The Peanuts Suite of Codes

A first overview for TSVV-G

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Why this suite matters

- Peanuts provides a coherent workflow for Alfvénic stability studies in realistic tokamak geometry.
- The key idea is to separate the problem into three tightly coupled layers:
  - equilibrium processing,
  - continuous spectrum and boundary condition calculation,
  - local eigenmode and kinetic stability analysis.
- For the present discussion, the three main ingredients are:
  - **equipe**: equilibrium post-processing,
  - **Falcon**: continuous spectrum and ballooning-space boundary conditions,
  - **DAS**: local eigenmode calculations in fluid or kinetic formulation.

Peanuts at a glance

Code	Primary role	Typical output
equipe	Process magnetic equilibrium in flux/Boozer-like coordinates	Metric coefficients, geometry along field lines, input coefficients for wave solvers
Falcon	Solve coupled SAW-ISW continuum problem in ballooning space	Floquet exponents, local dispersion curves, continuous spectrum, outgoing boundary conditions
DAS	Solve local eigenvalue problems in fluid or kinetic description	Mode frequency, growth rate, parallel structure, energetic-particle drive and damping diagnostics

Flux coordinates and ballooning-mode representation

- We use straight-field-line flux coordinates (r, θ, ζ), with magnetic surfaces labeled by $\psi = \psi(r)$.
- The equilibrium magnetic field is written as

$$\mathbf{B}_0 = F(\psi)\nabla\phi + \nabla\phi \times \nabla\psi,$$

- For a fixed toroidal mode number n, the fluctuation is written in ballooning-mode representation:

$$f(r, \theta, \zeta) = \sum_{m\in\mathbb{Z}} A_n(r) e^{i(n\zeta - m\theta)} \int d\vartheta\, e^{i(m-nq)\vartheta} \hat{f}_n(r, \vartheta).$$

$$= 2\pi A_n(r) \sum_{\ell\in\mathbb{Z}} e^{in\zeta - inq(\theta - 2\pi\ell)} \hat{f}_n(r, \theta - 2\pi\ell).$$

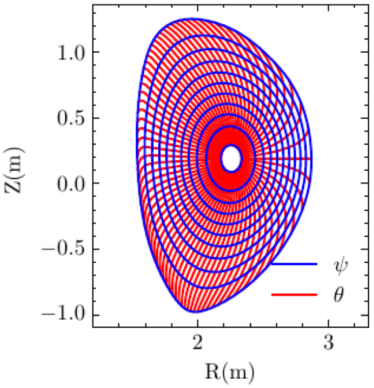
- In the local approximation, configuration-space operators are mapped into ballooning space as

$$\partial_\theta \mapsto \partial_\vartheta - inq, \qquad \partial_r \mapsto inq'(\theta_k - \vartheta), \qquad p(\theta) \mapsto p(\vartheta).$$

- This representation separates the slow radial envelope $A_n(r)$ from the short-scale parallel structure $\hat{f}_n(r, \vartheta)$.
- In the local calculation, the radial envelope is neglected, allowing fine radial/singular structures to be resolved efficiently in one-dimensional ballooning space.

equipe: purpose and output

- equipe is the equilibrium-processing layer of the suite.
- Starting from an MHD equilibrium, it reconstructs the geometric quantities needed along magnetic field lines.
- A key operational mode is miller, for calculations based on local Miller geometry.
- It bridges external equilibrium solvers and ballooning-space calculations.
- It provides the coefficients later used by Falcon and by local eigenmode calculations.
- In short, it turns an equilibrium file into physics-ready geometry.



Contour lines of ψ and θ coordinates in the processed equilibrium.

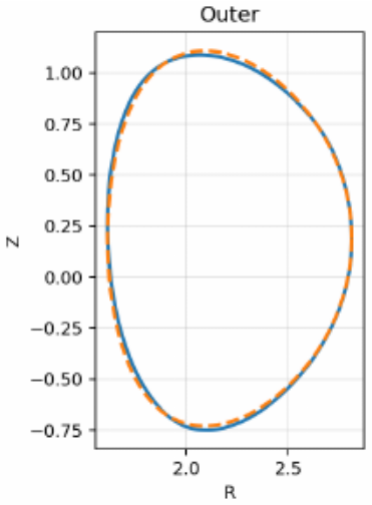
Miller Approximation

The local Miller model used by `equipe` starts from the standard parametrization:

$$R(r,\theta) = R_0(r) + r\cos\Big(\theta + \delta(r)\sin\theta\Big),$$
$$Z(r,\theta) = \kappa(r)\, r \sin\theta.$$

The geometry is controlled by:

- $R_0(r)$: shifted major radius,
- $\delta(r)$: triangularity,
- $\kappa(r)$: elongation.
- `equipe` fits these profiles surface by surface from the numerical equilibrium.
- Their radial derivatives enter the reconstructed metric coefficients.
- The result is a local analytic geometry for `Falcon` and `DAS`.

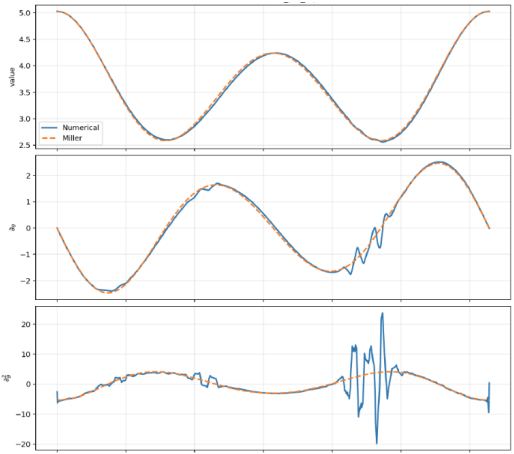


equipe in Miller Mode

- Read the fitted Miller profiles $R_0(r)$, $\delta(r)$, $\kappa(r)$ and their derivatives.
- Read $\psi(r)$ and $T(r)$ from the realistic geometry data.
- Rebuild one local Miller object on each radial surface and evaluate it on the same Boozer-angle grid.
- Reconstruct:

$$\nabla\psi^2,\ \nabla\theta^2,\ \nabla\theta\cdot\nabla\zeta,$$

$$\nabla\zeta^2,\ \nabla\psi\cdot\nabla\theta,\ \nabla\psi\cdot\nabla\zeta.$$



Potentials Entering the Equations

- Once the Miller geometry is reconstructed, it is propagated into the coefficients that appear in the continuum equations.
- In the full `Falcon` formulation, `Equipe` tracks the CFC potential arrays

$$f_1,\ f_{\mathrm{acc}},\ f_{31},\ f_{32}.$$

- We can verify that the Miller approximation preserves the geometric potentials seen by the ODE solvers
- `Falcon` and `DAS` standard usage relies on realistic geometry

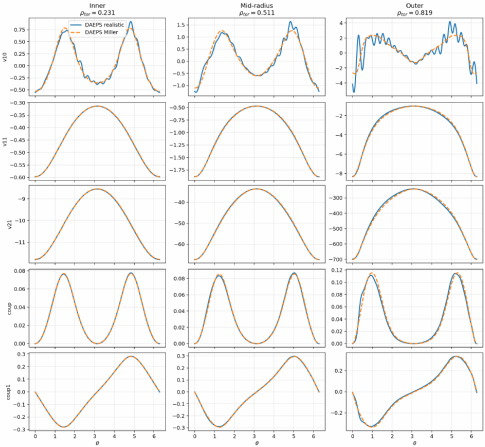


Figure 14: DAEPS fluid continuum potentials overplotted as functions of θ on three representative surfaces: realistic DAEPS input versus Miller-based DAEPS input.

Falcon: coupled SAW-ISW equations

In ballooning space, Falcon solves the coupled shear-Alfvén / ion-sound system:

$$\left(\partial_\vartheta^2 - \frac{\partial_\vartheta^2|\nabla r|}{|\nabla r|} + \frac{\omega^2 \mathcal{J}^2 B_0^2}{v_A^2}\right) g_1 = -(2\Gamma\beta)^{1/2} \frac{\mathcal{J}^2 B_0^2}{qR_0} \frac{s\vartheta}{|s\vartheta|} \kappa_g g_2,$$

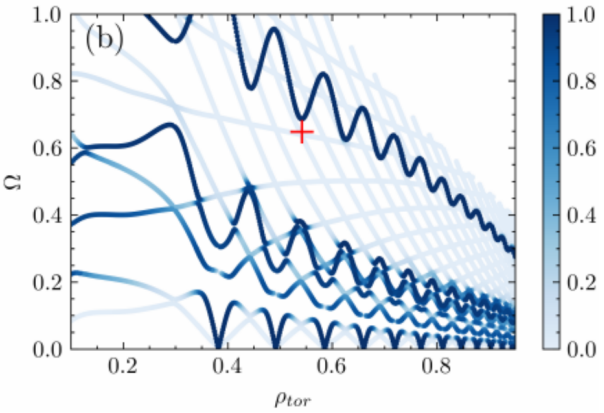
$$\left(\frac{\mathcal{J}^2 B_0^2}{q^2 R_0^2} + \frac{\omega_s^2}{\omega^2}\partial_\vartheta^2\right) g_2 = -(2\Gamma\beta)^{1/2} \frac{\mathcal{J}^2 B_0^2}{qR_0} \frac{|s\vartheta|}{s\vartheta} \kappa_g g_1.$$

- First equation: shear-Alfvén response
- Second equation: ion-sound/compressibility response

Falcon: full mode

Purpose

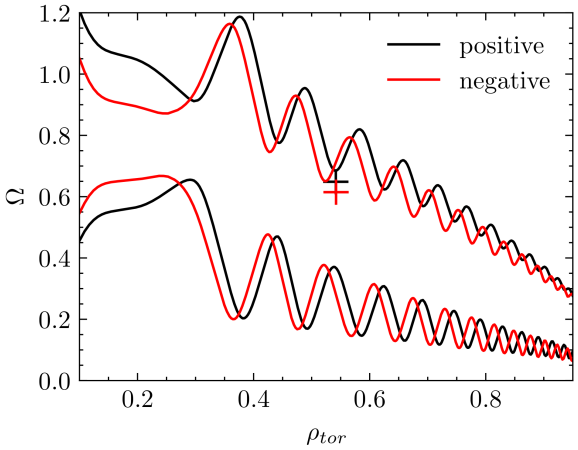
- Complete SAW–ISW continuum calculation.
- Resolves the full local dispersion curve.
- Captures the detailed gap structure and avoided crossings.
- Provides the most accurate reference calculation.



Falcon: slow-sound mode

Purpose

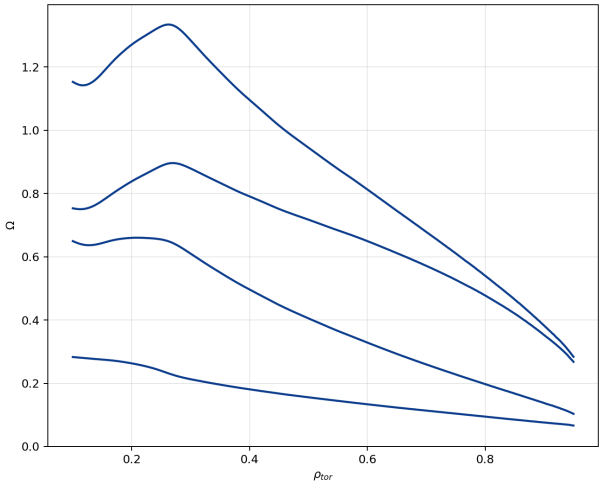
- Reduced formulation of the coupled SAW–ISW system.
- Retains the main coupling to the ion-sound branch.
- Focuses on the relevant slow-sound contribution to the gap.
- Faster than the full continuum calculation
- Incompressible calculation are available



Falcon: envelope mode

Purpose

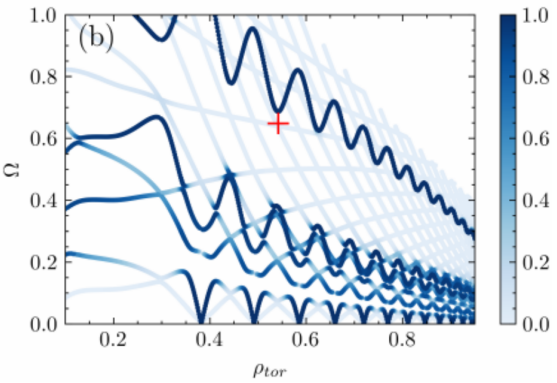
- Instantaneous estimate of the gap amplitude.
- It exploits the separation of scale used in the ballooning description
- Uses the slowly varying envelope of the dispersion curve.
- Does not resolve the detailed fast oscillatory structure.
- Well suited for rapid equilibrium scans.



Falcon: Alfvénicity

$$\mathcal{A} = \frac{\int_0^{2\pi} |g_1(\vartheta;\nu,r)|^2 \, d\vartheta}{\int_0^{2\pi} \left[|g_1(\vartheta;\nu,r)|^2 + |g_2(\vartheta;\nu,r)|^2 \right] d\vartheta}$$

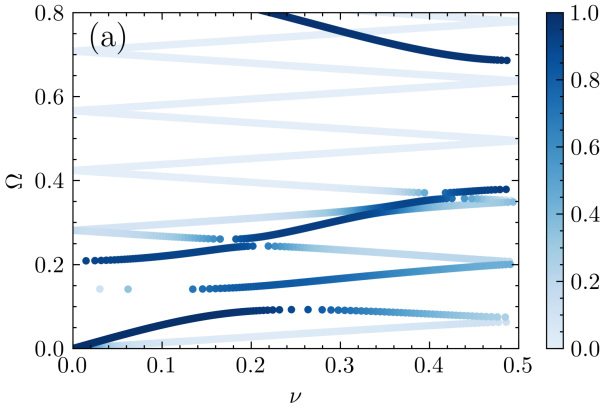
- $\mathcal{A} \simeq 1$: mostly SAW-polarized fluctuation.
- $\mathcal{A} \simeq 0$: mostly ISW/acoustic-polarized fluctuation.
- The color scale separates Alfvénic branches from acoustic branches.



Falcon: local dispersion curve

Falcon uses the ballooning-space solutions to build the local SAW–ISW dispersion relation.

- The dispersion curve is computed on a selected flux surface.
- It shows the allowed continuum branches as a function of frequency.
- The same information is used to choose the frequency window for the local eigenmode calculations.
- In the workflow, the dispersion curve is the bridge between geometry and the DAS eigenvalue calculation.



DAS: local eigenmode equations in the fluid case

In the fluid limit, DAS solves the local coupled SAW–ISW eigenmode problem in ballooning space:

$$\left[\partial_\vartheta^2 - \frac{\partial_\vartheta^2 \hat{\kappa}_\perp}{\hat{\kappa}_\perp} + \frac{\omega^2 \mathcal{J}^2 B_0^2}{v_A^2} - 8\pi \mathcal{J}^2 \frac{r B_0 P_0'}{q \hat{\kappa}_\perp \psi'} \mathcal{C}(\vartheta)\right] g_1(\vartheta) = -(2\Gamma\beta)^{1/2} \frac{\mathcal{J}^2 B_0^2}{q R_0} \mathcal{C}(\vartheta) g_2(\vartheta),$$

$$\left(\frac{\mathcal{J}^2 B_0^2}{q^2 R_0^2} + \frac{\omega_s^2}{\omega^2} \partial_\vartheta^2\right) g_2(\vartheta) = -(2\Gamma\beta)^{1/2} \frac{\mathcal{J}^2 B_0^2}{q R_0} \mathcal{C}(\vartheta) g_1(\vartheta).$$

$$\mathcal{C}(\vartheta) = \kappa_g \frac{\nabla\psi \cdot \hat{\kappa}_\perp}{\hat{\kappa}_\perp |\nabla\psi|} - \kappa_n \frac{r B_0}{q \hat{\kappa}_\perp |\nabla\psi|}.$$

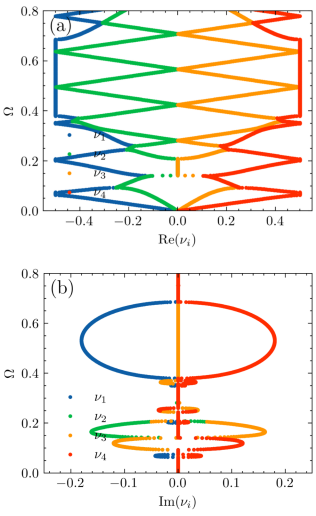
- First equation: shear-Alfvén response, including inertia and pressure-gradient curvature coupling.
- Second equation: ion-sound/compressibility response coupled through magnetic curvature.

DAS: dispersion curve and boundary conditions

- In the large-$|\vartheta|$ limit, the coupled SAW–ISW equations describe the radial singular response of the plasma, i.e. the local continuous spectrum.
- Because the equilibrium coefficients are 2π-periodic in ϑ, the asymptotic solutions are written in Floquet form:

$$x_i(\omega;\vartheta) = P_i(\vartheta)e^{i\nu_i\vartheta}, \qquad P_i(\vartheta+2\pi) = P_i(\vartheta).$$

- The selected Floquet solutions provide the outgoing or decaying boundary conditions for the TAE eigenmode calculation.



TAE eigenmode: matching and parallel structure

- The physical Floquet solutions are selected at the two sides of the ballooning domain:

$$\vartheta = \pm 2\pi\ell, \qquad \ell \gg 1.$$

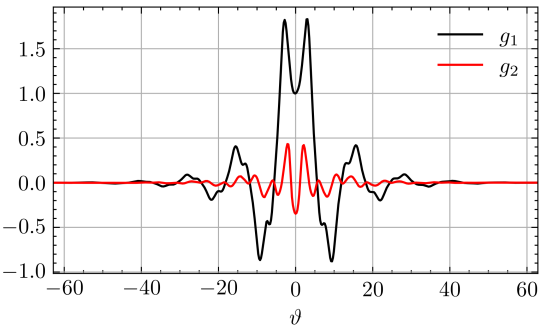
- The equations are integrated from both boundaries toward the center of the domain, $\vartheta = 0$.The left and right solutions must match at $\vartheta = 0$:

- This gives a homogeneous linear system,

$$M(\omega)w = 0,$$

- A non-trivial solution exists only when

$$D(\omega) \equiv \det M(\omega) = 0.$$



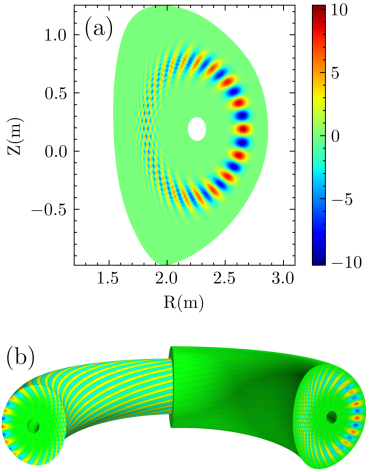
From local TAE structure to global reconstruction

- The eigenmode calculation gives the local parallel structure in ballooning space, $\hat{\Phi}_s(\vartheta, r_0)$.
- To visualize the global fluctuation, an analytical ground-state radial envelope is introduced around the reference surface r_0

$$A(r) = \exp\left[-\frac{(r-r_0)^2}{\Delta_r^2}\right]$$

$$\Delta_r^2 = \frac{2}{|nq'|}\left[\frac{\partial^2 D/\partial\theta_k^2}{\partial^2 D/\partial r^2}\right]^{1/2}_{r=r_0}$$

- The real-space field is reconstructed by combining $A(r)$ with the ballooning-space mode structure.

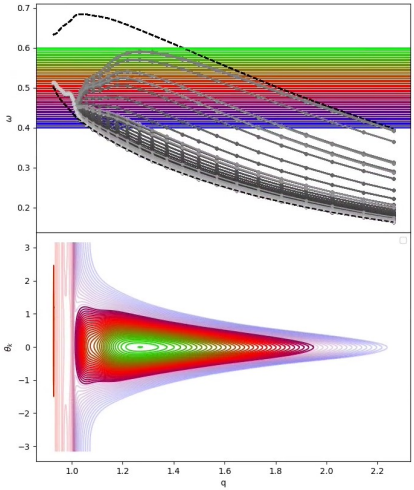


Global WKB closure and meso-scale envelope

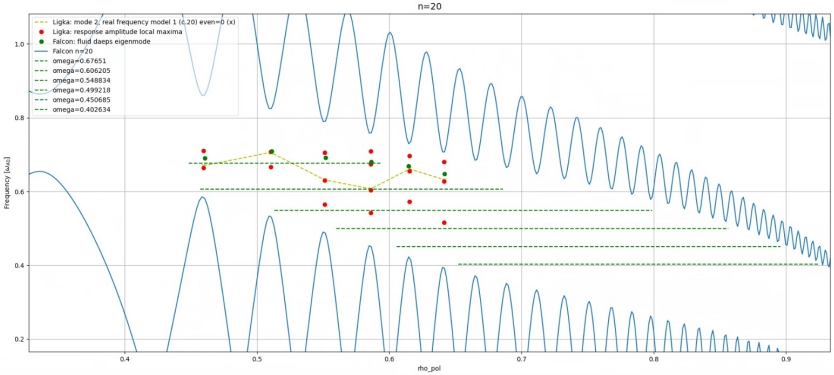
- A more complete approach determines the envelope from the global WKB eigenvalue problem.
- The original 2D problem is treated as two nested 1D problems:
  - local parallel structure in ballooning space,
  - slow radial envelope evolution.
- The local dispersion relation defines phase trajectories in (r, k_r) space:

$$D(\Omega, r, k_r) = 0.$$

- Global eigenfrequencies are selected by a quantization condition along closed WKB trajectories.



Comparison with LIGKA



- Both FALCON and DAS run on the EP-workflow, effectively providing the ballooning-based part of the workflow. Agreement with LIGKA provides an important cross-validation

DAS: kinetic vorticity equation

- In the kinetic formulation, the linear GK vorticity equation is coupled to the linear GK equation:

$$\left[\partial_\vartheta^2 - \frac{\partial_\vartheta^2 \hat{\kappa}_\perp}{\hat{\kappa}_\perp} + \frac{\omega^2 \mathcal{J}^2 B_0^2}{v_A^2} - 8\pi \mathcal{J}^2 \frac{r B_0 P_0'}{q \hat{\kappa}_\perp \psi'} \mathcal{C}(\vartheta)\right] \delta\hat{\Psi}$$

$$- 4\pi \mathcal{J}^2 \frac{r B_0}{q \hat{\kappa}_\perp \psi'} \mathcal{C}(\vartheta) \left\langle m_E \left(\mu B_0 + v_\parallel^2\right) \partial_r F_{0E} J_{0E}^2 \right\rangle_v \delta\hat{\Psi}$$

$$= - \frac{4\pi \mathcal{J}^2 B_0 \omega}{k_\vartheta c} \mathcal{C}(\vartheta) \sum_{s=i,e,E} \left\langle m_s \left(\mu B_0 + v_\parallel^2\right) J_{0s} \delta\hat{K}_s \right\rangle_v .$$

$$\left(\frac{v_\parallel}{\mathcal{J} B_0} \partial_\vartheta - i\omega + i\omega_d\right) \delta\hat{K}_s = i \frac{e_s}{m_s} Q F_{0s} J_{0s} \frac{\omega_d}{\omega} \frac{\delta\hat{\Psi}}{\hat{\kappa}_\perp}$$

- The left-hand side contains field-line bending, inertia, pressure-gradient curvature coupling, and the energetic-particle equilibrium pressure term.
- The right-hand side contains the kinetic compression response: energetic-particle drive and core-plasma damping.

DAS: solution of the kinetic eigenvalue problem

- The kinetic vorticity equation can be written schematically as

$$\left[\partial_\vartheta^2 + V_f(\omega;\vartheta)\right]\delta\hat{\Psi} = \mathrm{KC}\left(\omega, \delta\hat{\Psi};\vartheta\right),$$

  where V_f contains the fluid SAW potential and KC is the kinetic-compression term.
- without loosing generalization, we can introduce an iterative perturbative strategy:

$$\text{Lowest order:} \quad \left[\partial_\vartheta^2 + V_f(\omega_0;\vartheta)\right]\delta\hat{\Psi}_0 = 0,$$

$$\text{Iteration 1:} \quad \left[\partial_\vartheta^2 + V_f(\omega_1;\vartheta)\right]\delta\hat{\Psi}_1 = \mathrm{KC}\left(\omega_0, \delta\hat{\Psi}_0;\vartheta\right),$$

- At each step, the updated mode structure is used to recompute the kinetic response and the eigenfrequency.
- Equivalently, the problem can be discretized by a finite-element representation in ballooning space, leading to a nonlinear matrix eigenvalue problem:

$$\mathbf{A}(\omega)\,\mathbf{x} = \omega^2\mathbf{B}\,\mathbf{x}.$$

- The output is the complex eigenfrequency $\omega = \omega_r + i\gamma$ and the corresponding parallel mode structure $\delta\hat{\Psi}(\vartheta)$.

DAS: kinetic frequency shift and resonance structure

- The frequency shift is written perturbatively as

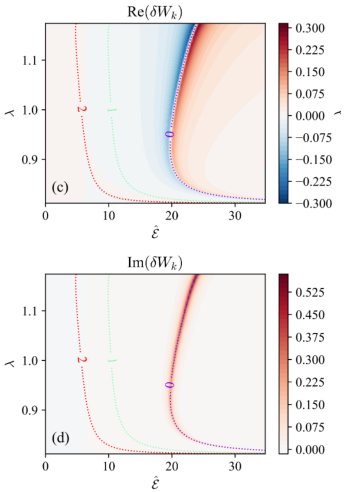
$$\frac{\Delta\omega}{\omega_0} = \frac{\delta W_k}{2\displaystyle\int_{-\infty}^{\infty} d\vartheta\, \frac{\omega_0^2 \mathcal{J}^2 B_0^2}{v_A^2} \left|\delta\hat{\Psi}_0\right|^2}.$$

- where the generalized kinetic potential energy is

$$\delta W_k = \int_{-\infty}^{\infty} d\vartheta\, \delta\hat{\Psi}_0^*\, \mathrm{KC}\left(\omega_0, \delta\hat{\Psi}_0; \vartheta\right).$$

$$\delta W_k = \int_{-\infty}^{\infty} d\vartheta\, \delta\hat{\Psi}_0^* \sum_{s=i,e,E} \frac{4\pi\mathcal{J}^2 B_0 \omega_0}{k_\vartheta c} \mathcal{C}(\vartheta) \left\langle m_s \left(\mu B_0 + v_\parallel^2\right) J_{0s} \delta\hat{K}_s \right\rangle_v .$$

- $\mathrm{Re}\,\delta W_k$ shifts the mode frequency.
- $\mathrm{Im}\,\delta W_k$ gives the balance between energetic-particle drive and core-plasma damping.



DAS + Falcon + equipe workflow

- For a given flux surface, `equipe` provides the local geometric coefficients needed by the ballooning-space calculation.
- `Falcon` is then used to compute the local dispersion curve and locate the relevant frequency gap.
- Once the gap is identified, `DAS` is used to search for the discrete eigenmode inside that gap.
- This gives a practical chained workflow:
  - `equipe`: geometry on the chosen surface,
  - `Falcon`: dispersion curve and gap structure,
  - `DAS`: eigenmode frequency, structure, drive, and damping.

Support to TSVV from Peanuts

- Fast continuous spectrum calculations are provided as a main diagnostic tool for ORB, both in `slow-sound` and in `full` mode.
- In particular, `envelope` calculations are essentially instantaneous.
- Local variations of the equilibrium are possible using Miller metric tensors.
- The whole suite of codes is available on the ITER cluster by means of the EP-workflow.