The hybrid kinetic MHD code VENUS-KMHD
Introduction and preliminary results

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Why a new kinetic MHD code?

Most existing **kinetic MHD** codes use either:

- a **spectral approach** (Landau approach) (`MARS-K`, `NOVA-K`, ...), an exact treatment is very cumbersome. A semi-analytical treatment made possible by approximations on the particle orbits is often used.
- a **time evolution** (`XTOR-K`, `JOREK`, ...). No approximation required on the particle orbits, but usually computationally more expensive.

Goal: write a new hybrid spectral kinetic MHD code using the Case-Van Kampen approach.

Cheng 1991, Liu 2009, Lütjens 2010, Bogaarts 2022

Landau vs Van Kampen approach: simplified example

Consider **1D Langmuir waves** in **slab geometry**. **Linearised Vlasov-Poisson** equations (distribution function $f(x, v, t) = F(x, v) + \delta f(x, t, v)$):

$$\frac{\partial \delta f}{\partial t} + v\frac{\partial \delta f}{\partial x} - \delta E\frac{\partial F}{\partial v} = 0, \qquad \frac{\partial \delta E}{\partial x} = -\int_{-\infty}^{\infty} \delta f \, dv', \qquad \delta f, \delta E \sim e^{i(kx-\omega t)}.$$

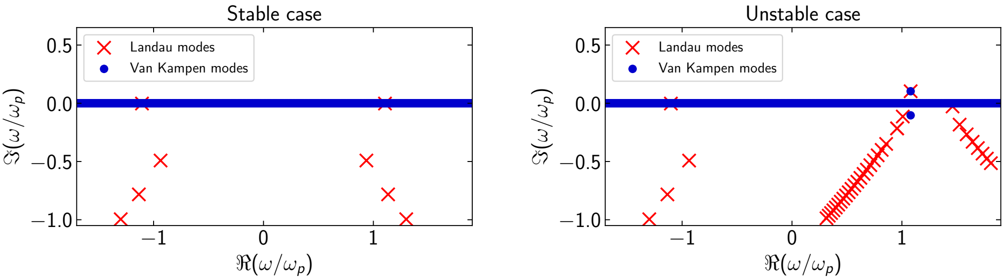
Landau approach: the system reduces to a **non-linear eigenvalue problem** for the perturbed electric field:

$$\left(1 - \frac{1}{k^2}\int_L \frac{1}{(v' - \omega/k)}\frac{\partial F}{\partial v'} dv'\right) \delta E(k) = 0.$$

Van Kampen approach: the system reduces to a **standard linear eigenvalue problem** for the perturbed distribution function:

$$kv\delta f(k, v) - \frac{1}{k}\frac{\partial F(v)}{\partial v}\int_{-\infty}^{\infty} \delta f(k, v')dv' = \omega\delta f(k, v).$$

Landau vs Van Kampen spectra: bump-on-tail instability



- The **stable Van Kampen spectrum** is **continuous** (here, it is discretised).
- **Unstable** Landau and Van Kampen modes **coincide**.
- In theory, **with finite collisionality**, **all the modes** (stable and unstable) **coincide** between the two approaches.

Van Kampen 1955, Case 1959, Ng 1999, Bratanov 2011

The Vlasov - Poisson equations as a standard eigenvalue problem

This problem can be written as a **generalised eigenvalue problem** $A\mathbf{x} = \lambda B\mathbf{x} = -i\omega B\mathbf{x}$:

$$\begin{pmatrix} ik & \int_{-\infty}^{\infty}(.)dv' \\ -\frac{\partial F}{\partial v} & ikv \end{pmatrix} \begin{pmatrix} \delta E \\ \delta f \end{pmatrix} = -i\omega \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \delta E \\ \delta f \end{pmatrix}$$

This problem is solved numerically by discretising the velocity space using **a regular grid**, $\{v_i\}_{i=1,\ldots,N} \in [-v_{max}, v_{max}]$, $v_1 = -v_{max}$, $v_N = v_{max}$, $\Delta v = v_{i+1} - v_i$.

$$\begin{pmatrix} ik & \frac{\Delta v}{2} & \Delta v & \ldots & \frac{\Delta v}{2} \\ -\frac{\partial F}{\partial v}(v_1) & ikv_1 & 0 & \ldots & 0 \\ -\frac{\partial F}{\partial v}(v_2) & 0 & ikv_2 & \ldots & 0 \\ \vdots & 0 & 0 & \ddots & 0 \\ -\frac{\partial F}{\partial v}(v_N) & 0 & 0 & \ldots & ikv_N \end{pmatrix} \begin{pmatrix} \delta E \\ \delta f(v_1) \\ \delta f(v_2) \\ \vdots \\ \delta f(v_N) \end{pmatrix} = \omega \begin{pmatrix} 0 & 0 & \ldots & 0 & 0 \\ 0 & i & 0 & \ldots & 0 \\ 0 & 0 & i & \ldots & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \ldots & i \end{pmatrix} \begin{pmatrix} \delta E \\ \delta f(v_1) \\ \delta f(v_2) \\ \vdots \\ \delta f(v_N) \end{pmatrix}$$

Can be solved with a **standard** linear eigenvalue solver.

Why a new kinetic MHD code?

Numerical approach inspired by the spectral version of gyrokinetic codes such as `GENE`:

- Using the Van Kampen (VK) approach leads to a **standard linear generalised eigenvalue problem**. **Easier** to handle **numerically** and **no approximation** needed on orbits.
- The spectral approach allows us to explore **subdominant modes**.
- This approach (VK) can easily be converted to a **time evolution** scheme, which can be extended to include **non-linear effects**.

Python code: solves a **generalised eigenvalue problem** $A\mathbf{x} = -i\omega B\mathbf{x}$ in three dimensions (r, θ, φ) (`VENUS-MHD`) or five dimensions $(r, \theta, \varphi, v_{||}, \mu)$ (`VENUS-KMHD`).

Discretisation using **finite elements** (B-Splines) in r direction, **finite differences** in $(v_{||}, \mu)$ and **Fourier modes** in θ (multiple m) and φ (single n) directions:

Jenko 2000, Roman 2010, Merz 2012, Lanthaler 2020, Bustos Ramirez 2022, Jeanquartier 2026 (submitted)

Kinetic MHD stability expressed as a generalised eigenvalue problem

Goal: following the **Van Kampen approach**, express the problem as:

$$A\mathbf{x} = -i\omega B\mathbf{x}.$$

A dummy equation is added to keep the eigenvalue problem **linear**:

$$\boldsymbol{\delta}\mathbf{u}_\perp = -i\omega\boldsymbol{\xi}_\perp.$$

The variables are $\boldsymbol{\xi}_\perp(r,\theta,\varphi)$, $\boldsymbol{\delta}\mathbf{u}_\perp(r,\theta,\varphi)$ and $\delta f^k(r,\theta,\varphi,v_\|,\mu)$. Formally:

$$\begin{pmatrix} A_{11} & 0 & A_{13} \\ 0 & 1 & 0 \\ q(.\times\mathbf{B})\cdot\mathbf{V_g} in\frac{\partial F}{\partial P_\varphi} & 0 & -\mathbf{V_g}\cdot\boldsymbol{\nabla} \end{pmatrix} \begin{pmatrix} \boldsymbol{\xi}_\perp \\ \boldsymbol{\delta}\mathbf{u}_\perp \\ \delta f^k \end{pmatrix} = -i\omega \begin{pmatrix} 0 & \rho & 0 \\ 1 & 0 & 0 \\ -q(.\times\mathbf{B})\cdot\mathbf{V_g}\frac{\partial F}{\partial\mathcal{E}} & 0 & 1 \end{pmatrix} \begin{pmatrix} \boldsymbol{\xi}_\perp \\ \boldsymbol{\delta}\mathbf{u}_\perp \\ \delta f^k \end{pmatrix}$$

Bustos Ramirez 2022

Benchmarks of VENUS-KMHD in cylindrical geometry

Thorough benchmarks of the new code were performed in **cylindrical geometry** (allows exact analytical results and lower numerical cost).

Example: benchmark against a **governing equation** for the low frequency limit $\omega \to 0$, kinetic hot ions.

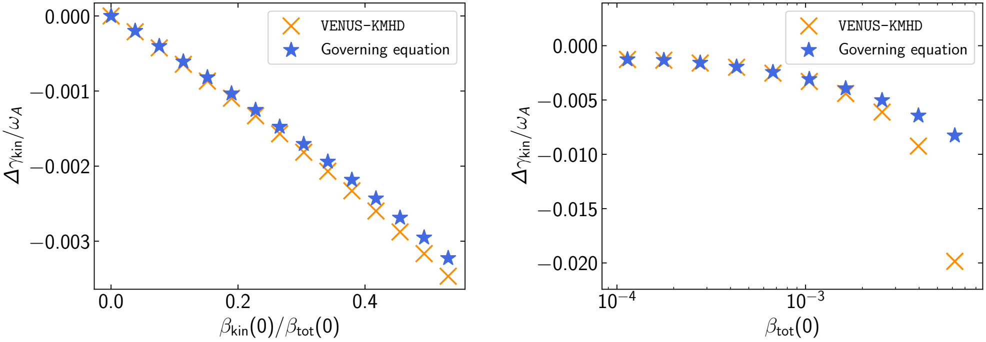
Using the radial and toroidal vorticity operators, one obtains a governing equation:

$$\frac{1}{r}\frac{d}{dr}\left[\left(\left(\frac{\gamma}{\omega_A}\right)^2 + \left(\frac{1}{q} - \frac{1}{q_s}\right)^2\right) r^3 \frac{d\xi_\perp^r}{dr}\right] - (m^2-1)\left(\left(\frac{\gamma}{\omega_A}\right)^2 + \left(\frac{1}{q} - \frac{1}{q_s}\right)^2\right)\xi_\perp^r$$
$$+\frac{\epsilon^2}{q_s^2}\left(\frac{3}{q_s} + \frac{1}{q(r)}\right)\left(\frac{1}{q(r)} - \frac{1}{q_s}\right)\xi_\perp^r + \frac{\epsilon\alpha}{q_s^4}\xi_\perp^r - \frac{\epsilon\alpha_{\mathrm{kin}}}{q_s^4}\xi_\perp^r = 0$$

Graves 2021, Jeanquartier 2026 (submitted)

Benchmarks of VENUS-KMHD in cylindrical geometry

Kinetic correction in the $\omega \to 0$ limit for a 1/1 internal kink mode.

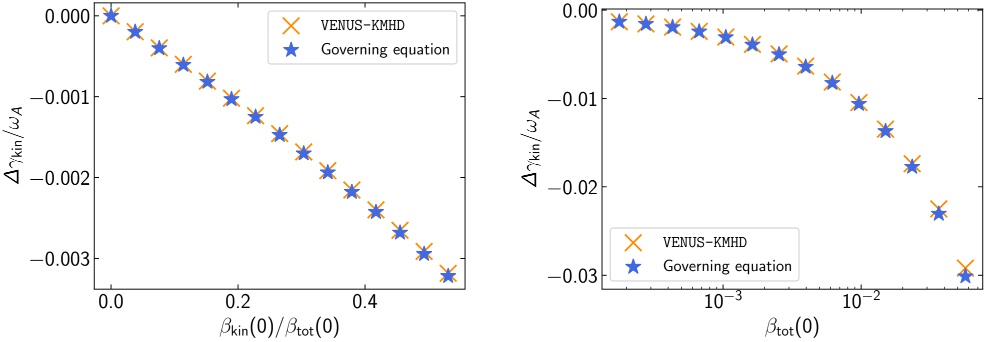


Some discrepancies are observed at high $\beta_{tot}(0)$.

Graves 2021, Jeanquartier 2026 (submitted)

Benchmarks of VENUS-KMHD in cylindrical geometry

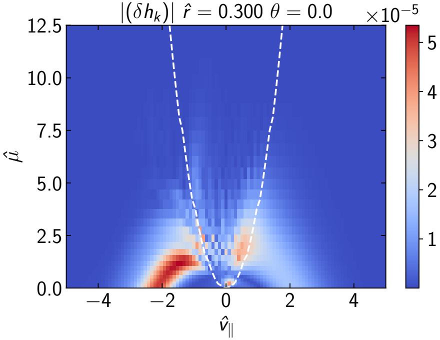
Without $\delta B_{\|}$ perturbations, a more consistent analytical solution can be obtained.

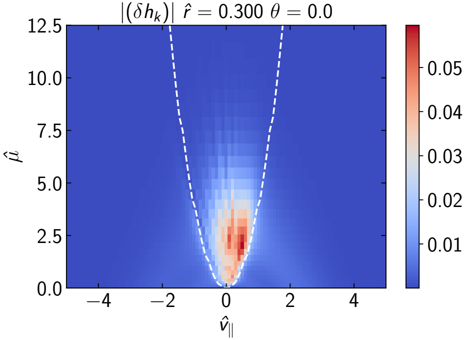


The same exercise was also done in the opposite limit ($\omega \to \infty$, not shown here).
Graves 2021, Jeanquartier 2026 (submitted)

Preliminary results in toroidal geometry

Thanks to the Van Kampen approach, the perturbed distribution for the kinetic species is simply part of the output. Left: $T_h \cong 300$ keV, passing particles dominate, right: $T_h \cong 15$ keV, trapped particles dominate.

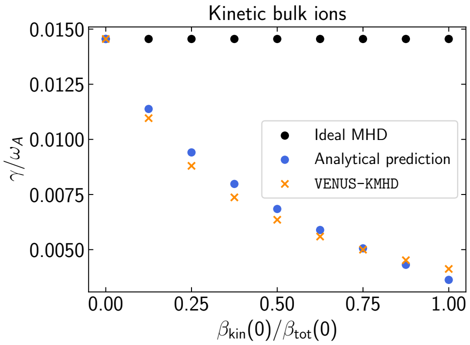




Preliminary benchmarks in toroidal geometry

Preliminary benchmark in the limit $\omega_* = -\left(\frac{q}{q_k}\frac{\partial F_k}{\partial \psi}\right) / \left(\frac{\partial F_k}{\partial \mathcal{E}}\right) \ll \omega$.

- Only the kinetic correction is computed analytically,
- Includes the kinetic correction to the inertia,
- Here for the case of kinetic bulk ions.



Conclusion and Perspectives

- `VENUS-KMHD` is a linear kinetic MHD code. It can be used in spectral mode (subdominant unstable modes can be obtained) or with a time evolution scheme,
- `VENUS-KMHD` was successfully benchmarked in cylindrical geometry,
- Preliminary results have been obtained in tokamak geometry.

Perspectives:

- Continue to **benchmark** the code against analytical results in the torus, starting with the **low frequency**, and **high frequency** (Kruskal-Oberman) limits,
- Benchmark `VENUS-KMHD` against existing kinetic MHD codes in tokamak geometry,
- Implement new physical effects, as discussed previously.