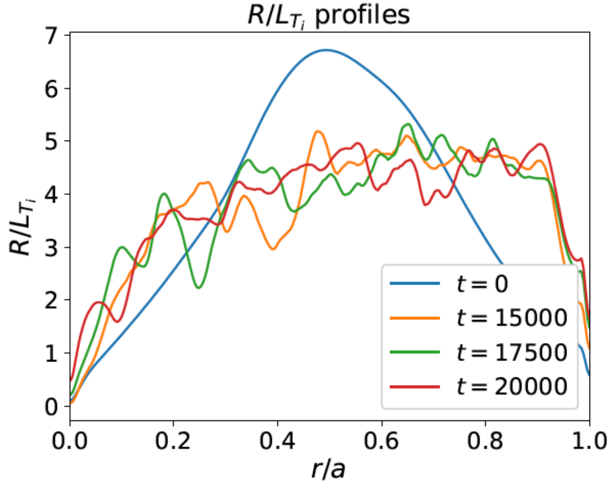


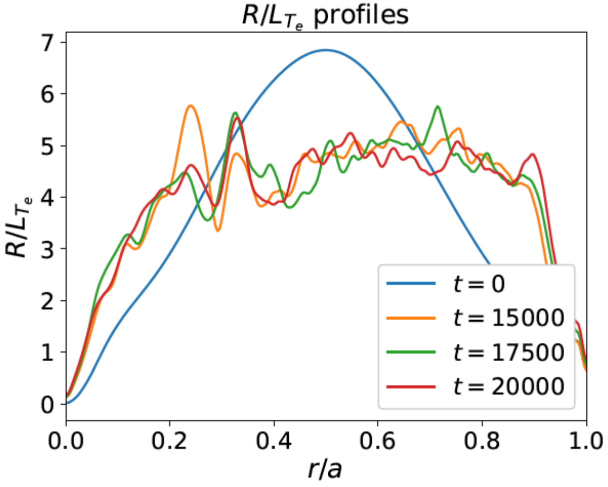
Linear Early-nonlinear Steady-state

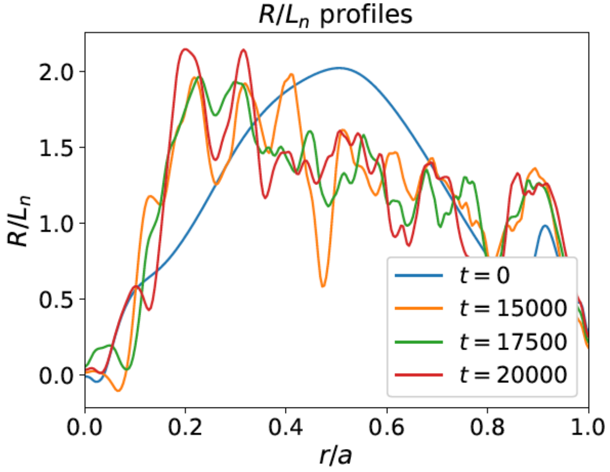
Nonlinear ITG simulation

- Profiles reaching steady-state
- Nonlinear flux-driven EM-ITG simulations with source+sink+collisions





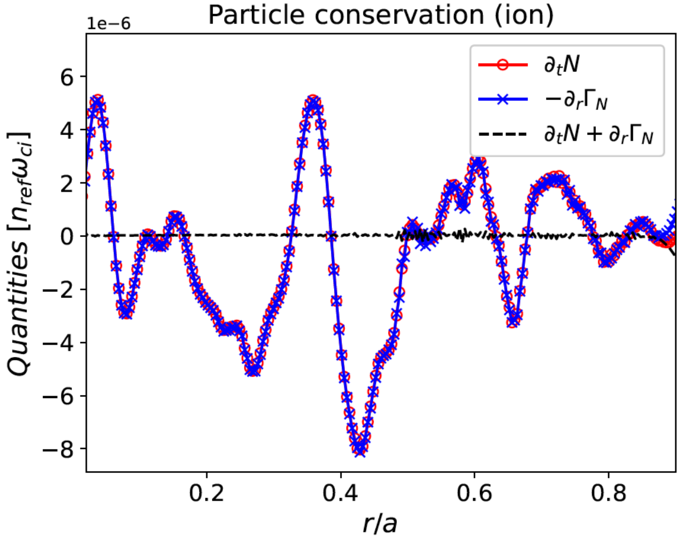




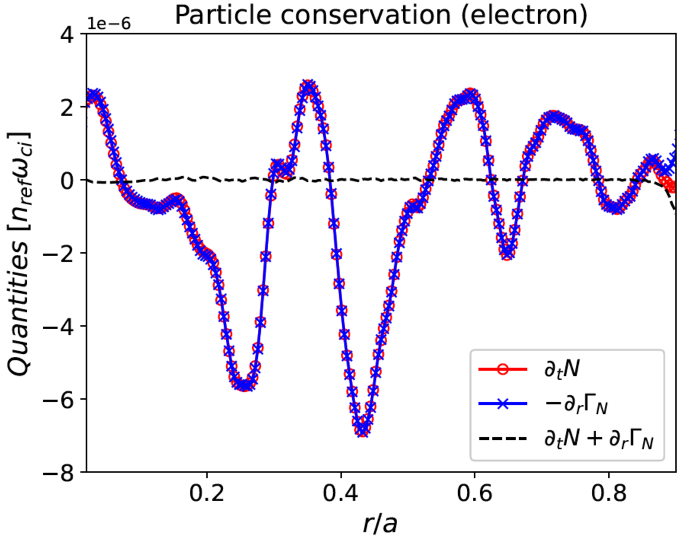
Nonlinear ITG simulation

- Good **conservation of particle** with < 1% error
- Good **conservation of energy and toroidal angular momentum** with < 1% error

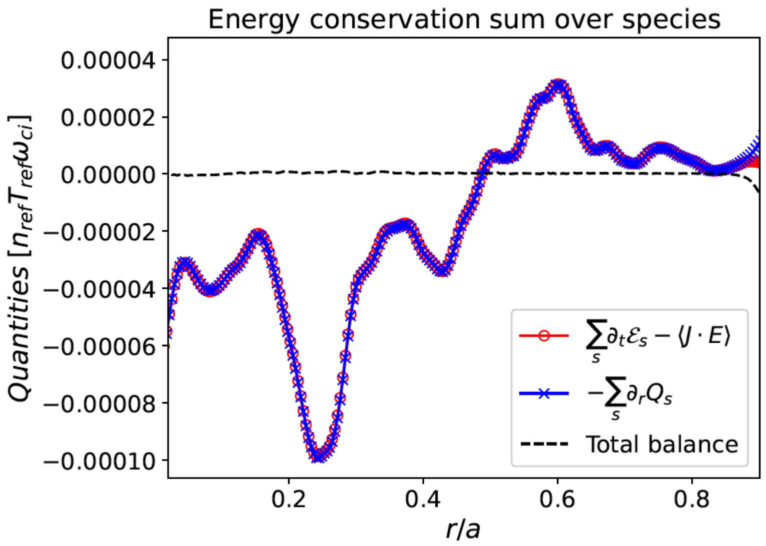
Ions



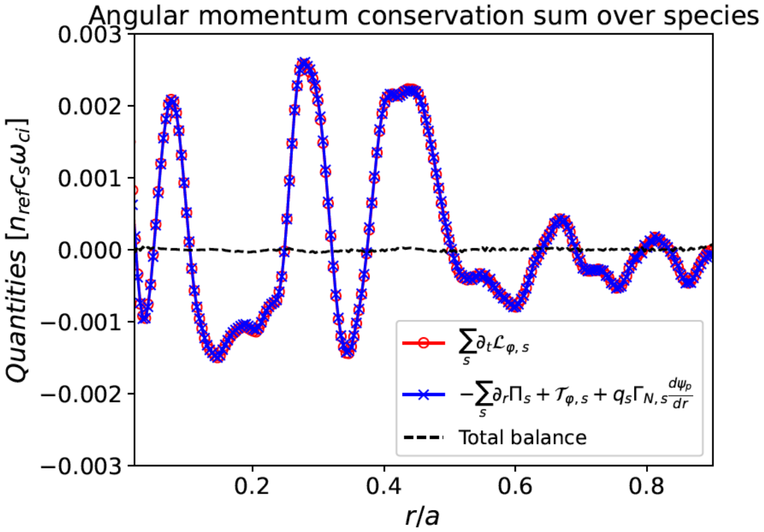
Electrons



Energy



Toroidal angular momentum



Ongoing work – CoM space diagnostics

- CoM grid generated using the EPCoM code (G. Brochard, CNRS);
- GYSELA F5D projection and diagnostics implemented in this work.

General Formula for CoM Distribution

$$F_{CoM} = \sum_{i,j,k,l,m}^{N} F_{GYSELA}(\psi_i, \theta_j, \varphi_k, v_{\parallel,l}, \mu_m) S(Z_{i,j,k,l,m} - Z_{CoM}) \frac{\Delta\Upsilon_{\psi,\theta,\varphi,v_\parallel,\mu}}{\Delta\Upsilon_{CoM}}$$

$$\Delta\Upsilon_{\psi,\theta,\varphi,v_\parallel,\mu} = J_{\psi,\theta,\varphi} J_{v_\parallel,\mu} \Delta\psi\Delta\theta\Delta\varphi\Delta v_\parallel \Delta\mu \qquad \Delta\Upsilon_{CoM} = J_{CoM}\Delta P_\varphi \Delta\lambda\Delta E$$

Methodology

1. **Compute CoM variables at each GYSELA point**

   For each $(\psi, \theta, \varphi, v_\parallel, \mu)$ grid point → evaluate (P_φ, λ, E) locally.

2. **Project GYSELA CoM values onto EPCoM grid**

   Locate each GYSELA CoM point inside EPCoM cell → compute tri-linear weight S → accumulate $F \times S$ onto EPCoM nodes.

3. **Apply volume correction**

   Multiply by $\Delta\Upsilon_{\psi,\theta,\varphi,v_\parallel,\mu}/\Delta\Upsilon_{CoM}$ to correctly weight each contribution.

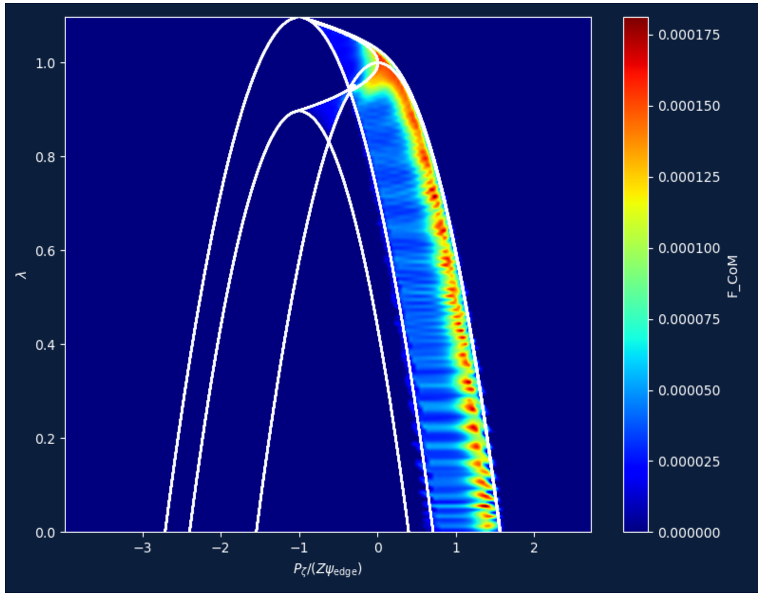
4. **Resolve degeneracy**

   Separate $v_\parallel > 0$ (co-passing) and $v_\parallel < 0$ (counter-passing).

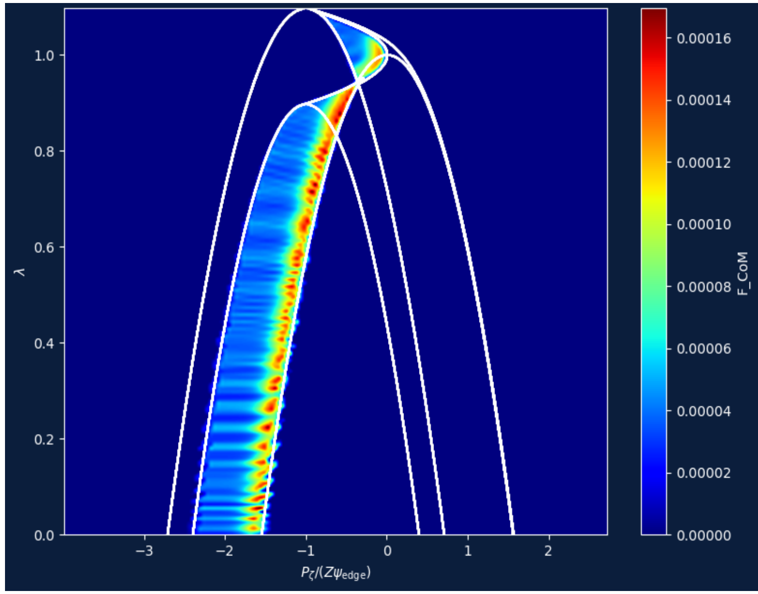
Credit: S Raj, G Brochard

GYSELA Results: Particle Regions in CoM Space

Co-passing ($v_\parallel > 0$)



Counter-passing ($v_\parallel < 0$)

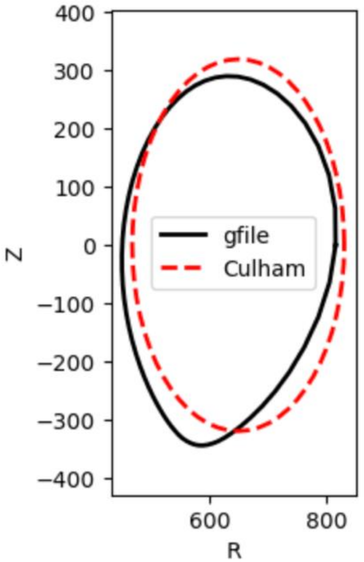


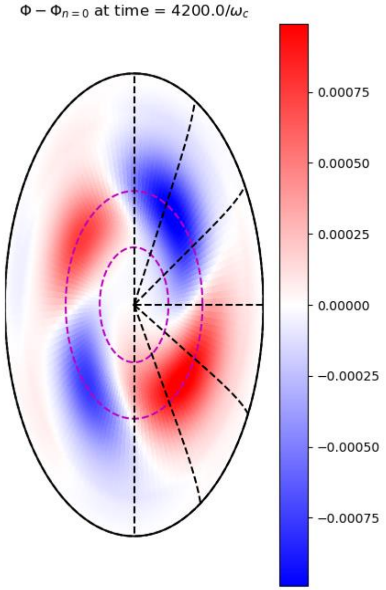
ITPA-EP TAE case, E=400keV

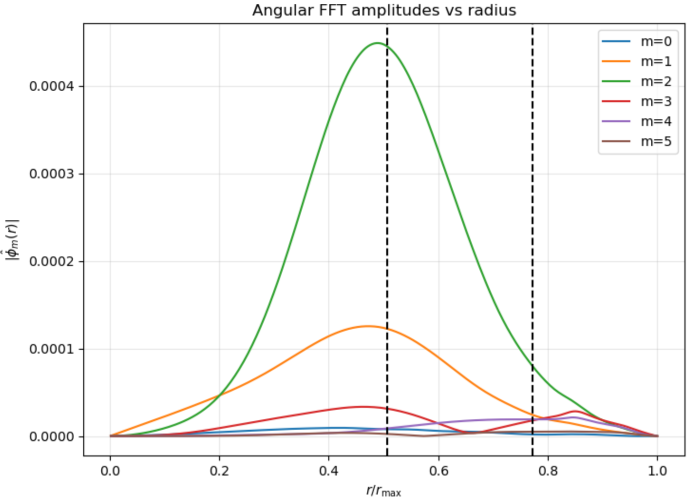
Ongoing work – NLED-AUG benchmark

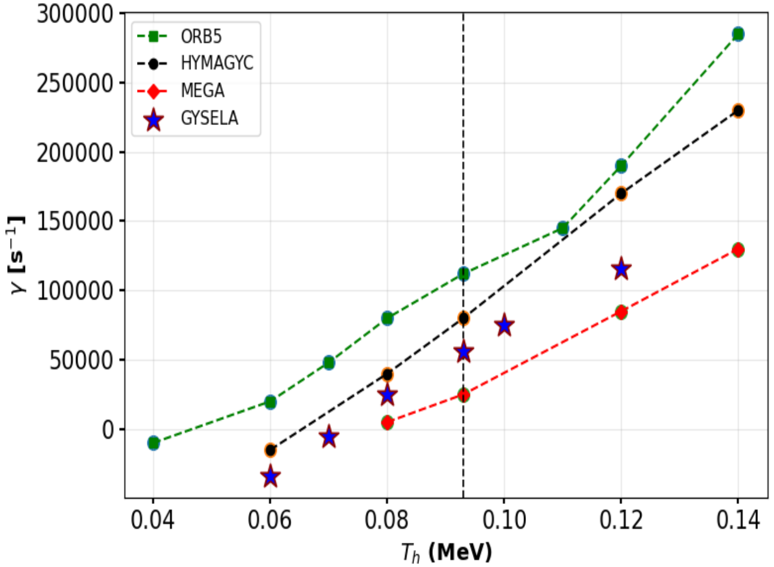
Credit: S Raj

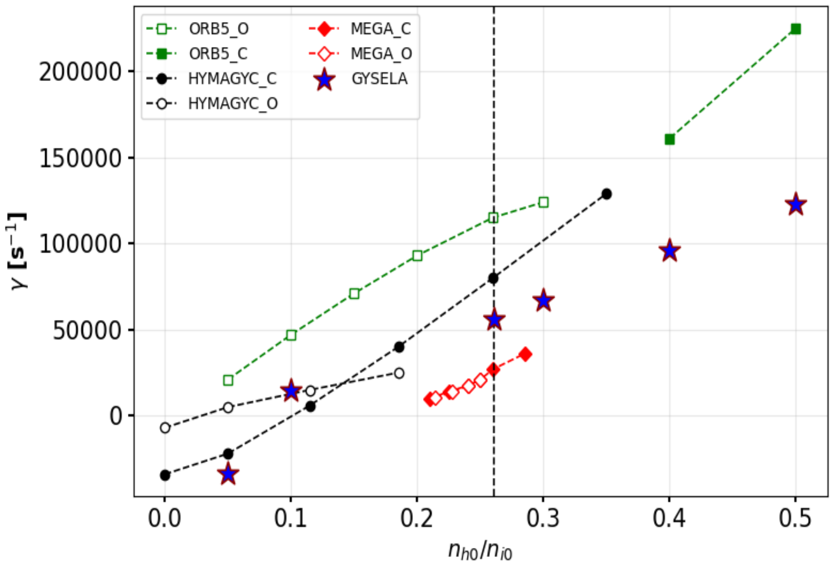
On-axis: ✓ Done
Off-axis: ⏳ Ongoing

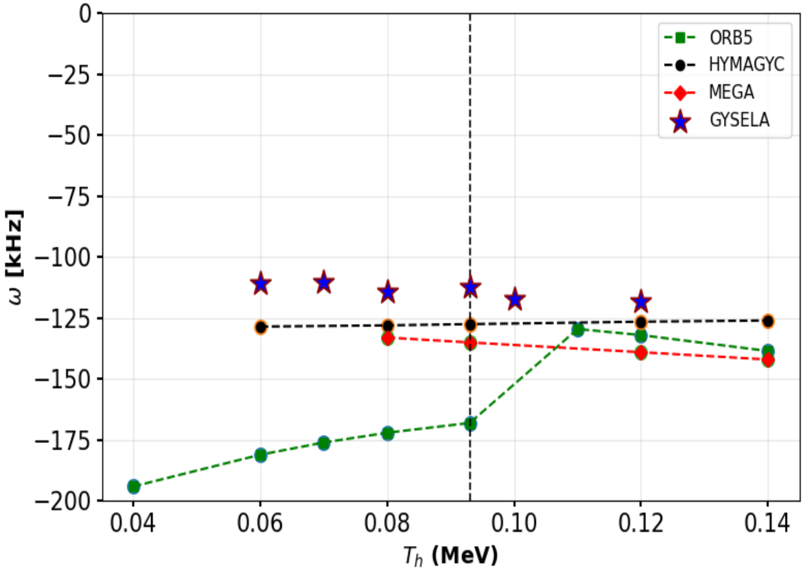


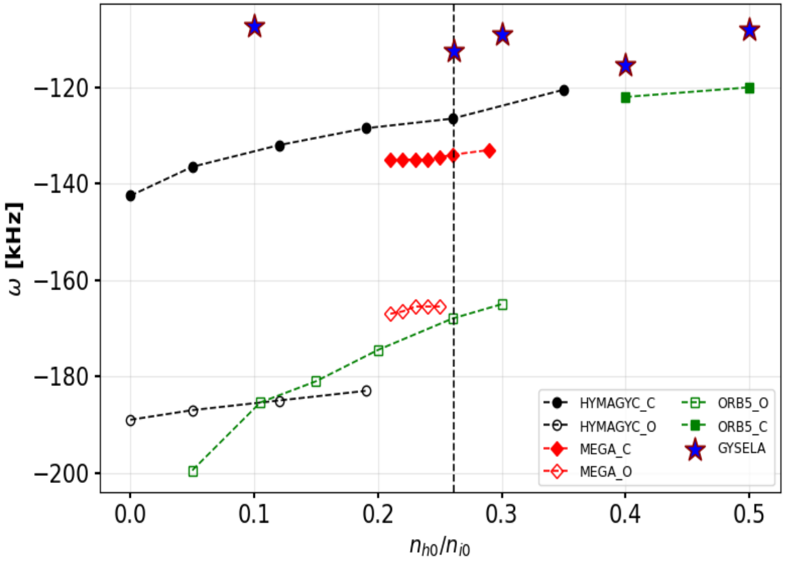












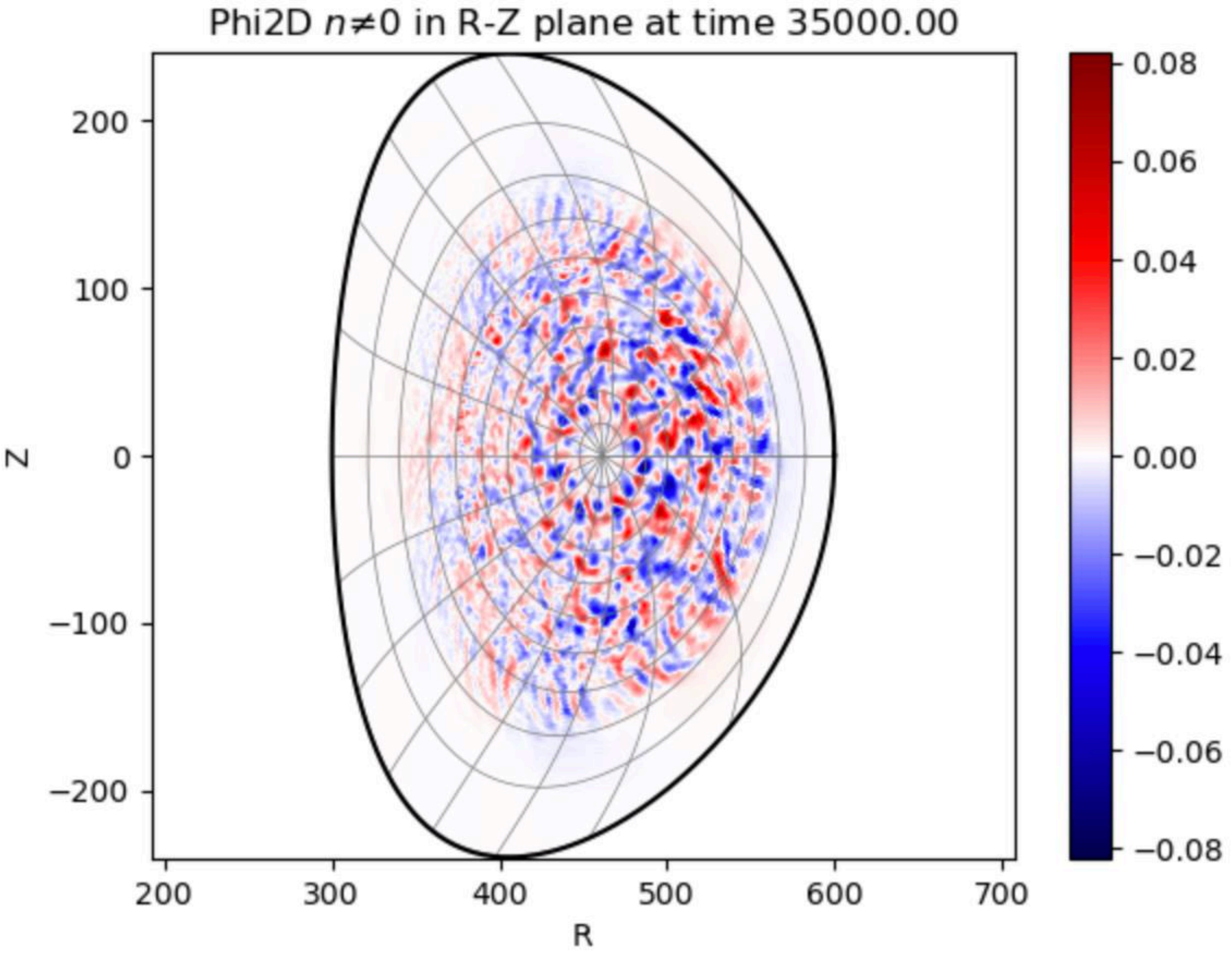
RSAE modes have been successfully observed with CULHAM geometry in GYSELA.

Ref: Vlad et al., Nucl. Fusion 61 (2021) 116026

Acknowledge: Ph. Lauber, IPP for benchmark input reconstruction.

Ongoing work – turbulence in non-circular geometry

- We are adding the capability of reading a Grad-Shafranov equilibrium
- Some benchmarks and fine-tunings are still needed



Right figure: Nonlinear EM ITG in CHEASE equilibrium

Summary

- We have developed and benchmarked the EM version of GYSELA.
- The future GYSELA-X++ will be based on the same algorithm.
- We are carrying out benchmarks for EP-driven instabilities.
- CoM space diagnostics and initialisation are under development.
- Various other physics studies with EM are underway (NTU + CEA + G. Brochard CNRS).
- Suggestions welcome